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STOCHASTIC FORAGE PRODUCTION

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THE RANGE STOCKING DECISION AND STOCHASTIC FORAGE PRODUCTION

Weather, domestic animals, wild animals, government policies, markets, and a host of other factors combine to make ranch production and income an uncertain event. In the midst of all this uncertainty, a rancher makes the crucial decision of how many animals to place on the range, that is, the stocking rate. Although prices certainly enter into long-run strategic decisions regarding entry, exit, and expansion, the rancher's stocking decision often is influenced more by the potential forage production rather than expected prices. The risk preferences of a rancher come into play when he or she evaluates the range of possible forage production levels and his/her preferences on having to buy hay in poor years versus having excess forage in good years. Stockers, which are bought, fed, and sold within a relatively short time period, are sometimes used in good forage production years. Even this seemingly short-run decision is, in practice, a fairly fixed decision due to tradition or commitments made months before the delivery and grazing take place. To help grasp the complexity of the long-run stocking decision, an economic model of this decision is developed which incorporates stochastic forage production caused by weather and will allow the incorporation of rancher's risk preferences.

In the first section of this paper, the range stocking problem is presented in more detail and past attempts to model the decision are discussed. Next, the model is developed and its results summarized. In the last section, potential extensions and uses of the model are examined.

THE RANGE STOCKING PROBLEM

The range stocking problem is formulated for a ranch in northern California which utilizes both private and public ranges and pastures in different geographic locations during different seasons. In order to concentrate on the stocking rate decision and the impact of weather on forage supplies, let us consider a rancher who raises calves and/or feeds stockers. This rancher already owns and leases rangeland and is not considering buying or leasing more land. Hay and other supplemental feeds are available on the ranch or from off-ranch sources. Hay and other crop production on the ranch are not included in the current model.

The most uncertain supply of forage is on those ranges with a Mediterranean-type climate. The upper mountain ranges are leased for summer grazing with limitations on the number of animal units and on the dates of grazing. Usually, the limitation is designed for lower than average forage production; thus, the amount available to the rancher is fairly certain due to these institutional restrictions. Forage production from irrigated pasture, an alternative source in the summer, varies little due to the certainty of water supply.

In California's Mediterranean-type climate, range forage growth starts in the fall after there has been a rainfall sufficient to germinate the seeds. Rapid fall forage growth occurs until the temperature becomes too cold at the beginning of winter, when growth slows. Rapid growth begins again in February or March when the temperatures begin to rise. Plants mature and produce seeds in April or May depending on how long the rains fall in the spring. After maturing, the plants dry out but remain standing through the summer and into the fall. Throughout this period, the forage decreases in both quantity and quality but is available as dry feed for livestock. In the fall, the rains begin again and the cycle repeats.

Weather variations cause different patterns of forage growth. In the fall, the amount of rainfall and time between seed germination and the beginning of cool winter temperatures are important determinants of the amount of growth that occurs initially. In the winter, temperatures may be so cold that growth actually declines or temperatures may be warm enough to have better than average growth. Usually, rainfall is not the limiting factor during winter. In the spring, the most important factor is the date at which temperatures begin to rise. However, there are rainfall patterns which can override the importance of the early warm temperatures.

Even though there is variation in the forage supply in all seasons, ranchers view the winter as their most critical season. A fall-calving cow herd has a rising demand for forage through the winter. The winter forage supply is usually the lowest level per land area compared to the other seasons and, thus, is the bottleneck for increasing the cow herd. Forage supply in other seasons can be utilized by buying stockers for short periods of grazing and then selling them on the market again.

The problem just described has several characteristics which drive the choice of economic model for analysis. The seasons divide the year into stages. Each of these seasons (i.e., stages) are separate due to the changes in the weather and the resulting impacts on forage growth patterns. Thus, even though forage growth is a continuous process, the year can be divided into discrete stages. The decisions are sequential through the year starting in the fall. At the beginning of each season, the rancher must decide whether to sell any remaining calves or to feed them for another season; once sold, the calves will not be available for further feeding and sale at a subsequent date. The most obvious characteristic is the stochastic nature of the forage production on the ranges in the Mediterranean-type climate. These characteristics (discrete stages, sequential decisions, and stochastic elements) drive the choice of model for analyzing the risk preferences of ranchers.

In previous work, Dean, Finch, and Petit (1966) analyzed alternative management strategies for ranchers in the face of uncertain forage supplies. They evaluated the expected returns from different stocking rates of both cows and stockers using linear programming. Rader and McCorkle (1966) incorporated weather uncertainty into their analysis of range improvements with comparison budgets but probabilities were not used to calculate expected returns. Wright and Dent (1969) argue that the actual range system has too many uncontrollable factors so simulation yields better results than programming; however, programming abilities have progressed since their article. Rae (1971a, 1971b) describes, exemplifies, and evaluates the use of stochastic programming for farm management applications. While Rae does not discuss range management specifically, the process and modelling he describes are very useful in this analysis. Trebeck and Hardaker (1972) found that integrating simulation and stochastic programming for whole farm planning under risk to be valuable in their example of beef production in Australia under weather conditions similar to those in this study. They used simulation to model some random elements and thus reduce the size of the matrix for computation. Linear programming is used by Weitkamp, et al., (1980) to analyze the effects of different forage production levels on ranch management. The probabilities associated with the forage production are not calculated so the expected returns can not be calculated.

Conner, et al., (1983) incorporates annual rainfall variation into the economic analysis of range improvement practices. Rainfall is divided into three categories--unfavorable, normal, and favorable. Normal is defined as that which occurs 50 percent of the time or within 20 percent of the historical mean. They compare the results of deterministic models with models which incorporate weather variation. The deterministic model consistently overestimates returns and provides less information on the variability of cash flows.

Pope and McBryde (1984) use a multi-period quadratic programming model to choose the optimal stocking rate, allowing for range improvement activities and the deterioration of forage supply over time. Their model is deterministic; it does not incorporate weather variation. Karp and Pope (1984) utilize Markov processes to analyze the range management under certainty, however, they do not include the uncertain elements present in ranch and range management. Rodriguez and Roath (1987) use dynamic programming to analyze short-term grazing decisions in Colorado; they do not include either cow herd decisions or the stochastic elements of forage and price uncertainty. Vantassell, et al., (1987) model the relationship between calf growth and environmental uncertainty (including rainfall and temperature); their approach and results are useful to guide the development of the coefficients in the planning model. Garoian, Conner and Scifres (1987) formulate a sequential two-year, three-stage discrete stochastic programming (DSP) model to estimate optimal brush burning schedules on Texas rangeland. Lambert (1988) develops a DSP model which analyzes a rancher's decision to graze or sell calves at two points in their growth cycle. Kaiser (1988) discusses the history and usefulness of DSP models for agricultural firms.

The iterative linear programming (LP) model developed by Olson, et al., (1987), incorporates weather variation for analysis of the long-range analysis of range stocking rates. The LP model is discrete and sequential, but does not incorporate stochastic forage production endogenously. The variation in prices and beef production can be incorporated into quadratic programming or MOTAD models, but the uncertainty of the resource constraints (e.g., forage supply by season) cannot be incorporated as easily. Dynamic programming, as used by Pope and McBryde or Rodriguez and Roath, and Markov processes, as used by Karp and Pope, do not allow for stochastic forage supplies. Stochastic dynamic programming and discrete stochastic sequential programming (DSSP) model (Rae; Apland and Kaiser (1984); Garoian, Conner and Scifres (1987)) are potential models to consider. The DSSP model is chosen for this analysis due to its ability to use programming algorithms and its applicability to the stocking problem and other agricultural problems (Kaiser, 1988).

FORMULATION OF THE DSSP MODEL

The LP model described by Olson, et al., (1987), is redefined as a DSSP model. In the winter and spring seasons, there are three forage production levels defined for the valley range: high, average, and poor (Table 1). The fall season has these three levels plus the potential combination of no rain until the cold starts resulting in no production.

Forage levels in each season and weather combination are based on George, et al., (1985). The summer season has fixed forage production due to institutional constraints or irrigation. Thus, there are 36 combinations of weather during the seasons of the year, that is, 36 possible sequences of random events. Probabilities are estimated from the procedures used in George, Olson, and Menke (1988).

Table 1. Valley forage production levels and probabilities by season used in the linear programming analysis.

Forage Production	-- Fall -- AUMs	prob.	- Winter - AUMs	prob.	- Spring - AUMs	prob.
No Prod.	0	(.08)	0	(.00)	0	(.00)
Poor	100	(.41)	100	(.43)	1000	(.26)
Average	500	(.48)	300	(.41)	2000	(.52)
High	1000	(.03)	500	(.16)	2500	(.22)

The objective of the DSSP model can be to maximize the expected value of the net returns from each of the sequences of random events or to maximize the rancher's utility derived from the expected returns and their variance. The maximization of expected returns is:

$$\text{Max: } \Sigma_{i=1}^{36} \alpha_i Y_i \qquad (1)$$

where:

α_i = the joint probability of the i^{th} random sequence of forage supplies occurring,

Y_i = the net return associated with the i^{th} random sequence of forage supplies during the seasons, and

$i,j = 1 \ldots 36$.

The equations which calculate the net return under each event are directly related to the original objective function of the LP model but now they feed into the objective function of the DSSP model. There are 36 net return equations (2) in this DSSP model.

$$- Y_i \; - \Sigma_{j=1}^{J} \Sigma_{t=1}^{T} C_{ji}^{R} R_{jti} - \Sigma_{t=1}^{T} C_{ti}^{H} H_{ti} - C_i^B B_i - \Sigma_{t=1}^{T} C_{ti}^{F} F_{ti}$$
$$+ \; \Sigma_{t=1}^{T} P_{ti}^{V} V_{ti} + \Sigma_{t=1}^{T} P_{ti}^{S} S_{ti} \quad \leq \; 0 \qquad (2)$$

where:

R_{jt} = an AUM of the j^{th} forage resource (e.g., range, pasture, or grazing permit) in the t^{th} season.

C_j^R = the cost per AUM on the j^{th} forage resource. This cost is assumed to be the same over all the seasons that a particular resource is used.

H_t = one ton of alfalfa hay purchased from another ranch enterprise or from off-ranch sources in the t^{th} season.

C_t^H = the cost per ton in season t.

B = the number of cows in the breeding herd.

C^B = the cost per cow per year excluding costs for range, pasture, grazing allotments, and hay costs and adjusted for the sale of cull cows and cull bulls and including the costs for a calf up to weaning (adjusted for conception and death rates).

F_t = the number of calves fed during the t^{th} season.

C_t^F = the cost per calf for feeding in the t^{th} season excluding costs for range, pasture, grazing allotments, and hay costs.

V_t = the number of calves fed sold at the end of the t^{th} season.

P_t^V = the calf price per head received at the end of the t^{th} season.

S_t = the number of stockers fed during the t^{th} season.

P_t^S = the net income per stocker received at the end of the t^{th} season after adjusting for all costs excluding range, pasture, grazing allotments, and hay costs.

i = the subscript denotes the i^{th} random sequence of states of nature.

Some activities are excluded a priori from the model in certain seasons. The fall pasture provides forage only in the fall season. The USFS lease and the irrigated pasture provide forage only in the summer. The valley range provides forage in the fall, winter, and spring seasons; this is the forage supply which has an uncertain supply in this model. Hay is purchased only in the fall, winter, and spring seasons. During the summer, the cattle are on the mountain leases or irrigated pastures and no hay is used. At the beginning of each season, the rancher decides to sell the calves or to feed them for the season. All remaining calves are sold at the end of the spring season and are not kept any longer. Stockers are bought, feed, and sold only in the spring season.

The model is formulated with the assumption of perfect knowledge of the past, but imperfect knowledge of the present and the future. That is, the rancher makes decisions at the beginning of each season knowing the weather events and forage levels of past seasons, but not knowing what will happen in the next and subsequent seasons. The one exception to this assumption is the hay purchase decision where we assume the rancher has perfect knowledge of the present; thus, he/she has continual access to hay and can purchase hay as needed according to forage needs.

The resource constraints, (3) through (7), are written without the i^{th} subscript to simplify reading. Each constraint is in the model 36 times for each of the potential forage production events.

The livestock cannot consume more that the total forage available in each range, pasture, or allotment. The forage may be produced in each season, carried over from the previous season, or carried to the next season:

$$R_{jt} + T_{j,t-1,t} - T_{j,t,t+1} \leq A_{jt} \qquad (3)$$

where all variables are as previously defined and:

A_{jt} = the amount of the j^{th} forage resource (measured in AUMs) produced in the t^{th} season and

$T_{j,t-1,t}$; $T_{j,t,t+1}$ = one AUM on the j^{th} range carried over from the previous season to the current season <u>or</u> from the current season to the next season.

For physical management reasons, all the forage produced in one season may not be transferable to the next season:

$$T_{j,t,t+1} \leq A^b_{jt} \qquad (4)$$

where A^b_{jt} = the maximum amount of AUMs which can be transferred from the t^{th} season to the next season.

Livestock nutritional needs are met in each season from forage produced that season, forage carried over from the previous season, or hay purchases:

$$\Sigma^J_{j=1} R_{jt} + 2.5H_t - Bb_t - F_tf_t - S_ts_t \geq 0 \qquad (5)$$

where all variables are as defined previously and:

b_t = the forage requirement per cow in the t^{th} season (AUMs),

f_t = the forage requirement per calf in the t^{th} season (AUMs), and

s_t = the forage requirement per stocker in the t^{th} season (AUMs).

The breeding cow activity, B, produces calves which are kept for replacement heifers, sold at weaning time in the fall, or fed for the fall season:

$$Ba_b - F_1 - V_1 \geq 0 \tag{6}$$

where all variables are as previously defined and:

a_b = the proportion of a calf weaned per cow. Adjustments are made for weaning and replacement rates.

Any calves which are fed during a season are either sold at the end of that season or fed for another season up to the end of the summer when all calves are sold. Adjustments are made for death rates during each season:

$$a_{t-1}F_{t-1} - F_t - V_t \geq 0 \tag{7}$$

where other variables are as previously defined and

a_t = the proportion of a calf produced by feeding 1 calf during the t^{th} season after adjusting for death rates.

The model also includes activities and constraints which allow the transfer of forages, cattle, and calves between seasons by paths which can be visualized as the branches of a decision tree. The model is constrained to <u>one</u> cow herd size for all seasons and states of weather. These resources which are used or produced in one season under one stochastic event are carried into the next season and the stochastic events which may occur. This is what causes the potential size constraint of the DSSP model.

RESULTS

The DSSP model is solved for the maximum expected value of the net returns with the General Algebraic Modeling System (GAMS). Potential returns under individual joint events range from $21,062 with no fall production and poor winter and spring production to $97,659 with high fall and winter production and either high or average spring production. The maximum expected net return considering the probabilities of all the events and the interrelationships between the seasons is $56,273. The potential return under average weather conditions is $70,433.

The optimal cow herd is 135 cows. The size of the cow herd is limited by the available forage on the irrigated pasture during the summer; the Forest Service lease is not used. Additional runs showed that (1) if hay prices were raised from $75 to $110 per ton, the cow herd would be reduced to 109 head and (2) if no stockers were allowed, the model would increase the cow herd to 242 and utilize part of the Forest Service lease and all of the pasture in the fall.

The 79 calves weaned at the beginning of fall (and not retained for replacement) are fed through to the beginning of the next summer for high or average fall forage production or are sold at the beginning of the winter season after the poor and no fall forage growth (Table 2). The number of stockers added during the spring forage season varies from 981 after an average fall and poor winter event to a high of 1,827 after high fall and winter production events. The need to buy hay under different weather conditions also changes.

Table 2. Results of the DSSP model which maximizes expected returns (hay priced at $75/ton).

	Stage I--No fall			Stage I--Poor Fall			Stage I--Average Fall			Stage I--High Fall		
	Stage II--Winter			Stage II--Winter			Stage II--Winter			Stage II--Winter		
	Poor	Ave	High	Poor	Ave	High	Poor	Ave	High	Poor	Ave	High
Hay Purchases (tons)												
Stage II (winter)	157	77	0	117	37	0	10	0	0	0	0	0
Stage III--High (spring)	0	0	0	0	0	0	0	0	0	0	0	0
Stage III--Ave (spring)	0	0	0	0	0	0	0	0	0	0	0	200
Stage III--Poor (spring)	400	400	400	400	400	400	400	400	400	400	400	600
Calf Sales in Stage II (winter)	78	78	78	78	78	78	0	0	0	0	0	0
Calves Fed in Stage II (winter)	0	0	0	0	0	0	78	78	78	78	78	78
Calf Sales in Stage III (spring)	0	0	0	0	0	0	0	0	0	0	0	0
Calves Fed in Stage III (spring)	0	0	0	0	0	0	76	76	76	76	76	76
Calf Sales after Stage III	0	0	0	0	0	0	75	75	75	75	75	75
Stockers (spring)	981	981	986	981	981	1053	911	1028	1161	1228	1361	1827

The DSSP model allows a rancher to consider simultaneously all the possible weather events and resulting forage production levels. Even without considering risk preferences, the inclusion of the joint probabilities into the simultaneous decision has caused the expected net returns to be less than the net returns under average conditions.

INCORPORATING RISK PREFERENCES INTO DSSP MODELS

The basic DSSP model just described can be modified to incorporate the impact of the rancher's risk preferences on the optimal stocking rate. Rae (1971a) describes various models for expressing expected utility. One method is to formulate the model as a quadratic risk programming model where the rancher's utility is derived from the expected returns and their variance. With the EV utility model, the DSSP model maximizes utility expressed as a function of expected net returns and the variance of those returns:

$$\text{Max: } \Sigma_{i=1}^{36} \alpha_i Y_i - \delta \, \Sigma_{i=1}^{36} \, \Sigma_{j=1}^{36} V_{ij} Y_i Y_j \tag{8}$$

where:

α_i = the joint probability of the i^{th} random sequence of forage supplies occurring,

Y_i = the net return associated with the i^{th} random sequence of forage supplies during the seasons,

δ = the coefficient of risk aversion,

V_{ij} = the covariance of joint events i and j or the variance of event i, and

i,j = 1 ... 36.

The variance of event i, $V_{ii} = \alpha_i(1-\alpha_i)$. The covariance of joint events i and j, $V_{ij} = -\alpha_i\alpha_j$ when $i \neq j$.

Solving the DSSP-EV model provided three points on the E-V frontier. These points and their corresponding risk coefficients are:

Risk Coefficient	Expected Return	Variance of Expected Return
0.0	56,273	3.2 E + 9
0.00001	50,000	2.5 E + 9
0.000015	33,484	1.1 E + 9

At risk coefficient of 0.00001, the cow herd remains at 135; however, more stockers are fed and more hay is purchased in the spring than in the maximum expected returns case. The increase in stocker numbers is being done to reduce the variance in returns.

Risk coefficient values between zero and 0.00001 caused the algorithm to degenerate and not obtain a solution. This and the small values that are required suggest that the expected returns-variance model may not be the appropriate utility model. However, both Lambert and Anderson, et al. (1977), suggest that it is not necessary to include nonlinear risk preferences in discrete stochastic programming models.

SUMMARY

A "rule of thumb" developed over the years is to stock the range at 80 percent of its average carrying capacity. The results of the DSSP model show that, even without endogenous risk preferences, the inclusion of forage variability and the associated probabilities result in lower expected returns compared to considering only average conditions. The DSSP model will be useful to ranchers, policy makers, researchers, advisors and others interested in analyzing the impact of public and private grazing policies and in improving the use of range resources rather than relying on "rules of thumb."

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