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A Statistical Analysis of Speculative Price Behavior

Claude S. Brinegar

Food Research Institute
Stanford University

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PREFACE

The substance of this monograph, which was completed as a doctoral dissertation in the Food Research Institute in 1954, is already known to that small group of scholars who have devoted their major research toward better understanding of the behavior of commodity-futures markets. It is published now because recent research into the workings of speculative markets indicates that its methodology and its findings should be made more generally available to students of those markets. It is also believed that Brinegar's study will help to fill an historical gap in the growing knowledge of the underlying structure of stock and commodity markets. Two specific developments have prompted this decision:

1. The publication of Paul H. Cootner's *The Random Character of Stock Market Prices* (Cambridge, Massachusetts) in 1964 and the Food Research Institute's *Symposium on Price Effects of Speculation in Organized Commodity Markets*, which was held in the autumn of 1967, have made it clear that the subject of speculative markets has become of increasing general interest.
2. Recent work by Holbrook Working, who initiated research along the lines pursued by Brinegar, depends in part on conclusions reached in the present study. General access to Brinegar's study should facilitate presentation of Working's latest findings.

Brinegar's work was the first to produce statistically significant evidence of "structure"—anything other than complete randomness—in the day-to-day changes of commodity-futures prices. Others had tried without success, including the distinguished British statistician M. G. Kendall in a paper published in the *Journal of the Royal Statistical Society* (London) in 1953. Moreover, the auto-correlations that Brinegar's results reveal as existing among the price changes are positive, whereas Working and most other interested economists had supposed that the chief auto-correlations would prove to be negative.

Brinegar deals with prices of three different commodities—wheat, corn, and rye—and gets essentially the same results for each. Not contented with overall results, Brinegar explores differences in price behavior according to the season of the year, and according to volatility of price movements.

This study represents the first use of Working's H-statistic, which became available only months before the research was undertaken. Inexperience with the new measure complicated the task of interpretation, and there was no fund of experience in statistical work that could be very helpful in guiding its use. It is worth noting, too, that the entire analysis was carried through without the kind of assistance that can now be rendered by modern high-speed computers.

Brinegar's dissertation rests in that tradition whereby recognizably important findings not fully understood at the time provide the stimulus to further study, and assume larger importance as they become better understood.

PREFACE

He has chosen to publish the manuscript essentially as it was submitted in 1954. Changes are limited to those that will improve readability, clarify obscurity, and to delete sections that now appear unnecessary. Revisions have not altered the original conclusions or methods in any material way.

WILLIAM O. JONES
Director

January 2, 1970

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Chapter 1

BASIC CONCEPTS OF SPECULATIVE PRICE ANALYSIS

The purpose of this study is to seek an objective, quantitative appraisal of the behavior of prices resulting from speculative trading on the organized commodity exchanges. As part of this appraisal, we endeavor to answer these two basic questions: In what respects and under what conditions do speculative prices fluctuate excessively, as various critics imply?[1] Or, on the other hand, in which respects do these prices reflect correct appraisals of all available information, as orthodox economic theory has long held?

The speculative prices examined are the Chicago "futures" prices for wheat, corn, and rye—three important, actively traded commodities on the principal speculative market for those commodities.[2] The intervals of price movement range from day-to-day changes to changes covering 16 weeks.[3]

The plan of attack follows directions suggested by Holbrook Working, in a paper published in 1949 (*48*, pp. 150–66).[4] There he formulated the concept of "ideal price behavior" and showed that testing actual price behavior against this "ideal" provided a method of developing quantitative answers to the basic questions cited above. The method of analysis is mainly by means of a new statistical test developed by Working, with the assistance of Phillipe Berthet and the author.

Working's concept of "ideal" price behavior is that it is such as would result if the price at all times correctly reflected all pertinent information available, thus making it the best possible current estimate of the price that would rule at some future date. If the price were not the best possible estimate, if known information had not been correctly evaluated, alert traders would be able to take advantage of this defect and correctly predict future price changes to their gain. Consequently, we say that in an ideal futures market the price of the May wheat future on, say, January 10 would represent the best possible estimate that could be made on that date of the price of spot wheat on the last trading day in the May future.

We do not, of course, expect the price of spot wheat in May to be the same as its price on January 10, for developments between January and May must be

[1] Typical examples are these (*18*, p. 13; *17*, p. 21):

> ... it is now coming to be agreed that organized (commodity) markets have defects of their own requiring correction. The excessive concentration by speculators on short-run profits to the neglect of long-run considerations makes prices undesirably volatile. Speculation, when acting in a price-stabilizing direction, should smooth the transition from one equilibrium point to the other ... ; but as the behavior of commodity markets in the past has amply shown, speculation also acts in a price-destabilizing direction—at any rate within certain ranges of price oscillation ....

[2] In the postwar years soybeans have become a very active speculative commodity at Chicago, occasionally attracting more speculative interest than any of the true grains.

[3] The work reported here was preceded by a considerable study of intra-day price movements, which provided useful information as background for the present study.

[4] This concept was further developed in a 1953 publication (*47*).

reflected in changing prices. What we do require is that these changes not be foreseeable on January 10, that is, that they contain no predictable elements. This consequence of the concept provides the key to statistical testing: ideal price behavior may be tested by determining to what degree futures prices contain behavior that departs significantly from behavior expected in a series that contains only random (and hence unpredictable) changes.

There are two main kinds of predictable price behavior to consider. First, there is the behavior where prices fluctuate excessively. This is the behavior implied or stated to exist by many of the sharpest critics of organized commodity speculation, particularly those who have followed and elaborated upon some of the ideas of Keynes.[5] Characterizing this behavior more concisely, we can say that price behavior departs from ideal when the prices move beyond the levels justified by the correct evaluation of all known information, requiring a subsequent correcting movement in the opposite direction. This form of behavior we describe by the term "price reaction."

The second form involves a price movement that is too gradual, a movement in which the price change warranted by new information does not occur as rapidly as it would in an ideal market. With this behavior we have an initial movement prompted by the new information, followed by a subsequent movement in the same direction, the new developments being only gradually reflected in changing prices. This behavior causes price changes to develop more slowly than in the ideal market, with the result that the movements prompted by new information move in a more gradual and continuous fashion. This form of behavior we describe as "price continuity."[6]

There is also the possibility of behavior where the prices contain some sort of a mixture of reaction and continuity. This behavior would be present, for example, if we found a price series that showed reaction for one-week intervals, while the intervals beyond two weeks contained continuity.

The concepts of reaction and continuity can be directly related to trading patterns of professional speculators. Since a market containing reaction is characterized by price changes going beyond the level warranted by the new information, a perceptive speculator could gain by timing his trading so that his sales would come after the prices had gone beyond the justified levels and by making his purchases on the compensating movement, or, if the price initially drops too low, the reverse action. To achieve success this perceptive speculator need not always sell at the top or always buy at the bottom. It would be sufficient for him to simply maintain his average purchases at the lower prices and his average sales in the upward swings, producing profits through heavy trading volume. The important point is that if the market contained significant reaction the speculators who were somehow aware of it could plan their trading strategy to profit by it.

[5] While Keynes himself devoted his main arguments against the speculation of the stock exchanges, his arguments can be transferred almost completely to commodity markets. Cf. *24*, pp. 153–64.

[6] This rather noncommittal term was chosen for two main reasons. First, it describes the condition broadly; that is, continuity of movement implies gradualness of response, resulting in a continually developing price movement, instead of the more prompt movement of the ideal market. Second, the term does not evoke "word-pictures" that might convey misleading ideas of what the behavior actually implies. This second reason caused us to reject two other possibilities—"trends" and "sluggishness."

Profiting from the presence of continuity requires a different approach. As indicated, a market with continuity contains price movements of a gradual sort, with the changes warranted by new developments being only slowly reflected in changing prices. In order for the astute speculator to profit in this type of market he must promptly and correctly judge when new information has and when it has not been properly evaluated, making his purchases in the early part of a gradual rise and his sales in the early part of a gradual decline.

Thus, the profitable trading pattern for continuity differs considerably from that for reaction. In the market containing reaction, the successful speculator may profit by trading simply on price swings, giving little consideration to the reasons for the swings. Each initial transaction is made with the expectation of a profit resulting from an anticipated price reversal. In the market with continuity, by contrast, the successful speculator must study pertinent market factors with care, properly foreseeing and evaluating new developments, and he must correctly predict to what degree, if at all, these developments are already reflected in price changes.

Chapter 2

HISTORICAL ATTEMPTS AT STATISTICAL APPRAISAL OF SPECULATIVE PRICE BEHAVIOR

Before turning to the methods of analysis used in this study, it is appropriate to briefly consider some of the prior statistical investigations into the behavior of speculative prices.

One of the earliest attempts at statistical appraisal was an 1896 study by Emery (*15*, pp. 123–43). This analysis, which formed the pattern of many succeeding studies, considered these two questions: (1) Does futures trading reduce price fluctuations?, and (2) How successful are futures prices at predicting the future level of spot prices? Emery's investigation of the first question was an appraisal of price variation before and after the introduction of futures trading. By considering the per cent by which the annual low price departed from the annual high price for cotton over three different decades, 1821–30, 1851–60, and 1885–94, the last during futures trading, he found a pattern of constantly declining fluctuations; a similar study of average monthly wheat prices at New York for four years before and four years after the introduction of futures trading, 1885–58 and 1890–93, revealed "in the main a smaller amount of fluctuations in the second period" (*15*, p. 127). Emery clearly realized the limitations of this evidence, cautiously concluding (*15*, pp. 127–28):

> The foregoing figures of price variation cannot, however, be accepted as an entirely accurate indication of the influence of speculation.... it is impossible to attribute a change of this nature unmistakably to speculation. The course of the price movements of today is a joint result of a joint development. The increased facilities of transportation and communication, the improvements in trade methods, and the speculative system, have all developed together. The results of all these forces working in concert is toward smaller variations in prices, but how much of the result can be attributed to any one cause it is perhaps fruitless to discuss.

His investigation of the question of the degree to which futures prices successfully predict the future level of spot prices[1] was based on an analysis of the per cent of error between a futures price and its spot price four or five months later. Here he treated not only cotton and wheat prices but also reported on an earlier study (covering 1850–90) of rye futures in Berlin. While his general conclusion was that "there has been a steady progress toward increased accuracy of prediction" (*15*, p. 133), there is again extreme caution, and the implication that such a fragmentary treatment really provides very little concrete information on the behavior of speculative prices.

[1] It should be noted that once we accept the idea that in an ideal market price changes are essentially random, the notion that futures prices should be good predictors of subsequent spot prices becomes of less importance.

Many later analysts, using basically similar methods, were less cautious. Boyle in 1920 compared the yearly price fluctuations (per cent by which the high exceeds the low price) for two futures-traded commodities, wheat and oats, with the price fluctuations for a non-traded, but similar commodity, barley (*6*, pp. 122–24). The comparison, covering 1899–1916, indicated that wheat and oats prices had smaller annual variations than barley prices. From this evidence, Boyle was led to say "it seems fair to conclude, therefore, that speculation in grain on the organized exchanges lessens price fluctuations" (*6*, p. 124). The principal weakness of this conclusion lies in ignoring the possibility that the supply and demand factors affecting barley might inherently produce wider price movements, with or without trading.[2]

Mills[3] commented briefly on the statistical methods used in Boyle's analysis, suggesting that the results are probably of little significance (*28*, pp. 60–62). After computing more refined measures of the variability of wheat, oats, and barley prices (using the mean deviation from the average price for the year to measure monthly variation, and the mean deviation of link relatives to measure annual variation) Mills concluded that "if we have in mind either average monthly variations or year-to-year changes, no such conclusion as that drawn by Dr. Boyle appears to be justified" (*4*, p. 62). Mills' results indicated that oats and barley prices had about the same variability and that wheat had considerably less.

A more elaborate method of attacking the question of price variability is found in the Federal Trade Commission's Report on the Grain Trade (*39*, pp. 26–29). The method used was an index of variability developed by Watkins (*43*), based on logarithmic principles.[4] Despite the elaborate technique, their results corresponded almost precisely with the findings of Mills, causing Mills to remark that "careful testing has not shown that this index (used by the Federal Trade Commission) possesses any advantage over the simpler measure of variability here employed..." (*28*, p. 63). The Commission's results are, of course, subject to the same basic weakness as the studies cited previously. In order to draw significant conclusions from comparisons of price behavior for futures-traded and other commodities, there must be some definite way of relating the findings to the existence or non-existence of futures trading. The investigators, while showing persistence, have failed to meet this requirement.

There have also been many detailed investigations into the second question considered by Emery—how successful are futures prices at predicting the future level of spot prices?[5] An entire chapter in the Federal Trade Commission's Report on the Grain Trade (*39*, pp. 217–37) is devoted to the topic "Futures Prices as Forecasts." The report, after devoting considerable attention to statistical appraisal of spot-future price spreads, reaches no positive conclusions. A more

[2] Boyle's analysis is cited as a "reasonable comparison" in a recent general text on futures trading (*4*, p. 71). In reproducing the statistics of Boyle the authors omit 1916, a year of extreme fluctuations in wheat prices.

[3] Mills incorrectly interprets Boyle's measure "as extreme price ranges expressed ... as percentages of the average price of each year" (p. 61).

[4] This measure of variability has apparently received little subsequent usage. In effect, it is the geometric mean of link relatives, adjusted for trend.

[5] The studies are all, of course, subject to the same objection as Emery's. Cf. footnote 1.

direct approach is found in a study by Killough (*25*), where, in a detailed study of the determinants of oats prices, he correlated spot prices of oats with previous futures prices. The results of the correlation analysis indicated that as the delivery month approaches, the futures price approaches the spot price. Such a conclusion is, of course, a logical outcome, and, as Killough recognized, it provides little information on the question of speculative price behavior.[6] Brief mention should also be made of a short study by Ashby (*3*, pp. 412–19) into the accuracy of cotton futures "forecasts." He based his analysis on futures prices eleven and five months before delivery, and classified by per cent of error of "forecast." His results, which are not relevant here, are subject to the same objection as the other related studies.

An ambitious study by White (*44*, pp. 329–42) into futures trading in hides deserves brief consideration. He devoted particular attention to the question of whether the introduction of futures trading in the New York Hide Exchange in 1929 had reduced the major fluctuations of spot prices. His principal analysis was based upon a comparison of averages, standard deviations, and coefficients of variation for weekly hide prices in the United States and Great Britain for six years (1923–28) prior to the introduction of futures trading, and six subsequent years (1930–35). On the basis of his analysis, and certain other statistical comparisons, he concluded that "there is no evidence that the introduction of futures trading in America has had any dampening effect on the fluctuations of spot prices—in fact, the evidence seems to point in the opposite direction" (*44*, p. 309). It is not necessary to comment at length on the limitations of this conclusion. The United States and Great Britain had entirely separate markets;[7] the price developments in America in the early thirties may have rested upon many different considerations; and, further, there is no particular reason to expect weekly price changes to reflect improved stability.

A short study by Malott (*26*, pp. 177–90) deserves recognition, for he considered the problem of fluctuations of futures prices in yet a different fashion, though one with the same limitations of the previously mentioned attempts. The particular question he considered was, "Does futures trading cause a dislocation of agricultural price trends, not found in the prices of commodities for which futures contracts are unavailable?" (*26*, p. 178). The method of analysis was the simplest of the group: he compared charts of the average monthly prices for wheat and cotton (heavy futures trading) with charts of the prices of wool, peanuts, beef, rice, and butter, all commodities with no direct futures trading (at least at the time of the study) for the years 1925–37. His conclusion was clearly tempered by an awareness of the limitations of visual comparisons: "... this study, incomplete and fragmentary as regards the whole subject of the relation of cash to futures prices on the organized commodity exchanges, discloses no

[6] In commenting on this and similar studies, Hoffman aptly observed: "The obvious shortcoming ... is that conditions change between the time of forecast and the time of fulfillment of the forecast. A perfect estimate may have been made in the month of May of probable prices in September; but since weather conditions cannot be foretold beyond the limits of a few days, it follows that this forecast must fail if growing conditions change" (*19*, pp. 437–38).

[7] This is not to imply, however, that the markets were necessarily independent, for they must reflect many similar basic factors. For a detailed analysis of the relationship between Chicago, Winnipeg, and Liverpool wheat futures prices see *7*.

trace of influence on prices attributable to futures contract trading or to speculation in futures contracts" (*26*, p. 190).

While there have been other investigations into the two questions discussed, little would be gained by considering them, for the methods of analysis and results are all closely analogous to the studies mentioned.[8]

An additional subject that has received considerable statistical attention, particularly by the Commodity Exchange Authority (CEA), is the question of the effects of the actions of the large-scale speculators on the behavior of speculative prices.[9] Although there have been a considerable number of such studies, published and unpublished, we can adequately illustrate the method and results by discussing the inquiry into wheat price changes in the first four months of 1925 (*31*). The study, made by the then Grain Futures Administration (now the CEA), was prompted by what appeared to be extreme price behavior. After selling at $1.10 per bushel in June 1924, wheat reached $2.00 by January 1925, and then abruptly broke to a low of $0.50 in March. This price break, like many others, was sharp enough to produce considerable feeling that excessive speculative activity had been responsible for at least part of the behavior.

To analyze the activities of the large speculators, the Grain Futures Administration made a detailed appraisal of the open interest and trading patterns of the largest speculators (those with open contracts of 100,000 bushels or above) during the months covered by the inquiry. While the holdings of the large speculators were found to change in relation to the price changes, there was, of course, no way of determining whether the speculators were causing the movements or were simply adjusting their positions as the prices moved. The results, consequently, were inconclusive.

Later studies (*13*; *14*; *32*; *33*) were equally inconclusive, for they were similarly hampered by inability to identify causal relationships. Yet, despite such inconclusive results, members of the CEA have used them to argue, from time to time, for legal limitations on the positions of speculators. For example, Paul Mehl wrote in 1934 "Excessive speculation by the public and large-scale operations of individuals or groups of traders sometimes pull prices out of line and cause fluctuations of considerable size which seriously disturb the market and do injury to producers and merchandisers of grain" (*27*, p. 485). He concluded "Any attempt made to prevent wide fluctuations necessarily must result in limiting the activities of speculators. By placing a limitation on the size of holdings of a speculator and thereby limiting the amount of trading that he may do in a single day, it is believed that greater market stability will result" (*27*, p. 495).

A similar, though more cautious, argument was advanced by Hoffman in 1937 (*21*, p. 300–309).[10] Basing his analysis on the same data as Mehl, plus supplemental inquiries, Hoffman concluded "There is . . . evidence to support the belief that the combined trading of these leading speculators is causal with ref-

[8] A similar appraisal of the value of these and similar investigations was made by Gerda Blau: ". . . the conclusiveness of such statistical investigations is limited because they must be based on the assumption of comparability of all other factors affecting the formation of prices in the two markets considered, or at the two periods considered, and this assumption does not, of course, correspond to the real facts . . ." (*5*, p. 25n).

[9] We exclude the effects of such illegal operations as deliberate "corners" and "squeezes." Such actions are largely now only of historical interest.

[10] A brief, and even *more* cautious, appraisal by the same author is found in *20*, pp. 43–60.

erence to price. . . ." (*20*, p. 307). Hoffman's evidence, however, was based upon inferences that are virtually impossible to substantiate statistically and his conclusions were appropriately tempered.

The conclusions of Mehl and Hoffman were commented upon by Working in the discussion following Hoffman's paper. After considering three possible, and mutually contradictory, hypotheses suggested by the correlations used by Hoffman, Working demonstrated that each was in accord with the statistical evidence, leading him to conclude:[11]

> Since the observed facts, as represented by the correlation coefficients, are consistent with all these suppositions, only one conclusion is possible: the facts chosen for observation are not suitable for discrimination among the hypotheses. The correlations cited really contribute no information on the question whether either large or small traders are good or bad influences on the market, from the standpoint of its function of registering sound price judgments.

We conclude this summary by noting two statistical studies that have provided some positive evidence regarding the behavior of speculative prices.

The first was an appraisal of stock market behavior similar to our inquiry into futures prices. This study appeared in two parts, the first by Cowles in 1933 (*9*, pp. 309–24), the second by Cowles and Jones in 1937 (*11*, pp. 280–94). The 1933 study examined the results of several thousand market forecasts made during the 4½ years from January 1928 to June 1932. After an appraisal of the recommendations of the forecasters, Cowles concluded that the forecasts were unsuccessful more often than they were successful, leading him to remark that, by comparison to his random experiments, "the least successful records are worse than what could reasonably be attributed to chance" (*9*, p. 324).

The 1937 study attempted to determine, by statistical methods, if stock prices exhibited measurable structure, or whether, as suggested by the first part, the price action was random in nature. The statistical technique employed was a non-parametric test based upon the ratio of sequences (i.e., successive changes in one direction) to price reversals. Defining an excess of sequences to reversals (as judged against the standard of a random-difference series) as market "inertia," the authors found that, in general, stock price indexes exhibited "inertia" from intervals of 20 minutes to about 1 year, with the tendency particularly strong around intervals of one month.[12]

Although the authors attempted no detailed analysis of this apparent structure, they did comment on its relation to market forecasting (*11*, p. 286):

> This evidence of structure in stock prices suggests alluring possibilities in the way of forecasting. In fact, many professional speculators, including in particular exponents of the so-called "Dow Theory" widely publicized by popular financial journals, have adopted systems based in the main on the principle that it is advantageous to swim with the tide.

[11] Discussion by Working following Hoffman's paper (*21*).

[12] In response to an inquiry from Working, Cowles in 1960 reexamined the 1937 data and found that the unusually strong inertia indication for one-month intervals was the result of inadvertently using data that contained averages. Monthly data without averages still showed inertia, but at a level more consistent with indications found in shorter and longer time intervals. Cf. *10*, pp. 909–15; *50*, pp. 916–18.

The second study is part of a broad investigation by Working into wheat price "cycles" (*46*, pp. 18–27; *49*). After studying major movements in wheat prices from 1884 to 1931, and subsequently through 1949, Working found that cycles starting with a rise of 17 cents or more (at 1913 price levels) from the low of any one day to the high of any day less than six weeks later had a high probability of promptly returning to about the initial level. Such movements he called "episodic" cycles.

Working did not identify the causes of these cycles, although he did suggest as his best hypothesis that they occur "largely because significant overseas news tends to be ignored or inadequately weighted in the American wheat market" (*48*, p. 158). His hypothesis is supported by the fact that he did not find such cycles in corn or oats prices, commodities with prices based mainly on domestic factors. In total, these cycles have occupied about one-eighth of the wheat price movements for the 50 years ending in 1940. The last identified cycle occurred in 1937.

In Chapter 6 we will relate the episodic cycles and the findings of Cowles and Jones to the conclusions of this inquiry.

Chapter 3

STATISTICAL METHODOLOGY

In statistical terms, the basic question considered in this study is that of determining to what extent and under what circumstances the behavior of actual speculative commodity prices differs from the behavior of a random difference series.

Several existing statistical techniques could be used to test this question. However, after considerable preliminary investigation it was concluded that all had serious shortcomings, at least when applied to speculative prices. This conclusion led to the development of the test used in this study.

The most widely known of the existing tests are those that are based on the various forms of the serial correlation coefficient.[1] In addition to three known forms,[2] there are two closely related approximations—"Von Neumann's ratio" (*40*, pp. 317–95)[3] and a statistic designated by Young (*52*, pp. 293–300) as "C." The serial correlation methods were rejected for these reasons:

1. The tests appear to be unusually sensitive to the underlying assumption that observations come from a universe with a reasonably constant standard deviation. As will be discussed later, speculative price changes are known to occasionally have widely varying standard deviations.[4]

2. Measurement of recurrent patterns with serial correlation techniques requires the assumption that the patterns are regularly spaced over specified time intervals. It is unlikely that any speculative price pattern would behave in such a regular way.

3. Extensive analysis with serial correlation techniques imposes extremely heavy computational burdens.

4. For a variety of reasons—some of which are listed above and some of which are probably not really understood—most large-scale published efforts to apply serial correlation methods have not produced satisfactory results.

There are several non-parametric tests available that avoid the limiting assumption of constant standard deviation, as well as other distributional assumptions. The most usable one appears to be the test devised by Cowles and Jones for their analysis of the structure of stock market prices (*11*). This test, which

[1] The term "serial correlation" is used to describe the correlation of terms within a single series. While other terminology has been used occasionally in the literature, as by Kendall and Tintner, this usage is consistent with the terminology of the majority of authors on the subject. Cf. *23, 29*.

[2] The three forms are (1) the basic coefficient initially proposed by Yule, defined analogously to the bivariate correlation coefficient; (2) the "circular" coefficient studied by R. L. Anderson, used to simplify the derivation of the sampling distribution; and (3) a further modification proposed by T. W. Anderson, used to eliminate the restrictive "circular" assumption. These variations are of importance primarily in small-sample tests for serial correlation, and their specific details need not concern us. Cf. *53*, pp. 1–69; *1*, pp. 1–13; *2*, pp. 88–116.

[3] For a discussion of the relation between Von Neumann's ratio and the serial correlation coefficient see *29*, p. 253. Tintner gives the equation relating the ratio and the circular serial correlation coefficient incorrectly, apparently misinterpreting the equation derived by T. W. Anderson (*2*, p. 115). In Tintner's equation (5), p. 253, the serial correlation coefficient should be T. W. Anderson's modified coefficient, not the circular coefficient studied by R. L. Anderson.

[4] Cf. *48*, p. 161.

is the ratio of sequences (i.e., successive changes in one direction) to price reversals, was discarded because it is both time-consuming and inefficient. Other nonparametric tests were also considered, but were also rejected on similar grounds.[5]

The method used in this study is based on a statistical test applied directly to the price ranges of the speculative prices.[6] Ranges were selected as the key component on these grounds:

1. A test based on ranges was found to be less sensitive than other tests to shifts in the standard deviations of the underlying observations.
2. Unlike the serial correlation test, a range test showed promise of being able to detect recurrent patterns that might occur anywhere within a specified time interval.
3. Since price ranges could be fairly easily obtained from published price records, computational burdens for a test based on ranges would not be too severe.
4. In other time series analysis (especially statistical quality control) ranges have been found to be powerful, easily handled analytical tools.

The statistical test is defined in terms of price ranges for given price segments. The segments are chosen so that each is composed of approximately the same number of price changes (designated "steps"). One group of segments, for example, might consist of the 65 four-week intervals in a five-year period, with the steps taken as the average number of price changes for four-week intervals during the period. The segments are further divided into a fixed number of intervals of approximately equal length, which are designed "subintervals" or "subsegments." In a single four-week segment, for example, the subintervals might consist of the two two-week intervals or the four one-week intervals.

Using the following notation:

n = number of segments, each containing approximately m steps,
k = number of subsegments per segment,
R_i = range of the i^{th} segment,
S_{ki} = sum of the ranges of the k subsegments of the i^{th} segment.

The statistic is defined[7]

$$H = \left(\frac{\sqrt{k}}{n} \sum_{i=1}^{n} \frac{R_i}{S_{ki}} \right) \Big/ B - 1, \tag{3.1}$$

[5] For two of the more useful ones see *42*, pp. 401–9, and *41*, pp. 378–88.

[6] While the author assisted Working in the mathematical background of bringing the ideas for the test to usable form, this work is too lengthy to be included in this study. As will be brought out subsequently, there is still additional mathematical and analytic analysis needed before all ramifications of the test are fully understood.

[7] The original 1954 calculations were made with the statistic H defined as follows:

$$H = \frac{\sqrt{k}}{n} \sum_{i=1}^{n} \frac{R_i}{S_{ki}} - B.$$

As a result of studies based on this original definition, it was found that it would be preferable, in certain circumstances, to modify the definition by dividing by B instead of subtracting it. The main reason for the change was to permit the interpretation of certain kinds of non-ideal behavior in terms of relative deviations rather than absolute deviations. Also, the change simplified comparisons between series which had (because of differing numbers of observation) different values of B. This revision, which has no significant effect on the conclusions reached with H as originally defined, was incorporated into this study in 1968 in order to provide better comparability with other related studies published since 1954.

The H statistic was one of two new statistics that were developed in 1951 and 1952, and designated by Working as G and H, following after F, because they were based on the principle of Fisher's analysis of variance. (Note added in 1968.)

where B is the expected value of

$$\frac{\sqrt{k}}{n} \sum_{i=1}^{n} \frac{R_i}{S_{ki}}$$

for a random-difference series,[8] so that for the "ideal" market, $E(H) = O$. An approximation of H that is substantially easier to compute is

$$H' = \frac{\sqrt{k}}{B} \frac{\sum_{i=1}^{n} R_i}{\sum_{i=1}^{n} S_{ki}} - 1. \tag{3.2}$$

Tables for computing values of B and approximate standard errors are given in the Appendix.

To determine how this statistic is used we shall consider its behavior when applied to artificial and experimental series containing known characteristics. To take an extremely simplified first example, let us assume that we have a single series divided into two separate intervals of m steps each, with one interval containing strong continuity throughout and the other strong reaction. We obtain H for these two separate intervals by first dividing them into k subintervals of m/k steps, and, for each interval, computing $\sqrt{k}\ R/S_k$. Since each ratio is divided by the same value of B, we can see in a general way how H indicates reaction and continuity by contrasting only the ratios.

The interval containing strong continuity will have its price movements in essentially one direction, for there will be little tendency toward fluctuations. For this interval the total range, R, will be nearly as large as the sum of the ranges of the k subintervals, for most of the movements within the subintervals will contribute to the range of the total interval. Thus, $\sqrt{k}\ R/S_k$ will be large, probably close to its maximum value, $\sqrt{k}$, occurrring when $R = S_k$.

On the other hand, if we assume that the interval containing strong reaction has, say, a tendency for a price rise and fall of about the same magnitude occurring approximately every m/k step (i.e., in each subinterval), there will then be k such oscillations dominating the interval. In this case, the range of the total interval will equal the range of each of the k subintervals, and the ratio, $\sqrt{k}R/S_k$, will be small, somewhere near its minimum value, $1/\sqrt{k}$, occurring when $R = R_1 .. = .. R_j .. = R_k$, where R_j is the range of the j^{th} subinterval.

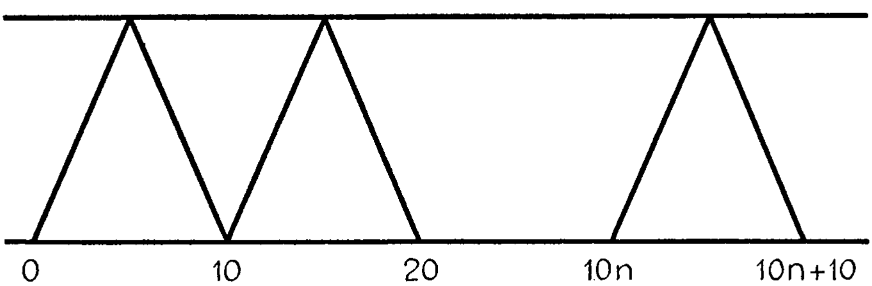
The preceding highly simplified example does not imply that by looking at price segments it is possible to distinguish one behavior from the other. We could easily select from a true random-difference series a great many intervals that appear to be good examples of continuity and reaction, yet these intervals would actually only be examples of what can occur in small samples under "ideal" behavior. A discriminating test requires the aggregation of characteristics of many separate intervals. While a single interval of a random-difference series may contain what appear to be examples of reaction or continuity, it would

[8] The term "random-difference series" is used to describe a series composed of random changes. Cf. *51*, pp. 11–24.

become more and more unusual, as the number is increased, for the majority to continue to exhibit any measurable tendencies in either direction. Consequently, while continuity and reaction can be broadly characterized in terms of recognizable price behavior, we can only reliably conclude that such behavior is present when H shows statistical significance, either through the repeated pattern of several related values (similar to the interpretation of a correlogram) or by the significant departure from zero of one or more values.

Our next example, though as artificial as the first, further illustrates the basic characteristics of H. Previously we considered how H indicated continuity and reaction in a fixed interval and subinterval length. We now undertake the more complex problem of applying H to a price series containing a combination of continuity and reaction. In this experiment we consider the results of varying m and m/k, the interval and subinterval lengths, fitting several values of H together to explore the characteristics of the entire series.

Let us assume that we wish to analyze price behavior for intervals of 1 to 16 days in a very large sample of consecutive daily close prices.[9] First, we divide the series into separate intervals of 1, 2, 4, 8, and 16 days.[10] Then, after determining the range of each interval, we apply equation (3.1) to obtain the possible values of H. Since each interval length can be grouped with subintervals formed from the shorter intervals these 5 segments produce 10 values.[11] Next, suppose this price series has a regular cyclical movement consisting of a uniform daily price increase for five days, followed by a uniform daily decrease for five days, with all changes of the same magnitude. Such a series would look like this:



Since the price changes are assumed constant, we can compute the ratio part of H—$\sqrt{k}\ R/S_k$—for the 10 combinations of intervals and subintervals without actually specifying the magnitude of the changes. These calculations are summarized in Table 3.1.

From this tabulation we obtain H by dividing by the appropriate values of B (computed from Appendix Table I) and subtracting 1 from the result. Table

[9] A series of daily closing prices differs from a complete price series in that it contains but one price for each trading day, whereas the complete series contains all the prices registered during the session, often several hundred. We will subsequently consider the statistical relationship between these series (Chapter 4).

[10] Although we could compute H for each interval length from 1 to 16 days, the computational requirements of such detailed analysis would be excessive. In practice, we have found that analysis based upon intervals increasing by multiples of two provides adequate coverage of most actual series.

[11] In general, we may say N intervals that increase by multiples of two can be combined to produce $N(N-1)/2$ values of H.

TABLE 3.1.—$\sqrt{k}R/S_k$ FOR VARIOUS INTERVALS AND SUBINTERVALS OF HYPOTHETICAL PRICE SERIES

Subinterval ("*days*")	Interval length ("*days*")			
	2	4	8	16
1	1.272	1.600	1.697	1.250
2	—	1.272	1.343	0.984
4	—	—	1.113	0.784
8	—	—	—	0.738

3.2 contains the complete set of H values for the ten combinations of intervals and subintervals.

TABLE 3.2.—H FOR VARIOUS INTERVALS AND SUBINTERVALS OF HYPOTHETICAL PRICE SERIES

Subinterval ("*days*")	Interval length ("*days*")			
	2	4	8	16
1	+.199	+.347	+.316	−.088
2	—	+.135	+.104	−.238
4	—	—	+.024	−.321
8	—	—	—	−.305

By comparing the individual indications of continuity and reaction contained in this table to the known characteristics of the hypothetical series (5-day trends, 10-day cycles), we can begin to see how a set of H values, each obtained from a different interval and subinterval combination, can be used to explain the characteristics of the entire series. This is more clearly evident if we rank the values in order of magnitude:

H	Interval and subinterval ("*days*")	
+.347	m = 4,	m/k = 1
+.316	m = 8,	m/k = 1
+.199	m = 2,	m/k = 1
+.135	m = 4,	m/k = 2
+.104	m = 8,	m/k = 2
+.024	m = 8,	m/k = 4
−.088	m = 16,	m/k = 1
−.238	m = 16,	m/k = 2
−.305	m = 16,	m/k = 8
−.321	m = 16,	m/k = 4

Considering continuity first, we see that the strongest indications occur for the intervals 4 and 8, with subintervals of 1. Thus, the 5-day trend (an extreme form of continuity) is indicated by the value based on the interval that is the nearest to the trend length, with subintervals of minimum length. A further indication that the maximum trend is between 4 and 8 is that for all intervals of 16, H abruptly becomes negative, indicating the presence of reaction in these longer intervals.

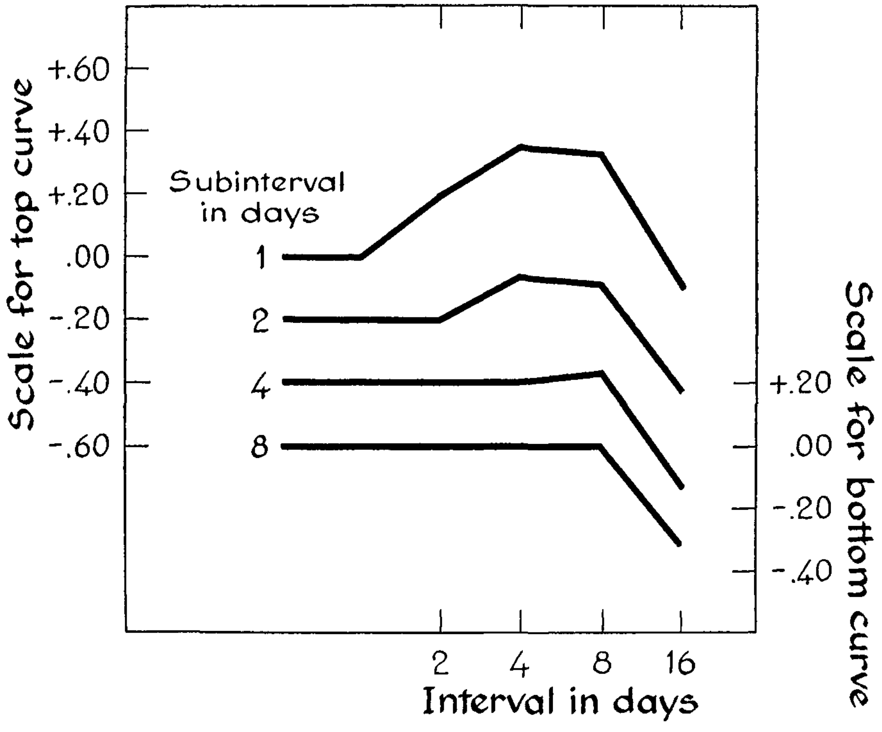
We can summarize the suggested conclusion as follows: for a series containing continuity of a fixed length (5-day trends in this example), the strongest indication occurs for the segment length that is closest to the length of the continuity, with subintervals taken as small as possible.

Turning to the evidence of reaction, we see that the strongest indications occur for intervals of 16 with subintervals of 4, 8, 2, and 1, in that order. Thus, the reaction pattern of this artificial sample (constant and recurring cycles) is reflected in values of H based on a long interval length, with the subintervals of a length equal to the interval length that gave the maximum indications of continuity.[12]

We can summarize this suggested conclusion as follows: if the reaction is in the form of a regular cyclical movement, its strongest indications will occur for interval lengths that include several cycles, with subintervals taken as one-half of the cycle length.

We conclude our analysis of this simple example by considering Chart 3.1. Here we show the values of H from Table 3.2 arranged so that the conclusions deduced from the table can be seen graphically. The top curve of the chart represents H for subintervals of 1 day, for intervals of 2, 4, 8, and 16 days; the second curve, subintervals of 2 days, for intervals of 4, 8, and 16 days; and so on. The vertical scales are shown for the top and bottom curves only. The other two

CHART 3.1.—*H* FOR VARIOUS SEGMENTS AND SUBSEGMENTS OF HYPOTHETICAL PRICE SERIES



[12] We are unable to determine on the basis of this highly artificial example exactly how long the interval must be to produce the maximum reaction indication. Our only justified conclusion is that it will occur for one of the longer interval lengths tested.

curves have the same scales with the origins shifted to the horizontal line opposite the subinterval designation.

The continuity is clearly evident in the top curve. *H* reaches a maximum for 4 days and then drops sharply on either side, strongly suggesting the presence of the 5-day trends. The same pattern is also reflected, but to a lesser degree, by the curve for subintervals of 2 days.

The reaction patterns are also evident. Since all *H* values for 8-day intervals are positive and all 16-day values are negative, it is clear that the continuity existing up to the 8-day intervals has been replaced by reaction. Although the chart does not enable us to easily distinguish which subinterval has given the maximum reaction indication, reference to Table 3.2 (or measurement in the chart) shows that subintervals of 4 days give a stronger reaction indication than 8 days. Thus, we deduce that the reaction pattern is a cyclical rise and fall, with the movement in each direction probably closer to 4 days than to 8.

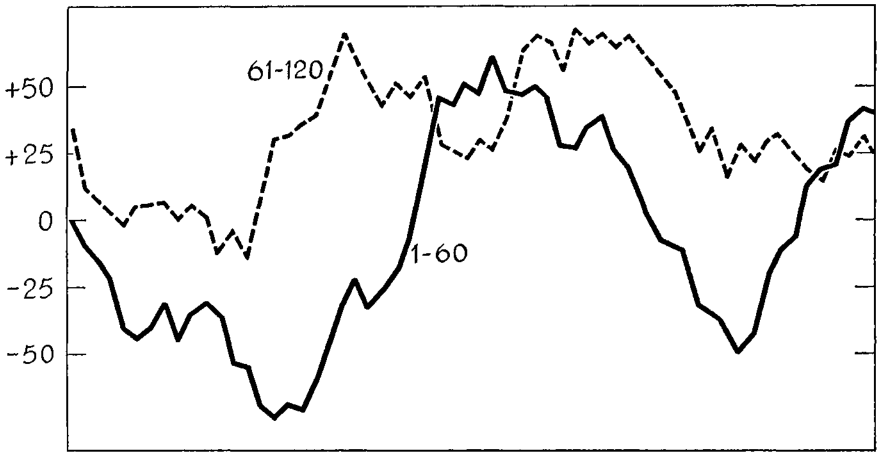
Although the conclusions indicated by Chart 3.1 are not too difficult to determine by simple inspection of the data, we will find in subsequent analysis that this graphic representation often provides the best way to analyze the behavior indicated by a full set of values of *H*.

We next apply *H* to an experimental series that contains known reaction and continuity tendencies of a sort that, when the experiment was designed, were thought likely to be present in actual prices. Prices were assumed to be subject to frequent movements of varying duration and amplitude that developed progressively and, after going too far, tended to be followed by partial reaction.

The experimental series embodying this concept (called a "trend-reversal" series) was constructed by drawing two digit random numbers from Tippett's tables (*30*). The first digit specified the duration of the trend (i.e., continuity) and the duration of the partial reversal. The second digit specified both the direction and the strength of the trend and the subsequent reversal. The following tabulation contains the specific instructions:

First digit	Trend duration	Duration of reversal	Second digit	Direction and strength of movement
1	1	1	0	−9
2	2	1	1	−6
3	3	2	2	
4	4	2	3	−3
5	5	3	4	
6	6	3	5	+3
7	7	4	6	
8	8	4	7	+6
9	9	5	8	
0	10	5	9	+9

CHART 3.2.—FIRST 120 TERMS OF EXPERIMENTAL TREND-REVERSAL SERIES



To illustrate the procedure, the number 54 would specify a trend-reversal movement of 5 consecutive decreases, each of 3 units, followed by a partial reversal of 3 consecutive increases, each of 3 units.

After the trend-reversal patterns were determined, they were added, term by term, to Working's random-difference series, until a total of 1,920 terms was reached. Chart 3.2, containing the first 120 terms, illustrates one portion of the trend-reversal series.

For convenience we consider each term as representing the closing price of one trading session. Characterizing the series broadly, we see that it contains equally likely trends varying from 1 to 10 days, with each trend followed by a reversal of about one-half the initial movement. The average trend is 5.5 days, the average reversal 3 days; the maximum trend is 10 days, the maximum reversal 5 days. As constructed, the series contains 226 randomly chosen "trend-reaction" movements, each occurring immediately after one another.

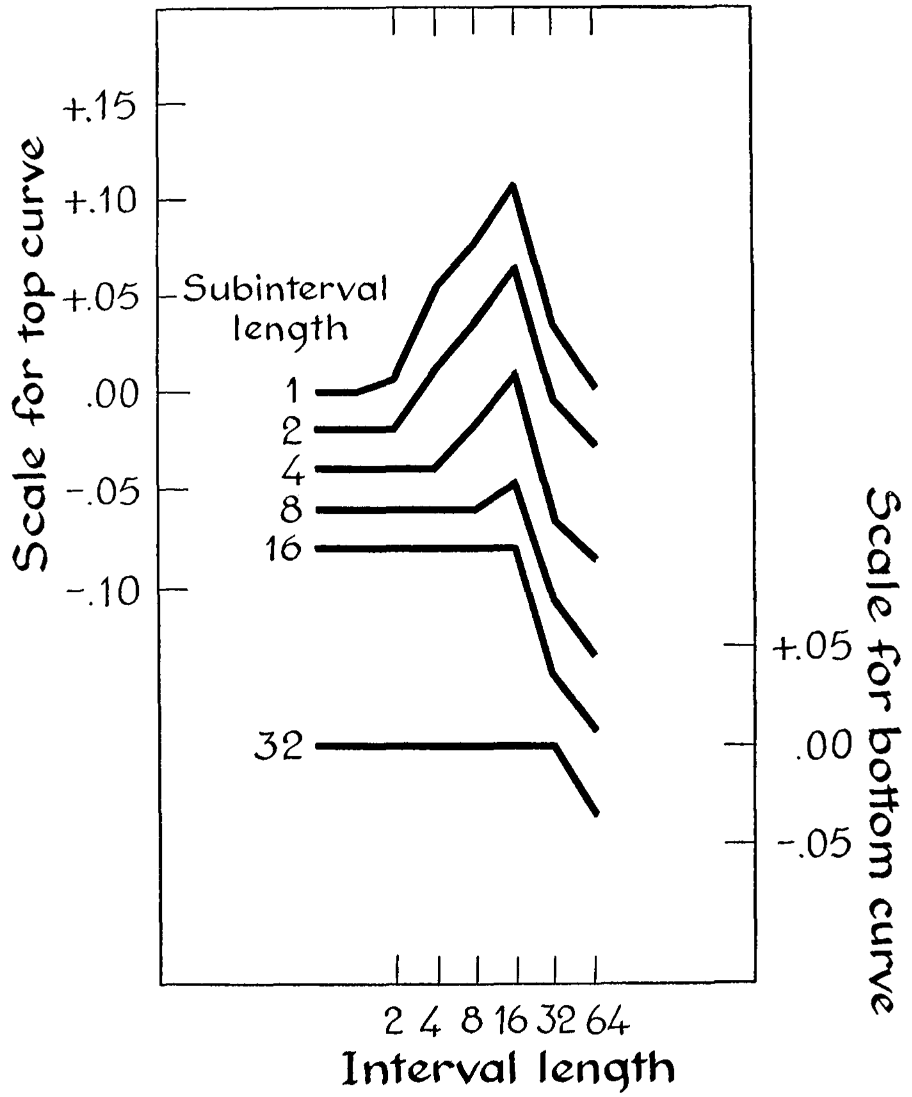
To apply *H* we divide the series into intervals of from 1 to 64 days (increasing by multiples of 2). This division produces 21 values, with sample sizes ranging from 960 for intervals of 2 days to 30 for intervals of 64 days. Table 3.3 contains these 21 values:

TABLE 3.3.—*H* FOR TREND-REVERSAL SERIES

Subinterval ("*days*")	Interval length ("*days*") 2	4	8	16	32	64
1	+.006	+.051	+.075	+.105	+.035	+.003
2	—	+.030	+.054	+.084	+.018	−.008
4	—	—	+.022	+.048	−.022	−.045
8	—	—	—	+.014	−.045	−.073
16	—	—	—	—	−.067	−.091
32	—	—	—	—	—	−.033

Chart 3.3 illustrates the values for the trend-reversal series:

CHART 3.3.—*H* FOR TREND-REVERSAL SERIES



Considering first the indications of variable continuity, we see that the largest values occur for the minimum subinterval length. For both 1- and 2-day subintervals, the strongest continuity is indicated for 16-day intervals, with the second strongest indication for 8-day intervals. Comparing these indications to the known characteristics of the series, we conclude that the strongest indication of variable continuity occurs when the intervals are taken as equal to, or slightly beyond, the length of the maximum continuity, with subintervals as small as possible.[18]

[18] Although we are unable to determine if the qualification "or slightly beyond" is a general one, it is clearly called for by the data of Table 3.3. While additional experimental series should enable us to determine whether it is due to some special characteristic of the trend-reversal series or is actually a general characteristic of *H*, computational limitations prevented us from constructing such a series.

It is noteworthy that although intervals of 4 and 8 days do show continuity, the strongest indication occurs not near the average trend length but instead near or slightly beyond the maximum length. This suggests that when variable trends exist, the continuity indications will continue to increase until the maximum length is reached.[14]

The maximum indication of reaction occurs for 64-day intervals and 16-day subintervals, an indication slightly exceeding the second strongest indication, occurring for the same interval with 8-day subintervals. This result is similar to that of the previous example, for in both cases the strongest reaction is indicated for one of the longer intervals with subintervals taken as equal to the interval that produced the maximum indication of continuity.

As in the case of the variable continuity, the indication occurs beyond the length of the maximum built-in reaction pattern, suggesting that the largest reversals particularly influence *H*. We thus conclude that for a series containing continuity followed by reversal, the strongest indication of reaction will be found in the longer intervals tested, with subintervals equal to, or slightly beyond, the length of the maximum reaction movement.

Although the behavior of some of the values of *H* might be interpreted as suggesting the presence of partial reaction, we are unable to reliably conclude that the statistic enables us to distinguish between complete and partial reaction patterns.

We complete this analysis of the basic experimental series by considering the interpretation of the significance tests of the continuity and reaction indications of Table 3.3, and the relation of these tests to our conclusions.

As noted above, the sample sizes vary according to the interval length. For the 1,920-term series, *H* for 2-day intervals has 960 segments, for 4-day intervals 480 segments, and so on down to 30 segments of 64 days. After computing the standard errors for the values of *H* in Table 3.3 (from Appendix Table II), we find the following are statistically significant (S.E. indicates "standard error"):

Interval length	Subinterval length	*H*	S.E.
	Over three standard errors		
m = 4	m/k = 1	+.051	±.016
m = 8	m/k = 1	+.075	±.022
m = 16	m/k = 1	+.105	±.030
	Over two standard errors		
m = 4	m/k = 2	+.030	±.013
m = 8	m/k = 2	+.054	±.019
m = 16	m/k = 2	+.084	±.030
m = 32	m/k = 16	−.067	±.030

This ranking shows that the values of *H* used to draw conclusions about variable continuity are highly significant, while only one of the reaction indi-

[14] The term "maximum length" needs qualifying. What is actually meant is "maximum length that occurs with sufficient frequency to merit recognition." Suppose, for example, the experimental series was constructed with the slight modification that one trend of 32 days was placed randomly somewhere in the 1,920 terms. While this one trend would be the unique maximum of the variable trends, it would have relatively little effect upon the values of *H*.

cations is significant. While such results seem to suggest that our conclusions about reaction are less reliable than those on continuity, we do not believe that this is the correct interpretation. There is an analogy in judging the overall significance of a correlogram.[15] In determining if a correlogram represents a harmonic series, for example, we do not test the serial correlation coefficients of each order separately, rejecting those near zero from consideration. Rather, it is the pattern of the entire set of coefficients that determines if the series is harmonic. While certain values might be significant, others must be close to zero in order for the series to be a true harmonic. Although it may be difficult to statistically conclude from a single correlogram just what the underlying structure of a series is, the conclusion would become increasingly significant if the same harmonic pattern clearly appears in repeated, independent samples of the series.

This is similar to the procedure followed in our subsequent analysis with H. If some values are statistically significant, we examine the entire set, attempting to determine from their pattern what kind of non-ideal behavior is indicated. Or, if none of the values of one group of samples is significant but the same pattern emerges from group after group (e.g., from wheat, corn, and rye for several different years), we also make the same kind of pattern analysis, relying on weight of overall evidence as our justification.

We next consider the use of the modified statistic, H', and its relation to H.

Rewriting the basic definitions in extended form brings out the principal difference between H and H':

$$H = \frac{\sqrt{k}}{Bn}\left(\frac{R_1}{S_{k1}} + \frac{R_2}{S_{k2}} + \ldots + \frac{R_n}{S_{kn}}\right) - 1\,, \tag{3.3}$$

$$H' = \frac{\sqrt{k}}{B}\left(\frac{R_1 + R_2 + \ldots + R_n}{S_{k1} + S_{k2} + \ldots + S_{kn}}\right) - 1\,. \tag{3.4}$$

Thus, H is an average of n separately formed ratios, while H' is the ratio of the total of the numerators to the total of the denominators.

Table 3.4 gives the values of H' obtained from the trend-reversal series and in Chart 3.4 the values of H and H' for this series are shown together.

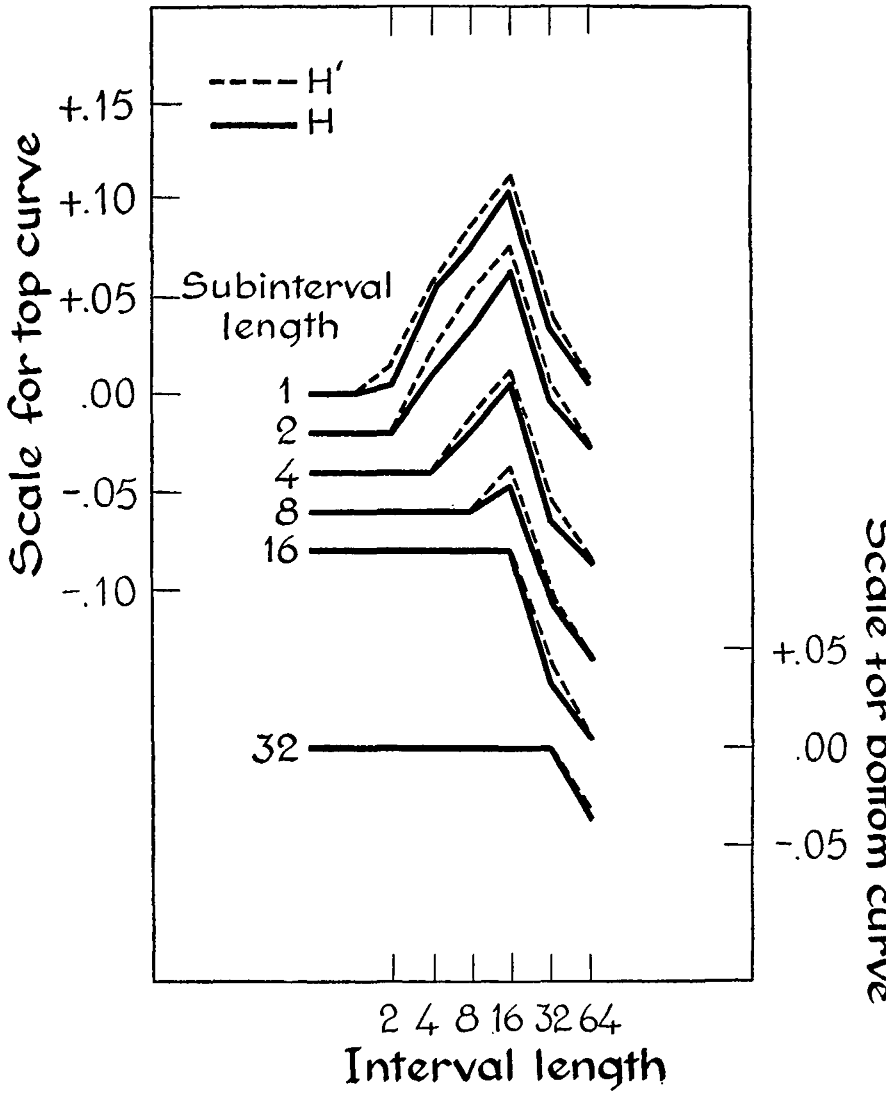
The chart shows that H and H' have produced very similar measurements of price behavior. In fact, if the two sets were shown in separate charts it would be difficult to visually distinguish between them.

TABLE 3.4.—H' FOR TREND-REVERSAL SERIES

Subinterval ("*days*")	Interval length ("*days*")					
	2	4	8	16	32	64
1	+.015	+.056	+.088	+.111	+.041	+.009
2	—	+.040	+.071	+.095	+.026	−.007
4	—	—	+.029	+.052	−.014	−.045
8	—	—	—	+.022	−.042	−.073
16	—	—	—	—	−.063	−.092
32	—	—	—	—	—	−.031

[15] See, for example, 22, pp. 396–413.

CHART 3.4.—COMPARISON OF H AND H' FOR TREND-REVERSAL SERIES



Any difference between corresponding values of H and H' derived from the same data can be seen to be largely attributable to a difference in weighting. Because of the difference in weighting, sampling fluctuations can affect the two statistics somewhat differently, hence the differences observed in this instance must be attributed at least in part to sampling fluctuations. They appear, nevertheless, to reflect also a significant characteristic of the experimental trend-reversal series, namely, the fact that some segments of the series contain longer and stronger trends than do other segments. Those segments in which continuity is particularly strong tend also to be segments in which "day-to-day" changes are larger than average. Consequently, they are given lighter weighting in H than in

H'. Knowing these characteristics of the series, we might have predicted that it would yield larger positive values of H' than of H.

We thus see that it can be desirable to compute both H and H' for any given series. Where choice must be made between the two, H' may be preferred because it is the simpler to compute. For example, to obtain the values for H in Chart 3.4 it was necessary to make 3,600 divisions and 3,600 additions. To obtain those for H', on the other hand, it was necessary to make 3,810 additions followed by only 21 divisions.

We can summarize our subsequent principal uses of H' as follows:

1. H' is used as a supplement to H to provide evidence on the particular circumstances under which certain behavior might be occurring.
2. H' is used as a simpler substitute for H when comparisons are to be made which are based on various prices classified in essentially the same manner. In such cases the problem of heavy weighting is avoided, since comparison is made only among like data.

We conclude this chapter with a brief comment on the problem raised by the fact that the standard deviations of the price changes are variable, whereas the mathematical basis for H and H' assumes a series of constant standard deviations.

An example of the sort of variability that can be found in actual speculative price changes is given in Table 3.5. Here we show the average weekly price ranges for wheat, corn, and rye futures for five separate years. Since price ranges are good measures of variability of price changes over fixed time intervals (such as one week, one month, etc.) these average weekly ranges illustrate the magnitude of the variability that can exist.

TABLE 3.5.—ANNUAL AVERAGE WEEKLY PRICE RANGES FOR DOMINANT WHEAT, CORN, AND RYE FUTURES AT CHICAGO, 1937–41

(Prices in eighths of a cent per bushel)

Year	Wheat	Corn	Rye
1937	50.1	36.4	43.6
1938	29.3	18.5	22.4
1939	28.6	19.1	26.4
1940	37.1	21.5	30.6
1941	33.3	19.3	28.4
Average	35.7	23.0	30.3

It is evident that considerable shifts in variability are possible. Each of the average ranges for 1938, for example, is only about one-half as large as the averages for the previous year. The table also illustrates that variability can remain moderately unchanged from one year to the next. Excluding 1937, the ranges for corn fluctuate within small limits. Other data, which we have not presented, show varying variability for longer intervals. Although we have determined no mathematical basis of either estimating or allowing for the effects of such variation on H, we have some indirect evidence that the effects are not too serious.

As observed in the previous section, H' is subject to much stronger influence from large price changes than H. But when we consider (in Chapter 5) the actual results of computing both H and H' for intervals of one to 16 weeks for years with widely differing characteristics, we still find a consistent relationship between H and H'. This relationship closely resembles that shown in Chart 3.4, where, it will be recalled, the values of H and H' were constructed from a series with relatively little variation in its standard deviation. This evidence strengthens our belief that the variable standard deviations in actual price changes have not significantly influenced either H or H'.

Chapter 4

SPECIAL TECHNICAL PROBLEMS

Before H and H' could be used to test actual speculative prices, two sets of special technical problems had to be solved. One arose from need to "splice" price series for different futures. This problem is treated in the later pages of this chapter. We begin with the more difficult problem raised by the fact that so-called "continuous" price quotations are in fact discontinuous, whereas the mathematical basis for calculating H and H' assumes a continuous series.[1] We will consider two independent methods of attacking this second problem. In the course of developing these methods we also analyze—and eventually decide upon—the use of simpler price series in place of the complete set of price quotations.

1. THE EFFECT OF INTER-SESSION PRICE CHANGES

Price quotations for commodity futures are essentially continuous from the beginning of a trading session until its end. There is then a break until the beginning of the next trading session, which is usually on the next calendar day, but is at least two days later when a Sunday or a holiday intervenes. Frequently prices during one trading session are all higher, or all lower, than the last price on the previous day, as in the example below:

Quotation	Price of Chicago May wheat future, in cents
Close, March 19, 1949	233½–234
High, March 20	233
Low, March 20	229⅛

In such a case the high price for a day regarded as beginning immediately at the close of trading on March 19 and extending to the close on March 20 must be taken as at least as high as 233½, the lower quotation of the previous closing range.[2] But if there had been trading during the interval between the two trading sessions, the price might have gone still higher. There seems reason, therefore, to suspect that these "interruptions" in the price series can introduce some kind of bias into H and H'.

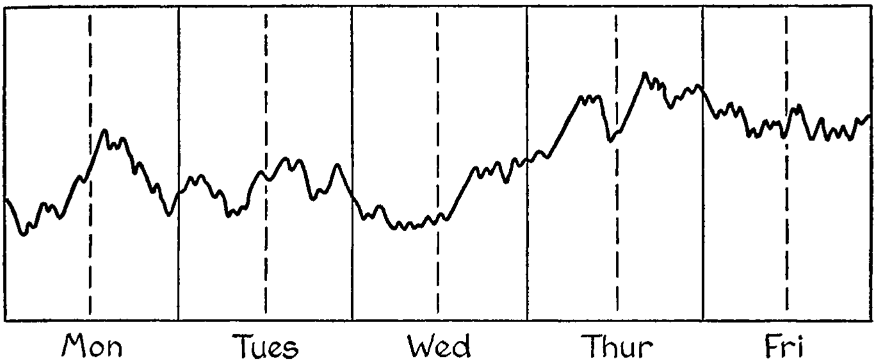
To illustrate let us assume that the first 270 terms of Working's random-

[1] The term "continuous quotations" is used to describe the series of prices recorded consecutively during the trading session. These prices usually change by the smallest unit permitted, such as one-eighth of a cent for wheat, corn, rye, and other commodities traded in bushels, and are usually thought of as being registered continually during the session. Actually, however, the price changes are often unevenly spaced throughout the session, and in very active markets the consecutive changes may become two or more times the minimum unit, reaching one-quarter or even one-half of a cent.

[2] In practice, we assume that the price must reach the midpoint of the closing range. Since the closing range represents two prices recorded almost simultaneously, we are unable to determine which of the two should properly be taken as the close. The closing ranges are usually only ⅛ or ¼ (½, as in this example, is fairly uncommon).

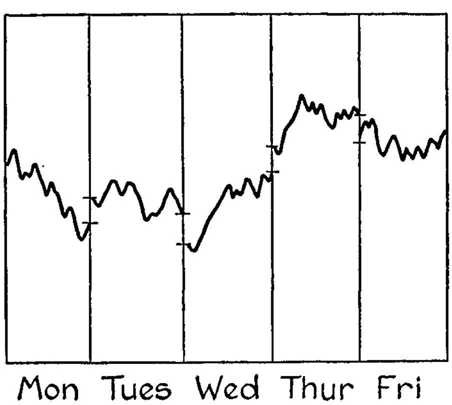
difference series (*51*) is a continuous series of speculative prices. These "prices," divided into five separate "days," are shown in Chart 4.1:

CHART 4.1.—HYPOTHETICAL PRICE SERIES



Next, we convert Chart 4.1 to a more realistic model of speculative prices by introducing overnight interruptions between each day's quotations. If we suppose that the first half of each day (divided by a vertical dotted line in Chart 4.1) contains prices occurring during the interval the market is normally closed, removal of these prices introduces interruptions similar to actual inter-session price changes. This revised series is shown in Chart 4.2:

CHART 4.2.—HYPOTHETICAL PRICE SERIES SHOWING THE EFFECT OF OVERNIGHT PRICE CHANGES



In this chart we have "compressed" the price movements by representing the first half of each day by its net change (i.e., differences between first and last price), omitting the intermediate movements.

This compression process shows that behind each interruption there is, at least conceptually, some sort of a series of price changes. While such a series is a conceptual reality,[8] in actual speculative markets this "hypothetical" series prob-

[8] In fact, the inter-session change is the net change of the "hypothetical" series. We use this information in our subsequent mathematical treatment of this problem.

TABLE 4.1.—RANGES FROM COMPLETE AND COMPRESSED SERIES*

"Day"	Complete series (Chart 4.1)			Compressed series (Chart 4.2)		
	High	Low	Range	High	Low	Range
Monday	80	−35	115	80	−18	98
Tuesday	37	−17	54	37	− 9	46
Wednesday	44	−34	78	44	−30	74
Thursday	108	33	75	108	33	75
Friday	103	36	67	103	36	67
Average			77.8			72.0
Five-day interval	108	−35	143	108	−30	138

* For method see the accompanying text.

ably exists only vaguely in the minds of the traders whose composite judgment has produced the opening price. By comparing the price ranges of each "day" in the two series, complete (Chart 4.1) and compressed (Chart 4.2), we can see more clearly how the inter-session interruptions can affect H and H'.[4]

Taking the "days" first, we see that the compressed series has three of its five ranges smaller than the corresponding ranges of the complete series. This indicates that each of these three subintervals had at least one of its extremes within the part of the series represented by the inter-session change. As a result, the average "daily" range for the complete series is 77.8 and for the compressed series, 72.0. Considering the range over the total interval, we see that the values for the complete and compressed series are 143 and 138. Thus, the total interval range was reduced by only 3 per cent, whereas the average of the subinterval ranges was reduced by nearly 8 per cent. Since the denominator was reduced by more than the numerator it is evident that the combination of these two effects would be to produce an upward bias in H and H'.

The first method that we use to attempt to estimate the general extent of this bias involves the basic equations used in the mathematical development of H and H'. This technique (designated for convenience the "method of reconstruction") makes use of the fact that the inter-session price changes can be interpreted as the net change resulting from a hypothetical series of price changes occurring while the market was closed. By using the "information" contained in these net changes (and assuming that price behavior over certain intervals does not depart significantly from the "ideal"), it should be possible to make a rough estimate of the per cent by which the overnight interruptions have reduced the close-to-close ranges relative to what they would have been in a continuously operating market.[5]

[4] Since the series contains negative values, we consider the values as representing deviations about the average price of the series.

[5] The general validity of the ideal market assumption is, of course, the basic question undertaken for inquiry. Because of later evidence that this assumption is invalid we subsequently use another method that is independent of this assumption. The development of the "method of reconstruction" is still included here because of its possible uses in other situations where the ideal market assumption can be shown to be sufficiently valid.

In a chain of m random unit steps, we have these relationships[6] for the mean and variance of the range, R_m, and the net change (without regard to sign), $|\Delta|_m$:

$$E(R_m) \cong 1.5958\sqrt{m} - 1, \tag{4.1}$$

$$\sigma^2 R_m \cong 0.2261m - 0.1804, \tag{4.2}$$

$$E|\Delta|_m \cong 0.7979\sqrt{m}, \tag{4.3}$$

$$\sigma^2|\Delta|_m \cong 0.3634m. \tag{4.4}$$

By the use of these relationships, we can devise an expression by which to estimate, for any given group of intervals of specified length, what the average price range would have been if quotations had been continuous through the gaps formed by the overnight interruptions. From our knowledge of the net price change over each gap, we can then estimate its corresponding contribution to the average price range. We will start with close-to-close price ranges, gradually extending the analysis to longer interval lengths. For convenience, we adopt the term "24-hour range" in place of the correct but more cumbersome "close-to-close range," recognizing that when a Sunday or holiday intervenes, the time interval from one closing price to the next may be 48 or even 72 hours.[7]

To obtain the needed equation we consider the two separate components of the 24-hour range: (1) the part contributed by the trading session, and (2) the part contributed by the inter-session change. Writing $E(R_c)$ for the expected "complete 24-hour range," we apply equation (4.1) to obtain this expression for the expected value of R_c:

$$E(R_c) = 1.5958\sqrt{\text{“}m\text{”}_{|\Delta|} + \text{“}m\text{”}_R} - 1, \tag{4.5}$$

where "m"$_{|\Delta|}$ represents the estimated average number of hypothetical price quotations producing the inter-session price change (the "hypothetical" series), and "m"$_R$ the average number for the trading session. The subscripts $_{|\Delta|}$ and $_R$ indicate that "m"$_{|\Delta|}$ and "m"$_R$ are to be estimated from the average inter-session change and average session range, respectively.[8]

Once we have estimated $E(R_c)$, we then estimate the per cent by which $\bar{R}$, the actual average 24-hour range, has been reduced because of the inter-session interruptions:

$$\text{Per cent reduction} = \frac{\bar{R} - E(R_c)}{E(R_c)} \times 100. \tag{4.6}$$

[6] Equations (4.1) and (4.2) are based upon theoretical analysis done independently by Daniels and Feller. Both give asymptotic formulas for the variance containing minor errors. The correct expression is the one obtained by Feller (but evaluated incorrectly):

$$\text{Variance } R = 4m(\log 2 - 2/\pi) \cong 0.22611m$$

Cf. *12*, pp. 244–51; *16*, pp. 427–32.

[7] Analysis of net changes from one session close to the next session open ("inter-session change") has indicated that the changes occurring during the longer periods of closing (i.e., weekends and holidays) do not differ appreciably from changes occurring between regular weekday sessions. Consequently, we can group all inter-session changes into one common classification.

[8] We use the session range to estimate the number of session quotations because, as the equations for the means and variances show, its coefficient of variation is substantially less than that of the session net change, thus providing a statistically more efficient estimate.

In estimating "m"$|\bar{\Delta}|$ and "m"$_{\bar{R}}$, the unknowns needed in (4.5), the intuitive method is to substitute $\bar{R}$ and $|\bar{\Delta}|$ for $E(R)$ and $E|\Delta|$ in (4.1) and (4.3), and solve the equations directly for m. Separate analysis (not given here) shows that these intuitive estimates are, in fact, maximum likelihood estimates.[9] Consequently, we adopt this procedure to estimate the needed values, using the following formula:

$$\text{From } \bar{R} \quad : \quad \text{“}m\text{”}_{\bar{R}} \quad = \quad \frac{(\bar{R}+1)^2}{(1.5958)^2} \tag{4.7}$$

$$\text{From } |\bar{\Delta}| \quad : \quad \text{“}m\text{”}|\bar{\Delta}| \quad = \quad \frac{|\bar{\Delta}|^2}{(0.7979)^2}\,.$$

After we have computed $m|\bar{\Delta}|$ and $m_{\bar{R}}$ we then estimate, by (4.5), the expected complete range. Then, applying (4.6), we obtain an estimate of the average per cent by which the denominator of H or H'—when based on 24-hour ranges—is reduced by the existence of the inter-session interruptions.

The method can be illustrated with a specific example based on the first thousand terms of Working's random-difference series. Dividing the 1,000 terms into segments of 20 steps, we have 50 such segments, with an average range of 63.78. Next, we introduce interruptions by removing the first 10 steps of each segment, leaving only the initial value. This means that in each segment we can determine for the first 10 steps only the net change, for the second 10 steps, the complete range, and for the entire segment a new reduced range. The averages of these values are as follows: $|\bar{\Delta}|$ for first-half $= 26.90$, $\bar{R}$ for second-half $= 40.67$, $\bar{R}$ for total segment $= 58.50$.

By the reconstruction procedure, we use these three values to estimate the complete range. Applying (4.5) and (4.7) we have[10]

$$\frac{\text{Est. Complete range}}{10} = 1.5958\sqrt{\left(\frac{2.69}{0.7979}\right)^2 + \left(\frac{4.68+1}{1.596}\right)^2} - 1\,, \tag{4.8}$$

which, upon reduction, gives the reconstructed estimate:

$$\text{Est. Complete range} = (1.5958\sqrt{21.45} - 1)10 = 63.89\,. \tag{4.9}$$

Comparing this reconstructed estimate of 63.89 to the actual value of 63.78, we see that the estimate is quite close. In percentage terms:

$$\text{Per cent discrepancy} = \frac{63.89 - 63.78}{63.78} \times 100 = 0.17\%\,. \tag{4.10}$$

Thus, the method of reconstruction has given an excellent estimate of the complete range—one based solely on information derived from the compressed segments.

[9] See, for example, *45*, pp. 133–42.

[10] Since our derivation of the method of reconstruction assumes a chain of m random *unit* steps, it is necessary in this example to divide the estimated complete range by 10, the standard deviation of the steps in the random-difference series.

TABLE 4.2.—COMPARISON OF ACTUAL AVERAGE AND "EXPECTED" 24-HOUR RANGES FOR WHEAT AND CORN, 1937–41 AND 1947–51, AND RYE, 1937–41

(Prices in eighths of a cent)

Commodity and years	Average values: Inter-session change (1)	Average values: Session range (2)	"Expected" average 24-hour range[a] (3)	Actual average 24-hour range[b] (4)	Per cent discrepancy (5)
1937–41					
Wheat	3.49	12.19	13.93	13.52	−2.94
Corn	2.18	8.02	9.02	8.79	−2.55
Rye	3.21	10.01	11.74	11.12	−5.28
1947–51					
Wheat	5.02	19.79	22.08	21.43	−2.94
Corn	4.72	17.40	19.67	18.96	−3.61

[a] For details of calculations of "expected" 24-hour ranges, see the accompanying text.

[b] Greatest difference among three prices: high and low for the trading session; and the previous closing price.

We next use the method to estimate the effect of inter-session changes on actual price ranges, using prices for wheat and corn for 1937–41 and 1947–51, and for rye for 1937–41. Table 4.2 contains the basic data.

The first two columns contain the averages of inter-session changes and session ranges for the three commodities. Column (3) shows the reconstructed estimate of the complete 24-hour range obtained by using the values of columns (1) and (2) in equations (4.7) and (4.5). Column (4) contains the actual average ranges (see the appended table note), and column (5) shows the per cent discrepancy between the actual and expected values.

Extending the calculations to cover longer intervals produces the result shown in Table 4.3:

TABLE 4.3.—PER CENT DISCREPANCY BETWEEN ACTUAL AND EXPECTED PRICE RANGES FOR INTERVALS OF 24 HOURS TO ONE WEEK*

Commodity and years	24 hours	One-half week	One week
1937–41			
Wheat	−2.94	+1.23	+2.29
Corn	−2.55	−1.81	−0.65
Rye	−5.28	−2.76	+2.12
1947–51			
Wheat	−2.94	+2.55	+5.72
Corn	−3.61	+3.83	+6.69

* Plus sign indicates actual range exceeds expected value; minus sign indicates actual range falls below expected value.

Moving from 24-hour to one-half week intervals we see that all of the percentages show a diminishing effect of the interruptions. For one-week intervals only one value (corn, 1937–41) remains even slightly negative, and all the others are positive.

The fact that for one-week intervals all of the percentages but one have become positive indicates that these intervals exceed their calculated "expected" values. These results are disturbing, for they suggest the presence of significant non-ideal behavior and the possibility that this method underestimates the effect of the overnight interruptions. Consequently, we next consider an alternative method that is essentially independent of the assumption of ideal behavior.

2. ABRIDGED PRICE SERIES

The term "abridged price series" is used to describe a series that contains only a portion of the total price data, so taken as to retain as much as possible of the information in the full series. An abridged series may be formed, for example, by taking only the closing price of each trading session or only the weekly closing price. While comparison of such a series to the continuous quotations would undoubtedly bring out noticeable differences in intervals as short as one week, in longer intervals the differences become of less significance, for the intraday price movements are soon overshadowed by the inter-day changes. For a sufficiently long interval of time the two series would soon become virtually indistinguishable.

An example of abridged price data can be seen in Table 4.4, where we show average ranges for the complete and three abridged series for the dominant wheat future at Chicago, 1947–51, for intervals of one to 16 weeks.

As brought out above, the inter-session interruptions affect the ranges and subranges for the complete series because they introduce discontinuities into an otherwise continuous series. The abridged series avoids this difficulty—provided only that the abridgement is taken over reasonably uniform time intervals that exceed the trading session. For this reason, comparison of values of H and H' obtained from an abridged series with unadjusted values obtained from the complete series provides a method of measuring the effect of the inter-session inter-

TABLE 4.4.—AVERAGE PRICE RANGES FOR COMPLETE AND THREE ABRIDGED SERIES, WHEAT, 1947–51, INTERVALS FROM ONE TO SIXTEEN WEEKS

(Prices in eighths of a cent)

Interval (*weeks*)	Complete[a]	Daily close	Wed.–Sat. close[b]	Saturday close[b]
1	57.1	42.7	34.8	29.3
2	82.4	66.4	58.2	50.2
4	120.6	102.9	92.3	84.4
8	183.9	163.6	151.1	140.6
16	274.7	251.8	235.3	227.8

[a] As published, without adjustment by "method of reconstruction."

[b] Saturday trading was discontinued in 1952. A similar series could, of course, be constructed from Tuesday–Friday closing prices or Friday closing.

ruptions that is essentially independent of the presence or absence of non-ideal behavior.

As part of the detailed calculations behind the analysis reported upon in Chapter 4, *H* was computed for both the complete and daily close price series for wheat and corn for 1937–41 and 1947–51 and for rye for 1937–41. The differences between *H* obtained from these two price series—one affected by the interruptions and one not—for intervals of one to 16 weeks and subintervals of 24 hours to four weeks, are shown in Table 4.5.

These differences in *H* computed by the two methods show generally different amounts of bias for each commodity and for each group of years. For short intervals and subintervals (e.g., at least up to two weeks), with one exception, it is evident that both the numerator and denominator of the ratios computed from the continuous quotations are being affected by the interruptions. This can be seen in the gradual increases of the differences as the intervals increase for fixed subinterval lengths. For long interval lengths (e.g., 8 to 16 weeks) the differences for fixed subintervals are reasonably constant, indicating that only the bias of the subinterval range is still present. The generally constant behavior of the values for each subinterval length—that is, a gradual increase and then a leveling off at a maximum value—provides good evidence that his analysis is correctly measuring only the effects of the overnight interruptions.

Table 4.6 summarizes the estimates of the per cent discrepancy in the continuous quotation price ranges that can be obtained by graphic smoothing of the data in Table 4.5.

The following two main conclusions can be drawn from these results:

1. The estimates in Table 4.6 indicate that the price ranges for the continuous quotations are reduced by the overnight interruptions by considerably larger percentages—and for longer intervals—than suggested by the "method of reconstruction" (summarized in Table 4.3). We thus conclude that the method of reconstruction cannot be reliably used as a means of adjusting for the bias resulting from the overnight changes. Evidently, the assumption of ideal behavior is not adequately satisfied.

2. The varying per cent discrepancies in Table 4.6 for different commodities and time periods make it evident that no general, single adjustment can be made. In order to compute *H* and *H'* from the continuous quotations for commodities and years other than those covered by Table 4.6 it appears that it is necessary to compute the statistic for both the continuous quotation and the daily close series, and then estimate the extent of the bias by the procedure used in Table 4.5. However, since this procedure amounts, in essence, to forcing the values from the continuous quotation into close agreement with those from the daily close series it is apparent that the analysis might as well be limited to the values from the daily close series.[11] This is the procedure adopted subsequently in this study. No further analysis is attempted with the continuous quotations series.

[11] The main advantage of computing *H* or *H'* from the continuous price series—and one that motivated the above unsatisfactory effort to find a way to adjust for the overnight interruptions—is its smaller sampling error for a given time period. If only a very limited amount of price data were available the procedure used in Table 4.5 might be worthwhile in order to take advantage of this smaller error. For this present study, however, sufficient data are available to permit reliable conclusions from the daily close series.

TABLE 4.5.—DIFFERENCES BETWEEN H COMPUTED FROM COMPLETE QUOTATIONS AND FROM DAILY CLOSING PRICES, SUBINTERVALS OF 24 HOURS TO FOUR WEEKS*

Years and commodity	Total interval (*weeks*) 1	2	4	8	16
		A. Subintervals of 24 Hours			
1937–41					
Wheat	+.051	+.068	+.067	+.069	+.060
Corn	+.021	+.090	+.112	+.117	+.128
Rye	+.082	+.136	+.142	+.160	+.158
1947–51					
Wheat	+.083	+.076	+.085	+.107	+.115
Corn	+.073	+.086	+.101	+.124	+.143
		B. Subintervals of 1/2 Week			
1937–41					
Wheat	+.017	+.029	+.026	+.023	+.015
Corn	+.009	+.060	+.046	+.054	+.061
Rye	+.059	+.074	+.066	+.101	+.094
1947–51					
Wheat	+.027	+.019	+.022	+.038	+.038
Corn	+.025	+.032	+.039	+.053	+.074
		C. Subintervals of 1 Week			
1937–41					
Wheat	—	+.014	+.008	+.002	−.007
Corn	—	+.025	+.029	+.041	+.047
Rye	—	+.016	+.027	+.032	+.043
1947–51					
Wheat	—	+.002	+.012	+.015	+.021
Corn	—	+.006	+.015	+.017	+.021
		D. Subintervals of 2 Weeks			
1937–41					
Wheat	—	—	−.002	−.001	−.013
Corn	—	—	+.021	+.019	+.018
Rye	—	—	+.016	+.025	+.014
1947–51					
Wheat	—	—	+.010	+.006	−.001
Corn	—	—	+.009	+.008	+.016
		E. Subintervals of 4 Weeks			
1937–41					
Wheat	—	—	—	+.003	−.008
Corn	—	—	—	+.001	−.001
Rye	—	—	—	+.010	+.002
1947–51					
Wheat	—	—	—	−.013	−.016
Corn	—	—	—	+.004	+.010

* Differences shown are H (com.) − H (dc), where H (com.) is a value of H calculated from the complete series of recorded quotations, without adjustment for inter-session interruptions of quotations (hence biased upwards), and H (dc) is a corresponding value of H calculated from daily closing prices.

TABLE 4.6.—APPROXIMATE PER CENT DISCREPANCY IN PRICE RANGES CAUSED BY OVERNIGHT INTERRUPTIONS IN CONTINUOUS QUOTATIONS*

Commodity and years	Subinterval				
	24 Hours	1/2 Week	1 week	2 Weeks	4 Weeks
1937–41					
Wheat	− 6.5	−2.0	—	—	—
Corn	−12.5	−6.0	−4.5	−2.0	—
Rye	−16.0	−9.5	−4.0	−2.0	−0.5
1947–51					
Wheat	−11.0	−4.0	−2.0	−0.5	—
Corn	−14.0	−7.0	−2.0	−1.0	−0.5

* Derived by graphic smoothing of data in Table 4.5.

3. THE BASIC PRICE SERIES

The final problem to be considered in this chapter is that of selecting the specific price series to use in our detailed study of price behavior. At any given time in the wheat, corn, and rye markets there are usually at least three and often as many as five futures being traded. Trading in the May future, the most active of all futures, usually starts in late summer and continues until May 21; trading in the July future runs from early fall until July 21; trading in the December future from late spring or early summer until December 21. In addition, trading in the secondary futures, March and September, often reaches levels of considerable importance. Such a constant shifting of activity from future to future raises a problem about how to obtain a single price series that properly reflects the most important market characteristics.

After analyzing the various alternatives, we selected a method based on "splicing" separate futures into a single series. In this method each future is used for the price series during the months for which that future is approximately dominant.[12] As the future is replaced by the succeeding dominant one a special adjustment is made to join (or "splice") the two different futures into a single series.

Two restrictions underlie our selection of futures for the basic price series. First, we wish to select futures that are approximately dominant; and, second, in order to minimize the number of times the futures have to be joined, it is desired to use each for some multiple of four weeks.

To illustrate variations in actual intervals of dominance, we show in Table 4.7 the dates the principal wheat and corn futures became dominant in each of five years.

By appraisal of this and more extensive listings of similar dates for other years,

[12] The term "dominant" is used in the same sense as by the Commodity Exchange Authority (see, for example, *38*, p. 80). A future is dominant when its open contracts (contracts entered into and not yet liquidated by an offsetting transaction nor fulfilled by delivery) exceed those of any of the other futures, and remain greater.

TABLE 4.7.—COMPARISON OF DATES THAT MAY, JULY, AND DECEMBER FUTURES FOR WHEAT AND CORN BECAME DOMINANT AT CHICAGO, 1934–38*

Year	Wheat			Corn		
	May	July	Dec.	May	July	Dec.
1934.......	Oct. 17	April 17	July 24	Nov. 3	April 21	Aug. 4
1935.......	Oct. 1	April 25	Aug. 6	Nov. 5	April 27	Aug. 12
1936.......	Sept. 19	April 24	Aug. 3	Nov. 30	April 15	Aug. 8
1937.......	Oct. 29	April 23	Aug. 10	Nov. 21	April 5	Aug. 3
1938.......	Nov. 4	April 20	Aug. 10	Nov. 24	March 23	Aug. 22

* Data from wheat from *38,* Table 40, p. 80; data for corn from *35,* Table 37, p. 80.

we were able to select starting dates for each of the futures that fulfill, reasonably well, both restrictions.[13] These dates are summarized in Table 4.8. As expected, May constitutes the bulk of the basic price series, covering over 6½ months, with July and December sharing the remainder equally. This division was found to provide the most satisfactory balance between the intervals the futures are actually dominant and the requirement of using each for some multiple of four weeks.

Two special points deserve mention. First, the date on which a future becomes dominant does not represent the sudden ascension of this future to a position of importance. Almost without exception each future is nearly dominant for several weeks before it finally becomes dominant. During this period of "near dominance" it could properly be used in a basic series. The second point concerns the intervals used for the July and December futures. While the July future begins at about the appropriate time, it runs until the first week of July, somewhat beyond its apparent period of dominance. This is necessary in order to shift directly to the December future, without briefly using September. While the September future is usually dominant for a portion of both June and July, the interval of its dominance varies considerably from year to year, and, at best, would be satisfactory for only four or eight weeks. We believe that the need for minimizing the number of "splices," coupled with the late activity of the July future and the early activity of December, justified the direct step from July to December, omitting September entirely.

The method used to join the futures was as follows: (1) for the week in which the futures are joined the average daily spread (at the close) between the two futures was computed; (2) this average spread (or "adjustment factor") was then

TABLE 4.8.—SUMMARY OF FUTURES USED TO FORM BASIC PRICE SERIES

Future	Approximate starting date	Interval (*weeks*)
May	Week of October 1	28
July	Week of April 15	12
December	Week of July 5	12

[13] Actually, for economy of computing we imposed a third restriction: each commodity must have the same future start on the same date. Although equivalent dates for rye are not readily obtainable, inspection of open contract statistics for different years indicates that approximately the same pattern prevails as for wheat and corn. Consequently, we consider these three commodities as one group.

used to bring subsequent prices to a comparable basis with the prices of the previous future.

We can illustrate with an actual example: Suppose, in switching from December wheat to May wheat in the week of October 1, 1951, we need to calculate an eight-week price range. The actual four-week ranges before and after the date of the splice and the average price spread for the week of October 1 are as follows (in cents):

Four-week December range		*Four-week May range*		*Average spread*
High	247 2/8	High	258 3/8	+4 1/8
Low	242 4/8	Low	247 7/8	

The "spliced" eight-week range is then computed by subtracting 4 1/8 cents from each of the May prices and determining the extremes of the December and adjusted May ranges:

"Spliced" Eight-week range[14]	
High	(254 2/8)
Low	242 4/8

This adjustment gives an eight-week range, part of which is based on the May price and part on the December price.

To form the basic price series we carried out this procedure three times for each year that was studied. At every "splice" caution was used to insure that the price spread existing at that time was typical of the spreads immediately before and after the splice. Of the 62 splices made, none was found to require any special adjustments.

The reliability of the splicing method can be demonstrated by a brief experiment. Since each future is used for an interval that is a multiple of four weeks, we can compute H and H' for intervals of four weeks for a series containing no splices. Then, by starting the grouping of the same series two weeks earlier, we can form an alternative series that requires 15 splices in each five-year period. Since the basic data for each series would be practically the same, the spliced statistic should differ appreciably from the unspliced one only if some sort of a bias were introduced by the splices.

Carrying out this analysis for wheat, 1937–41, we obtain the values given in Table 4.9. This table shows that the values obtained from the series with 15 out of

TABLE 4.9.—COMPARISON OF H' FOR FOUR-WEEK INTERVALS, OBTAINED FROM SPLICED AND UNSPLICED PRICE SERIES, WHEAT, 1937–41, SUBINTERVALS OF 24 HOURS TO TWO WEEKS

(Daily close series)

Subinterval	Unspliced	With 15 splices	Difference
24 hours	+.075	+.071	−.004
One-half week	+.064	+.061	−.003
One week	+.061	+.060	−.001
Two weeks	+.031	+.030	−.001

[14] We show the high in parentheses to indicate that it is not an actual, registered price, but rather one estimated by means of price spreads.

64 intervals spliced are in good agreement with the values obtained from the unspliced series. The differences, although all in one direction, suggest no significant bias. If a bias were present, it would affect only the four-week ranges, producing a constant difference between the two sets. But since the small differences vary with the subintervals we conclude that a reliable basic series can be formed by joining different futures by the method described above.

Chapter 5

THE BEHAVIOR OF SPECULATIVE PRICES

This chapter summarizes the detailed results of applying H and H' to actual speculative prices. Analysis in this chapter will be limited mainly to statistical identification of non-ideal behavior. Economic implications and possible causes will be considered in the summary chapter.

The statistical tests are directed towards answering two parts of the basic question: (1) Are there measurable departures from ideal behavior that exist under most conditions for all speculative commodities?, and (2) Are there measurable departures from ideal behavior that exist only under special circumstances and conditions? Computational limitations force us to limit the bulk of our statistical analysis to the first question; while some attention is devoted to the second part, we reach only tentative conclusions in this area, hopefully pointing the direction for further research. As mentioned previously, all analysis is based on values of H and H' computed from daily closing prices.

The basic calculations were made using the speculative prices of wheat and corn for two five-year periods, 1937–41 and 1947–51, and for rye for 1937–41 (rye futures were traded intermittently during the period 1947–51). Subsequent to the basic calculations, separate calculations were made for wheat prices for the 12-year period, 1924–35. This last set of calculations was done in order to confirm the results of the other calculations for an independent time period.[1]

The values of H and H' obtained from the above calculations are summarized in the following 12 tables. Values that are significant at the 5 per cent level are indicated by one asterisk; those significant at the 1 per cent level by two asterisks. H and H' are shown jointly in order to see if there is noticeable evidence of departure between H and H' resulting from the heavier weighting in H' produced by periods of large price movements. The six charts that follow the tables

TABLE 5.1.—H FOR DAILY CLOSING PRICES, WHEAT, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	−.003	+.016	+.057	+.085	+.078
1/2 week	−.016	+.011	+.055	+.090*	+.077
1 week	—	+.013	+.052	+.086	+.079
2 weeks	—	—	+.022	+.046	+.043
4 weeks	—	—	—	+.010	+.009
8 weeks	—	—	—	—	−.001

* Maximum departure of 1.63 standard errors above expected value. None of the indexes is significant at the 5 per cent level.

[1] Computational limitations forced us to exclude subintervals of 24 hours for the period 1924–35.

TABLE 5.2.—H' FOR DAILY CLOSING PRICES, WHEAT, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	+.008	+.037	+.075*	+.082	+.072
1/2 week	+.002	+.036	+.064	+.080	+.071
1 week	—	+.031	+.061	+.076	+.068
2 weeks	—	—	+.031	+.046	+.038
4 weeks	—	—	—	+.014	+.005
8 weeks	—	—	—	—	−.009

* Maximum departure of 1.80 standard errors above expected value. None of the indexes is significant at the 5 per cent level.

TABLE 5.3.—H FOR DAILY CLOSING PRICES, WHEAT, 1947–51, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	−.028	−.020	+.007	+.067	+.118
1/2 week	−.022	−.012	+.006	+.069	+.129
1 week	—	−.001	+.003	+.070	+.136
2 weeks	—	—	+.003	+.076	+.149*
4 weeks	—	—	—	+.061	+.121
8 weeks	—	—	—	—	+.050

* Indicates significance at the 5 per cent level.

TABLE 5.4.—H' FOR DAILY CLOSING PRICES, WHEAT, 1947–51, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	−.006	−.005	+.027	+.104	+.161*
1/2 week	−.001	+.005	+.028	+.108*	+.172*
1 week	—	+.003	+.029	+.108*	+.172*
2 weeks	—	—	+.027	+.107*	+.171*
4 weeks	—	—	—	+.078	+.139*
8 weeks	—	—	—	—	+.056

* Indicates significance at the 5 per cent level.

TABLE 5.5.—*H* FOR DAILY CLOSING PRICES, CORN, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

	Total interval (*weeks*)				
Subinterval	1	2	4	8	16
24 hours	+.005	+.005	+.030	+.026	+.007
1/2 week	−.004	+.015	+.034	+.041	+.028
1 week	—	+.014	+.035*	+.037	+.026
2 weeks	—	—	+.007	+.005	−.003
4 weeks	—	—	—	−.024	−.028
8 weeks	—	—	—	—	−.009

* Maximum departure of one standard error above expected value. None of the indexes is significant at the 5 per cent level.

TABLE 5.6.—*H′* FOR DAILY CLOSING PRICES, CORN, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

	Total interval (*weeks*)				
Subinterval	1	2	4	8	16
24 hours	−.005	+.037	+.079	+.046	+.045
1/2 week	+.010	+.058*	+.091*	+.066	+.066
1 week	—	+.045*	+.080*	+.055	+.055
2 weeks	—	—	+.036	+.012	+.011
4 weeks	—	—	—	−.023	−.025
8 weeks	—	—	—	—	−.001

* Indicates significance at the 5 per cent level.

TABLE 5.7.—*H* FOR DAILY CLOSING PRICES, CORN, 1947–51, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

	Total interval (*weeks*)				
Subinterval	1	2	4	8	16
24 hours	+.003	+.004	+.075	+.211**	+.257**
1/2 week	−.017	−.010	+.052	+.195**	+.235**
1 week	—	−.002	+.056	+.204**	+.236**
2 weeks	—	—	+.047	+.194**	+.250**
4 weeks	—	—	—	+.116*	+.151*
8 weeks	—	—	—	—	+.017

* Indicates significance at the 5 per cent level.
** Indicates significance at the 1 per cent level.

TABLE 5.8.—H' FOR DAILY CLOSING PRICES, CORN, 1947–51, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	+.019	+.043	+.139**	+.267**	+.341**
1/2 week	−.012	+.016	+.100*	+.233**	+.306**
1 week	—	+.016	+.113**	+.247**	+.320**
2 weeks	—	—	+.087*	+.218**	+.291**
4 weeks	—	—	—	+.121**	+.187*
8 weeks	—	—	—	—	+.058

* Indicates significance at the 5 per cent level.
** Indicates significance at the 1 per cent level.

TABLE 5.9.—H FOR DAILY CLOSING PRICES, RYE, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	+.005	+.031	+.052*	+.067	+.048
1/2 week	−.019	+.011	+.030	+.053	+.034
1 week	—	+.014	+.041	+.062	+.049
2 weeks	—	—	+.011	+.013	+.003
4 weeks	—	—	—	−.009	−.020
8 weeks	—	—	—	—	−.026

* Maximum departure of 1.26 standard errors above expected value. None of the indexes is significant at the 5 per cent level.

TABLE 5.10.—H' FOR DAILY CLOSING PRICES, RYE, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
24 hours	−.002	+.042	+.070	+.083	+.075
1/2 week	−.014	+.035	+.052	+.078	+.071
1 week	—	+.045*	+.066	+.087	+.080
2 weeks	—	—	+.022	+.042	+.035
4 weeks	—	—	—	+.020	+.012
8 weeks	—	—	—	—	−.008

* Indicates significance at the 5 per cent level.

TABLE 5.11.—*H* FOR DAILY CLOSING PRICES, WHEAT, 1924–35, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF ONE-HALF TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
1/2 week	−.012	−.010	+.015	+.060	+.103*
1 week	—	−.015	+.004	+.055	+.097*
2 weeks	—	—	+.007	+.054	+.105*
4 weeks	—	—	—	+.053*	+.097*
8 weeks	—	—	—	—	+.018

* Indicates significance at the 5 per cent level.

TABLE 5.12.—*H′* FOR DAILY CLOSING PRICES, WHEAT, 1924–35, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF ONE-HALF TO EIGHT WEEKS

Subinterval	Total interval (*weeks*) 1	2	4	8	16
1/2 week	+.001	+.001	+.008	+.079*	+.117*
1 week	—	−.011	+.001	+.070*	+.108*
2 weeks	—	—	+.011	+.084*	+.122*
4 weeks	—	—	—	+.070*	+.107*
8 weeks	—	—	—	—	+.035

* Indicates significance at the 5 per cent level.

CHART 5.1.—*H* AND *H′* FOR WHEAT, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

(*Daily closing prices*)

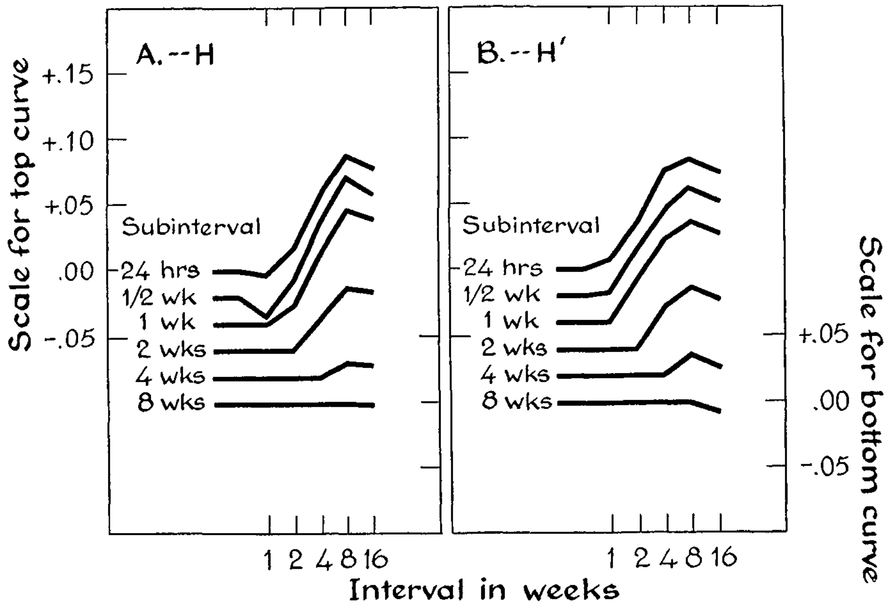


Chart 5.2.—*H* and *H′* for Wheat, 1947–51, Intervals of One to Sixteen Weeks, Subintervals of 24 Hours to Eight Weeks

(*Daily closing prices*)

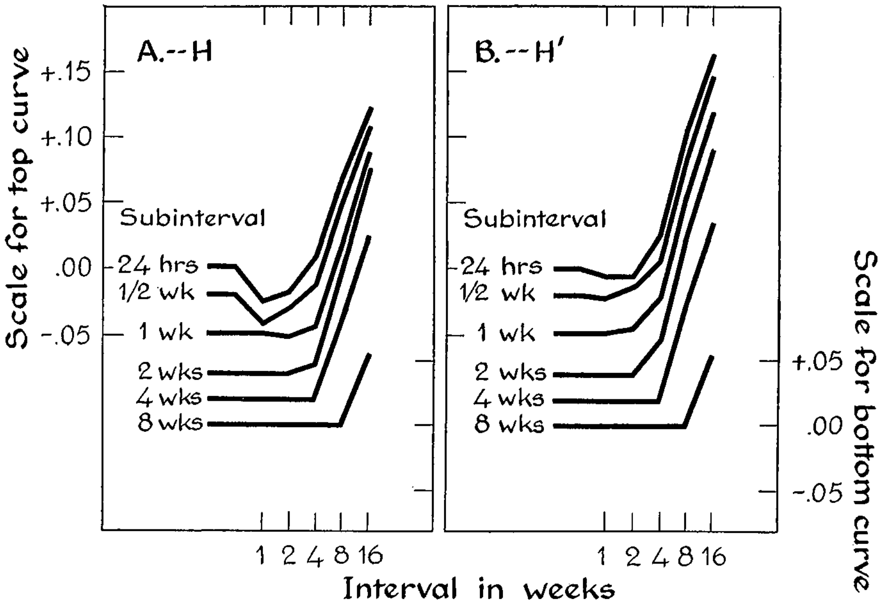


Chart 5.3.—*H* and *H′* for Corn, 1937–41, Intervals of One to Sixteen Weeks, Subintervals of 24 Hours to Eight Weeks

(*Daily closing prices*)

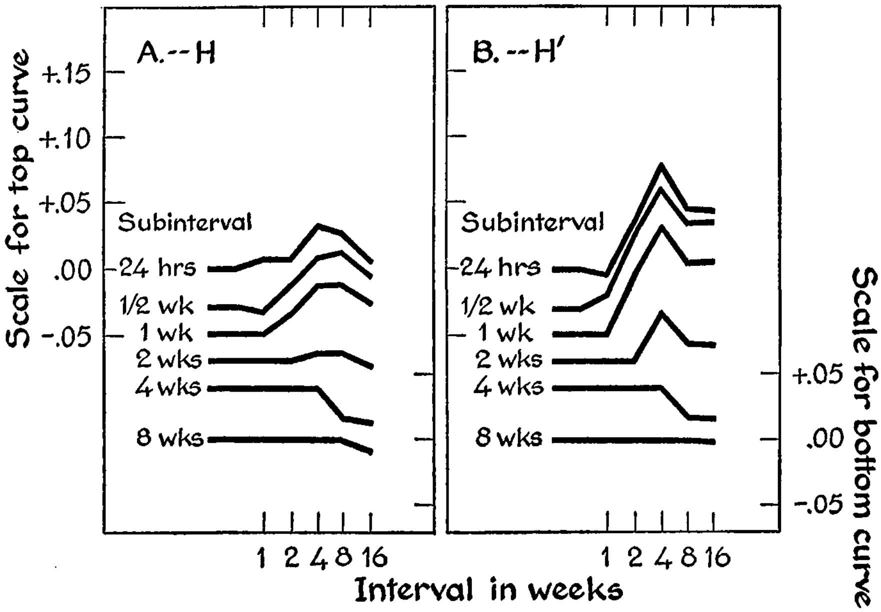


CHART 5.4.—*H* AND *H′* FOR CORN, 1947–51, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

(*Daily closing prices*)

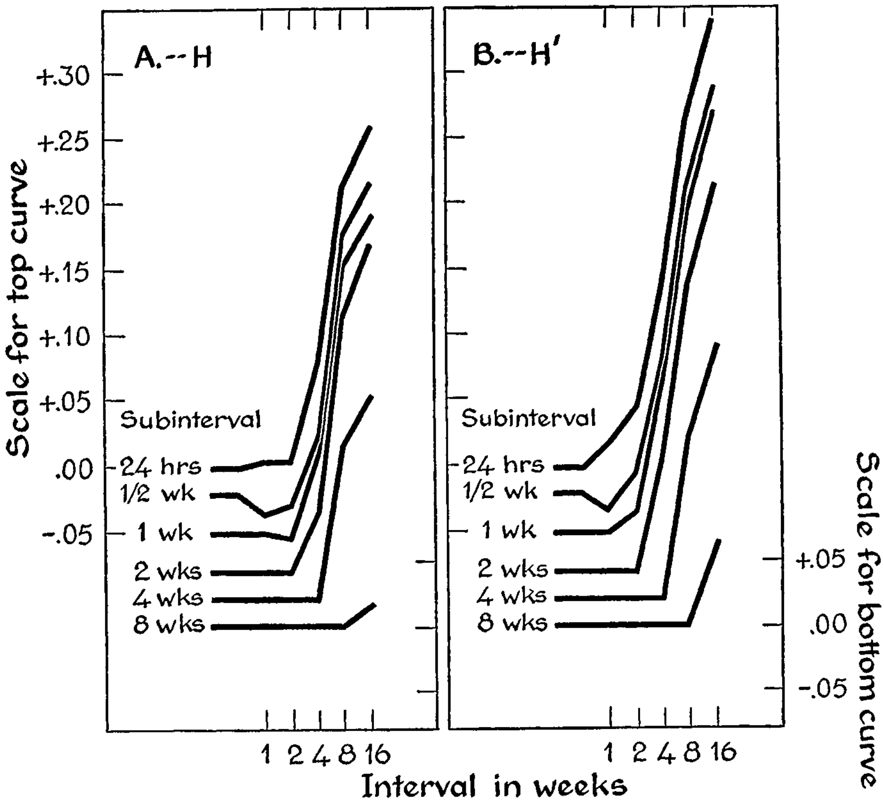


CHART 5.5.—*H* AND *H′* FOR RYE, 1937–41, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF 24 HOURS TO EIGHT WEEKS

(*Daily closing prices*)

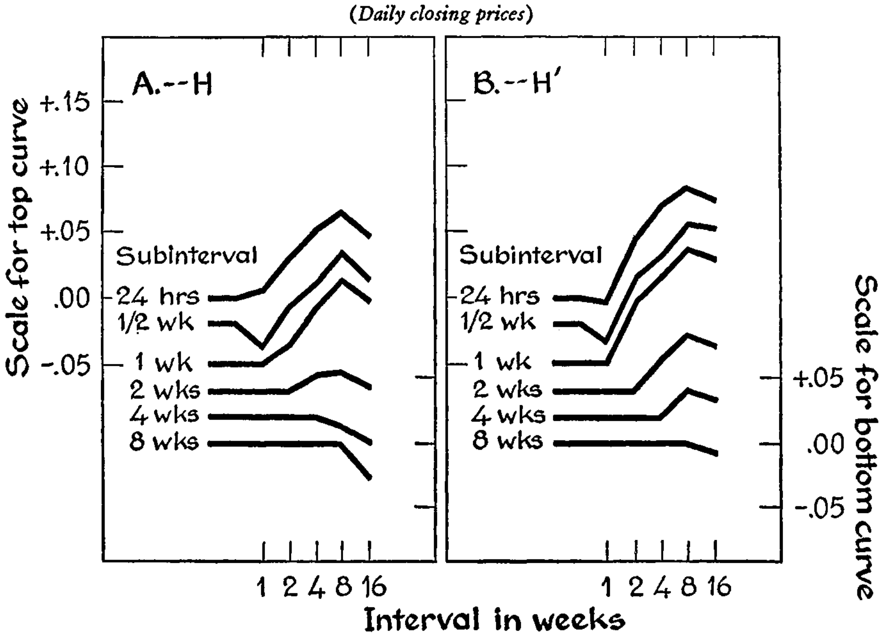
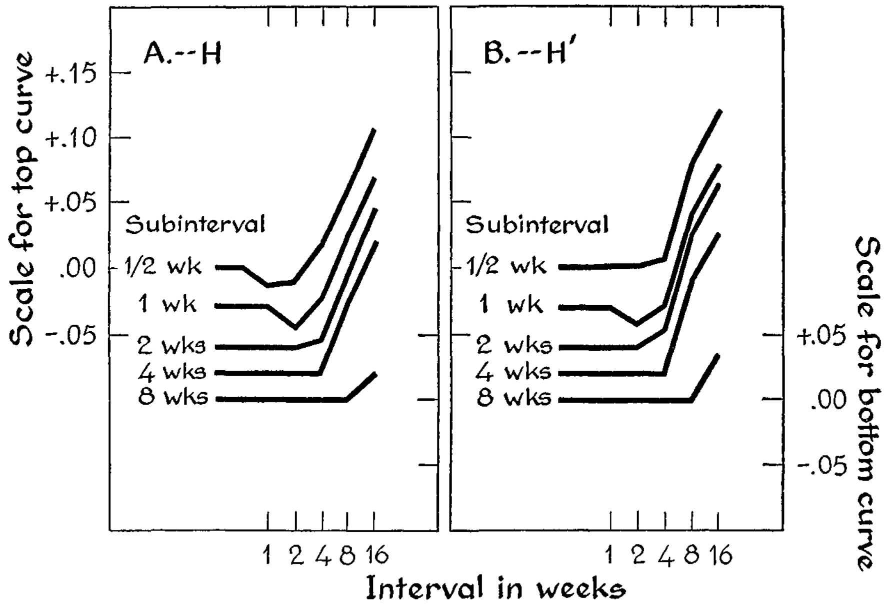


CHART 5.6.—H AND H' FOR WHEAT, 1924–35, INTERVALS OF ONE TO SIXTEEN WEEKS, SUBINTERVALS OF ONE-HALF WEEK TO EIGHT WEEKS
(*Daily closing prices*)



show the values of H and H' graphically, using the plotting method illustrated in Chart 3.3.

Statistical Findings

The most striking feature brought out by the data in these tables and charts is the clear indication that all these prices contain significant continuity tendencies. Although the amounts vary from commodity to commodity and from time period to time period, the basic continuity pattern is clearly evident in all 12 sets of data. The strongest continuity indication (about 25–30 per cent more than expected) occurs for corn in the period 1947–51 and the weakest (about 3–5 per cent more) for corn for 1937–41. While three of the 12 sets do not contain values that are statistically significant at the 5 per cent level, the general similarity of the 12 patterns—plus the presence of statistical significance in 9 sets—provides sufficient weight of evidence to warrant the above general conclusion.

The duration of the continuity varies. Each of the three commodities for the years 1937–41 shows a continuity tendency starting at about two weeks and rising steadily to a peak somewhere around eight weeks.[2] Values for the other time periods—1947–51 and 1924–35—start rising at about four weeks and continue rising until the end interval examined—16 weeks—is reached. For these values we cannot determine when the maximum indication occurs.

[2] The analysis of the trend-reversal series in Chapter 3 brought out that the maximum indication of variable continuity may come somewhat beyond the limit of the actual continuity.

The second important conclusion from these data is the evidence of the presence of a slight reaction tendency for short intervals—apparently up to one to two weeks. While none of the individual negative values of H is statistically significant, we believe that the repeated appearance of small negative values for short subintervals provides strong evidence of the existence of a slight reaction tendency. For the years 1937–41 the tendency is generally shorter and weaker than it is during the years 1947–51 and 1924–35. Quantitatively, the reaction tendency appears to reach a peak strength of not over 2 per cent more than expected; its weaker indications are less than 1/2 of 1 per cent more than expected.

The third important point that can be drawn from the data in the tables and charts is the conclusion that the continuity tendencies are especially associated with the periods of larger price movements.[3] This characteristic can be seen most clearly in the side-by-side comparisons of the two sections in each chart. In every case, as the continuity indications increase with increasing interval length the values of H' rise more rapidly than do those of H. Further evidence on this conclusion will be developed in the next section.

Price Behavior Under Special Conditions

Our remaining analysis is directed at trying to determine if there are any special circumstances and conditions where the above listed general patterns are particularly present. Because of computational limitations, we are able only to indicate preliminary findings. Despite their preliminary nature, it was believed that those findings should be presented in the hope that they might serve as a guide to needed additional research.

Amount of Price Movement

The first special classification is the one suggested by our general conclusion that the continuity patterns are particularly associated with periods of large price movements. Although there are several ways of undertaking a classification based on "amount of price movement," we restrict ourselves to one that is easily understood and simply computed. Our analysis of general behavior, it will be recalled, was based on complete sets of H and H', contrasting the behavior of one set with that of another. But when we restrict ourselves to special classifications, we can no longer properly consider a complete set, for while one grouping may represent, say, those 16-week intervals with price movements in the upper quartile, upon regrouping to 1-week intervals the new group might fall in only the second or third quartile. Consequently, we confine this analysis to intervals of 4 weeks, each divided into four 1-week subintervals.

The method of classification is the grouping of values of H into four classes, depending upon the size of S_k, the sum of the subranges in the denominator of the basic ratio, $\sqrt{k}\ R/S_k$.[4] Thus, those values of H obtained from periods of the largest 25 per cent of price movements, as measured by the sum of the subranges, are in one class, those in the second largest in another class, and so on.

[3] It will be recalled that in Chapter 3 it was concluded that H' gives heavier weighting to periods of large price movements than does H.

[4] Working has found that S_k and $\sqrt{k}\ R/S_k$ are essentially independent.

TABLE 5.13.—*H* FOR SELECTED COMMODITIES, 1937–41 AND 1947–51, INTERVALS OF FOUR WEEKS, SUBINTERVALS OF ONE WEEK, CLASSIFIED INTO QUARTILES ACCORDING TO "AMOUNT OF PRICE MOVEMENT"

(Daily closing prices)

Years and commodity	Quartile grouping[a]			
	1	2	3	4
1937–41				
Wheat	+.118	−.013	−.020	+.114
Corn	−.082	+.035	+.115	+.072
Rye	−.006	−.030	+.095	+.104
Total	+.010	−.003	+.063	+.096*
1947–51				
Wheat	−.081	−.018	+.058	+.038
Corn	−.045	−.030	+.126	+.174*
Total	−.061	−.024	+.092	+.106*
10-year total	−.019	−.011	+.075*	+.101**

[a] The lowest quartile is indicated by 1.
* Indicates significance at the 5 per cent level.
** Indicates significance at the 1 per cent level.

Table 5.13 summarizes the results of this classification for wheat, corn, and rye, for 1937–41, and for wheat and corn, for 1947–51.

Considering the totals for 1937–41 and 1947–51, we see the same general pattern in both periods, a pattern that supports the previous general conclusion. Significant continuity tendencies are especially associated with the periods of large price movements (i.e., the highest quartile), with the other periods showing no significant non-ideal behavior for this interval. For the postwar years, and for corn in the prewar years, we also have a weak suggestion that periods of small price movements contain reaction.

Considering the commodities separately, we see that, except for wheat for 1937–41, each has produced the same general pattern. Wheat's contrary behavior is puzzling, but since each value is based on only 16 observations, this result may be only a sampling fluctuation. It is also noteworthy that both the prewar and postwar values for corn show the greatest variation between quartiles. This behavior suggests not only a difference between the prices of corn and those of the other two commodities, but also suggests the probable explanation for the fact that H' for corn departs from equivalent values of H to a greater degree than for the other two commodities (especially noticeable in Charts 5.3 and 5.4).

Direction of Price Change

The second special classification is based on the direction of the net price change. As in the previous table, we confine ourselves to 4-week intervals, each divided into four 1-week subintervals. Table 5.14 contains the values of H thus classified for wheat, corn, and rye, for 1937–41, and for wheat and corn for 1947–51.

TABLE 5.14.—*H* FOR SELECTED COMMODITIES, 1937–41 AND 1947–51, INTERVALS OF FOUR WEEKS, SUBINTERVALS OF ONE WEEK, CLASSIFIED ACCORDING TO DIRECTION OF FOUR-WEEK NET PRICE CHANGE

(*Daily closing prices*)

Years and commodity	Price decline: Number of intervals	Price decline: *H*	Price increase: Number of intervals	Price increase: *H*
1937–41				
Wheat	30	+.044	34	+.056
Corn	31	+.044	33	+.026
Rye	36	+.043	28	+.038
Total	97	+.044	95	+.040
1947–51				
Wheat	23	−.055	41	+.031
Corn	25	−.022	39	+.107*
Total	48	−.038	80	+.068*
10-year total	145	+.018	175	+.053*

* Indicates significance at the 5 per cent level.

Considering the three commodities in the prewar years, it is evident that there is no significant difference between price behavior when the price has increased and when it has decreased. In the postwar years, however, the values for both wheat and corn differ between price increases and decreases, indicating, in the total, significant continuity during periods of price increases. It should also be noted that in the postwar years there are nearly twice as many intervals of price increase as there are intervals of price decrease.

While it is still an unanswered question as to whether or not price behavior in the postwar years was significantly different between periods of rising and falling prices, these results suggest that there was a difference. When these values are considered in connection with those of Table 5.13, we see that continuity patterns in the postwar years (especially corn) were particularly restricted to the periods of large price movements that occurred while the market was rising. While our limited statistical evidence does not warrant general conclusions, we can see a distinct possibility that such conclusions might be demonstrated.

Classified by Quarter of Crop-year

The third special classification is suggested by observations of trade and other analysts that speculative price movements are often particularly pronounced during certain parts of the crop-year, especially the first two or three months.[5] To make such an analysis, we group the values of *H* for wheat, corn, and rye, for 1937–41, according to crop-year quarters. As before, the data are for 4-week intervals, each divided into four 1-week subintervals. The results of this classification are shown in Table 5.15.

[5] The crop-year for wheat and rye is July–June; for corn, October–September.

TABLE 5.15.—*H* FOR SELECTED COMMODITIES, 1937–41, INTERVALS OF FOUR WEEKS, SUBINTERVALS OF ONE WEEK, CLASSIFIED ACCORDING TO QUARTER OF CROP-YEAR[a]
(*Daily closing prices*)

Commodity	Crop-year quarter			
	1	2	3	4
Wheat	+.089	+.070	−.035	+.076
Corn	+.112	−.047	−.001	+.075
Rye	+.069	+.009	+.026	+.059
Total	+.090*	+.011	−.004	+.070

[a] The crop-year for wheat and rye is July–June; for corn, October–September.
* Indicates significance at the 5 per cent level.

Although the pattern shown by each of the commodities contains no individually significant values, considered as a group these values provide clear evidence that prices registered in the first and last quarters of the crop-year contain the strongest continuity. This is an interesting result, and, if supported by additional statistical evidence, would indicate that strong continuity patterns exist not only at the beginning of each crop-year, but also during the two or three months before the formal beginning (i.e., at the end of the previous crop-year).

This observation corresponds with previous findings by Working (*46*, pp. 16–18). In the course of analyzing wheat price cycles, he found that price movements starting about in April seemed to constitute a more distinct period of price movement than those starting in July. After careful study of wheat futures prices for 47 years, he concluded that "Both the unity within the periods and the contrasts between adjacent periods give significance to the designation of the April–March period as the wheat 'price-movement season' " (*46*, p. 16). Although it could not be definitely established, Working implied that this behavior was the result of the market anticipating supply and demand conditions of the new crop-year three or so months in advance of its actual beginning.

While we likewise are unable to establish the significance of these results, we believe that it indicates the likelihood that corn and rye prices also exhibit a "price-movement season" starting two or three months prior to the actual beginning of the crop-year. As previously observed, the continuity patterns seem to be particularly associated with periods of large price movements, periods that could be expected to be concentrated a few months before and a few months after the formal beginning of the crop-year.

Net Position of Large-Scale Traders

The final classification is prompted by a recurring question about speculative prices: What effects do the actions of large-scale traders have on price behavior? The method of classification is a grouping based upon the average daily net position of large-scale traders (i.e., with open interest of 200,000 bushels or more) expressed as a per cent of average daily total open interest. As before, we restrict ourselves to intervals of 4 weeks, each divided into four 1-week subintervals. The analysis is for wheat and corn prices for the five-year period, 1937–41. Table 5.16

TABLE 5.16.—*H* FOR WHEAT AND CORN, 1937–41, INTERVALS OF FOUR WEEKS, SUBINTERVALS OF ONE WEEK, CLASSIFIED ACCORDING TO AVERAGE NET POSITION OF LARGE-SCALE SPECULATORS[a]

(*Daily closing prices*)

Average net position as per cent of total open interest	Number of intervals	*H*
WHEAT		
Short to long 5.0%	25	+.014
Long 5.1% to 8.0%	20	−.004
Long 8.1% to 17.0%	19	+.154*
CORN		
Short to long 10.0%	25	+.020
Long 10.1% to 19.0%	21	+.022
Long 19.1% to 36.0%	18	+.071

[a] Large-scale speculators are those with net positions of 200,000 bushels or more. See footnote 6, p. 49, for sources of information.
* Indicates significance at the 5 per cent level.

contains the values obtained by this classification, grouped into three classes of approximately equal sample size.[6]

Since we have classified the large-scale traders according to their total net position, rather than total aggregate position, this table illustrates price behavior when the large-scale speculators express substantial agreement about the future course of prices. For a more complete appraisal of the effects of these speculators we should also classify them according to aggregate position, a classification that would give some indication of price behavior when they hold, in total, a large or small share of all open contracts.

The result shown in Table 5.16 is essentially the same for both wheat and corn. What these values seem to indicate is that when the large-scale speculators hold a net long position that is a large share of all open contracts the prices contain significant continuity tendencies, whereas when they hold a net short or small net long position, price behavior does not depart significantly from ideal.

Does this result mean that when the large-scale traders have a heavy net long position, they produce continuity tendencies, or does it mean that when the market contains these tendencies the large-scale traders find it especially attractive? There is no way to answer this question on the basis of our fragmentary evidence, and we raise it mainly to indicate the two possible interpretations. We should point out, however, that the previous conclusion (from Table 5.13) that periods of large price movements contain significant continuity tendencies provides an additional clue. If the large-scale speculators find an active market (i.e., with large price movements) particularly attractive, it seems likely that they enter the market because it contains continuity, instead of somehow producing the tendencies as they accumulate or change their positions.

[6] The information on daily positions of the large-scale traders was collected from several government publications. In chronological order, by commodity, the sources are *38*; *36*; *37*; *35*; *34*.

Although we are unable to establish the validity of any of the above suggested explanations, there is one positive conclusion that can be drawn from the results of Table 5.16: These values have provided no support for the statements of those who claim that large-scale speculators are responsible for "excessive" price fluctuations. The statement of Paul Mehl (27, p. 485), as an example, that "any attempt made to prevent wide fluctuations necessarily must result in limiting the activities of speculators," cannot be considered valid in light of our results.

Chapter 6

SUMMARY AND CONCLUSIONS

The foregoing analysis has shown that speculative prices contain two general kinds of non-ideal behavior: (1) there is a clear continuity tendency in longer intervals, and (2) there is a slight reaction tendency in short intervals. Although we can only roughly measure the duration of these tendencies, the reaction tendency appears to be concentrated around intervals of 1 or 2 weeks and the continuity tendency, while varying from year to year, is concentrated somewhere between 4 and 16 weeks.[1] In varying degrees, these two characteristics were found to exist in the speculative prices of three separate commodities over widely spaced time periods.

Possible causes of the small reaction tendency include the rapid in-and-out actions of those day traders (called "scalpers") who buy and sell on the smallest price fluctuations, but always end the session with little or no open commitments.[2] Another cause might be the actions of the traders who trade actively throughout the session, but on somewhat larger price fluctuations than the strict scalpers. A third possibility might be the trading pattern of the brokerage-house traders—the non-member and often small-time speculators. But whatever the cause, we believe that the existence of this slight tendency provides only weak support for those who argue that futures trading results in "excessively fluctuating" prices.

Turning to the more important general characteristic—the tendency for the longer intervals to contain continuity of movement—it is our belief that this behavior stems from the composite actions of all traders registering (through the process of buying and selling) their divergent opinions about the future course of prices. Although these traders, as a group, may be doing their best to try to foresee the significance of all available information,[3] their collective abilities simply may not measure up to the level required by the ideal market. The type of behavior that we have observed may well represent the closest approach to ideal that is obtainable. It may well be that if the market were any more "perfect" it would not contain enough profit-making opportunities to sustain the speculative interest needed to keep it going.

Since many professional traders make money with a fair degree of consistency,

[1] Since this inquiry is limited to intervals of 16 weeks, we have not determined if continuity exists beyond this maximum interval length.

[2] There is independent evidence that the trading patterns of the day traders do not produce excessive price fluctuations. After an extensive study of the actual trading records of several very active day traders, Working concluded (47, p. 331):

> All of the evidence converges toward the conclusions that: (1) the price movements that day traders are able to anticipate with even moderate reliability are usually small relative to the total price range for the day; (2) the reliability of their judgment is rather low; and (3) the over-all effect of their trading operates strongly toward "smoothing" the course of prices.

[3] Some employ others to do it for them.

it seems clear that they are taking advantage of some sort of non-ideal behavior. If these traders are profiting from the existence of continuity it would be necessary for them to successfully determine the significance of a considerable amount of new information before it has been entirely evaluated by the market. This belief finds support in the writing of many close students of market behavior. Hoffman, for example, in describing sources of market information, commented (*19*, pp. 251–52):

> This . . . body of market reports and news is gathered by wire from all possible sources, relayed through news agencies, brokerage firms and individuals to thousands of interests who buy and sell accordingly. Much of the information is of little or no practical value but, in an effort to gain insight into probable price changes, it is ferreted out where there is the merest possibility of its being of market importance. All shades of news values are thus thrown together to be weighted by traders and in turn to be reflected through the composite opinion of all in the price.
>
> In this process the elements of time and judgment are all-important. All traders do not receive items of information at the same time. Some rely upon trade journals and periodical reports, others upon telegraphic reports and the newspapers. Just at what time important information is received often depends upon how much trouble and expense a trader is in a position to incur. For substantially the same reasons the degree of judgment displayed varies widely between trading interests. As a result, a continuous flow of widely varying market information finds its way to a single center, there to be interpreted at varying intervals of time by individuals with varying degrees of judgment.

Baer and Saxon, in a similar discussion, observed (*4*, p. 102):

> The inexperienced speculator, who takes a flyer in some commodity future on a tip or on the basis of random news items he has read, would be astonished at the range of information deemed desirable for a commodity exchange to procure for the benefit of its members who attempt to analyze future market trends scientifically.
>
> Information is gathered by the exchanges from government reports, periodically issued on current crop conditions at home and abroad, from numerous private commercial agencies which serve exchanges, and from special correspondents of the exchanges in markets throughout the world, as well as from their own members and from various producers', dealers', and manufacturers' associations operating in the same fields.

Another example is by Working (*48*, p. 155):

> Many traders in futures markets give a great part of their attention to acquiring information which has not become generally available, and thus, has not been reflected in market expectations. These traders, or the commission houses which serve them, gather crop information ahead of its reporting by public agencies; they study the weather reports and seek to predict effects of the weather on the crops, thus trying to base expectations on crop developments which have not yet occurred; and they have even employed a long-range weather forecaster to predict the weather several weeks ahead.

Such concerted effort on the part of a certain group of traders supports the opinion that continuity is the result of the market only gradually accepting and acting upon new information, as opposed to the rapid, correct, and near-unanimous evaluations required in the ideal market.[4]

Although not as firmly established as the two general conclusions listed above, our work in classifying the data according to various external conditions has provided some further clues in understanding price behavior:

1. Continuity appears to be especially pronounced during periods of large price movements. This suggests that when significant developments are affecting prices these developments may not be reflected in changing prices as rapidly as they should.
2. Reaction may be present during periods of small price movements. This suggests that during the absence of important price-changing news the market may develop a slight tendency to fluctuate excessively.
3. Periods of price increases may contain more continuity than periods of price decreases. This pattern may be tied to the nature of the news causing the movement. Price increases may be associated more with large changes than are price decreases.
4. Periods just before and just after the start of the crop-year contain more continuity than other parts of the year. This suggests the presence of a "price-movement season" in speculative prices.
5. Periods when large-scale speculators had large net positions were also the periods of the strongest continuity tendencies. It remains an unanswered question, however, as to whether the large speculators were responsible for the continuity or were acting to take advantage of a situation that offered the promise of profits. External evidence causes us to favor the latter conclusion.

Each of these "special classification" conclusions needs further analysis to determine their overall reliability and generality.

Before concluding, we should briefly note the relationship of our conclusions to those of two other statistical inquiries.

In Cowles' and Jones' appraisal of stock market behavior[5] various indexes of stock prices were found to contain what was termed "inertia," especially over long periods. From their discussion and definitions, it is clear that this inertia would be measured by H and H' in terms of continuity of movement. This suggests that both stock market and commodity futures prices contain a similar underlying structure. This is an area that merits further investigation.

The second study is the investigation of wheat price cycles by Working that resulted in the identification of the "episodic" price cycle (*46*; *49*). Since our tests do not extend far enough to cover the interval length of these long cycles, it was not unexpected that we found no evidence of them. Also, these cycles may not occur with sufficient frequency to show up in a statistical test such as H and H'.

Our final observation is this: All in all, we think that our statistical results

[4] It is interesting to contrast these descriptions of the extreme efforts of speculators to foresee new developments with Keynes' description of stock market speculators as those who simply attempt to "outwit the crowd," and who devote their intelligences to "anticipating what average opinion expects the average opinion to be." Cf. *24*, pp. 155–56.

[5] The study appeared in two parts (*9*; *11*).

have strengthened the belief that speculative prices reflect rational appraisals of all available information and that they do provide a valid economic function. This conclusion—with its substantial quantitative backing—echoes Working's 1953 observation (47, p. 332):

> During the twenty years of publication of *Wheat Studies* by the Food Research Institute, members of the research staff periodically studied the recent fluctuations of wheat prices and sought to interpret them as warranted by current developments, or unwarranted. For much of that period we sought three times a year to appraise price prospects for the next several months. Everyone concerned with these efforts gained a great respect for the rationality of the price behavior observed.

* * * * *

Postscript

My 1954 acknowledgments included the following: "The research project has been under the general direction of Professor Holbrook Working, associate director of the Food Research Institute. The author would like to express grateful appreciation for the training received during this three-year association. The values derived from Professor Working's guidance, insight, and patience will benefit the author far beyond the normal boundaries of academic training." The years since 1954 have made me appreciate, more than ever, my extreme good fortune for the opportunity to train under Professor Working.

Also, to repeat another 1954 acknowledgment, I wish to note that my wife, Elva, helped with this study in a great many ways. Her contributions were significant and deeply appreciated.

C.S.B.

APPENDIX

Table Notes

Definition of H and H':

$$H = \left(\frac{\sqrt{k}}{n} \sum_{i=1}^{n} \frac{R_i}{S_{ki}} \right) \Big/ B - 1 , \quad H' = \left(\frac{\sqrt{k} \sum_{i=1} R_i}{\sum_{i=1}^{n} S_{ki}} \right) \Big/ B - 1 .$$

Where:

n = Number of segments, each containing approximately m steps.

k = Number of subsegments per segment.

R_i = Range of the i^{th} segment.

S_{ki} = Sum of the ranges of the k subsegments of the i^{th} segment.

To compute B, take two values of X from either Table I (steps of constant size) or from Table II (normally distributed steps), one of the values of X being X_m, corresponding to the value of m, the number of steps per segment, and the other being $X_{m/k}$, corresponding to m/k, the number of steps per subsegment. Then calculate $B = X_m / X_{m/k}$.

The standard error for H or H' is obtained by this expression:

$$\sigma_H \cong \sigma_{H'} \cong \frac{1}{\sqrt{n}} \left(0.298 + \frac{0.187}{\sqrt{m}} \right) \sqrt{1 - 1/k}$$

Test H or H' as a normal deviate.

When accuracy to three decimal places is desired, and m (or m/k) is large, interpolation should be in terms of $1/\sqrt{m}$ (or $1/\sqrt{m/k}$); an auxiliary column is supplied in the tables for this purpose.

To test H or H' computed from continuous quotations, we use Table I, assuming an average of 200 quotations per trading session. This average is based on samples of quotations obtained from several different years. To test H or H' computed from the series of daily closing prices, we use Table II, with m taken as the number of trading sessions in the interval.

APPENDIX TABLE I.—GENERAL TABLE FOR USE IN COMPUTING *B* WHEN STEPS ARE OF CONSTANT ABSOLUTE SIZE

m or *m/k*	*X*	*m* or *m/k*	*X*	*m* or *m/k*	*X*	$1/\sqrt{m}$ or $1/\sqrt{m/k}$	*m* or *m/k*	*X*
1	1.000	11	1.331	21	1.397	.105	90	1.495
2	1.061	12	1.340	24	1.408	.094	112	1.505
3	1.155	13	1.349	26	1.415	.084	143	1.515
4	1.188	14	1.357	30	1.427	.073	188	1.525
5	1.230	15	1.364	34	1.436	.062	257	1.535
6	1.250	16	1.371	39	1.446	.052	372	1.545
7	1.276	17	1.377	45	1.456	.041	582	1.555
8	1.290	18	1.382	53	1.466	.031	1,054	1.565
9	1.307	19	1.387	62	1.475	.021	2,316	1.575
10	1.318	20	1.391	74	1.485	.011	8,637	1.585
						.000	∞	1.596

APPENDIX TABLE II.—GENERAL TABLE FOR USE IN COMPUTING *B* WHEN STEPS HAVE APPROXIMATELY NORMAL DISTRIBUTION

m or *m/k*	*X*	*m* or *m/k*	*X*	*m* or *m/k*	*X*	$1/\sqrt{m}$ or $1/\sqrt{m/k}$	*m* or *m/k*	*X*
1	0.798	11	1.296	21	1.378	.101	98	1.495
2	0.966	12	1.309	22	1.385	.091	121	1.505
3	1.052	13	1.320	25	1.396	.081	152	1.515
4	1.113	14	1.330	30	1.414	.071	198	1.525
5	1.159	15	1.339	34	1.425	.061	269	1.535
6	1.194	16	1.347	39	1.436	.051	384	1.545
7	1.223	17	1.354	43	1.444	.041	595	1.555
8	1.246	18	1.361	50	1.455	.031	1,040	1.565
9	1.265	19	1.367	58	1.465	.021	2,066	1.575
10	1.282	20	1.373	68	1.475	.011	8,248	1.585
				81	1.485	.000	∞	1.596

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