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UNCERTAINTY AND DIVERSIFICATION IN INTERNATIONAL TRADE

“Diversification” has become a commonplace goal of economic policy in less developed countries. They seek to escape their heavy dependence on one or two products for the bulk of their export earnings, and thus to avoid the costs in human welfare and to development objectives of sharp fluctuations in export receipts. This paper sketches some reasons for the interest in diversification, explores the consequences for production and trade of efficient diversification, and indicates briefly some policies that might be used to guide private investors to socially optimal diversification. It is divided into seven parts. In Section I we suggest some of the costs which a country might bear as a result of wide fluctuations in export earnings. It represents a non-rigorous attempt to establish legitimate grounds for a less developed country to shun fluctuations in export earnings.

Section II indicates how the traditional two-commodity model of international trade should be modified to take account of uncertainty in the prices at which international trade takes place when there are social costs, such as those indicated in Section I, to unforeseen fluctuations, i.e., when there is aversion to risk. The model is then extended to cover uncertainties in home production as well. In Section III the influence of risk aversion on foreign trade is extended to cover many commodities. This represents an application of Markowitz-Tobin portfolio theory to international trade. While the discussion is couched in terms of fluctuations in commodity prices, the analysis is easily generalized to cover other sources of instability, and this extension is made in a technical appendix. Section IV presents some casual empirical evidence which suggests that opportunities for effective diversification do exist.

A public program to diversify the economy is appropriate only if there is some discrepancy between social and private costs due to uncertainty, or if for some reason private investors are unable to diversify adequately even in response to their own aversion to risk. Section V suggests several reasons why the social costs associated with unforeseen fluctuations in earnings are likely to diverge from private costs. Section VI points out several policy measures to achieve the

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socially optimal degree of diversification. Section VII offers a brief summary and concluding remarks.

The argument throughout is framed with a producer and exporter of primary products in mind, but many of the observations apply to other countries as well.

I. THE COSTS OF INSTABILITY

That export earnings of most less developed countries do fluctuate widely is not open to serious question. A comprehensive IMF study showed that during the period 1948–1958, for example, the average annual fluctuation in export earnings for primary producing countries was 9 per cent, compared with 6 per cent for the industrial countries. For some countries the annual fluctuation is much higher, rising to as high as 21 per cent in the case of the Sudan.¹ Even if fluctuations in the earnings of less developed countries were no wider than those of developed countries, in many instances they would be larger relative to income. In any case the need to moderate such fluctuations in the less developed countries may be urgent because their impact on economic welfare and on economic growth may be more substantial and less easily offset than is the case for developed countries. Compensatory fiscal and monetary policy is still in a primitive stage of development in many less developed countries, so the means to offset fluctuations are weak; and the impact of export fluctuations on important growth sectors of the economy is substantial.

Before proceeding to the central argument of the paper, it is worth reviewing—even if only in general terms—the disadvantages which are allegedly associated with wide fluctuations in earnings, especially export earnings.² These costs arise to some extent even if fluctuations in export earnings are accurately foreseen, as anyone who has observed the elaborate food preparation required to sustain an agricultural family from harvest to harvest will appreciate. Real resource costs in the form of inventories and storage facilities, etc.—or interest costs where borrowing takes place—are incurred even when fluctuations are correctly anticipated. However large these costs may be, the costs are surely still larger if the magnitude and the direction of the fluctuations are not known.

The costs of unforeseen fluctuations may be broken roughly into three groups: those resulting directly from actual movements in export earnings, those resulting from reactions by the parties directly affected by these movements, and those resulting from defensive attempts by firms and individuals to avoid these uncertainties through means at their own disposal.

The principal element in the first group of costs is the variation in the level and the distribution of money incomes to which fluctuations in export receipts usually give rise. Most individuals probably regard fluctuations in income as undesirable

¹ See 5. The figures represent average deviations from five-year moving averages.

Recently a number of writers have pointed out that the size of fluctuations in export earnings is not closely related either to heavy reliance for earnings on only one or two export products or on one or two market areas. Commodity concentration accounts for only a small part of export instability, and geographical concentration is, if anything, inversely correlated to export instability. But this observation does not weaken the case for diversification; as we will show below, it merely indicates that diversification must be undertaken carefully. See 3, ch. 5; 9, ch. 2; and 8.

² We should note here, however, that although these costs are almost universally regarded as substantial the development literature is surprisingly vague in specifying them with any precision.

per se, particularly if the future magnitude and direction of such swings are unknown and are largely outside their control. For an individual at a low level of income, unforeseen declines in income may even mean starvation. The precise impact of fluctuations in export earnings on incomes will of course depend on the structure of the markets for commodities and labor in the country in question. Where small freeholders produce most of the export crop, as with rubber production in parts of Malaya and cocoa production in Ghana, fluctuations in exports reduce labor incomes directly. Where exports are produced on plantations or in mines, the impact of fluctuations in earnings falls partly on business (including foreign business),³ partly on wages and employment.

A similar cost associated with instability of export earnings is the disturbance it creates in the public sector. Typically a substantial fraction of government revenues derives from taxation of foreign trade. If trade fluctuates, so do revenues. When export earnings fall (which usually leads to a reduction in revenues from imports, too), either public services must be curtailed or the government must raise alternative funds by other taxes or by deficit finance. When the latter route is taken it helps to stabilize incomes, but at the expense of aggravating the deficit in the balance of payments.

The reactions of the party directly affected by a change in earnings, whether it be an individual, firm, or government, are likely to transmit the fluctuation to other parts of the monetized economy. Decreases in expenditures by one sector affect other sectors' receipts, leading to a reduction in income larger than the original one. In the absence of substantial reserves or borrowing facilities abroad, the loss of export earnings may require a retrenchment of imports, both public and private. With imports of "luxury" items usually already held to a minimum in countries with development ambitions, the loss of imports can hamper development plans by delaying the acquisition of needed capital goods or industrial materials. Delays in getting parts or equipment can increase the costs of an investment many times and postpone its returns, turning what would have been a profitable venture into an unprofitable one.

For these various reasons, therefore, most less developed countries may have good grounds, in the language of modern portfolio theory, for being "risk averse" in their export transactions.

The costs of fluctuations in earnings will induce individuals to shun high variability activities. Subsistence farmers will be discouraged from shifting to cash crops, even with high average yield per acre and per man-hour, because the possibility of starvation in a year of low receipts has an unacceptable finality to it. Would-be entrepreneurs may be discouraged from borrowing for investment even when there is institutionalized lending, for the gloomy prospect of overbearing debt in bad years may outweigh the glittering prospect of high returns in good ones. Uncertainty about future earnings, in short, will generally affect the composition and may lower the total of private investment.⁴ These private re-

³ In Chile the brunt of fluctuations in copper prices is apparently absorbed by the profits of foreign-owned firms, so the transmission of instability to the indigenous economy is minimal. See 10. Similarly, if import requirements move in sympathy with export receipts—a possibility we examine in some detail below—the impact of fluctuations in export earnings is correspondingly less.

⁴ Sir Sidney Caine has suggested that large fluctuations in earnings *raise* rather than lower total investment, for investment stimulated in boom periods more than offsets the decline in investment in

sponses to uncertainty will mitigate the costs considered above, but they may not do so in a socially optimal manner. Possible discrepancies between private and socially optimal responses to uncertainty will be discussed below in Section V.

Some, but not all, of these costs can be reduced or eliminated if the country maintains substantial foreign exchange reserves or has access to adequate borrowing facilities abroad. In that case, for the price of such funds, the costs of drastic curtailment of imports can be avoided, and the country can pursue (if it is able to do so) a stabilizing fiscal policy without immediate concern about its balance of payments. Although borrowing or the use of exchange reserves enables a country to spread fluctuations in its foreign exchange earnings over several periods, in general it cannot completely remove the need for some adjustment of consumption and/or investment in response to a loss of exchange earnings. Moreover, the investment of a portion of the national wealth in low return foreign exchange in order to mitigate the effects of variability in export earnings, like subsistence production by the nation's farmers, is an action which has as its cost the foregone (higher) expected return on riskier investments.

Specifying the "costs" of uncertainty with any precision is exceedingly difficult even for a single country; and of course these costs will differ from country to country. In the formal argument that follows we therefore resort to the customary academic escape from concreteness by postulating a social utility function which incorporates the assumption of risk aversion. This is conveniently done by assuming a utility function which is quadratic in its arguments (see the Appendix).

II. UNCERTAINTY IN A TWO-COMMODITY WORLD

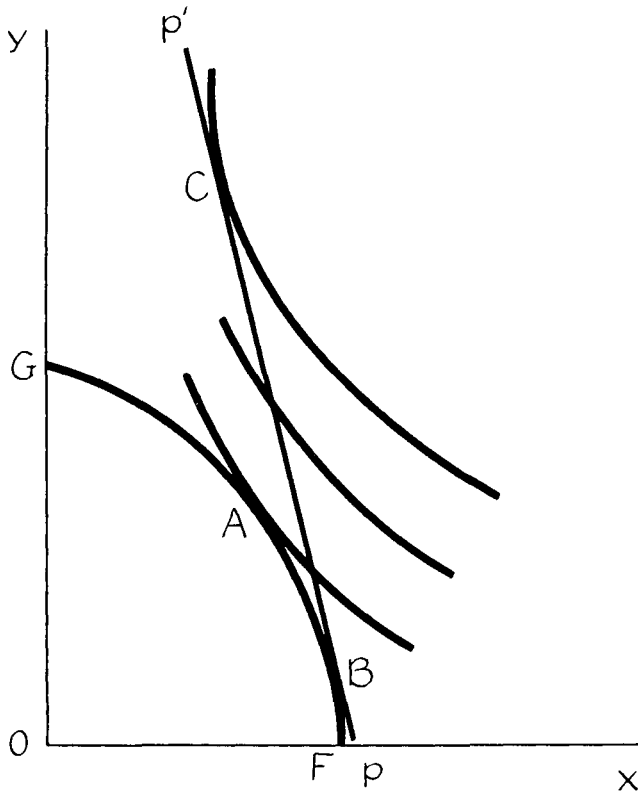
Classical trade theory fails to recognize the implications of risk aversion for the profitability of specialization and foreign trade. A few writers on trade theory have acknowledged in passing that uncertainty will influence the degree of specialization, but the formal theory has proceeded on the assumption that production costs and trading possibilities are known with certainty—or, alternatively, that there is no lapse of time between investment for production of a given product and its exchange for imports of foreign goods. This section amends the simple two-commodity trade model to incorporate uncertainty with respect to the prices at which goods can be exchanged internationally. It also briefly discusses the consequences of uncertainty in domestic production.

Figure 1 reproduces the familiar diagram for showing the advantages of trade. *FG* is the production frontier available to the country in question, which can produce any combination of the two goods, *x* and *y*, bounded by the frontier. Only points on the frontier are efficient, and, in the absence of trade, optimal production is at *A*, where the frontier is tangent to an indifference curve for the community.

If, however, the country has the option of trading on a world market at a

periods of slump. This outcome is possible, but it implies a kind of "profit illusion" on the part of investors which, if present to the degree required to raise total investment, is also likely to result (*ex post*) in a misdirection of investment. Thus the higher level would have to compensate for lower efficiency. See 2, 1.

FIGURE I



price ratio p indicated by the slope of the line pp' , it can improve its position by shifting the composition of its production from A to B , specializing in x , and exporting the surplus of production over consumption (x_c) in exchange for imports (px_c) of y . The new pattern of consumption is C , which results in an improvement in welfare as compared with A .

The conclusion of traditional trade theory that the country should and/or will shift its production from A to B along the production frontier assumes either that movements along the production frontier can be made quickly and costlessly to take advantage of changes in prices as they arise or that the price p is known with certainty.

Neither assumption is valid in practice. In general, once product-specific investments are made, the possibilities for substitution between final products are greatly reduced. Specialization in a product often involves investment in production facilities a substantial period of time before actual production takes place. Investment decisions today affect future output, not present output, and they must be based on some estimate of (uncertain) future prices.

The presence of uncertainty modifies the descriptive and normative conclusions of neoclassical trade theory. This can be shown easily by an extension of

Figure 1. Suppose for simplicity that the costs of shifting resources are negligible, but that one year must elapse between the decision to produce and maturity of the investment. Output decisions today must be determined, not by today's prices, but by prices believed to prevail a year from now. Suppose further that there is no uncertainty about the characteristics of domestic demand for production; the only uncertainty resides in the prices at which exports will exchange for imports on the world market.

The price line pp' in Figure 1 must now be reinterpreted to indicate the *expected* price; price is a random variable with a known expected value (\bar{p}) and standard deviation (σ_p). Decision-makers in the country must now choose between several riskless options (consuming any combination of x and y on FG , among which A is optimal) and a number of uncertain ones. Any decision to trade by exporting an amount x_e involves an uncertain outcome, but with each proposed exchange we can associate an expected value (\bar{y}) and a standard deviation (σ_y) for imports of y . If production and trading decisions must be made together, before the actual exchange price is known,⁵ we can construct the locus of consumption possibilities for given values of \bar{p} and σ_p .

Expected consumption (\bar{y}) of y equals $y_p + \bar{p}x_e$, where y_p is the amount of y produced domestically, and x_e is the amount of x exported at a price expected to be \bar{p} . The consumption of $x = x_p - x_e$ is known with certainty once the decision has been made to produce x_p and export x_e . The consumption of y has a standard deviation $\sigma_y = x_e \sigma_p$. Thus on the basis of decisions concerning x_p and x_e , we can plot the opportunity locus of consumption "bundles" (x, \bar{y}, σ_y).

This surface is sketched in Figure 2. The opportunity locus is generated by considering each point on the production frontier FG and allowing varying amounts of x to be exported. In each case raising exports by Δx_e will increase \bar{y} and σ_y proportionally, by $\bar{p}\Delta x_e$ and $\sigma_p\Delta x_e$ respectively. Thus the opportunity locus is a surface of parallel straight lines starting from a point on the production frontier FG and tracing out the curve GHI in the $\bar{y} - \sigma_y$ plane. Points on the curve GHI represent the \bar{y} obtained by exporting all of the x produced at the corresponding production points on FG . The maximum value of \bar{y} (point H) can be obtained by producing at B , where the expected price line is tangent to the production frontier.

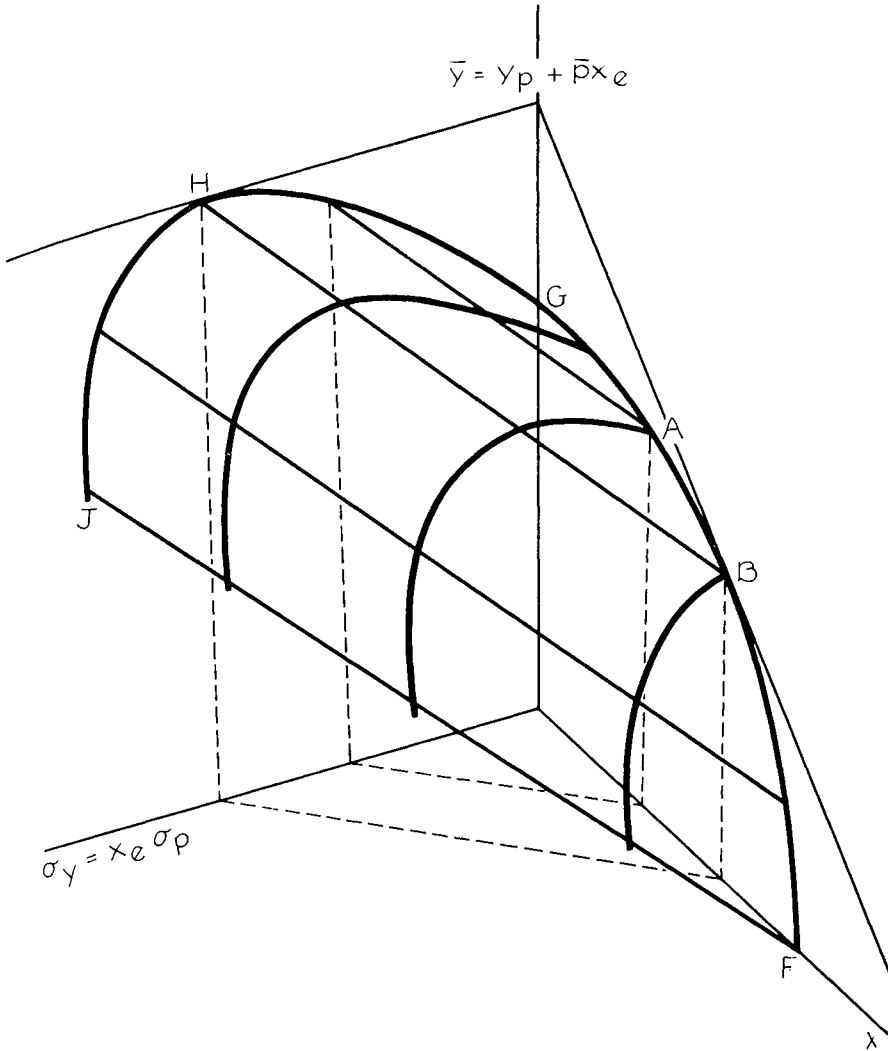
The optimal decision for production and trade is determined by the tangency of the opportunity locus with an indifference surface reflecting the trade-offs between consumption of x , expected consumption of y , and the undesirable variation in y .⁶ From this point of tangency σ_y and the expected amount of x and y to be consumed can be read off directly. The decision variables can then be derived as $x_e = \sigma_y/\sigma_p$, and $x_p = x + x_e$.

If there is no "risk aversion," i.e., if the country attaches no cost to variations in y , then the indifference surface will be parallel to the σ_y axis, and the optimal point will be on the line BH ; the choice of production will be B , as it was in the

⁵ Regime I in the Appendix. This regime would arise if a country agrees under contract to supply specified quantities of this export product in the future, but at world prices prevailing at the time of delivery. An alternative model, in which a country must make its production decisions in the present but can decide how much to trade after world prices are known, is considered briefly below and as Regime II in the Appendix.

⁶ See the Appendix, Model I, Regime I, for the marginal conditions.

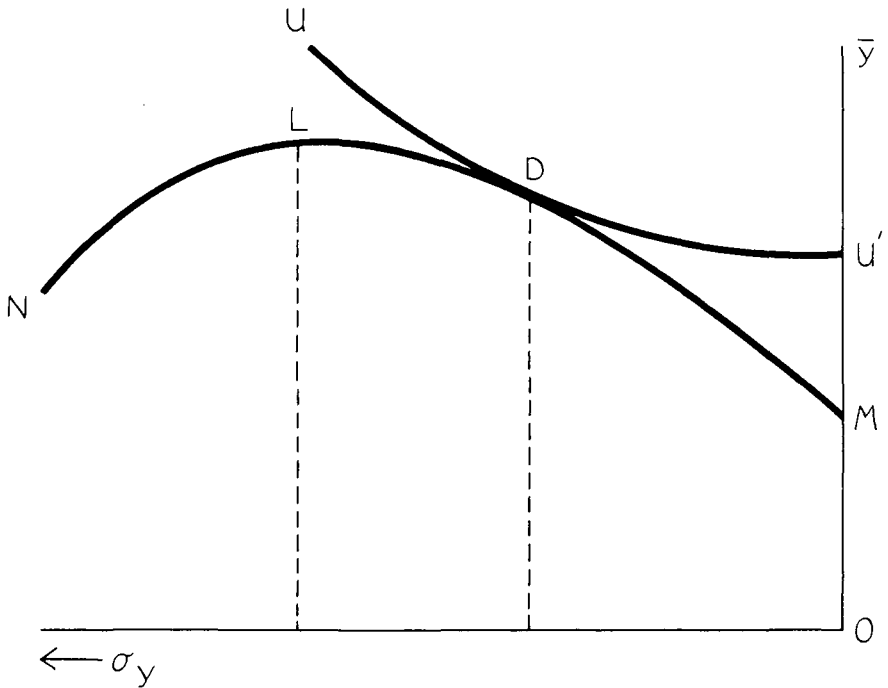
FIGURE 2



absence of uncertainty about world prices. In that case, all that is relevant is the projection of BH on the $x - \bar{y}$ plane, and the analysis reduces to that indicated in Figure 1.

If, however, substantial disadvantages, such as those outlined earlier, attach to fluctuations in export earnings, increasing uncertainty (a rise in σ_y) would require compensation in the form of higher expected values for y or x in order to leave the country as well off—i.e., the indifference surfaces will rise as σ_y increases. Since for any given x on the opportunity locus, \bar{y} first rises, then reaches a maximum and finally declines as σ_y increases, some indifference surface must be tangent to it. The optimum degree of specialization—indicated by the point of tangency—will under these circumstances be less than when there is no uncer-

FIGURE 3



tainty about future prices, i.e., the optimum point on the production frontier FG will lie between A and B rather than resting at B , as in the case of no uncertainty.⁷ The greater the variation of world prices and the greater the aversion to uncertain prices, the closer will the optimum production point be to A , approaching it asymptotically as risk aversion or uncertainty becomes larger and larger.⁸

The influence of risk aversion on the selection of the optimal amount of trade can be seen in Figure 3, where for a given x the opportunity locus and the indifference surface between \bar{y} and σ_y are shown as MN and UU' respectively. M is a typical point on the production frontier FG . The maximum expected consumption of y occurs at L (a point on line BH in Figure 2). If risk aversion were absent (i.e., if UU' were a horizontal line), L would be the point of tangency between the indifference curve and opportunity locus. Risk aversion is reflected in the positive slope to UU' as σ_y increases. This will typically result in a point of tangency such as that at D , calling for a smaller σ_y , hence (given σ_p) a smaller value of trade than is indicated at L .

The presence of uncertainty in domestic production as well as in trading price modifies these results. In that case trade may serve to *reduce* the uncertainty in consumption. Paradoxically, therefore, it may be desirable to engage

⁷ See Appendix, Model I, Regime I.

⁸ It is worth noting, however, that in this model strict autarky is never warranted on the basis of risk and risk aversion. The utility surface must be perpendicular to the $x - \bar{y}$ plane. At A , the expected world price is so much more favorable than the marginal rate of transformation between x and y that some trade will always raise utility.

in trade even when the average price of the imported commodity would have made trade unattractive in a world of certainty. If, for example, the domestic production of y is highly uncertain, exporting x in exchange for y may provide a means of decreasing the variance in the consumption of y , even when the world price of y itself is subject to variation. Uncertainty in the production of x , on the other hand, may reinforce the effect of uncertainty in p , causing production of x to be less than in the world of certainty (see Appendix, Regime I, Case B).

We have assumed so far that production and trade decisions must be made before the trading price is known. If the country has the opportunity to decide how much to trade *after* it knows the price at which it must trade (Regime II in the Appendix), it will obviously be better off than if it must decide beforehand how much to trade, since it has more information in optimizing. In this case the marginal rate of substitution between x and y in consumption can be equated to the known price ratio. What is less obvious is that this freedom to choose the amount exported after the price is known also influences the choice of an optimum pattern of production.⁹ The precise character of this influence is much more complicated than in the regime just considered, however, and no generalization can be made without knowing more both about the distribution of p and about the character of the utility function.

III. UNCERTAINTY WITH MANY COMMODITIES

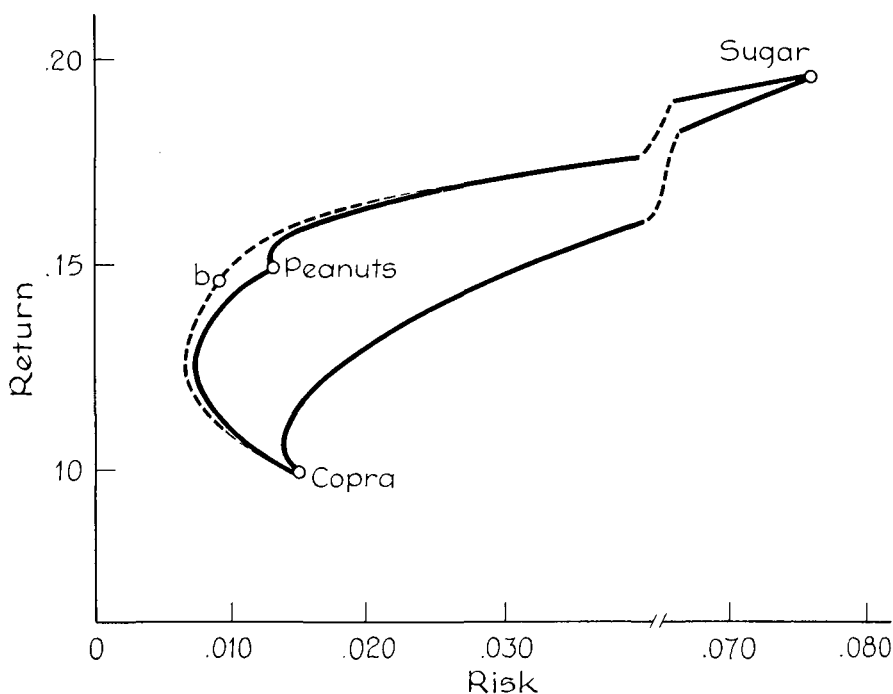
The foregoing discussion has introduced uncertainty in world markets into the traditional two-commodity analysis of international trade and has suggested that the theory must be amended to allow for uncertainty. In practice, of course, countries face a wide range of choice in making their investment decisions. This section extends the analysis to include many commodities, encompassing both actual and potential exports and products which displace imports.

To illustrate most clearly the effect of uncertainty on choice among many commodities, we assume constant costs (= constant rates of return) for simplicity, an assumption which will be relaxed below. We also assume that expected utility depends on the distribution of expected rates of return and deviations from expected rates of return, or, simply, returns and risks,¹⁰ rather than on commodity bundles as in Section II. These risks and returns may be defined in terms of domestic currency or, where the distinction is appropriate, foreign exchange. Investment in the production of any commodity, whether for export or import substitution, may be represented in the same way, each with its associated risk and return. The risks can arise from fluctuations in world commodity prices, from variations in weather, from work stoppages, or from a variety of other causes. For concreteness and simplicity we focus here on fluctuations

⁹ Theil makes the same point in a different context (see *II*, pages 121–131).

¹⁰ See Appendix, Model II. "Rate of return" must be defined appropriately for the problem at hand. The term derives from the genesis of this analysis in the selection of an optimum investment portfolio. More broadly, it may be applied to the return to any factor of production. For the individual businessman or farmer, it is the rate of return to his investment of capital, land, and effort; for an economy chronically short of foreign exchange, it may be the return in foreign exchange to alternative uses of scarce domestic resources. In this section, we will interpret the rate of return in this last sense.

FIGURE 4



in prices on world markets as the source of uncertainty, but this restriction is unnecessary and a more general formulation is given in the Appendix, Model II.

All feasible investments can be represented in a diagram in terms of their respective risks and returns.¹¹ For example, Figure 4 plots the prospects of three different investments called copra, sugar, and peanuts. Consider first sugar and peanuts. As drawn, sugar yields a higher rate of return than peanuts, but it is also subject to greater uncertainty.¹² It will be clear from the analysis in Section II that selecting the investment with the highest expected rate of return will not be optimal if it is associated with substantial risk and if risk aversion is high. However, combining several investments into a "portfolio" introduces a new possibility: advantage can be taken of any differences in the pattern of export prices, e.g., for peanuts and sugar. In other words, the covariation between prices of various exports becomes important.

If the returns on two investments are perfectly positively correlated, always rising and falling together, then a straight line joining the two points describes the yield-risk characteristics of all combinations of the two investments. If, on the other hand, yields on the two investments are virtually uncorrelated, the yield-risk

¹¹ This technique is developed by James Tobin (see 13, 14).

¹² In order to construct Figure 4 some assumption must be made about the returns on various activities. In this example the returns in foreign exchange per unit of domestic cost were assumed to be .2, .15, and .1 for sugar, peanuts, and copra, respectively. It was also assumed that risk is due entirely to variation in export prices. To obtain the risk associated with unit domestic expenditure on each activity, the coefficient of variation observed for world markets, from Table 2, below, was multiplied by the assumed return on each activity.

characteristics of all combinations of the two investments are shown by a curved line like the one joining peanuts and sugar in Figure 4. Lack of correlation reduces the risk associated with any given expected yield on the portfolio. Where returns are perfectly negatively correlated, with one yield always high when the other is low and vice versa, then the risk-yield characteristics of various combinations of the two activities would be indicated by two straight line segments joining the two initial points with a common point on the vertical axis (zero risk) showing a return higher than peanuts but lower than sugar.¹³

It is possible to reduce the risk associated with any portfolio of investments by adding investments with returns not highly positively correlated with those already in the portfolio. Thus a country may stabilize its export earnings by diversifying into exports which have uncorrelated or (preferably) inversely correlated movements in world prices. It may even make sense for a country to invest in a low yield-high risk export industry, if its price pattern has a high negative correlation with the prices of other products. This possibility is seen in Figure 4 by considering combinations of peanuts and copra. An investment in peanuts has higher yield and lower risk than a similar investment in copra, and thus the former investment is preferable on both counts. However, if the returns to peanuts and copra are highly inversely correlated, total risk can be reduced by investing partly in copra. The curved line connecting peanuts and copra in Figure 4 assumes $\rho = -.47$ for peanut and copra prices, taken from Table 2.

The choice of a particular point on the efficiency frontier is governed, of course, by the utility function. Utility is maximized by selecting a point on the efficiency frontier which is tangent to an indifference curve; the common slope will indicate the rate of substitution, or "trade-off," between risk and return. If there is no risk aversion, then the indifference curves are horizontal and maximum utility is achieved by investing all resources, in the case of Figure 4, in sugar.

The analysis of Figure 4 can be extended, with some complication but without any new principle, to encompass more than two investment opportunities. For all possible combinations of investments, there will be an "envelope" which represents the "efficiency frontier" facing the country; all other combinations will be dominated by some combination on this frontier.¹⁴ The shape of the envelope,

¹³ The risk-yield characteristics of any portfolio S allocated to two investments x and y in the proportions a and $(1 - a)$ are given by the following relationships:

Expected yield of S ,

$$r_s = ar_x + (1 - a)r_y$$

Variance of the yield of S ,

$$\sigma_s^2 = a^2\sigma_x^2 + (1 - a)^2\sigma_y^2 + 2a(1 - a)\rho\sigma_x\sigma_y,$$

where r_i = the expected rate of return on investment i , σ_i^2 = the variance of the return on i , and ρ is the correlation coefficient between the returns on the two investments. Setting $\rho = 1$ and $\rho = -1$, respectively, gives the curves described in the text.

For a portfolio S_n of n investments,

$$r_s = \sum_{i=1}^n a_i r_i$$

and

$$\sigma_s^2 = \sum_{j=1}^n \sum_{i=1}^n a_i a_j \rho_{ij} \sigma_i \sigma_j,$$

where a_i is the proportion of the portfolio in investment i , $\sum a_i = 1$, and $\rho_{ii} = 1$.

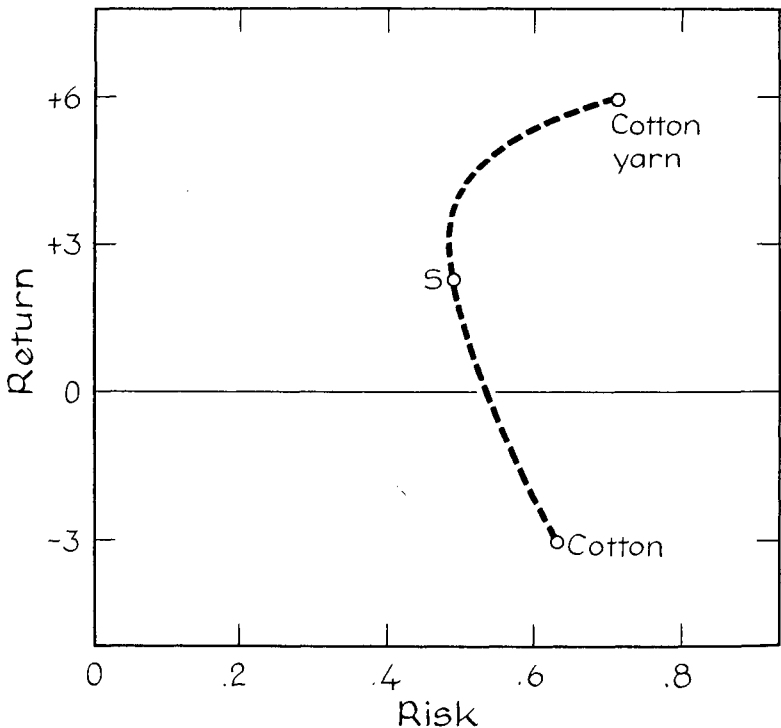
¹⁴ See *I*⁴, ch. 3. The dotted curved line in Figure 4 indicates the "efficiency frontier" for the three commodities shown.

and the possibility for reducing risk by diversifying investments, is governed by the covariance matrix of returns on all the various investments. The more small or negative correlations there are in realized returns to the various investments, the greater the opportunity for reducing risk through diversification.

The price movements of intermediate and final products often reflect movements in the prices of important inputs; the return to the *process* of fabrication is much less variable, or varies with a different pattern. Countries may therefore be able to reduce the variability of their foreign exchange requirements if they enlarge their *imports* of products whose prices are highly correlated with the prices of their exports. A country with extensive processing, such as Japan, experiences considerably less difficulty from wide fluctuations in export earnings than might appear at first sight because of sympathetic movements in import requirements and import prices. Similarly, a country can often reduce fluctuations in earnings by processing industrial materials produced domestically rather than exporting them in the raw state. Even where the processing is relatively high in cost, the gains from reduced risk may compensate for the low yield.¹⁵

Figure 5 illustrates the influence of variation and covariation in input prices

FIGURE 5



¹⁵ Some of the more highly fabricated products in Table 1 show lower variability, relative to price, than their less highly fabricated inputs. Because of sympathetic movements in the prices of inputs and outputs, the variability of return to the fabrication *process* is much lower still.

on the risk-yield characteristics of a processing industry. Cotton-cotton yarn is considered for concreteness; data on price variation are from Table 1. We can represent the spinning process(es) by plotting the gross receipts and their variation for cotton yarn, i.e., *before* deducting costs and variations in cotton inputs. Cotton inputs can then be represented separately with their own variation and with a negative "yield," since they must be purchased. Not surprisingly, prices of cotton and cotton yarn are highly correlated positively; but for the analysis of processing industries a positive correlation behaves analytically like a negative correlation for the case of two final product industries. Thus the opportunity locus is bowed to the left.¹⁶ If the input-output relationship is technologically fixed (e.g., 1.2 pounds of cotton are required to make one pound of cotton yarn), then the risk-yield characteristics of the processing industry are found by finding the point *S* on the curve (drawn for $\rho = +.75$) representing that fixed "mix" between cotton and cotton yarn. If prices are highly correlated, the variability of the processing industry will be smaller than that for either product, as shown. This point *S* then represents a possible investment, and should be added (after normalization to the basis on which rates of return are computed, such as domestic costs) to the array of activities considered, such as those in Figure 4.¹⁷

IV. THE VARIABILITY OF EXPORT PRICES

A diagram like Figure 4 indicates how risk, or unforeseen variations in earnings, can be reduced through diversification. But the benefits from diversification depend intimately on variations in return to different investments being "out of step." Unhappily, returns to exports of many primary products are positively correlated, largely because earnings for all industrial materials are heavily influenced by the strength of overall demand in the industrial countries and hence tend to fluctuate together with fluctuations in output and demand in the major countries. Diversifying from the production of copper to the production of aluminum in the 1950s, for example, would not have reduced by much the instability of export earnings.

However, for a number of commodities the major source of instability is on the supply side. Demand for tropical beverages such as coffee, tea, and cocoa does not fluctuate much with the business cycle, for example, but prices do fluctuate considerably due to variations in the weather and to long-run supply cycles. Primary products can be grouped roughly according to whether the principal source of price instability arises from fluctuations in demand or supply. Diversification to reduce risk should involve choosing commodities on both lists.

A third list can be made of imported commodities, largely manufactures but

¹⁶ This effect can be shown algebraically. Set P_1 = the price of the input, p_2 = price of the output, and p_p = the price paid to factors engaged in the "process." Then for unit output $p_p = p_2 - a_{12}p_1$ where a_{12} represents the physical input-output coefficient for the process, assumed constant. Then

$$\text{Var}(p_p) = \text{Var}(p_2) + a_{12}^2 \text{Var}(p_1) - 2a_{12} \text{Cov}(p_2, p_1).$$

The Appendix, Model II, provides a more general formulation.

¹⁷ The analysis so far has assumed constant costs, or rather constant rates of return. The presence of rising costs complicates the analysis, since the risk-yield characteristics of any particular investment then depend on the magnitude of the investment. Conceptually, however, the analysis is similar, and in principle the envelope of efficient investments could be represented in the three dimensions expected yield (r), risk (σ), and volume of output (q). The case of rising costs is introduced algebraically in the Appendix. The presence of rising costs permits the use of taxes and subsidies to guide private entrepreneurs to the optimal degree of diversification, as will be explained in Section V.

TABLE 1.—VARIATION AND COVARIATION IN WHOLESALE COMMODITY PRICES
IN WORLD TRADE, 1951-1963*

Commodity pair	Correlation coefficient ρ	Normalized standard deviations		Normalized covariance times 1000 $\frac{1000 \rho \sigma_1 \sigma_2}{\mu_1 \mu_2}$	Mean price (U.S. dollars per short ton, except as noted)	
		$\frac{\sigma_1}{\mu_1}$	$\frac{\sigma_2}{\mu_2}$		μ_1	μ_2
Substitutes in production:						
Bacon-rice	-.01	.04	.23	— .078	617	148
Beef-butter	-.15	.16	.11	— 2.5	580	816
Beef-rice	-.80	.16	.23	—29.	580	148
Beef-wheat	-.53	.16	.10	— 8.3	580	64
Beef-wool	-.86	.16	.32	—43.	580	1300
Cocoa-coffee	+.65	.28	.24	44.	680	909
Copra-peanuts	-.47	.15	.09	— 6.7	160	384
Copra-rubber	+.24	.15	.28	15.	160	620
Corn-wheat	+.48	.16	.10	7.7	50	64
Cotton-peanuts	+.44	.21	.09	8.4	584	384
Lumber-woodpulp	-.49	.02	.22	— 2.6	85	169
Peanuts-abaca	-.58	.09	.25	—13.	384	358
Rice-jute	-.16	.23	.26	— 9.5	148	220
Rice-rubber	+.12	.23	.28	7.9	148	620
Rice-sugar	+.10	.23	.38	8.7	148	81
Rice-tea	-.74	.23	.13	—23.	148	1070
Wheat-wool	+.54	.10	.32	17.	64	1300
Fabricating processes:						
Copra-coconut oil	+.85	.15	.14	18.	160	264
Corn-bacon	+.11	.16	.04	.73	50	617
Cotton-cotton yarn	+.75	.21	.12	18.	584	1473
Cotton yarn-fabric	+.87	.12	.23	23.	1473	16 ^a
Jute-burlap	+.78	.26	.20	41.	220	11 ^a
Lumber-furniture	+.31	.02	.08	.57	85 ^b	... ^c
Pig iron-steel	-.19	.12	.11	— 2.4	559	756
Steel-steel products	+.79	.11	.12	11.	756	... ^c
Wheat-flour	+.20	.10	.08	1.6	64	123
Wool-wool yarn	+.80	.32	.25	78.	1300	3020
Other:						
Aluminum-copper	+.44	.12	.15	8.1	479	609
Bacon-beef	-.16	.04	.16	— 1.0	617	580
Beef-hides	-.04	.16	.12	— .68	580	430
Bicycles-tea	-.18	.04	.13	— 1.0	... ^c	1070
Coconut oil-palm oil	+.33	.14	.12	5.4	264	216
Coffee-tea	+.12	.24	.13	3.8	909	1070
Rice-steel	-.93	.23	.11	—23.	148	756

* Subscripts on the means and standard deviations designate the first and second commodities in the listed pairs. Standard deviations have been "normalized" by dividing each by the mean value of the variable. For means, foreign prices converted to U.S. dollars at official exchange rates.

Computed from data in United Nations, *Monthly Bulletin of Statistics*, and Germany, Statistisches Bundesamt, *Statistisches Jahrbuch für die Bundesrepublik Deutschland*.

^a Dollars per hundred yards.

^b Dollars per thousand board feet.

^c Variations taken from an index of prices; mean prices not available.

also including some semi-manufactured industrial materials. Produced at home, some of these products are subject to quite different risks from the first two groups. Import-replacing activities may even be fully dominated by export activities which have both higher yield and lower risk, yet still have some role in a sensible investment strategy if their yields vary inversely with the yields on profitable export industries.

We mentioned earlier that fluctuations in export earnings do not seem to be closely related to concentration of export earnings on one or two primary products. One explanation for this is that countries tend to produce a range of products which have similar characteristics, e.g., their exports all depend primarily on industrial production in the major countries or they are all subject to the same vicissitudes of rainfall and sunshine. These countries, though ostensibly "diversified," have not diversified properly; simply adding commodities to the list of exports is not sufficient. Some of them may have diversified properly through import substitution, of course, but this would not be reflected in the figures on export earnings and hence would not influence the finding of Massell and others.¹⁸

That earnings on all the export products of primary producing countries are not highly correlated is suggested by the data in Tables 1 and 2. Table 1 shows, for pairs of commodities, the variation and covariation in world prices between 1951 and 1963, using annual averages of commodity prices compiled by the United Nations. Column 1 records the correlation coefficient between prices of the two products in each row. The next two columns give the coefficient of variation (normalized standard deviation) for the first and second commodity in each pair, respectively. Column 4 gives the normalized covariance, and the final two columns enter the mean values of the first and second set of prices, respectively, converted where necessary to U.S. dollars at official exchange rates.

Table 2 provides a matrix of price correlation and covariance data for twelve commodities over the same period. Correlation coefficients are shown above the diagonal, normalized standard deviations along the diagonal, and normalized covariances below the diagonal. This is the sort of matrix which any particular country wanting to diversify might construct. Exclusive emphasis on price variations is of course not justified; the sources of variation in export earnings extend well beyond variations in world prices. According to estimates by MacBean, fluctuations in export earnings for most individual countries are due more to variations in domestic supply than to variations in world prices.¹⁹ For any individual country, therefore, it would be important to construct a matrix such as that in Table 2 for variations in domestic output. Such data would be specific to one country or to a group of contiguous countries.

¹⁸ See 5; 3, ch. 5; 9, ch. 2; 8. The main explanation for Massell's findings (see fn. 1, page 258) is that many highly industrialized countries export a wide range of goods—and hence show low export concentration ratios—but capital goods, which are notoriously sensitive to slight changes in the growth in world demand and output, bulk large in their exports. The industrialized countries, which hold substantial international reserves, can on the whole absorb fluctuations in export earnings with much less difficulty than can the less developed countries. If industrial countries are excluded from Massell's sample, the correlation between fluctuation in export earnings and export concentration ratio might well rise. Coppock points out, for example, that among broad groupings of primary producing countries alone, there does seem to be a systematic connection between instability of export earnings and concentration of exports on a few products. See 3, page 104.

¹⁹ See 9, page 332.

TABLE 2.—WHOLESALE PRICES OF 12 COMMODITIES: CORRELATIONS AND COVARIANCES, 1951-1963*

	Rice	Maize	Coffee	Cocoa	Sugar	Bacon	Peanuts	Copra	Coconut oil	Cotton	Cotton fabric	Bicycles
Rice	.23	.88	.47	.48	.10	-.01	.26	.09	.51	.86	.65	-.26
Maize	33.	.16	.67	.50	.15	.11	.35	.06	.50	.94	.71	-.55
Coffee	26.	27.	.24	.65	-.15	-.13	.43	-.13	.16	.58	.24	-.77
Cocoa	31.	23.	44.	.28	-.19	-.31	.39	.36	.57	.53	.12	-.51
Sugar	8.7	9.6	-14.	-20.	.38	.44	-.02	-.07	-.03	.08	.21	.47
Bacon	-.078	.73	-1.3	-3.4	6.7	.04	-.09	-.13	-.02	-.15	-.09	.14
Peanuts	5.6	5.3	9.8	10.	-.75	-.34	.09	-.47	-.26	.44	-.19	-.26
Copra	3.2	1.5	-4.8	15.	-4.1	-8.2	-6.7	.15	.85	.09	.30	-.15
Coconut Oil	17.	11.	4.9	22.	-1.8	-.85	-3.4	18.	.14	.48	.49	-.35
Cotton	41.	32.	29.	30.	6.2	-1.2	8.4	2.9	14.	.21	.73	-.51
Cotton Fabric	34.	26.	13.	7.3	18.	-.90	-3.9	10.	16.	34.	.23	-.25
Bicycles	-2.5	-3.8	-7.8	-6.0	7.5	.23	-1.0	-.95	-2.0	-4.5	-2.4	.04

* Correlation coefficients are above diagonal; normalized covariances times 1000 are below. Normalized standard deviations are along the diagonal. See Table 1 for sources of data.

Fluctuations in world prices represent a source of potential variation for all trading countries. The analysis of these fluctuations included here is primarily meant to be suggestive of the possibilities for diversification rather than the basis for actual prescription. The variances and covariances reported in Tables 1 and 2 were computed from historical data without removal of trends. What is really wanted, of course, are estimates of the variation and covariation in future prices. To obtain estimates in which one would have any confidence would require a commodity by commodity study. In the case of particular commodities there may be information which leads one to suspect that past price variation is likely to be unrepresentative of future price variation. Similarly, in the case of some commodities (e.g. light manufacturing) it may be thought that the price trend is predictable *ex ante*, in which case it would be more appropriate to measure the variance around trend. The figures in Tables 1 and 2 are therefore introduced merely as concrete illustrations, not as results of a serious attempt at the required estimation.

Table 1 places commodity pairs into three broad groups: those for which there is a fairly high elasticity of substitution in production, those which involve processing one product into the other, and a miscellany of others, including joint products, close substitutes in consumption, and so on.²⁰ On the basis of these data, it would appear that countries such as Argentina and Australia did well to produce beef and wheat or beef and wool, since price movements for these commodities have been inversely correlated. Lumber and woodpulp also made a good combination. "Diversifying" from wheat to wool or from coffee to cocoa, however, represented largely diversification in name only, since price movements for these pairs of commodities broadly paralleled one another.

In selecting commodities for diversification it is not sufficient to look at the expected covariation in receipts; it is necessary also to look at expected costs and returns. Even for extreme risk aversion there is some sacrifice in expected return which is so great as to make diversification into the high cost product unwarranted. Rates of return for any given product line will of course vary greatly from country to country, and they must be estimated for each country. These rates of return influence the risk each country takes in making a selection of activities.

V. DIVERGENCES BETWEEN PRIVATE AND SOCIAL RISKS

As we have seen, the introduction of uncertainty into the analysis of international trade problems modifies some of the traditional normative and descriptive propositions of trade theory. It does not *ipso facto* produce a case for policy measures to induce diversification in primary producing countries, even when the uncertainty faced is very large. Such a case depends on a failure by private decision-makers to adjust appropriately to high variability in export or other earnings.

Many of the disadvantages of high risk redound to the private investor. If he

²⁰ In Tables 1 and 2 bicycles were inserted as a proxy for all the light manufacturing of final goods which many less developed countries have fostered. Price variations in these products have been very modest in size, and as a result covariances with other products are low, indicating some potential reduction of risk.

can, he will take steps himself to diversify; and if he has chosen not to diversify, it may indicate that the reduction in risk which would accompany diversification is not sufficient to compensate for the reduction in expected rate of return. If the opportunities confronting an individual investor are the same as those confronting the economy, intervention is justified only if his estimates of risk and return, and the costs he attaches to risk, differ from the costs, returns, and risks to society. There are a number of reasons, however, for conjecturing that private investors do not take into account the full social costs of high variability in earnings, especially export earnings. Private investors look out for their own interests; social objectives are often different and considerably broader in scope than those of private investors. Discrepancies between actual private and socially optimal diversification may arise, first, because individuals are unable to diversify adequately even when they want to, and, second, because their perception of returns and risks may differ from the returns accruing to or the risks incurred by society.

Imperfect capital markets, universally present in less developed countries, impede individual diversification beyond what can be managed directly by the investor. Thus, in terms of Figure 4, an individual whose risk-return preferences would call for some combination of investments on the dotted envelope may be obliged to confine his activity to growing peanuts, for example.

Inadequate facilities for diversification and inadequate information about such facilities induce risk-averse investors to diversify in ways which are not always appropriate for the economy as a whole. The most frequent form of hedging against uncertainty in less developed countries—but also until very recently widespread in Europe as well—is to engage in diversified subsistence agricultural production. The hazards of being faced with high food prices in years of low income are sufficiently disastrous in many parts of the world to induce farmers and nonfarmers alike to retain land for home use. This risk-averting behavior helps perpetuate extremely low productivity in agriculture.

Even when diversification into a wide range of activities is available to the individual investor, however, he may not diversify optimally from a social point of view. In framing their investment decisions, investors will take into account only the disadvantages of instability in receipts to themselves, not the disadvantages to other groups. There are three reasons for supposing that this makes insufficient allowance for the social costs of variation in returns, especially when export receipts are involved.

First, as indicated earlier, a part of the variation in receipts may be borne by workers or other factors, rather than by the investors.²¹ The effect of variability on investors' profits can be cushioned by passing part of it on to groups which have no direct role in making the investment decisions. Yet the social and psychic costs of variations in wage incomes and employment are likely to be high, and they ought to be reckoned in any investment strategy.

Second, swings in export receipts influence both government revenues and the

²¹ Some economists may prefer to call this another case of market imperfection. With perfect labor markets, labor's preferences with respect to uncertainty in the wage rate and employment would presumably be reflected in the wage rate.

country's capacity to import. They thus reduce the ability of government to carry on public expenditure programs efficiently and the ability of private investors to realize their planned import requirements. The resulting delays and disruptions will reduce the efficiency of public and private investment, but the individual investor will not take these social costs into account in making his own decisions.

Third, where the investor has international interests, his diversification may leave any one country with excessive variation in earnings. Large international firms take a global view of their investments, not a national one. But balanced investment decisions at a global level will typically leave some participating countries with a higher risk than exists for the portfolio as a whole. Here the interests of the foreign investor and that of the host, or debtor, country may diverge sharply.

For all these reasons there is likely to be a substantial discrepancy in many less developed countries between private behavior toward risky investments and that which would be socially optimal.²² Some sort of policy interference with the allocation of resources may therefore be desirable to help guide investment and output decisions toward the socially optimal degree of diversification. If in Figure 4, for instance, the risk-yield opportunity frontier facing the country is the dotted envelope and the point of tangency with the social indifference curve (not drawn) is at \bar{b} , but private investors underestimate the costs of risky investments and invest more in sugar than is indicated at \bar{b} , then government measures to influence the pattern of investment will be warranted. Several types of measure can be used to encourage diversification: taxes and subsidies, guarantees and insurance, direct controls over investment, and encouragement of capital markets. The merits of each depend upon the particular circumstances of the country.

VI. MEASURES TO ENCOURAGE DIVERSIFICATION

Where effective markets exist and where entrepreneurs respond readily to market incentives, use of simple taxes and subsidies to guide private decisions toward the optimal portfolio of investments for the country as a whole may be effective. If production is subject to increasing costs, then a system of specific taxes and subsidies—taxes and subsidies per unit of output—can be used to “distort” the yields on various investments with a view to penalizing high risk projects and rewarding low risk ones to achieve the socially desirable combination. If we ignore processing industries, the appropriate tax (or subsidy) to levy on industry i is

$$t_i = - (n_s - n_p) [q_1^* \sigma_{1i} + q_2^* \sigma_{2i} + \dots + q_i^* \sigma_{ii} + \dots + q_n^* \sigma_{ni}],$$

where t_i is the tax (subsidy) per unit of i , $(n_s - n_p)$ is the discrepancy between

²² Tobin has argued, for the United States, that individual investors will tend to invest less than is socially optimal because they face individual risks—inroads by competitors, loss of financial control of the firm or farm, etc.—which do not represent social risks. *Total* private investment in less developed countries may be limited for similar reasons. We are here primarily concerned with the possible misallocation of investment among various alternatives, not with the total. However, the ability of investors to pass on some variations in demand to other elements of society, such as workers or government, suggests that in some instances too *much* private investment will take place. See 12, pages 13–14.

social and private risk aversion,²³ q_j^* is the socially optimal output of q_j , and σ_{ij} is the covariance of prices for products i and j .

If social risk aversion exceeds private risk aversion, $-(n_s - n_p) > 0$, and each product should be taxed in proportion to its own variance ($\sigma_{ii} = \sigma_i^2$) and to the covariance of its prices with the prices of other products. If these covariances are sufficiently negative, it may be appropriate to subsidize the product—as in the case of copra in Figure 4. Allowance for processing industries complicates the tax formula somewhat. Along the lines of the analysis in Section III, a high correlation between a product's price and those of its principal components reduces the risk in the processing function; thus such correlations reduce the tax (or increase the subsidy) appropriate for the processing industry.²⁴ Where a desirable industry competes with imports, tariffs could be used instead of subsidies, although tariffs have the undesirable by-product of reducing domestic demand.²⁵

Policy-makers wishing to influence private investment may take advantage of the fact that investors respond to changes in risk as well as changes in prospective yield. Socially desirable investment can be encouraged by reducing risk, for example, by establishing a minimum guaranteed rate of return or by providing insurance. A system of progressive profits taxation lowers the yield to investors, but it lowers even more the risk they face, since tax revenues absorb a more than proportionate share of any fluctuations in profit. Commodity marketing boards in less developed countries often stabilize returns to producers and this stabilization may induce them to reallocate resources from the subsistence sector to money crops.

By the same token, investments in certain lines could be discouraged with the objective of diversifying the country's structure of output by policy measures designed to *increase* the risk producers face. Thus, marketing boards might deliberately exaggerate, rather than damp, fluctuations in world prices in their payments to producers.

Reliance on simple taxes and subsidies to achieve a socially optimal portfolio of investments requires that individual investments be subject to rising costs or that private investors attach some aversion to risk and are able to diversify their own investments. If private risk aversion is absent, for example, investors will select projects with the maximum yield after allowance for taxes and subsidies; and in a constant cost world they will all choose the same product. Similarly, if investors are unable to diversify individually, each will still concentrate on one product which satisfies best his trade-offs between yield and risk, after taking taxes and subsidies into account.

²³ With risk aversion, $n < 0$. If private entrepreneurs underrate the costs of risk, $n_s < n_p$ and $(n_s - n_p) < 0$. For a derivation of this formula, see the Appendix, Model II.

²⁴ See Appendix, Model II.

²⁵ Use of these taxes and subsidies to achieve the socially optimal "portfolio" of output assumes that private entrepreneurs are trying to maximize the same kind of objective function (utility function) that society is, except for differences in risk aversion. In particular, it assumes that market prices for all products and factors reflect their true social cost, and that the exchange rate is in equilibrium so that profit maximizing behavior leads, in the short run, to maximum national income. Much of the literature on economic development emphasizes precisely the discrepancies between private and social costs, e.g., for labor or foreign exchange. The discussion in Section III above presupposes equilibrium in the foreign exchange rate, for instance. Where such discrepancies exist, the taxes and subsidies appropriate for diversification must be modified accordingly.

Here other measures are needed to achieve socially optimal diversification. One possibility is the use of government direction over investment and output decisions. Such direction is needed because there is no system of market influences which will guide private investors to the optimal *composition* of output; all private investors will choose the best opportunity of the moment, and risk-reducing diversification will be lost.

A long-run alternative to government direction lies in improvement of local capital markets and other financial institutions, so that risk-averting individuals are able to diversify their own investments. The difficulty for individual investors of managing a wide range of activities can be overcome if individuals have the opportunity to hold some of their assets in equities of businesses which they do not manage. Special efforts are required to create facilities for the peasant farmer to "diversify" in ways other than those he has traditionally followed. Efficient capital markets in urban centers are not likely to reach him. Agricultural investment banks and credit unions are important links between the financial markets and the peasant, enabling him to extend his investments beyond those permitted by his own immediate management capability and providing an outlet for savings other than reinvestment in the land.

The improvement of financial institutions and capital markets, while important, will not by itself assure a socially optimal composition of output because of the discrepancies between private and social costs of instability mentioned earlier. But by permitting individuals to diversify, capital markets improve the effectiveness of measures designed to influence private investment decisions.

VII. SUMMARY AND CONCLUSION

The argument of this paper can be briefly stated as follows: wide variations in export receipts leading to fluctuations in national income are on balance costly to primary producing countries in terms of social cohesion, efficient allocation of resources, and economic growth. The pure theory of international trade has not incorporated uncertainty about the prices at which trade will take place (or the quantities which will actually be available for exchange); it rests on assumptions concerning the mobility of resources and knowledge about the future which reduce questions of uncertainty to negligible importance. In the real world, however, lack of perfect knowledge about the future combined with a time lag between investment and returns to investment give uncertainty a very great importance in influencing economic behavior. In Section II we indicate how the descriptive and normative theory of trade can be modified to take this uncertainty into account. We then consider how an economy can diversify in such a way as to reduce the variation in its receipts by taking into account divergences in the price and output variation for different products.

Finally, we consider various ways in which the social cost of fluctuations may differ from the private costs, or at least the private costs of those making investment decisions. Private diversification may not be optimal from a social point of view.

Where such discrepancies do exist, various policy measures can be used to achieve the desired degree of diversification. These measures include the appropriate combinations of taxes and subsidies (or tariffs); and they include various

measures, such as insurance and guarantees, to influence the risks incurred in investment. In some circumstances, only direct interference in the choice of outputs will lead to the socially optimal composition of output.

It has long been recognized in the theory of international trade, even by free traders, that protection may be warranted when "externalities" are present—when the investment in question produces desirable external effects which cannot be captured by the investor in his profits.²⁶

The case for protection here also rests on externalities; what we have added is merely a *new dimension*, not analyzed explicitly in the literature on the theory of trade, in which such externalities may occur. That dimension is uncertainty. The private investor may reckon the expected rate of return in a way which provides the appropriate basis for social calculation—private costs and benefits may correspond to social costs and benefits in this dimension—but still may make the wrong decision from a social viewpoint because he underrates the social costs of high variability in receipts.

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²⁶ Tariff protection is almost always inferior to taxes or subsidies on production or consumption, however, at least before allowing for costs of administration. See 6.

APPENDIX

Model I.

This appendix demonstrates formally some of the statements in the text for a particular class of utility functions. Suppose a country's preferences for two commodities, x and y , can be represented by a quadratic utility function in the two goods:

$$(1) \quad U = ax + by + kxy + mx^2 + ny^2; \quad x \leq \frac{2na - bk}{k^2 - 4mn}, \quad y \leq \frac{2nb - ak}{k^2 - 4mn}$$

where x and y represent consumption of the two goods and where $a > 0$, $b > 0$, $m < 0$, $n < 0$ and $4mn - k^2 > 0$ provide the desired properties of positive but diminishing marginal utility in x and y in the range under consideration. n reflects the degree of risk aversion in y , and m the degree of risk aversion in x . k influences the degree of substitution ($k < 0$) or complementarity ($k > 0$) between x and y . We define a production frontier $y_p = F(x_p)$, $F_1 < 0$, $F_{11} < 0$ indicating the maximum amount of y the country can produce for each specified production of x . The country can also obtain y by exporting x , receiving p units of y for each unit of x exported.²⁷ Thus the consumption of the two goods is:

$$(2) \quad \begin{aligned} x_c &= x_p - x_e \\ y_c &= y_p + px_e \end{aligned}$$

where the subscript c refers to consumption, p to production, and e to export. Substituting (2) in (1) gives:

$$(3) \quad \begin{aligned} U &= a(x_p - x_e) + b(y_p + px_e) + k(x_p - x_e)(y_p + px_e) \\ &\quad + m(x_p - x_e)^2 + n(y_p + px_e)^2 \end{aligned}$$

The country's objective is to maximize (3) (or its expectation) subject to the production frontier.

Certainty

In the absence of uncertainty, maximizing U requires:

$$(4.a) \quad \frac{\partial U}{\partial x_p} = a + bF_1(x_p) + ky_c + kx_eF_1(x_p) + 2mx_c + 2ny_cF_1(x_p) = 0$$

$$(4.b) \quad \frac{\partial U}{\partial x_e} = -a + pb + kpx_c - ky_c - 2mx_c + 2npy_c = 0$$

These equations indicate that x_p and x_e should be chosen so that:

$$(5.a) \quad F_1(x_p) = -\left(\frac{a + ky_c + 2mx_c}{b + kx_e + 2ny_c}\right) = -\frac{\partial U}{\partial x} / \frac{\partial U}{\partial y}$$

$$(5.b) \quad p = \left(\frac{a + ky_c + 2mx_c}{b + kx_e + 2ny_c}\right) = \frac{\partial U}{\partial x} / \frac{\partial U}{\partial y}$$

²⁷ x is always chosen to represent the commodity exported, i.e., $x_e < 0$ is not admissible.

Thus $F_1(x_p) = -\bar{p}$, the usual condition for equilibrium.

The amount traded (from 5.b and 2) will be:

$$(6) \quad x_e = \frac{a + ky_p + 2mx_p - \bar{p}(b + kx_p + 2ny_p)}{2(n\bar{p}^2 - k\bar{p} + m)} = \frac{\frac{\partial U(x_p, y_p)}{\partial x} - \bar{p} \frac{\partial U(x_p, y_p)}{\partial y}}{2(n\bar{p}^2 - k\bar{p} + m)}$$

Uncertainty: Regime I

Case A. Uncertainty in price.

In the presence of uncertainty, we assume the country desires to maximize the expected value of utility, $E(U)$. If uncertainty resides only in the price at which x exchanges for y in world markets, and if the country must decide in advance how much of each commodity to produce and how much x to trade, the expected utility from (3) is:

$$(7) \quad E(U) = a(x_p - x_e) + b(y_p + \bar{p}x_e) + k(x_p - x_e)(y_p + \bar{p}x_e) \\ + m(x_p - x_e)^2 + n(y_p + \bar{p}x_e)^2 + nx_e^2\sigma_p^2$$

where \bar{p} is a random variable with expected value $E(\bar{p}) = \bar{p}$ and variance σ_p^2 . Thus

$$E(y_c) = \bar{y}_c = y_p + \bar{p}x_e \quad \text{and} \quad \sigma_{y_c} = x_e\sigma_p$$

To make the comparison with the certainty case meaningful, we will assume that \bar{p} equals the known price in the certainty case. Maximizing $E(U)$ with respect to x_p and x_e requires:

$$(8.a) \quad \frac{\partial E(U)}{\partial x_p} = a + k\bar{y}_c + 2mx_c + F_1(b + kx_c + 2n\bar{y}_c) = 0$$

$$(8.b) \quad \frac{\partial E(U)}{\partial x_e} = -(a + k\bar{y}_c + 2mx_c) + \bar{p}(b + kx_c + 2n\bar{y}_c) + 2nx_e\sigma_p^2 = 0$$

which gives:

$$(9.a) \quad F_1(x_p) = -\left(\frac{a + k\bar{y}_c + 2mx_c}{b + kx_c + 2n\bar{y}_c}\right) = -\frac{\partial U(x_c, \bar{y}_c)}{\partial x} / \frac{\partial U(x_c, \bar{y}_c)}{\partial y}$$

$$(9.b) \quad \bar{p} = \frac{a + k\bar{y}_c + 2mx_c - 2nx_e\sigma_p^2}{b + kx_c + 2n\bar{y}_c} = \frac{\left[\frac{\partial U(x_c, \bar{y}_c)}{\partial x} - 2nx_e\sigma_p^2\right]}{\frac{\partial U(x_c, \bar{y}_c)}{\partial y}}$$

From (9) it is clear that in this case $\bar{p} > -F_1(x_p)$ so long as $x_e > 0$ (since $2nx_e\sigma_p^2$ is negative). In other words, the presence of price uncertainty means that the country picks a point on the production locus corresponding to less production of x and more of y than in the certainty case.

It can be seen that exports will be less in the uncertainty case than in the certainty case by differentiating the system of equations (8) with

respect to σ_p^2 :

$$(10) \quad \begin{bmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{dx_p}{d\sigma_p^2} \\ \frac{dx_e}{d\sigma_p^2} \end{bmatrix} = \begin{bmatrix} 0 \\ -2nx_e \end{bmatrix}$$

where

$$V_{11} = \frac{\partial^2[E(U)]}{\partial x_p^2}, \quad V_{12} = \frac{\partial^2[E(U)]}{\partial x_p \partial x_e}, \quad V_{22} = \frac{\partial^2[E(U)]}{\partial x_e^2}$$

From the concavity of $E(U)$ we know that the determinant of $[V_{ij}]$ is positive and $V_{11} < 0$. Hence, by applying Cramer's Rule,

$$(11) \quad \begin{aligned} \text{sign } \frac{dx_e}{d\sigma_p^2} &= \text{sign} (-2nx_e) V_{11} \\ &= \text{sign} (nx_e) < 0 \quad \text{for } x_e > 0 \end{aligned}$$

Increases in price variance thus decrease exports; exports are therefore lower in the uncertain case than in the certain case.

Case B. Uncertainty in domestic production and in prices.

We will assume that decisions are made on the basis of expected production of x and y and that actual output of x and y differ from expected output by a multiplicative random variable:

$$\begin{aligned} x_p &= \bar{x}_p \cdot \mu & \bar{\mu} &= 1, & \sigma_\mu^2 &= w^2 \\ y_p &= F(\bar{x}_p) \cdot \varepsilon & \bar{\varepsilon} &= 1, & \sigma_\varepsilon^2 &= \eta^2 \end{aligned}$$

For simplicity we will assume also that μ , ε and p are independently distributed. In that case the expected utility from (3) is:

$$(12) \quad \begin{aligned} E(U) &= a(\bar{x}_p - x_e) + b(\bar{y}_p + \bar{p}x_e) + k(\bar{x}_p - x_e)(\bar{y}_p + \bar{p}x_e) \\ &+ m(\bar{x}_p - x_e)^2 + n(\bar{y}_p + \bar{p}x_e)^2 \\ &+ m\bar{x}_p^2 w^2 + n(\bar{y}_p^2 \eta^2 + x_e^2 \sigma_p^2) \end{aligned}$$

Maximizing $E(U)$ with respect to \bar{x}_p and x_e requires:

$$(13.a) \quad \frac{\partial E(U)}{\partial \bar{x}_p} = \frac{\partial U}{\partial x}(\bar{x}_c, \bar{y}_c) + F_1 \frac{\partial U}{\partial y}(\bar{x}_c, \bar{y}_c) + 2m\bar{x}_p w^2 + 2n\bar{y}_p \eta^2 F_1 = 0$$

$$(13.b) \quad \frac{\partial E(U)}{\partial x_e} = -\frac{\partial U}{\partial x}(\bar{x}_c, \bar{y}_c) + \bar{p} \frac{\partial U}{\partial y}(\bar{x}_c, \bar{y}_c) + 2nx_e \sigma_p^2 = 0$$

If there is no uncertainty in the production of y (i.e., $\eta = 0$) the condition may be written

$$(14) \quad -F_1 = \bar{p} + \frac{2(nx_e \sigma_p^2 + m\bar{x}_p w^2)}{\frac{\partial U(\bar{x}_c, \bar{y}_c)}{\partial y}} < \bar{p} \quad \text{since } m, n < 0$$

In this case then, production of x is unambiguously less in the presence of uncertainty.

If, at the other extreme, there is uncertainty in the production of y but not in x , the condition can be written:

$$(15) \quad \frac{-F_1}{\bar{p}} = \left(\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial x} - 2nx_e\sigma_p^2} \right) \left(\frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial y} + 2n\bar{y}_p\eta^2} \right)$$

The first bracketed expression on the right is less than unity, while the second bracketed expression exceeds unity. In this case the uncertainties in prices and in the production of y work in opposite directions. If uncertainty in the production of y dominates, $-F_1 > \bar{p}$ and more x will be produced and exported than in the certainty case.

In both cases, increasing the variance of prices of export goods decreases the amount of the export goods produced and traded.

Uncertainty: Regime II

An alternative uncertainty regime permits the country to select the amount to be traded *after* the trading price is known, but requires a production decision solely on information about the expected value and variance of price. This case is much more complicated analytically than Regime I, for the amount traded, x_e , becomes a random variable related to the random variable p . As before, we require maximization of $E(U)$. From (3), where both p and x_e are random variables:

$$(12) \quad E(U) = a[x_p - E(x_e)] + b[y_p + E(px_e)] \\ + k[y_p x_p - y_p E(x_e) + x_p E(px_e) - E(px_e^2)] \\ + m[x_p^2 - 2x_p E(x_e) + E(x_e^2)] \\ + n[y_p^2 + 2y_p E(px_e) + E(p^2 x_e^2)]$$

Differentiating this with respect to x_p , setting $\frac{\partial E(U)}{\partial x_p} = 0$ and manipulating:

$$(13) \quad F_1(x_p) = - \left(\frac{a + k[y_p + E(px_e)] + 2m[x_p - E(x_e)]}{A} \right) \\ + \frac{(a + ky_p + 2mx_p) \frac{\partial E(x_e)}{\partial x_p} - (b + kx_p + 2ny_p) \frac{\partial E(px_e)}{\partial x_p}}{A} \\ - \frac{m \frac{\partial E(x_e^2)}{\partial x_p} + k \frac{\partial E(px_e^2)}{\partial x_p} - n \frac{\partial E(p^2 x_e^2)}{\partial x_p}}{A}$$

where

$$A = \left[b + k[x_p - E(x_e)] + 2n[y_p + E(px_e)] \right] = \frac{\partial U(\bar{x}_c, \bar{y}_c)}{\partial y}$$

The first part of the right hand side of equation (13) bears a family resemblance to (5.a) and (9.a). Unfortunately it is not possible to determine the sign of the second two parts unambiguously without having specific information about the distribution of p , and without specifying values for the various parameters in the utility function. Trading decisions, of course, are much simpler in this model. Since the price and amount produced are known at the time the trading decision is made, equation (4.b) applies; the price is equated to the ratio of marginal utilities.

*Model II*²⁸

The difficulties which attend regime II can be avoided if we can write the utility function of the community or country in terms of a single variable instead of two.

In the text we have argued that developing countries may have particular interest in focusing on the implications of investment and production decisions for net foreign exchange earnings. Alternatively, a country might be concerned with the level of (and variance in) money income. In order to preserve the notion of risk aversion we assume that the country's preferences with respect to relevant variables are represented by a utility function which takes the form:

$$(14) \quad U = \pi + \frac{1}{2}n(\pi - \bar{\pi})^2$$

where π represents actual net earnings, $\bar{\pi}$ represents expected net earnings, and $n < 0$ is a measure of risk aversion, as before.

Equation (14) implies a gain in utility for increases in earnings but a reduction in utility for "surprises"—positive or negative deviations in actual earnings from expected earnings. This form of a quadratic utility function has the advantage that it can provide for risk aversion without any assumption concerning "diminishing marginal utility of income." It is "surprises" which reduce the utility of gambling on risky ventures, not asymmetries between the utility of gains and losses.

Net earnings are made up of gross earnings less costs. A "small" country can consider world prices beyond its influence, so

$$(15) \quad \pi = q'p - C(q)$$

where p is a vector of world prices, q is a vector of the country's gross production of various commodities, and where $C(q)$ represents total costs, a function of production levels q .

In order to simplify the analysis, and to make explicit the fact that the production and sale of some commodities involves the purchase of others, we assume $C(q) = q'Ap + C_0(q)$ where $A = [a_{ij}]$ represents the physical inputs of commodity j required for a unit output of commodity i , $a_{ii} = 0$, and where $C_0(q) = \sum_i c_{0i}(q)$ represents all costs independent of variations in p , e.g. wages plus fixed costs. With this assumption, (15)

²⁸ The model that follows is based on the pioneering work of Markowitz and Tobin. See [7] and [13], as well as [4, pp. 239-257], where an analytically similar problem is treated.

becomes:

$$(16) \quad \begin{aligned} \pi &= q'p - q'Ap - C_0(q) \\ &= q'(I - A)p - C_0(q) \\ &= q'Bp - C_0(q) \end{aligned}$$

where $B = I - A$. Now $q'B$ in (16) is the vector of net sales (or purchases) of commodities which correspond to a decision to produce q . That is, a decision to produce q_i can be regarded as a decision to operate a "process" which requires for net output of one unit of q_i inputs of a_{ik} units of each of the other commodities. The "prices" of the "processes" q are Bp .

Uncertainty in net earnings may arise from a number of sources. First let us consider the case where the only uncertainty resides in the prices at which goods can be traded. We will return later to a more general formulation. In that case:

$$(17) \quad \text{Var } \pi = q'B\Omega B'q$$

where Ω is the covariance matrix of prices p , and $B\Omega B'$ may be interpreted as the covariance matrix of the "prices" for the "processes" corresponding to production decisions on the q_i .

Returning to (14), we now wish to choose the outputs q so as to maximize expected utility:

$$(18) \quad E(U) = E(\pi) + \frac{1}{2}nE(\pi - \bar{\pi})^2 = \bar{\pi} + \frac{1}{2}n \text{Var } \pi$$

where $\bar{\pi} = q'B\bar{p} - C_0(q)$ and $\text{Var } \pi$ is given by (17). Maximization of (18) requires:

$$(19) \quad \frac{\partial \bar{\pi}}{\partial q_i} + n(B\Omega B')_{iq} \leq 0; \begin{cases} 0 & \text{for } q_i > 0 \\ < 0 & \text{for } q_i = 0 \end{cases}$$

where $(B\Omega B')_i$ is the i^{th} row of $B\Omega B'$. Note that

$$\frac{\partial \bar{\pi}}{\partial q_i} = \sum_k b_{ik}p_k - \frac{\partial C_0}{\partial q_i}$$

where $\sum_k b_{ik}p_k$ can be regarded as the price of the i^{th} process, $b_{ii} = 1$, $b_{ik} = -a_{ik}$. Thus condition (19) reduces to the usual profit maximizing condition that the price of a process equal its marginal cost when $\Omega = 0$.

The discussion of Section III of the text can be interpreted as showing how to represent the best combinations of risk and return, where risk is measured by the square root of (17) and returns are measured by Bp per unit of domestic cost if all the components of C_0 represent purely domestic costs.

If there are competitive markets and an equilibrium exchange rate, the decisions of private entrepreneurs can be regarded as maximizing (18), where n is set equal to n_p (indicating the degree of private risk aversion). In that case taxes can be used to influence private decisions by affecting costs.

$$(20) \quad C'_0(q) = C_0(q) + tq$$

where t is a vector of flat rate taxes per unit of output. Taking taxes into account, profit maximizing private entrepreneurs will try to maximize $\bar{\pi} - tq + \frac{1}{2}n_p \text{Var } \pi$, which will lead to the conditions

$$(21) \quad \frac{\partial \bar{\pi}}{\partial q_i} - t_i + n_p(B\Omega B')_i q \leq 0; \quad \begin{cases} 0 & \text{for } q_i > 0 \\ < 0 & \text{for } q_i = 0 \end{cases}$$

We want to choose taxes (and subsidies) to guide private decisions toward the socially optimal composition of output. Choosing n in (19) to indicate social risk aversion (n_s) and subtracting (21) leads to

$$(22) \quad t = -(n_s - n_p)B\Omega B'q^*$$

where q^* is the solution of (19). For $B = I$ this gives the expression in the text.

Uncertainty in net earnings can arise from a number of sources other than uncertainty in world prices. q itself may be a random variable. In that case, production decisions may be regarded as specifications of the expected output of the various commodities. It is also possible to regard $C_0(\bar{q}) = \sum c_{0i}(\bar{q})$ as a random variable reflecting fluctuations in costs not directly related to fluctuations in prices or randomness in q .

Thus we can specify the following relationships, in vector notation:

$$(23) \quad \begin{array}{llll} p = \bar{p} + u & E(u) = 0 & \text{Cov}(u) = \Omega & \text{Cov}(u, \varepsilon) = 0 \\ q = \bar{q} + \varepsilon & E(\varepsilon) = 0 & \text{Cov}(\varepsilon) = \Theta & \text{Cov}(u, \eta) = 0 \\ c_0 = \bar{c}_0 + \eta & E(\eta) = 0 & \text{Cov}(\eta) = \Phi & \text{Cov}(\eta, \varepsilon) = Z \end{array}$$

With these assumptions we can rewrite (17)²⁹:

$$(24) \quad \text{Var } \pi = \bar{q}'B\Omega B'\bar{q} + \bar{p}'B'\Theta B\bar{p} + \delta'\Phi\delta - 2\bar{p}'BZ\delta$$

where δ is a vector of ones.

The first term on the right gives the contribution to the variance in returns made by the variance of prices ($B\Omega B'$ is the covariance matrix for the "prices" of the "processes" q). The second term is the contribution of the variance of output. The third term is the contribution of variance in the C_0 's and the last term the contribution due to the covariance of output and the c_0 's.

²⁹ This expression is an approximation which ignores higher order terms in the computation of the variance of the product-term.