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THE QUIDDITY OF THE COBWEB THEOREM[†]

In a recent article on the cobweb theorem, Waugh discussed its central position among dynamic systems, especially those of the recursive type, and sought to generalize the basic model in a number of ways (22). He cited Moore's demonstration that current production of cotton depended on the price in the previous year as an instance of implicit use of the cobweb model. Evidently, dependence of production on past price, a form of recursiveness, was taken by Waugh to be the distinguishing characteristic of the cobweb model. But can recursiveness appropriately be identified with this model, or might there be other recursive models which are essentially unrelated to it except through sharing that attribute? If there are, it would clearly be incorrect to subsume them under the classification of cobweb model.

Many instances can be cited in which a single feature of the cobweb model, for example its assumption that expected price equals current price, was adduced as evidence of the operation of the complete cobweb mechanism in a price or production cycle. The model seems to be so intriguing, and so persuasive, that it is uncritically accepted on meager grounds. Many economists have seemed to assume that the model has been empirically demonstrated in agriculture, where its archetype is the hog cycle.¹ At the same time, economists working with agricultural price and production data have tended to assume that the cobweb theorem has been developed theoretically in a completely satisfactory way and that its range of application is known. The cobweb enjoys a position similar to that of the normal distribution of errors, of which it has been said that the mathematicians believe in it because they think it is an empirical law, and the scientists believe in it because they think it is a mathematical theorem. We shall argue that neither the theoretical nor the empirical case for the cobweb is as clear-cut as supposed. Not all recursive models can be said to be related in any fundamental way to the cobweb theorem. In particular, there is a basically different model, which I have termed harmonic motion, that provides a more likely explanation of the hog cycle and many other agricultural production cycles. The two models share a number of attributes, and indeed have been widely confused.

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¹ There is a fascinating parallel between this situation and that in E. B. White's *Charlotte's Web*, where Charlotte, the spider, wrote trenchant comments on her web in the pigpen doorway, which were mistakenly attributed to Wilbur, the pig, who did no more than to inspire them.

Despite the enthusiastic acceptance of the cobweb by most economists, there has been since the model was first introduced a progressively sharpened line of criticism directed toward the nature of the supply curve, or the planned output curve, in the model. Since it lends theoretical support to the alternative recursive model, we will consider this criticism in some detail.

CRITICISM OF THE TRADITIONAL COBWEB THEOREM

We will assume that the traditional cobweb theorem is familiar, and will not attempt a formal restatement. Instead we refer the reader to Waugh's paper or to Ezekiel's original article (8).² Many subordinate sections of Ezekiel's paper, including an extended discussion of the limitations of the model, deserve careful study. We will extract only that which is needed for present purposes. The theorem gets its name from the set of horizontal and vertical lines that are festooned on a set of supply and demand curves. Producers plan production in one period on the basis of the then current price, but the resulting production level may lead to a different price, and hence altered plans in the subsequent period. The alternations of price and production may increase, decrease, or remain constant, depending on the relative slopes of the demand and supply curves. Most criticism of the model has centered around the nature of the supply curve, which will also be our chief concern.

Ezekiel defines his supply curve as representing "the number of units of the commodity which holders (or producers) of the product stand willing to sell within the period at varying prices" (8, p. 257). He chooses to adjust both the demand and supply curves for carryover stocks. Since many products can be stored, treating supply and demand in this way adds to the realism of the model. One might suppose that storage operations would tend to lessen cobweb fluctuations, but they introduce no essential theoretical difference, since storage operators could have expectations similar to those of producers. With this inconsequential adjustment, then, Ezekiel's curve is equivalent to Marshall's short-run supply curve.

But as Buchanan has pointed out (3), the supply curve in the cobweb theorem is completely reversible, which is more to be expected of a long-run than of a short-run supply curve. Yet by its very nature, the supply curve in the cobweb must be short-run. A corollary pointed out by Buchanan is that since expectations will not be realized and since plans envision normal profits, there will be alternately aggregate profits and losses. The losses must exceed excess profits over time and the cumulated losses must eventually affect response, except in the convergent case. We should therefore expect all cases to be more convergent than the crude theory shows. Buchanan lists a number of other reasons for doubting the realism of the cobweb model, many of which are concerned with the apparently excessive rigidity and mechanical nature of the model.

An aspect of short-run supply related to irreversibility is its dependence on past levels of output. This was not considered directly by Buchanan, and indeed

² A method of achieving constant amplitude cycles by introducing a nonlinear supply curve due to Leontieff, is discussed by Samuelson (18). The "traditional" cobweb is not necessarily coextensive with Ezekiel's model in every respect, but the two are virtually identical.

it does not seem to be widely recognized in other economic writings. J. M. Cassels, also emphasizing the irreversibility of short-run supply curves, and their time dependence or length of run, stated the proposition clearly (4):

It seems to me, therefore, that each supply curve must be regarded as relating to an established level of output and should be recognized to have two distinct parts, one representing expansion beyond that output and the other representing contraction below it.

Probably Marshall's treatment of supply curves has contributed to the neglect of this point (16). Marshall seems always to regard the supply schedule as the number of units of the good which will be supplied within the period at varying prices, assuming that producers have been producing an equilibrium quantity at equilibrium prices for a long enough time to be fully adjusted to these conditions. This assumption is not made explicit, since Marshall was primarily concerned to explain the determination of long-run normal price.

This aspect of short-run supply curves is very germane to the cobweb model, as Gustave Åkerman has clearly shown (1). He prefers to regard market supply as not perfectly inelastic, but he retains the concept of expected price as equal to price in the previous period. Åkerman also assumes that the disturbance that initiates the fluctuation is a single permanent shift (increase) in demand. The most important original feature of his model is that the short-run supply curve shifts to new positions dictated by achieved levels of output. His cobwebs appear to be spun by a skew or shear spider. The essential ideas, for our purposes, lie in the clear distinctions made between the market supply curve, the short-run supply curve, and the long-run supply curve, and in the shifts in the short-run supply curves induced by expanding levels of previous production. Åkerman concludes that shifts in the short-run supply curve, the most relevant of the three curves, will almost certainly lead to convergence more quickly than the traditional (and fundamentally erroneous) cobweb theorem would suggest, for instead of reverting to the original short-run supply curve, producers stop short at a point which is much closer to the ultimate equilibrium point.

Åkerman's example has some nonessential features, and by no means exhausts the possibilities. The initiating disturbance might be a temporary change in demand or supply which returns to normal after only one period. The original equilibrium position would be reinstated, and shifts in the short-run supply curve may be less pronounced than in Åkerman's example, in which there is a permanent shift to a new long-run equilibrium. Also, the slopes of the demand and supply curves may be such that without the shifts in the short-run supply curve Ezekiel's divergent model, Case 2, would obtain, so one would conceivably still have fluctuations of constant or increasing amplitude. Another way to reinstate the constant amplitude and divergent cobwebs is to permit extrapolative price expectations, in which case expected prices are farther from equilibrium than are realized prices, and the extrapolations compensate for the shifts in the supply curves. Those commodities with elastic short-run supply curves and for which relatively permanent supply changes are readily made might well be those in which extrapolative price expectations occur. The actual importance of this possible factor is certainly open to question, however.

Åkerman showed that shifts in the short-run supply curve should lead to more rapid convergence. He felt that with the traditional cobweb assumption of the equality of expected price and previous price (or rather, current price during the planning period) divergence would be unlikely. We have shown that depending on other features of the model all forms of behavior postulated for the cobweb model are still possible. But Åkerman's shifting supply curves remain of great interest and importance to the theory, since they show the dependence of supply response on the dynamic situation. We have already reached a point where static supply parameters, such as slope or elasticity, are no longer wholly instrumental in determining the dynamic supply response, and indeed the static parameters may be unmeasurable and of little interest.

In its crudest form the cobweb was clearly set off from other dynamic models. But the theoretical criticisms of Buchanan and Åkerman have shown that the traditional model is illogical or at least inconsistent with more fundamental theory. The modifications forced by the criticism involve greater dynamism, i.e., greater attention must be paid to the emerging situation within which the model operates. We need to redefine the essential nature of the cobweb in the light of these modifications and extensions.

For this purpose, we propose the following set of criteria for the cobweb: (a) demand is a function of current price; (b) supply is a function of achieved level of output in the previous period (depending on the nature of the adjustments required to achieve this level of production) and of expected prices that depend heavily on current price; and (c) decisions implied by the demand and supply functions are relatively inflexible and irrevocable, and are resolved by market-clearing adjustments of price.

This set of criteria is a good deal less explicit than the corresponding set for the traditional theorem, and indeed it must be considered in the light of the full discussion above to be understood unambiguously. But it does convey the central idea of the cobweb, which has been scarcely affected by the modifications: demand decisions and supply decisions are made in two distinct periods separated by the length of time required to effect production. Furthermore, the demand and supply decisions are more or less adequately represented as having reference to a demand curve and a supply curve.

One test of a classification is its power to exclude. Any model involving administered prices cannot be subsumed in the cobweb category. The model of adaptive expectations is excluded by criterion (b), but extrapolative or reactive expectations are included, provided the extrapolation is not very large in absolute value. The harmonic motion model, which we will discuss next, is excluded. A wide range of model structures can still be accommodated, and this is probably all to the good, in view of the basic position the cobweb has assumed in economic thought.

The cobweb model is not uniquely specified by a single attribute. Other models, including Metzler's inventory cycle, depend on the fixed production lag. (See for example 17.) Two of the earliest writers, Cohen and Barker (6), appeared to equate the cobweb theorem with the expectation model, expected price equals current price, but this attribute is shared by the model that produces harmonic motion.

HARMONIC MOTION

We have stated that one essential feature of the cobweb model is that decisions about levels of output are made on the basis of expected prices with reference to a supply curve, with the important reservation that the supply curve can be permitted to shift as relatively durable inputs are injected into or withdrawn from the production process in adjusting production. For the cobweb to be a good representation of reality, movements along the supply curve in response to varying prices must be relatively more important than movements occasioned by shifts in the supply curve.

Numerous studies of agricultural price and production cycles indicate, however, that producers do not in fact decide to produce a given level of output in response to an expected price, but rather decide to change the current rate of production in response to current prices, or current levels of profit. These studies will be cited in the following section. They repeatedly show that when net profits exceed equilibrium, or zero, producers expand output, and when they are below equilibrium producers contract output. With the further assumption that the strength of the response is proportional to deviation from equilibrium this model produces harmonic motion. When this response can be adjusted more or less continuously to varying price throughout the production period, the resulting fluctuations are quite different from those generated by the cobweb model, even though this response pattern has been loosely identified with the cobweb by many writers. We shall not insist on a complete absence of expectations in forming the harmonic motion model, but rather assert that reference to a supply curve might well be supplanted by a decision rule which is highly conservative, in the sense of involving little explicit prediction of future events. Or, adopting a position more sympathetic to producers' powers of prediction, we could alternatively assert that producers are constrained from altering production more than by a given amount or percentage and need current and expected profits to overcome the obstacles to higher or lower rates of production.

The harmonic motion model can be set forth as a set of equations:

Equation (1) states that quantity demanded is a linear function of current price. Equation (2) states that the time rate of change of planned production is a linear function of price (as departure from trend). Equation (3) states that there is a fixed production lag of length v, and equation (4) is the market clearing equation. Note that all relationships except that in equation (2) are also true of the cobweb theorem, and even this equation embodies the same expectation model as does the cobweb. Producers plan changes in production on the basis of *current* profitability, evidently assuming that current prices will continue. Under certain assumptions about the values of some of the parameters, the set of equations can be solved to obtain autoregressive differential equations in p and q, whose solutions are:

(5)
$$P_t = A\cos(\frac{2\pi t}{4\nu} + d_p)$$

and

(6)
$$q_t = A\cos(\frac{2\pi t}{4\nu} + d_q)$$

where the period is four times the production lag, and d_p and d_q are phase angles that differ by $\frac{\pi}{2}$ radians. Equation (6) refers to *planned* production.

The cobweb theorem yields cycles of constant amplitude if the slope of the demand curve is the negative reciprocal of the slope of the supply curve. Similarly, the cosine curves are solutions of the set of equations specifying the present model only if bc = 1. The slope of the demand curve can be set equal to one by a suitable choice of units, but the supply response parameter, c, will not obviously be made equal to one thereby. But if it is assumed that c is greater than one at equilibrium, and decline as the cycle increases in amplitude, a critical amplitude will be reached, at which c will equal one, and the cycle will tend to exhibit this amplitude.⁸ The argument is similar to one advanced by Goodwin to explain the persistence of cycles (10). In order that the demand relation be realized, price must be π radians out of phase with realized production. This is achieved in the model by a phase shift of $\frac{\pi}{2}$ radians caused by the assumption that rate of change of q depends on p, coupled with the fact that

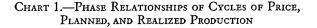
$$\frac{d}{dt}\cos\theta = \sin\theta = \cos(\theta + \frac{\pi}{2}),$$

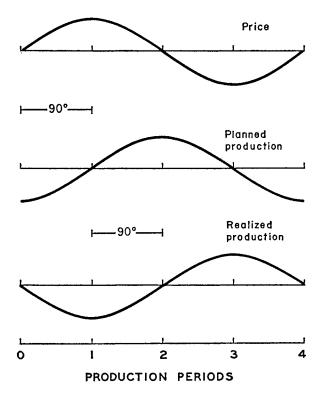
and an additional phase shift of $\frac{\pi}{2}$ radians caused by the production lag. Hence, if $v = \frac{\pi}{2}$, the period must equal 4v. The relationships between price, planned

production, and realized production are shown in Chart 1.

Many economists have assumed a response pattern like the one used here but only Lorie noted that this led to a cycle four times the production lag in length (15, pp. 46 ff.). There have been a number of studies of the hog and cattle cycles since the publication of Lorie's work, most of which ignore his finding even though some of them quote the study in which his results appear. Engineers might have been expected to recognize the hog cycle mechanism as harmonic motion induced by "feedback," but economists apparently were predisposed to recognize it as a form of cobweb because they were unwilling or unable to dispense with the basic concept of a supply curve, which lies at the heart of the cobweb theorem. Thus the economists studying in this area have tended to subsume too large a class of phenomena in the cobweb classification while adhering

³ The hog cycle has exhibited a number of cycles of nearly constant amplitude in the United States in the postwar period. (See 14.)





to a rigid concept of the model *per se*. The result has been that the theoretical construct of the cobweb theorem probably has done as much to mislead empirical research workers as it has done to direct them to a true understanding of the phenomena under study.

COBWEBS AND CONFUSION IN AGRICULTURAL PRODUCTION CYCLE ANALYSES

In the cobweb model, especially without the Åkerman modification, the quantity supplied is a function of price during the period when production plans were formulated. In the harmonic motion model, *change* in planned level of quantity supplied is a function of price in that same period. Failure to distinguish between the two models has led to a great deal of error and confusion.

One area in which the cobweb theorem has been employed repeatedly is the hog cycle. Indeed one of the first papers on the cobweb dealt with the hog cycle in Germany (12). Many further references to the German literature are given in a recent article by Dr. G. Schmitt (19). Ezekiel also used the hog cycle, as manifested in the hog-corn ratio and hog production series from 1901 to 1935, and seemed to regard it as approximately a cobweb, though he was very mindful of the limitations of the model (8, p. 271). Coase and Fowler, on the other hand,

completely rejected the cobweb theorem as the explanation of the pig cycle in Great Britain (5). The chief difficulty in accepting the cobweb as the explanation of the hog cycle has been that the hog cycle is usually about four years long (slightly less in some countries), whereas, in view of the 12-month production period for market-weight hogs, the cycle should according to the cobweb theorem be two years long. Harlow has gone so far as to postulate a further lag of one year between the formation of breeding intentions, based on then prevailing prices, and their execution (13). He also considers Ezekiel's extension of the cobweb to include a two-year supply response. Although he referred to Lorie's study, Harlow did not take note of the alternative explanation of the four-year cycle given there.

A number of studies have shown that change in quantity produced is the actual variable associated with price (or the hog-corn ratio in the case of hogs), and not absolute level of production. Thus, Wells showed a positive correlation between changes in annual receipts at Chicago and the Chicago hog-corn price ratio one and two years previously (23, p. 19). In Canada, Schrader used percentage change in hog production as the dependent variable and hog and barley prices as independent variables in a regression analysis that yielded good results (20, p. 10). Very recently, French and Bressler showed a very close relationship of percentage change in lemon plantings as a linear function of five-year average net returns, and cited this as evidence of the cobweb theorem-with various elaborations and modifications (9, p. 1025). By contrast Suits and Koizumi show that "the systematic annual oscillations of prices and outputs shows the onion market to be almost a textbook example of an agricultural cobweb system" with constant supply elasticity, by computing the regression of quantity produced in period t on price in period t-1 (21). They estimated the regression coefficient by taking first differences of both variables, and used a true cobweb model.

Brandow has provided a clear formulation of a model quite similar to the harmonic motion model of the hog cycle (2, p. 3):*

The changes in farrowing caused by variation in prices are changes around the level of the preceding year rather than around the long-term average. A "normal" hog-corn price ratio induces farmers to hold hog production about where it was the year before even though that may have been above or below "normal."

Brandow does not call the model he uses a cobweb, nor does he discuss the relationship of his model to it. He appears to feel that fall farrowing decisions are dependent on spring decisions, whereas we prefer to regard the decision process as continuously dependent upon current price, in the harmonic motion model. The scope for action is limited in months of low production, however. Harlow followed Brandow in seeking to make hog production decisions discrete, and further emphasized autocorrelation of decisions in order to explain the four-year cycle (13, p. 842). Both writers acknowledge some scope for adjustment throughout the year, and it might be noted that hog production is becoming increasingly uniform.

⁴ According to Brandow, the hog-corn ratio during October-December is the most important variable determining total production throughout the year.

The continuity of decision processes is less evident in the case of crops. A number of "annual" crops actually are produced in a number of regions and are subject to important production adjustments several times during a year. These crops include onions and potatoes, which have been supposed to exhibit cobweb fluctuations (21; 11, pp. 178 ff.). The main growing region, wherein only one crop can be grown, tends to dominate, however, which probably makes the discrete (cobweb) model appropriate. It would be possible to develop a discrete model similar to harmonic motion by substituting a difference equation for the differential equation in the model, relating change in production to departure of price from equilibrium. The resulting model would be difficult or impossible to distinguish from the cobweb; note for example that the period of the cycle would be twice the production lag, as in the cobweb theorem. Conceptually, however, it would still be a distinct model.

A sufficiently close approach to continuity in decision making probably occurs in tree crops. At least four or five years are required for lemon trees to reach bearing age; for other perennial tree crops, the nonbearing period may be even longer. Therefore even if it should be true that trees are usually set out at a particular season of the year, the decision making process for three crops is best viewed as continuous and not discrete. Hence, the harmonic motion model, leading to a cycle with a period four times the gestation period in length, say 20 or so years, is more to be expected than the cobweb period of 10 years. The findings of French and Bressler are consistent with this hypothesis, and indeed provide one of the most dramatic verifications of the model (9, pp. 1025, 1030).

Confusion of the harmonic motion model with the cobweb theorem can lead to errors of interpretation that extend beyond the identification of the cycle mechanism *per se*. For example, Dean and Heady infer a change in the elasticity of supply of pork from an increase in the amplitude of the hog cycle (7, pp. 845-60). I am unable to substantiate the increase in amplitude, using monthly data, but even presuming this the inference may not be warranted. Even if the harmonic motion model is left aside, the Åkerman modification of the cobweb theorem precludes imputing values to the slope or elasticity of the supply curve in the Marshallian sense. If the harmonic motion model is accepted, the imputation of supply elasticity is utterly meaningless.

SUMMARY AND CONCLUSIONS

The cobweb theorem in its original formulation was based on a static model of supply and demand which was placed in a dynamic setting. Buchanan and Åkerman showed that dynamic factors necessarily alter the static model in fundamental ways. The traditional cobweb theorem must give way to a modified version that incorporates more of the dynamic elements such as fixity of inputs, achieved levels of output (or inputs), and realized profits or losses. Provided that the decisions of producers depend on expected price referred to a supply curve, the cobweb theorem would appear to survive as a basic dynamic model. But if producers adjust production on the basis of other decision rules, such as that of the model of harmonic motion, for example, the cobweb theorem does not apply.

Economists have tended to identify the cobweb theorem with any model in

which there is a fixed production lag and in which production decisions are predicated on the assumed continuation of present prices in the planning period. We have shown that these attributes are shared by the harmonic motion model which is nonetheless fundamentally different, in that the decision is to change the level of production, rather than to produce an amount determined with reference to a short-run supply curve. The most obvious consequence of the difference in model specification is that it produces a cycle four times the production lag in length, rather than twice the lag, but the fundamental distinction lies in the nature of the assumed response of planned production to expected price.

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