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INPUT-OUTPUT METHODS FOR LABOR MARKET ANALYSIS AND PROJECTION

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Abstract

This report presents a review of input-output methods for labor market analysis in Minnesota. For the computational examples, 1972 U.S. and Minnesota input-output tables were used, including employment and income statistics from U.S. Department of Commerce and U.S. Bureau of Department of Labor. This is the first in a series of reports on Minnesota industry structure and performance in the past decade and its outlook for the 1980's and 1990's in job productivity, skill requirements, and income generation.

Summary

Basic procedures of input-output analysis are presented with the use of data from 1972 U.S. and Minnesota input-output tables. Existence of detailed industry statistics on sales, purchases, value added, and employment for both the U.S. and the State of Minnesota has made possible extension of input-output methods to the analysis of Minnesota industry, structure, performance, and prospects which heretofore would not have been feasible because of the lack of detailed Minnesota and corresponding U.S. industry statistics.

Four principal topics are presented, starting with the input-output concept, its origins and acceptance, and its basic assumptions. This introduction is followed by a delineation of steps in building a computable input-output model for labor market analysis. Reasons for highly detailed industry and highly aggregated area data are discussed, along with implications of using less detailed industry groupings and less aggregated area groupings. The theory and practice of input-output analysis in collecting and preparing industry data and calibrating, documenting and validating the interindustry transactions tables is examined, also.

The model-building discussion is followed by an examination of its use, especially the interpretation of the input-output coefficients which are derived from the interindustry transactions tables. Output multipliers, both demand-type and supply-type, are derived, with illustrations of their use in labor market analysis.

Finally, U.S. statistical series on employment and income are related to the input-output data. Steps in deriving various input-output coefficients are illustrated in the Appendix.

The input-output method starts with the product and income accounts which depict the total income originating from remuneratively productive activities, i.e., value added, as equal to the domestic final product, plus net exports. This identity is expressed by the form,

DFP + (EXP - IMP) = VA

which, for the U.S., is expressed quantitatively, in billion dollars, by the equality,

1,186.2 + (72.8 - 76.2) = 1,182.8.

The Minnesota final product differs slightly compared to the U.S. final product in its distribution among the principal product categories, as shown below:

Final Product Category	Dom. Purchases U.S. (bil.$)	Dom. Purchases Minn. (mil.$)	Total Purchases U.S. (bil.$)	Total Purchases Minn. (mil.$)
Pers. Cons. Exp.	729.7	10,945	738.1	12,995
State and Local Gov.	68.1	1,179	150.7	2,863
Federal Gov.	49.5	371	102.1	1,105
Gr. Priv. Cap. Form.	184.9	2,836	184.9	3,475
Change in Bus. Inv.	17.9	386	10.4	343
Total Domestic Product	1,050.1	15,617	1,186.2	20,780

Personal consumption expenditures were 62.5 percent of the total in Minnesota and 62.2 percent of the total in the U.S. Both state and local government purchases and business capital outlays also were larger in Minnesota than the U.S. -- 18.4 percent vs. 18.4 percent, and 13.8 percent vs. 12.7 percent, respectively. Only federal government purchases were smaller in Minnesota than the U.S. -- 5.3 percent vs. 8.6 percent.

Differences in external trade also occurred in 1972 between Minnesota and the U.S., as shown below:

Trade Category	U.S. (bil.$)	Minnesota (mil.$)
Competitive exports from U.S.	57.9	652
Competitive imports to U.S.	-56.8	-411
Minn. net exports to RON		7,183
Minn. net imports from RON and U.S. noncomp. imp.		
Intermediate inputs	-5.1	-4,279
Final inputs	-10.1	-3,281
Total	-14.1	-36

A negative balance of trade was estimated for both Minnesota and the U.S.

Minnesota gross state product is readily estimated from the preceding data, as follows (in million dollars):

20,780 + (7,835 - 7,871) = 20,744

Interindustry and intersectoral transactions of the Minnesota and U.S. economies are summarized in a 10-industry breakdown of the producing sectors of the two economies. High levels of imports for some industries in the Minnesota economy of course reduce the internal interdependence, and, thus, the input/output multiplier values, which are derived from the Leontief inverses, are reduced, also.

Employment and income data also are summarized for the 10 industry representation of the U.S. economy. They are presented here for the 80-industry breakdown reported in the Survey of Current Business. These data show vastly differing conpensation levels and hours worker per week in the U.S. economy. Detailed industry statistics for states and regions are essential, therefore, to avoid compounding changes in industry composition with changes in industry productivity and earnings, especially where these changes depart from national patterns.

INPUT-OUTPUT METHODS FOR LABOR MARKET

ANALYSIS AND PROJECTION

Wilbur R. Maki

New approaches to labor market analysis and projection have been formulated, tested and proposed by labor market analysts in the U.S. Bureau of Labor Statistics and the Minnesota Department of Economic Security.[1/] This paper for the Seminar on Input-Output Analysis complements these new approaches by extending conventional input-output methods to the study of labor market structure, growth and change.

The purpose of this extension of input-output methods to labor market analysis and projection is two-fold: it serves as a demonstration of the strengths and weakness of input-output methods in small area economic analysis and projection and it serves as a test of each of the several approaches in providing reliable and useful information on future state and substate employment prospects. This purpose is pursued under five topical headings, starting with the problem focus of labor market analysis and projection and followed by individual steps in model building, data interpretation, and economic impact analysis and forecasting. This discussion concludes with examples of case studies on the use of input-output methods in labor market analysis and projection. First, however, the input-output concept, its origins and acceptance, and its basic assumptions are discussed.

1/ See, particularly, the discussion of state and substate employment projections and projection methods in the recent update of the 1985 industry and occupational employment projections for the State of Minnesota and the Minneapolis-St. Paul SMSA (31, p. 94).

Input-Output Concept and Its Origins

Professor Wassily Leontief of Harvard University, winner of the Nobel Prize in Economics for his work in input-output analysis, is usually thought of as the founder of input-output economics. Input-output economics is a branch of economics, and also of econometrics. It emphasizes the structure of an economic system and the measurement of this structure for purposes of macro-economic analysis, particularly the effects of changes in the final demands for goods and services on a particular industry with references to its sales and purchases.

Leontief published the first input-output table of the American economy in 1936 (14). John Maynard Keynes had already rekindled interest in aggregative economics. With the Great Depression as an appropriate setting for the ensuing discussion of Keynes' General Theory, the second revolution in economic thought launched by Leontief was initially a quiet one. Significant work in this new area did not occur until the 1940's when Leontief, continuing with his own efforts in input-output analysis, was joined by his colleagues and others in demonstrating new applications of the input-output approach, especially in the study of aggregate economic impacts (3 ,4 ,15, 16,17,18). Much of the work was supported by the U.S. Bureau of Labor Statistics. In 1944, the first practical application of the input-output approach was demonstrated in estimating the effects of shifting from war to peace on employment (36).

Within the next two decades, national, and even regional, input-output models had become commonplace. Phil Borque, in his survey of state and regional input-output models published in 1970, all but 38 states were included among those listed as having work completed or in process (2). Minnesota was included in this list twice -- once for the 1966 Itasca County input-output model completed by Jay Hughes and a second time for the 1963

Minnesota input-output model completed by Henry Hwang and Wilbur Maki as part of a Souris-Red-Rainy River Basin Planning Commission study (5,11). Today, more than half of the substate development regions in Minnesota, and even three counties -- Itasca, Mower and Pennington -- are represented by their own input-output tables.

The core of the Leontief input-output system is the input-output table in which individual industry purchases are represented by columns and individual industry sales are represented by rows, as in Table 1.1. For this example, an 85-industry 1972 U.S. input-output table was collapsed into three industry sectors, three primary input-output sectors, three final demand sectors, and a rest-of-world sector (to account for exports from,and imports to, the U.S.).

Summary data from the 1972 U.S. Input-Output Table are used to illustrate the derivation of input-output tables with reference to the underlying assumptions for these procedures. In Table 1.1, three producing sectors are listed -- a primary sector of agriculture and mining, a secondary sector of construction and manufacturing, and a composite tertiary sector of all non-commodity, services-producing industries. In this illustration the three industry groups produced a gross output of $1,966 billion. Interindustry transactions were $1,046 billion, or slightly more than 50 percent of industry gross output. By definition, gross output is equal to gross outlay for each industry.

The complete input-output table can be quartered, with the intermediate purchases in Quadrant I, the final purchases and exports in Quadrant II, the primary inputs and imports in Quadrant III, and the interinstitutional transactions in Quadrant IV. The export and import sectors balance the external trade and payments accounts of the economy as represented by the tables. Thus, the individual entries in Table 1.1 are represented algebraically by the form,

Table 1.1. Illustrative Input-Output Table: Intermediate and Final Purchases of Specified Industry Output and Primary Inputs by Industry and Non-Industry Sectors.

Sector	Intermediate Purchases: Goods: Agr. & Mining	Intermediate Purchases: Goods: Constr. & Mfg.	Intermediate Purchases: Services	Intermediate Purchases: Total	Final Domestic Purchases: Personal Consump. Expend.	Final Domestic Purchases: Govern-ment	Final Domestic Purchases: Business Invest-ment	Final Domestic Purchases: Total	Rest-of-World: Comp. Exports	Rest-of-World: Comp. Imports	Gross Output
					(billion dollars)						
Producing Industry:											
Goods											
Agr., Mining	25	67	9	101	7	-1	3	9	6	-6	110
Const., Mfg.	18	359	94	463	217	80	180	477	38	-46	932
Services	18	141	192	351	506	39	20	564	14	-5	924
Total	61	559	295	915	730	118	203	1,050	58	-57	1,966
Primary Inputs:											
Emp. Comp.	11	264	305	508	5	132	0	137	0	0	718
Ind. Bus. Tax	3	18	90	111	0	0	0	0	0	0	111
Prop.-Type Inc.	35	89	231	355	0	0	-8	-8	0	0	354
Total	49	371	626	1,046	5	132	-8	130	0	0	1,183
Rest-of-World:											
Noncomp. Imp.	0	2	3	5	7	4	0	10	1	-16	0
Dummy Ind.	0	0	0	0	-4	0	0	-4	14	-4	3
Gross Outlay	110	932	924	1,966	738	253	195	1,186	73	-76	3,148

$$\sum_j X_{ij} = \sum_i X_{ij}, \qquad \text{Eq. (1.1)}$$

for each row and its corresponding column.

While the row total equals the column total for the producing industries, the primary input rows and final purchases columns are not necessarily equal. Equality is achieved by including exports and imports in the balancing equations. For these three rows and columns, the aggregate value of primary inputs is equal to the aggregate value of final purchases, plus net exports, in the form,

$$\sum_{i=4}^{10} \sum_{j=1}^{3} X_{ij} = \sum_{i=4}^{10} \sum_{j=5}^{7} X_{ij} + \left(\sum_{i=4}^{10} X_9 - \sum_{i=4}^{10} X_{10} \right), \qquad \text{Eq. (1.2)}$$

or,

Total Value Added = Total Final Product + Net Exports.

Substituting from Table 1.1, the balance equation is now represented by the numerical entries as follows:

718 + 111 + 354 = 738 + 253 + 195 + (73-76)

1,183 = 1,183 (in billion dollars)

Thus, the total value added of $1,183 billion is exactly equal to the gross final purchases of $1,186 billion, minus net imports of $3 billion.

The concept of input-output analysis as an extension of national income and product accounting is suggested by the entries in Table 1.1. Because interindustry transactions, i.e., purchases and sales represented in Quadrant I, balance out, they would not be included in the summary product and income accounts. Without the interindustry transactions, however, input-output analysis would not be possible.

Acceptance of Input-Output Approach

Wide acceptance of the input-output approach in economic impact analysis and forecasting stems, in part, from the input-output concept itself -- its inclusiveness, adaptability, and fundamental simplicity. An input-output table depicts the economic transactions of all remuneratively employed economic units. It can be disaggregated from a small number of large industry groups to many, but smaller, industry groups and their transactions with many, but also smaller, final demand sectors and primary input sectors. Yet, despite the apparent complexity of the economic structures represented by input-output tables, the manipulation of data in the analytical framework is the essence of simplicity in preparation and application. A competently prepared input-output table packs a great deal of useful economic information in small space.

Easy access to the input-output approach makes input-output data and methods prime candidates for well-earned skepticism about their acceptability for specific economic impact and policy analysis applications. While multiplier analysis is now widely associated with the input-output approach, much more than the derivation of multipliers, or the uncritical, uninformed use of multipliers, is involved. If input-output multipliers were the essence of this approach, it rightly would deserve widespread rejection rather than acceptance.

Widespread acceptance of the input-output method is based on its competent and judicious use in economic analysis and forecasting. It deals with short-term effects of industry-specific or sector-specific changes in output demand on all industry and sectors in a given place and time. It sorts out these effects, usually in terms of changes in output, but it can show these effects in terms of changes in income, employment, capital stock, and investment (10,30). It can be used to show the effects of changes in input supplies as well as output demands (6,7). It also provides a method for dealing with data omissions and for achieving forecast consistency (24,33). And it can be used in a small area as well as a national or global geographic setting (17). It still is, however, primarily a method for short-term impact analysis and forecasting, although it is now being extended to long-term development planning (22,24,33).

Basic Assumptions of Input-Output Analysis

Preparation and use of input-output tables is guided by its basic assumptions -- linearity, homogeniety and constancy of input-output relationships. Each industry is represented by a linear and a homogeneous production function with fixed input proportions. Graphically, output is represented as a straight-line function of input, starting from a "zero" origin. In its conventional formulation, the economy is demand-driven. Neither capital nor labor are limiting resources. These assumptions are further illustrated in the preparation and use of the input-output data in Table 1.1.

First, a set of input-output coefficients was derived for each of the four quadrants in Table 1.1. Production coefficients were derived from Quadrant I data while consumption coefficients were derived from Quadrant II. In the conventional input-output table, neither Quadrant III nor Quadrant IV coefficients are needed. The four sets of coefficients,which are summarized in Table 1.2, thus show the proportion of the total purchases of each industry or sector which is acquired from each "producing" (i.e., row) industry or sector.

The input-output coefficients in Table 1.2 show certain proportions of total outlays of each industry allocated to each producing industry, primary-input sector and rest-of-world sector. Thus, for the agriculture and mining industry group, the 22.727 cents of each $1 of total outlay is allocated to its industry group (primarily for feed, livestock and similar transfers from one enterprise to another). Total agriculture and mining industry purchases from producing industries were 55.455 cents per $1 total outlays. Outlays for

Table 1.2. Illustrative Input-Output Table: Intermediate and Final Purchases of Specified Industry Outputs and Primary Inputs per $1 of Total Purchases.

Sector	Intermediate			Final Demand			Rest-of-World	
	Goods		Services-Prod.	Personal Cons. Exp.	Govern-ment	Business Invest.	Comp. Exports	Comp. Imports
	Agr. & Mining	Constr. & Mfg.						
				(dollars)				
Producing Industry:								
Agr., Mining	0.22727	0.02189	0.00974	0.00949	-0.00395	0.01538	0.08219	-0.07895
Constr., Mfg.	0.16364	0.37661	0.01073	0.29404	0.31621	0.92308	0.52055	-0.60526
Services	0.16364	0.15129	0.20779	0.68564	0.15415	0.10256	0.19178	-0.06579
Total	0.55455	0.59979	0.31926	0.98916	0.46640	1.04103	0.79452	-0.75000
Primary Inputs:								
Emp. Comp.	0.10000	0.28326	0.33009	0.00678	0.52174	0	0	0
Ind. Bus. Tax.	0.02727	0.01931	0.09740	0	0	0	0	0
Prop.-Type Inc.	0.31818	0.09549	0.25000	0	0	-0.04103	0	0
Total	0.44545	0.39807	0.67749	0.00678	0.52174	-0.04103	0	0
Rest-of-World:								
Noncomp. Imp.	0	0.00215	0.00325	0.00949	0.01581	0	0.01379	-0.21053
Dummy Ind.	0	0	0	-0.00542	0	0	0.19178	-0.05263
Gross Outlay	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	-1.00000

primary inputs accounted for the remaining 44.545 cents of outlays.

The construction and manufacturing industry group differed from agriculture and mining in its much lesser backward linkage with agriculture, its much higher internal linkage, its much larger outlays for employee compensation, and its much smaller allocation of total outlays to property-type income, i.e., corporate profits and propripriretorial income. The services-producing industry group also differed from agriculture and mining in its lower overall interindustry transactions per $1 gross outlay and its much higher allocation of value added to employee compensation, even with a nearly as high an allocation to property-type income.

The distribution of final product purchases also differed sharply among the three final product sectors. Personal consumption expenditures were largely for services, government expenditures were largely for employee compensation, while business investment expenditures were largely for manufactured (durable) goods, and construction materials and services. Competitive exports and competitive imports (i.e., commodities produced domestically which contrasts with noncomparable imports) were largely manufactured goods.

The Leontief inverse, the matrix of industry-specific demand multipliers, is derived from a set of input-output coefficients like those in Table 1.2. The demand multipliers are represented by the (I-A) inverse in the form,

$$X = [I-A]^{-1} Y, \qquad \text{Eq. (1.3)}$$

where the [I-A] matrix is obtained from the technical coefficients in Table 1.2. The technical coefficient, a_{ij}, is represented by the ratio,

$$a_{ij} = \frac{X_{ij}}{X_j}$$

where, X_{ij} = total value, in dollars, of i-th industry output purchased by j-th industry.

A system of equations can be specified which describe the input-output relationships of economy, as in the form,

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 = X_1 \qquad \text{Eq. (1.4)}$$

$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + Y_2 = X_2$$

$$a_{31}X_3 + a_{32}X_2 + a_{33}X_3 + Y_3 = X_3,$$

where the $a_{ij}X_j$'s and Y_i's now represent the intermediate and final demand, respectively, for the i-th industry output, X_i. The three-equation system can be represented also in the algebraic form,

$$(1 - a_{11})X_1 \quad a_{12} \cdot X_2 \quad a_{13} \cdot X_3 = Y_1 \qquad \text{Eq. (1.5)}$$

$$a_{21} \cdot X_1 \quad (1 - a_{22})X_2 \quad a_{23} \cdot X_3 = Y_2$$

$$a_{31} \cdot X_1 \quad a_{32} \cdot X_2 \quad (1 - a_{33})X_3 = Y_3$$

The set of technical coefficients in Table 1.2 can be represented as an [I-A] matrix to correspond with the coefficients preceding the X_j's in Equation (1.5).

Derivation of the Leontief inverse from the technical coefficients in Table 1.2 is illustrated in Appendix A with the use of simple matrix procedures. Results of using these procedures are summarized in Table 1.3. This table contains the individual demand multipliers in the Leontief inverse. The multipliers can relate a given change in final purchases, say of Y_1, to a corresponding change in each of the three commodities with the form,

$$\Delta X = [I-A]^{-1}\Delta Y, \qquad \text{Eq. (1.6)}$$

where $[I-A]^{-1}$ is the Leontief inverse and ΔY and ΔX are the specified demand and derived output changes, respectively.

Individual demand multipliers are illustrated by three columns of coefficients in Table 1.3. These coefficients were derived from the technical coefficients in Table 1.2. They show the consequences of large internal

Table 1.3. Illustrative Input-Output Table: Total Effect of a $1 Change in Final Demand for Specified Industry Output.

Sector	Goods: Agr. & Mining	Goods: Constr. & Mfg.	Services
		(dollars)	
Agr., Mining	1.33662	0.16323	0.03741
Constr., Mfg.	0.40867	1.70565	0.22424
Services	0.35411	0.35945	1.31252
Total	2.09940	2.22833	1.57427

linkage in large demand multipliers. The construction and manufacturing industry group, which had the largest total for its individual technical coefficients, also has the largest demand multiplier. For example, a $1 increase in final demand for construction and manufacturing output results in a $2.23 increase in overall industry output. Of this total effect, $1.71 is due to output change in the construction and manufacturing industry as a result of additional intra-industry requirements for achieving a sufficiently high increase in output to satisfy both the $1 increase in final demand and the 71 cent additional increase in intra-industry purchases.

The basic assumptions of linearity, homogeneity and constancy impose important constraints on the use of the input-output approach in labor market analysis and forecasting. While input-output relationships may not change (that is, only the levels of inputs and outputs change, not their proportions), the degree of import dependency of a small area may change. More or less of an industry's inputs may be acquired from outside the labor market, thus changing the degree of internal, backward linkage and, also, the value of its demand multiplier. For small areas, particularly, the rule of constancy is inapplicable, unless changes in import levels are included in the derivation of the input-output coefficients. A similar qualification applies, also, in the use of the consumption coefficients in overall area analysis and forecasting.

Additional limitations in the use of the input-output approach stem from industry-specific technological and price changes. The computer industry, which is part of the non-electrical machinery industry in the Standard Industrial Classification System, has undergone rapid transformation of both its technology and price structure. Indeed, the price of computers fell at the same time that energy prices rapidly outpaced other price increases.

For Minnesota, particularly, the contrasting price experiences of the computer and the petroleum industries resulted in sharp changes in interindustry relationships. The rule of constancy in input-output relations was seriously violated during the 1970's as computer prices dropped relative to petroleum prices. Minnesota exports computers, but imports its petroleum. Its terms of trade thus worsened, except for the output-increasing effects of new computer technology and its widespread business applications. As prices dropped, utilization increased, partly because of substitution of new computers for old ones and partly because of new uses for new computers. With these and similar distortions of input-output relationships, great care and expertise must be exercised in the appropriate use of the input-output approach in small area impact anlaysis and forecasting.

Various computational procedures have been developed for dealing with the constraints imposed by the basic assumptions of input-output analysis. These procedures are discussed in later sections of this report. First, however, a problem focus for input-output approaches is delineated and discussed. Model building steps are related to the problem focus. They include the delineation of study area; industry and sector classification; model specification; data collection and preparation; model calibration, documentation and verification; and model validation and acceptance. Data interpretation is discussed next. In this section, the different parts of input-output tables of the U.S. and Minnesota, and their inverses, are examined, including industry sales and purchases; value added and final purchases; imports and exports; and direct and indirect effects. Finally, applications of the input-output approach in economic impact analysis and projection are presented.

MODEL BUILDING

Model-building involves a series of steps starting with a definition of the problem and a delineation of the geographical problem area. The model building steps parallel the building of a decision information system in which local and national macro-economic data and analytical and forecasting methods are related to public sector planning and management. In such a system, the input-output model and the model builder in essence convert data into information which the model user translates into specific decision-information. Model-builder and model user thus collaborate in the deployment of the information system output for decision making purposes. They may collaborate, or at least exchange views, in earlier stages of model building, for example, the problem delineation.

Problem and Area Delineation

The problem focus in model building is identified as a primary consideration in deciding whether or not the input-output approach is ideally the appropriate one. Many problems require no more than the trained and experienced judgement of a practicing economic consultant. Others may require some quantification, but nothing more sophisticated than a single equation model with less than a half dozen variables. Some problems are less tractable. They call for more sophisticated approaches, but even then, neither the trained and experienced judgement of the practicing economist nor the quantification provided by a simple, single-equation model can be discarded. Effective use of the input-output approach depends on the parallel development of proven economic analytical competences.

The input-output approach is most suited for large areas with much internal linkage, or to small, growing areas which are in the process of becoming increasingly interdependent as a result of population and income growth and industry proliferation. The Upper Midwest Region (as defined by the Minneapolis Federal Reserve Bank), the State of Minnesota, the seven-county Metropolitan Council Region, and the eight-county Arrowhead Region plus Douglas County, Wisconsin are regions with much internal interdependence. Of the four regions, only the Arrowhead Region is declining rather than growing, but its internal interdependence is nonetheless increasing. Many smaller areas, of course, are growing in both total economic activity and internal linkage.

A problem focus in areas of strong internal linkages which emphasizes the measurement of industry-specific and sector-specific economic effects of changes in demand and supply, or related governmental policy and climatic conditions, is one obviously tailored to an input-output approach.

Further delineation of Minnesota substate regions for the input-output approach could start with a grouping of existing substate development regions. For example, Regions 1, 2, 4, 6W and 8 form a dominantly agricultural economic region, while Regions 6E, 7E, 7W, 9 and 10 form a transitional agricultural-industrial region (Figure 2.1). Indeed, the metropolitan core region, Region 11, may be joined by Regions 7E and 7W to form an extended metropolitan focal region. Finally, Regions 2 and 5 could be grouped with Region 3 to form a natural resources-based urban-industrial economic region. With a minimum of four substate input-output models, economically different substate regional groupings can be related directly to U.S. output markets and input sources, as well as to each other.

Further regional subdivisions can be achieved within the three larger regional groupings outside the Metropolitan Council Region. The input-output approach could apply even to subregions. The use of substate regional groupings would facilitate, rather than preclude, the preparation and use of small area input-output tables. Both model calibration and validation procedures, for example, could be more readily implemented by starting with

Figure 2.1. Substate Planning and Development Districts,

Minnesota, 1978.

Table 2.1. Relation of Minnesota 214-Industry Classification Code to Selected U.S. and Minnesota Industry Classification Codes: Series B, 1981.

Minnesota 214-Industry Code No.	Title	BLS 154-Ind.	USDC 496-Ind.	Minnesota 55-Ind.	Minnesota 95-Ind.	SIC Code (1972 Edition)
1.	Dairy farm prod.	pt.1	1.01	pt.1	pt.1	0241,pt.0191,pt.0259,pt.0291
2.	Poultry & eggs	pt.1	1.02	pt.1	pt.1	025(exc.0254 & pt.0259),pt.0191,pt.0219
3.	Meat animals & prod.	2	1.0301,.0302	pt.1	pt.1	021(exc.pt.0219),27,pt.0191,pt.0219,pt.0259,pt.0291
4.	Food,feed grain	4	2.0201,.0202	pt.2	pt.2	pt.011,pt.0139,pt.0191,pt.0219,pt.0259,pt.0291
5.	Vegetables	pt.5	2.0501	pt.2	pt.2	0134,0161,pt.0119,pt.0139,pt.0191,pt.0219,pt.0259,pt.0291
6.	Sugar crops	pt.5	2.0502	pt.2	pt.2	0133,pt.0191,pt.0219,pt.0259,pt.0291
7.	Oil-bearing crops	pt.5	2.0600	pt.2	pt.2	0116,pt.0119,pt.013,pt.0173,pt.0219,pt.0259,pt.0291
8.	Other crops	3,pt.5	2.01,2.0203,.03, .04,.07,.0503	pt.2	pt.2	pt.0119,pt.0139,pt.0191,pt.0219,pt.0259,pt.0291
9.	Forest. & fish. prod.	6	3.00	pt.3	3	081-4,091,097
10.	Agr.,for.,fish.serv.	7	4.00	pt.3	4	0254,07(exc.074),085,092
11.	Iron ore mining	8	5.00	4	5	101,106
12.	Copper ore mining	9	6.01	6	pt.6	102
13.	Other nonfer. ores	10	6.02	5	pt.6	103-105,pt.108,109
14.	Coal & peat mining	11	7.00	pt.7	7	1111,pt.1112,1211,pt.1213
15.	Oil & gas extract.	12	8.00	pt.7	8	131,132,pt.138
16.	Stone & clay	13	9.00	pt.7	9	141-145,pt.148,149
17.	Chem. & fert.	14	10.00	pt.7	10	147
18.	New resid. build.	15	11.01	pt.8	pt.11	pt.15,pt.16,pt.17
19.	New nonres. build.	16	11.02	pt.5	pt.11	pt.15,pt.16,pt.17
20.	New public utility	17	11.03	pt.8	pt.11	pt.16,pt.17
21.	New highway const.	18	11.04	pt.8	pt.11	pt.16,pt.17
22.	All other const.	19	11.0501,2,5,7	pt.8	pt.11	pt.15,pt.16,pt.17
23.	Well drilling,min.ex.	20	11.0503,4,6,8	pt.8	pt.11	pt.108,pt.1112,pt.1213,pt.138
24.	Maint. & repair	21	12.0100-.0215	pt.8	12	pt.15,pt.16,pt.17
25.	Complete guided mis.	23	13.01	pt.9	pt.13	3761
26.	Other ordnance	22	13.02-.07	pt.9	pt.13	348, 3795
27.	Meat packing	pt.24	14.0101	pt.11	pt.15	2011
28.	Sausages & other	pt.24	14.0102	pt.11	pt.15	2013
29.	Poultry dressing	pt.24	14.0103	pt.11	pt.15	2016
30.	Poultry & egg proc.	pt.24	14.0104	pt.11	pt.15	2017
31.	Creamery butter	pt.25	14.02	pt.11	pt.14	2021
32.	Cheese,nat. & proc.	pt.25	14.03	pt.10	pt.14	2022
33.	Cond. & evap. milk	pt.25	14.04	pt.10	pt.14	2023
34.	Ice cream & froz. des.	pt.25	14.05	pt.10	pt.14	2024
35.	Fluid milk	pt.25	14.06	pt.10	pt.14	2026
36.	Canned fr. & veb.	pt.26	14.09	pt.10	pt.14	2033
37.	Frozen fr. & veg.	pt.26	14.13	pt.10	pt.14	2037,8
38.	Other pres. fr. & veg.	pt.26	14.08,.10	pt.10	pt.14	2032,2034,2035
39.	Fresh,froz.,pres.fish	pt.26	14.07,.11	pt.10	pt.14	2091,2092
40.	Flour & other grain	pt.27	14.1401	pt.12	pt.16	2041
41.	Cereal preparations	pt.27	14.1402	pt.12	pt.16	2043
42.	Blended & prep. flour	pt.27	14.1403	pt.12	pt.16	2045
43.	Dog,cat & other pet	pt.27	14.1501	pt.12	pt.16	2047
44.	Prepared feeds,n.e.c.	pt.27	14.1502	pt.12	pt.16	2048
45.	Rice milling	pt.27	14.16	pt.12	pt.16	2044
46.	Wet corn milling	pt.27	14.17	pt.12	pt.16	2046
47.	Bread,cake & rel. pr.	pt.28	14.1801	pt.10	pt.14	2051
48.	Cookies & crackers	pt.28	14.1802	pt.10	pt.14	2052
49.	Sugar	29	14.19	pt.13	pt.14	2061-3
50.	Confect. & rel.	30	14.20	pt.10	pt.14	2065-7
51.	Alcoholic beverages	31	14.21	pt.13	pt.17	2082-2085
52.	Soft drinks	pt.32	14.22	pt.13	pt.17	2086
53.	Flavoring ex. & syr.	pt.32	14.23	pt.13	pt.17	2087
54.	Fats & oils	pt.33	14.24-.27,.29	pt.14	pt.14	2074-7,2079
55.	Misc. food prod.	pt.33	14.28,.31,.32	pt.10	pt.14	2095,2097-9
56.	Tobacco manuf.	34	15.01-.02	pt.13	18	21
57.	Fabric & thread	35	16.01-.04	pt.14	19	221-224,226,228
58.	Floor coverings	36	17.01	pt.14	pt.20	227
59.	Misc. text. prod.	37	17.02-.10	pt.14	pt.20	229
60.	Hosiery & knit	38	18.0101-.0300	pt.14	pt.21	225
61.	Apparel mfg.	39	18.04	pt.14.	pt.21	23(exc.239),39995
62.	Fabricated text.	40	19.01-.0306	pt.14	22	239
63.	Logging	41	20.01	pt.15	pt.23	241
64.	Sawmills & plan. mills	pt.42	20.02	pt.15	pt.23	2421
65.	Hardwood flooring	pt.42	20.03	pt.16	pt.23	2426
66.	Special prod. sawmills	pt.42	20.04	pt.16	pt.23	2429
67.	Millwork & cabinets	pt.43	20.05	pt.16	pt.23	2431,4
68.	Veneer & plywood	pt.43	20.06	pt.16	pt.23	2435,6

the larger regional groupings rather than individual substate development regions, or individual counties. Such a hierarchical approach would reduce data disclosure problems for small area studies and also reduce data costs while increasing the probability of user acceptance because of more readily implemented model calibration and validation methods.

Model and Sector Classification

The extent of industry and sector disaggregation depends on the geographical area and its immunity from problems of industry disclosure. For example, a densely populated multi-county area would have economic data reported for many more individual industries than a sparsely populated multi-county area.

Starting with the State of Minnesota, a 214-industry breakdown of industry output, employment and income, as specified in Table 2.1, is readily implemented. Currently, such a breakdown is available, not only for the State, but, also, Regions 2, 3 and 11. These industry breakdowns were devised specifically for the mineral-related and forest-related studies now being completed at the University of Minnesota.

In addition to the 214-industry breakdown, a potential 12-sector breakdown is available for the differentiating of final product by recipient sector. The 12 sectors are listed as follows:

Household: personal consumption expenditures.

Government: state and federal, with four state (education; welfare, and sanitation; safety; and other general governmant) and two federal (national defense and nondefense) sectors.

Business Investment: gross private capital formation and change in business inventories.

Rest-of-World: competitive exports;competitive imports; and exports from state or region to rest of nation.

Table 2.1. Relation of Minnesota 214-Industry Classification Code to Selected U.S. and Minnesota Industry Classification Codes: Series B, 1981.

Minnesota 214-Industry Code No.	Title	BLS 154-Ind.	USDC 496-Ind.	Minnesota 55-Ind.	Minnesota 95-Ind.	SIC Code (1972 Edition)
1.	Dairy farm prod.	pt.1	1.01	pt.1	pt.1	0241,pt.0191,pt.0259,pt.0291
2.	Poultry & eggs	pt.1	1.02	pt.1	pt.1	025(exc.0254 & pt.0259),pt.0191,pt.0219
3.	Meat animals & prod.	2	1.0301,.0302	pt.1	pt.1	021(exc.pt.0219),27,pt.0191,pt.0219,pt.0259,pt.0291
4.	Food,feed grain	4	2.0201,.0202	pt.2	pt.2	pt.011,pt.0139,pt.0191,pt.0219,pt.0259,pt.0291
5.	Vegetables	pt.5	2.0501	pt.2	pt.2	0134,0161,pt.0119,pt.0139,pt.0191,pt.0219,pt.0259,pt.0291
6.	Sugar crops	pt.5	2.0502	pt.2	pt.2	0133,pt.0191,pt.0219,pt.0259,pt.0291
7.	Oil-bearing crops	pt.5	2.0600	pt.2	pt.2	0116,pt.0119,pt.013,pt.0173,pt.0219,pt.0259,pt.0291
8.	Other crops	3,pt.5	2.01,2.0203,.03, .04,.07,.0503	pt.2	pt.2	pt.0119,pt.0139,pt.0191,pt.0219,pt.0259,pt.0291
9.	Forest. & fish. prod.	6	3.00	pt.3	3	081-4,091,097
10.	Agr.,for.,fish.serv.	7	4.00	pt.3	4	0254,07(exc.074),085,092
11.	Iron ore mining	8	5.00	4	5	101,106
12.	Copper ore mining	9	6.01	6	pt.6	102
13.	Other nonfer. ores	10	6.02	5	pt.6	103-105,pt.108,109
14.	Coal & peat mining	11	7.00	pt.7	7	1111,pt.1112,1211,pt.1213
15.	Oil & gas extract.	12	8.00	pt.7	8	131,132,pt.138
16.	Stone & clay	13	9.00	pt.7	9	141-145,pt.148,149
17.	Chem. & fert.	14	10.00	pt.7	10	147
18.	New resid. build.	15	11.01	pt.8	pt.11	pt.15,pt.16,pt.17
19.	New nonres. build.	16	11.02	pt.5	pt.11	pt.15,pt.16,pt.17
20.	New public utility	17	11.03	pt.8	pt.11	pt.16,pt.17
21.	New highway const.	18	11.04	pt.8	pt.11	pt.16,pt.17
22.	All other const.	19	11.0501,2,5,7	pt.8	pt.11	pt.15,pt.16,pt.17
23.	Well drilling,min.ex.	20	11.0503,4,6,8	pt.8	pt.11	pt.108,pt.1112,pt.1213,pt.138
24.	Maint. & repair	21	12.0100-.0215	pt.8	12	pt.15,pt.16,pt.17
25.	Complete guided mis.	23	13.01	pt.9	pt.13	3761
26.	Other ordnance	22	13.02-.07	pt.9	pt.13	348, 3795
27.	Meat packing	pt.24	14.0101	pt.11	pt.15	2011
28.	Sausages & other	pt.24	14.0102	pt.11	pt.15	2013
29.	Poultry dressing	pt.24	14.0103	pt.11	pt.15	2016
30.	Poultry & egg proc.	pt.24	14.0104	pt.11	pt.15	2017
31.	Creamery butter	pt.25	14.02	pt.11	pt.14	2021
32.	Cheese,nat. & proc.	pt.25	14.03	pt.10	pt.14	2022
33.	Cond. & evap. milk	pt.25	14.04	pt.10	pt.14	2023
34.	Ice cream & froz. des.	pt.25	14.05	pt.10	pt.14	2024
35.	Fluid milk	pt.25	14.06	pt.10	pt.14	2026
36.	Canned fr. & veb.	pt.26	14.09	pt.10	pt.14	2033
37.	Frozen fr. & veg.	pt.26	14.13	pt.10	pt.14	2037,8
38.	Other pres. fr. & veg.	pt.26	14.08,.10	pt.10	pt.14	2032,2034,2035
39.	Fresh,froz.,pres.fish	pt.26	14.07,.11	pt.10	pt.14	2091,2092
40.	Flour & other grain	pt.27	14.1401	pt.12	pt.16	2041
41.	Cereal preparations	pt.27	14.1402	pt.12	pt.16	2043
42.	Blended & prep. flour	pt.27	14.1403	pt.12	pt.16	2045
43.	Dog,cat & other pet	pt.27	14.1501	pt.12	pt.16	2047
44.	Prepared feeds,n.e.c.	pt.27	14.1502	pt.12	pt.16	2048
45.	Rice milling	pt.27	14.16	pt.12	pt.16	2044
46.	Wet corn milling	pt.27	14.17	pt.12	pt.16	2046
47.	Bread,cake & rel. pr.	pt.28	14.1801	pt.10	pt.14	2051
48.	Cookies & crackers	pt.28	14.1802	pt.10	pt.14	2052
49.	Sugar	29	14.19	pt.13	pt.14	2061-3
50.	Confect. & rel.	30	14.20	pt.10	pt.14	2065-7
51.	Alcoholic beverages	31	14.21	pt.13	pt.17	2082-2085
52.	Soft drinks	pt.32	14.22	pt.13	pt.17	2086
53.	Flavoring ex. & syr.	pt.32	14.23	pt.13	pt.17	2087
54.	Fats & oils	pt.33	14.24-.27,.29	pt.14	pt.14	2074-7,2079
55.	Misc. food prod.	pt.33	14.28,.31,.32	pt.10	pt.14	2095,2097-9
56.	Tobacco manuf.	34	15.01-.02	pt.13	18	21
57.	Fabric & thread	35	16.01-.04	pt.14	19	221-224,226,228
58.	Floor coverings	36	17.01	pt.14	pt.20	227
59.	Misc. text. prod.	37	17.02-.10	pt.14	pt.20	229
60.	Hosiery & knit	38	18.0101-.0300	pt.14	pt.21	225
61.	Apparel mfg.	39	18.04	pt.14.	pt.21	23(exc.239),39995
62.	Fabricated text.	40	19.01-.0306	pt.14	22	239
63.	Logging	41	20.01	pt.15	pt.23	241
64.	Sawmills & plan. mills	pt.42	20.02	pt.15	pt.23	2421
65.	Hardwood flooring	pt.42	20.03	pt.16	pt.23	2426
66.	Special prod. sawmills	pt.42	20.04	pt.16	pt.23	2429
67.	Millwork & cabinets	pt.43	20.05	pt.16	pt.23	2431,4
68.	Veneer & plywood	pt.43	20.06	pt.16	pt.23	2435,6
69.	Struct. wood,n.e.c.	pt.43	20.0701	pt.15	pt.23	2439
70.	Prefabricated wood	pt.43	20.0702	pt.15	pt.23	2452
71.	Wood preserving	pt.43	20.0800	pt.15	pt.23	2491
72.	Wood pellets & skids	pt.43	20.0901	pt.15	pt.23	2448
73.	Particleboard	pt.43	20.0902	pt.15	pt.23	2492
74.	Wood prod.,n.e.c.	pt.43	20.0903	pt.15	pt.23	2499
75.	Wood containers	44	21.00	pt.16	24	244(exc.2448)
76.	Wood household furn.	pt.45	22.01,.02	pt.16	pt.23	2511,2512,2517,2519
77.	Other household furn.	pt.45	22.03,.04	pt.16	pt.23	2514,2515

Table 2.1. Relation of Minnesota 214-Industry Classification Code to Selected Minnesota and U.S. Industry Classification Codes: Series B, 1981 (continued).

Minnesota 214-Industry Code No.	Title	BLS 154-Ind.	USDC 496-Ind.	Minnesota 55-Ind.	Minnesota 95-Ind.	SIC Code (1972 Edition)
78.	Wood office furn.	pt.46	23.01	pt.16	pt.26	2521
79.	Other furn. & fix.	pt.46	23.02-.07	pt.16	pt.26	2522,2531,254,259
80.	Pulp mills	pt.47	24.01	pt.17	pt.27	261
81.	Paper mills	pt.47	24.02	pt.17	pt.27	262
82.	Paperboard mills	pt.47	24.03	pt.17	pt.27	263
83.	Conv. paper prod.	pt.47	24.04,.05,.07	pt.17	pt.27	264
84.	Build. Paper & bd.	pt.47	24.0602	pt.17	pt.27	266
85.	Paperboard contain.	48	25.00	pt.17	28	265
86.	Newspaper print.& pub.	49	26.01	pt.18	pt.29	271
87.	Period. & book	50	26.02-.04	pt.18	pt.29	272-274
88.	Misc. Print. & pub.	51	26.05-.08	pt.18	pt.29,30	275-279
89.	Ind. inorg. & org. ch.	52	27.01	pt.19	pt.31	281(exc.28195),2865,2869
90.	Agricultural chem.	53	27.02-.03	pt.19	pt.31	287
91.	Misc. chem. prod.	54	27.04	pt.19	pt.31	2861,289
92.	Plastic & rubber	55	28.01,.02	pt.19	pt.32	2821,2822
93.	Synthetic fibers	56	28.03-.04	pt.19	pt.32	2823,2824
94.	Drugs	57	29.01	pt.19	pt.33	283
95.	Cleaning & toilet	58	29.09-.03	pt.19	pt.33	284
96.	Paints	59	30.00	pt.19	34	285
97.	Petroleum ref.	pt.60	30.01	pt.20	35	291,299
98.	Paving & asp. mix.	pt.60	31.02,.03	pt.20	36	295
99.	Tires & in. tubes	61	32.01	pt.21	pt.37	301
100.	Misc. rub. prod.	62	32.02,.03,.05	pt.21	pt.37	302-306
101.	Plastic prod.	63	32.04	pt.21	pt.37	307
102.	Leather tan. & ind.	64	33.01	pt.21	38	311
103.	Footware & other	65	34.01-.0305	pt.21	39	313-319
104.	Glass	66	35.01-.02	pt.22	40	321-323
105.	Hydraulic cement	pt.67	36.01	pt.22	pt.41	324
106.	Brick & clay tile	pt.68	36.02	pt.22	pt.41	3251
107.	Other struct. clay	pt.68	36.03	pt.22	pt.41	3253,3255,3259
108.	Pottery & rel. prod.	69	36.06-.09	pt.22	pt.41	326
109.	Concrete,exc. block	pt.67	36.11	pt.22	pt.41	3272
110.	Concr. block	pt.67	36.10,.12	pt.22	pt.41	3271,3273
111.	Lime & gypsum	pt.67	36.13,.14	pt.22	pt.41	3274,3275
112.	Misc. stone & clay	70	36.15-.22	pt.22	pt.41	328,329
113.	Bl. furn. & steel	pt.71	37.0101	pt.23	pt.42	3312
114.	Electromet. prod.	pt.71	37.0102	pt.23	pt.42	3313
115.	Steel wire & rel.	pt.71	37.0103	pt.23	pt.42	3315
116.	Cold fin. steel	pt.71	37.0104	pt.23	pt.42	3316
117.	Steel pipe & tubes	pt.71	37.0105	pt.23	pt.42	3317
118.	Iron & steel found.	pt.72	37.0200	pt.23	pt.42	332
119.	Iron & st. forg.	pt.72	37.0300	pt.23	pt.42	3462
120.	Metal heat treat.	pt.72	37.0401	pt.23	pt.42	3398
121.	Pri. met. prod.n.e.c.	pt.72	37.0402	pt.23	pt.42	3399
122.	Primary copper	pt.73	38.0100	pt.24	pt.43	3331
123.	Other prim. cop.	pt.73	38.07,.10,.12	pt.24	pt.43	3351,3357,3362
124.	Pri. alum. & prod.	74	38.04,.08	pt.25	pt.43	3334,3353-5,3361,28195
125.	Other pri. nonfer.	75	38.02,.03,.05,.06,.09,.13,.14	pt.25	pt.43	3332,3333,3339,334,3356,3369,3463
126.	Metal containers	76	39.01-.02	pt.26	44	341
127.	Heat. & plumb. fix.	77	40.01-.02	pt.26	pt.45	343
128.	Fabricated metal	78	40.03-.09	pt.26	pt.45	344
129.	Screw machine prod.	79	41.01	pt.26	pt.46	345
130.	Metal stampings	80	41.02	pt.26	pt.46	3465,3466,3469
131.	Cutlery & gen. hdw.	81	42.01-.03	pt.26	pt.47	342
132.	Other febr. metal	82	42.01-.11	pt.26	pt.47	347,349
133.	Engines	83	43.01	pt.27	48	351
134.	Farm machinery	84	44.00	pt.27	49	352
135.	Const. & mining mach.	85	45.01-.03	pt.27	50	3531-3533
136.	Materials handling	86	46.01-.03	pt.27	51	3534-3537
137.	Metalworking mach.	87	47.01-.04	pt.72	52	354
138.	Special inds. mcha.	88	48.01-.06	pt.27	53	355
139.	Gen. industrial	89	49.01-.07	pt.27	54	356
140.	Machine shops	90	50.00	pt.27	55	359
141.	Electronic computing	pt.91	51.0101	pt.27	pt.57	3573
142.	Calculating & acctg.	pt.91	51.0102	pt.27	pt.57	3574
143.	Office machines	92	51.02-.04	pt.27	56	3572,3576,3579
144.	Service ind. mach.	93	52.01-.05	pt.27	58	358
145.	Electrical trans. eq.	94	53.01-.03	pt.28	pt.59	361,3825
146.	Electrical ind. appar.	95	53.04-.08	pt.28	pt.59	362
147.	Household appl.	96	54.01-.07	pt.28	60	363
148.	Electric light.	97	55.01-.03	pt.28	61	364
149.	Radio & TV sets	98	56.01-.02	pt.28	pt.62	365
150.	Telephone & tel. eq.	99	56.03	pt.28	pt.62	3661
151.	Radio & comm. equip.	100	56.04	pt.28	pt.62	3662
152.	Electron tubes	pt.101	57.01	pt.28	pt.63	3671-3
153.	Semiconductors	pt.101	57.02	pt.28	pt.63	3674
154.	Other electr. comp.	pt.101	57.03	pt.28	pt.63	3675-9
155.	Misc. electr. eq.	102	58.01-.05	pt.28	64	369
156.	Motor vehicles	103	59.01-.03	29	65	371
157.	Aircraft	104	60.01-.04	30	66	372,3764,3769

Table 2.1. Relation of Minnesota 214-Industry Classification Code to Selected Minnesota and U.S. Industry Classification Codes: Series B, 1981 (concluded).

Minnesota 214-Industry Code No.	Title	BLS 154-Ind.	USDC 496-Ind.	Minnesota 55-Ind.	Minnesota 95-Ind.	SIC Code (1972 Edition)
158.	Boat building	105	61.01-.02	pt.31	pt.67	373
159.	Railroad equip.	106	61.03	pt.31	pt.67	374
160.	Motor cycles	107	61.05	pt.31	pt.67	375
161.	Other transp. eq.	108	61.06-.07	pt.31	pt.67	3792,3799,2451
162.	Eng. & sci. instr.	pt.109	62.01	pt.32	69	3811
163.	Mech-measuring dev.	pt.109	62.02	pt.32	pt.68	3823,3824,3829
164.	Auto. temp. controls	pt.109	62.03	pt.32	pt.68	3822
165.	Surg. & med. inst.	pt.110	62.04	pt.32	pt.68	3841
166.	Surg. appl. & supp.	pt.110	62.05	pt.32	pt.68	3842
167.	Dental eq. & supp.	pt.110	62.06	pt.32	pt.68	3843
168.	Opt. instr. & lenses	pt.111	63.01	pt.32	pt.70	383
169.	Ophthalmic goods	pt.111	63.02	pt.32	pt.70	385
170.	Photogr. equip.	112	63.03	pt.32	pt.70	386
171.	Watches & clocks	113	62.07	pt.32	pt.68	387
172.	Jewelry & silver.	114	64.01	pt.33	pt.71	391,3961
173.	Mus. instr. & sport	115	64.02-.04	pt.33	pt.71	393,394
174.	Other misc. mfg.	116	64.05-.12	pt.33	pt.71	395,396(exc.3961),399(exc.39996)
175.	Railroad transport.	117	65.01	35	73	40,474,pt.4789
176.	Local transit & int.	118	65.02	36	74	pt.41
177.	Truck transport.	119	65.03	37	75	42,pt.4789
178.	Water transport.	120	65.04	pt.34	pt.72	44
179.	Air transport.	121	65.05	38	76	45
180.	Pipeline trans.	122	65.06	pt.34	pt.72	46
181.	Transp. services	123	65.07	pt.34	pt.72	47(exc.474,pt.4789)
182.	Commu. exc. radio,TV	124	66.00	pt.39	77	48(exc.483)
183.	Radio & TV broad.	125	67.00	pt.39	78	483
184.	Electric util.	126	68.01	40	79	pt.491,pt.493
185.	Gas utilities	127	68.02	41	80	492,pt.493
186.	Water & san. serv.	128	68.03	42	81	494-497,pt.493
187.	Wholesale trade	129	69.01	43	82	50,51(exc. Mfgrs., sales off.)
188.	Retail trade	pt.130	69.02	44	pt.83	52-57,59,7396,8042
189.	Banking	131	70.01	pt.45	pt.84	60
190.	Credit agenc. & brok.	132	70.02-.03	pt.45	pt.84	61(exc.pt.613),62,67
191.	Insurance	133	70.04-.05	pt.45	pt.84	63,64
192.	Owner-occ. real est.	134	71.01	----	----	not applicable
193.	Real estate	135	71.02	46	85	65,66,pt.1531
194.	Hot. & lodg. pl.	136	72.01	pt.47	pt.86	70(exc. dining)
195.	Personal rep. serv.	137	72.02	pt.47	pt.86	72(exc.723,724),762-4,pt.7699
196.	Barber & bea. sh.	138	72.03	pt.47	pt.85	723,724
197.	Misc. bus. serv.	139	73.01	pt.48	pt.87	73(exc.731,7396),769(exc.pt.7699)
198.	Advertising	140	73.02	pt.48	pt.87	731
199.	Misc. prof. serv.	141	73.03	pt.48	pt.87	81,89(exc.8922)
200.	Eating & drink. plac.	pt.130	74.00	pt.44	pt.83	58,pt.70
201.	Auto. repair	142	75.00	49	88	75
202.	Motion pictures	143	76.01	pt.50	pt.89	78
203.	Amuse. & recr. serv.	144	76.02	pt.50	pt.89	79
204.	Doctors' & dent. serv.	145	77.01	pt.51	pt.90	801-803,8041
205.	Hospitals	146	77.02	pt.51	pt.90	806
206.	Other med. serv.	147	77.03	pt.51	pt.90	074,8049,805,807-9
207.	Educ. services	148	77.04	pt.51	pt.90	82
208.	Nonprofit org.	pt.149	77.05	pt.51	pt.90	84,86,8922
209.	Social serv.	pt.149	77.06-.09	pt.51	pt.90	8321,8331,8351,8361,8399
210.	Post Office	150	78.01	pt.52	pt.91	4311
211.	Other fed. ent.	151	78.02-.04	pt.52	pt.91	pt.491,pt.613,several others
212.	Local gov. trans.	152	79.01	pt.53	pt.92	pt.41
213.	Other state-loc. ent.	153	79.02-.03	pt.53	pt.92	pt.491, several others
214.	Scrap,used & sec.	---	81.00	pt.54	pt.95	
Rows:						
215.	Total inter. inputs	---	T.I.I.			---
216.	Dummy Industry	---	82,83,84,85			---
217.	Noncom. imp.	---	80.00			
218.	Value added,total		V.A.			
219.	Total indus. output		T.I.O.			
220.	Employee comp.	---	88.00			---
221.	Indirect bus. taxes		89.00			---
222.	Prop.-type income		90.00			---
Columns:						
215.	Total inter. use	---	T.I.U.			---
216.	Pers. cons. exp.	---	91.00			---
217.	Gross priv. cap.form.	---	92.00			---
218.	Change in bus. inv.	---	93.00			---
219.	Exports	---	94.00			---
220.	Imports	---	95.00			---
221.	Fed. gov.	---	96.00,97.00			---
222.	State-local	---	98.00,99.10-.30			---
223.	Total fin. demand	---	T.F.D.			---
224.	Total com. output	---	T.C.O.			---

For current studies, the four state sectors are combined into one sector and the two federal sectors are combined into a second government sector.

A second industry breakdown is available for Minnesota that parallels the 85-industry breakdown of published U.S. input-output tables (34), but with disaggregation of petroleum refining, food products, nonelectrical machinery manufacturing, and public utilities industries which results in a 95-industry listing. In addition, a 75-industry breakdown is available for general-purpose studies. This breakdown uniquely delineates economically important Minnesota industries.

Use of different industry and sector classification systems is guided by knowledge of the basic input-output assumptions and their implications for both the model builder and the model user. More or less homogeneous economic activities are grouped together on the assumption that their input requirements per unit of output will remain constant. If the activity composition in an industry changes, the assumption of constancy may be violated. Similarly, for small area studies, the import requirements per unit of output must remain constant for the input-output multiplier values to hold. Input substitution within an industry group, however, would not contradict the constancy assumption as long as the input requirements per unit of output remain unchanged. When the basic input-output assumptions no longer hold, new input-output tables must be constructed which may require a re-classification of a region's economic activities to form more homogeneous groupings of industries and final demand sectors. Public disclosure rules and data limitations, of course, will force compromises which may require frequent updating of the input-output tables. Time and money costs of maintaining and updating state and substate regional input-output tables become an important consideration in the acceptance of the input-output approach for labor market studies.

Model Specification

An input-output table is based on an input-output model, as shown in Eq. (1.5), which is now specified in the matrix form,

$$X[A-I] = Y, \qquad \text{Eq. (2.1)}$$

where, X = individual industry outputs in dollars;

$[I-A]$ = matrix of individual input-output (i.e., technical) coefficients, a_{ij}'s, subtracted from an identity matrix, I;

Y = final demand for individual industry outputs in dollars.

The input-output coefficient, a_{ij}, was defined earlier as the purchases of i-th industry output per \$1 of all purchases by the j-th industry.

A three-industry (I-A) matrix is presented in the Appendix (p.), where its derivation and use in the input-output approach is also indicated. The (I-A) matrix is inverted to obtain the Leontief input-output model of the form,

$$X = [I-A]^{-1}Y , \qquad \text{Eq. (2.2)}$$

where, $[I-A]^{-1}$ = Leontief inverse of demand multipliers which show the total effects -- direct and indirect -- of a one-unit change in industry-specific final demand, Y, on all industry as specified by the individual elements and their total in each column of the Leontief inverse.

All final demand sectors are treated alike with respect to their effects on individual industry outputs. A one-unit increase in the final demand is the same whether the increase occurs in household purchases or government purchases.

Input-output model specification thus requires identification of at least three components as listed in Equation (2.2) -- industry gross outputs, X; final demands for industry gross outputs, Y; and all interindustry transactions, which are shown by a matrix, $[I-A]^{-1}$, of input-output multipliers. This model specification represents the input-output approach as demand based. A change in final demand, ΔY, "drives" the input-output model, thus yielding estimates of corresponding changes in industry outputs, which are indicated by the vector, ΔX, shown earlier in Eq. (1.6).

An alternate specification of the input-output model is given by the form,

$$X(I-C) = V, \qquad \text{Eq. (2.3)}$$

where, $(I-C)$ = matrix of individual disbursement coefficients, c_{ij}, subtracted from an identity matrix, X.

V = value of individual industry primary inputs and imports in dollars.

The c_{ij} coefficient represents the value of disbursements of the i-th industry to the j-th purchasing industry or sector per $1 of total i-th industry disbursements of gross output. Only the diagonal disbursement and technical coefficients would be the same from a given interindustry transactions table. Off-diagonal values would differ (because the denominators of the two ratios would differ for a given X_{ij}). Thus, the inverse of the (I-C) matrix is multiplied by the change in primary inputs and imports to obtain the corresponding change in industry outputs, as indicated by the form,

$$\Delta X = [I-C]^{-1}\Delta V \qquad \text{Eq. (2.4)}$$

In this formulation of the input-output model, a change in industry input supply, rather than output demand, accounts for the corresponding changes in industry outputs (6,7). The input-output model is now supply-constrained rather than demand-constrained and, hence, increases in output will depend upon increases in input supply rather than output demand.

Both the demand-constrained and supply-constrained versions of the input-output approach can be represented totally in terms of output changes by dividing each column and row in the inverse by its corresponding diagonal coefficients. Thus, a series of output multipliers are obtained in place of the demand and supply multipliers specified in Eq. (2.2) and Eq. (2.4), respectively. The new output and input multipliers are specified in the two forms,

$$\Delta X - [I-\hat{A}]^{-1} \Delta XO \qquad \text{Eq. (2.5)}$$

and,

$$\Delta X = [I-\hat{A}]^{-1} \Delta XI \qquad \text{Eq. (2.6)}$$

where, $[I-\hat{A}]^{-1}$ = matrix of adjusted b_{ij}'s obtained by dividing each column of b_{ij}'s by its diagonal element;

$[I-\hat{C}]$ = matrix of adjusted d_{ij}'s obtaining each row of d_{ij}'s by its diagonal element;

ΔXO = given (direct) change in specified industry gross output;

ΔXI = given (direct) change in specified industry gross outlay;

ΔX = derived total (direct and indirect) change in specified industry output.

Therefore, in the two adjusted matrices each diagonal element is equal to unity, and each off-diagonal element also is smaller than its original value.

The adjusted output (i.e., $\hat{b}_{ij}$) multipliers constrast with adjusted input (i.e., $\hat{d}_{ij}$) multipliers in the direction of causality, whether demand-originating or supply-originating. A one-unit change in total output due to a change in output demand results in direct and indirect effects on other industry outputs in proportion to the given industry's backward linkages with other industries in the state. Thus, the larger the local backward linkages, the larger the output multiplier, and the larger the total output change. On the other hand, a one-unit change in total output due to a change in primary input or import supply results in direct and indirect effects on other industry outputs in proportion to the given industry's forward linkages with other industries in the state. Thus, the larger the local forward linkages, the larger the input multiplier, and the larger the total output change.

The input-output relationships specified in the first six equations are static representations of state or regional industry structure. They refer to industry input and output changes in response to changes in specified demand and supply constraints in a given time period. Additional variables, and their relationships with the exogeneous input-output variables, V and Y, must be specified in a dynamic, forecasting model of the state or regional economy depicted by the series of six equations. The additional variables and their relationships are discussed in the last two chapters. Implementation of the static input-output model is discussed next.

Data Collection and Preparation

Two distinctly different methods -- one direct (see, ref. 9,11), the other indirect (see, ref. 12,23) -- and varying combinations of these two methods (see, ref. 10,20), have been used in preparing state and regional input-output tables. The direct method makes use of business, household and government surveys in the estimation of individual industry sales and purchases, and individual sector disbursements and receipts. Usually, surveys include high proportions of all large establishments and low proportions of small establishments. The number of households is small, also, while all government units are likely to be surveyed.

Size of sample is dictated by size of industry, desired accuracy of estimates, and total survey budget. For most studies, the primary survey costs are much too high to warrant use of survey data only in the preparation of state or regional input-output tables.

The indirect method makes use of existing published and unpublished statistics of business, household and government activities. Much of these data is obtained from reporting requirements of state unemployment insurance programs and state sales and income tax laws. The U.S. Department of Commerce also publishes detailed annual statistics of employment and income for each state. Comparable statistical series are available for the entire U.S., also. Thus, ratios of state employment or income to corresponding U.S. employment or income can be derived for use in allocating U.S. industry gross outputs to individual states.

A University of Minnesota two-region input-output computer program is available for making use of state and national statistical series, along with U.S. input-output tables, in the preparation of U.S. two-region input-output tables (12,21, 22,25,26,27). This is an efficient, special purpose computer program which fully utilizes existing data series in the implementation of indirect input-output estimation procedures.

Combined direct and indirect input-output estimation methods make use of both survey data (covering mostly manufacturing industries and large establishments in selected non-manufacturing industries) and existing comparable area input-output tables. This method, while less costly than a completely survey-based estimation procedure, is much more costly than the indirect estimation procedures and, also, less complete in its implementation of the import sector for both intermediate and final purchasing sectors. Neither the direct nor the combined methods usually provide import matrices (i.e., tables of specific local industry purchases from specific out-of-state or out-of-region industries) for a state or region to serve as a source of additional information for later adjustments which incorporate changes in individual industry exports and imports . Updating of input-output tables based on combined estimation methods is difficult without access to import matrices for deriving the effects of specified input-output changes on import requirements and input-output relationships.

Implementation of the input-output approach is usually in terms of the convention established by Leontief, namely, that producers' prices apply to all industry gross output, except in the case of the wholesale and retail trade group where only the trading margins are included. In the alternate formulation of the input-output approach, the originating industry of all goods which are resold would be identified in an input-supplying industry in Quadrant I. In this formulation, all imports from rest-of-nation would be received by a purchasing industry and, hence, included in Quadrant III. In the conventional input-output formulation, however, imports of goods for resale are shown under the appropriate final purchasing sector (as would the originating local industry of all final purchases), and they are entered in both Quadrant III and Quadrant IV.

Calibration, Documentation and Verification

Implementation of the input-output model is followed by its calibration, documentation and verification -- the most important steps for model acceptance and application (28). Calibration usually refers to parameter and variable adjustments which allow the model forecasts to track actual events. For example, if the input-output model is based on 1972 data it may not forecast 1977 or 1980 industry output levels because of the structural effects of post-1972 price increases. A calibration procedure is available to adjust the 1972 input-output coefficients to 1977 or 1980 prices relationships which results in improved forecast accuracy (see, p.). Documentation refers to the exact listing and identification of specific data sources and computational procedures for replicating the working model and its results by another model builder or user. Verification, finally, is the reality-testing part of model specification. It refers to the logical fit of the model and the overall conformance of model implementation with model specification.

Model calibration is the first step following model implementation. It includes the initial comparisons of model forecasts with actual events. For example, if 1973 final demands were given in 1972 dollars, then Eq. (2.2) would be used to forecast 1973 industry gross outputs in 1972 dollars. Similarly, other post-1972 forecasts would be prepared and, also, compared with actual industry output levels -- all in 1972 dollars. Large differences between forecast and actual output levels would be examined for probable sources of structural change. These differences may be tolerable insofar as they more or less balance for the economy and also yield acceptable levels of aggregate industry output and value added. Input-output ratios may be adjusted for some industries when these adjustments improve both individual industry and aggregate industry forecasts.

Preparation of the U.S. and Minnesota 1977 input-output tables was based on a two-step calibration procedure, starting with forecasts of 1977 U.S. industry final demands, given acual 1977 industry output levels, and the adjustment of these forecasts to actual 1977 national gross product and export and import levels. This step involved recomputation of input-output coefficients. The 1977 industry output levels, in 1972 dollars, were then adjusted to 1977 price levels and a second new interindustry transactions table was created. This step resulted in further changes in input-output relationships and, hence, required another recomputation of input-output tables. The first part of the two-step procedure would be repeated for the post-1977 period,for example, in the preparation of 1978 industry output forecasts, based on 1978 given or forecast final demand levels, and these forecasts would be compared with actual 1978 industry output levels. Again, differences between forecast and actual output levels are likely, but these differences may balance and the aggregate forecast levels of economic activity may compare closely with actual levels.

Additional post-1977 forecasts would be prepared to more completely determine the extent of individual industry and aggregate industry differences between the forecast and the actual series and the acceptability of these differences, if any, as measures of forecast accuracy and tests of model reliability. Both the additional and the initial series of comparisons are part of model validation, which is discussed next. The correspondence of actual computer programs and the initially specified input-output model and its assumptions would be verified, and also validated, if the two were identical. The verification step focuses on model implementation and its conformance with model specification; in short, whether or not the model is, indeed, what it purports to be.

Validation and Acceptance

Next to documentation, verification and validation are considered the most important steps in model acceptance (28). Validation differs from verification by its focus on reality and the conformance of model assumptions and forecasts with actual events. It addresses the issue of reasonableness of fit between the forecast and the actual event.

A model may be re-calibrated, because of the perceived lack of forecast reasonableness, as in the case of the 1972 U.S. input-output model (which was re-calibrated when used to forecast post-1977 industry output changes). Certain tests of forecast reasonableness are introduced in the validation step as a basis for deciding whether or not model refitting and re-calibration is necessary and desirable. These tests are discussed later in the discussion of model use.

The final test of model adequacy is its acceptance by the model builder and model user. Model rejection may be due to any one of the steps towards model acceptance, or it may be rejected because of its lack of timeliness and/or its high development, maintenance, and utilization costs. The latter constraints to model acceptance are considered also with reference to model use in impact analysis and forecasting.

Validation of an input-output model is less difficult than validation of the dynamic forecasting system cited earlier of which the input-output model is a part (23). Even with the input-output model, validation procedures may require indirect, rather than direct, approaches (28). For example, alternatively a small area model may be used to prepare a reference forecast series for comparison with the input-output-based results. Large unexplained differences between the two sets of forecasts would signal a need to re-evaluate the reliability of both models, and especially the input-output model.

The six topical areas of model building discussed in this section deal with the design, implementation, assessment, and acceptance of the input-output model in labor market analysis and forecasting. The six areas are interrelated to one another. Ultimately, model acceptance depends on feedback from decision maker to model user and from model user to model builder. Because of interaction between model user and model builder, feedback starts in early stages of model building, indeed, with problem and area delineation. The final stages of model building are most important, however, because of the progressive and accumulative nature of the model building process itself. Feedback from decisions makers to model builder may not convey fully the lack of model acceptance, and the reasons for it. Familarity with the decision making processes in which model forecasts become involved thus becomes an additional pre-condition of successful model building.

DATA INTERPRETATION

Data interpretation refers to activities surrounding the use of model output in decision making. The model builder interprets the input-output findings for the model user, who in turn interprets them for the decision makers. Neither the data input nor the data output are self-explanatory; they require competent and careful interpretation if they are to be used effectively in model building or in model use.

Direct and Indirect Effects

The demand and supply multipliers obtained from the (I-A) and (I-c) inverses are used in calculating individual industry output effects of given changes in final demand or primary inputs and imports. Whether or not the particular use of input-output multipliers is appropriate is a question, again, of interpretation, in this case, of the multiplier relationships with particular demand and supply variables.

The multiplier effect in the conventional demand-centered input-output analysis results from its linkages with local input-supplying industries. For example, in the case of the agriculture and mining industry group, the total multiplier of 2.09940 (see,Table 1.3) is due to the internal linkages of this industry and its "backward" linkages with the construction and manufacturing industry group and the services industry group. The direct linkages account for 0.55455 dollars of purchase per $1 total purchases (see, Table 1.2). Thus, the indirect linkages much account for the remaining 1.54485 dollars of the 2.09940-dollar total effect. In summary, the direct and indirect effects included in the total multiplier for the agriculture and mining industry group are distributed among the three industry groups as follows:

Industry	Direct	Indirect	Total
Agr., Mining	0.22727	1.10935	1.33662
Constr., Mfg.	0.16364	0.24503	0.40867
Services	0.16364	0.19047	0.35411
Total	0.55455	1.4485	2.09940

Inclusion of the household sector with the interacting local industries sharply increases the individual multiplier values. First, the Type II total multiplier for the agriculture and mining industry group is nearly twice as large as the Type I multiplier -- 4.19668 as compared with 2.09940 (see, Appendix, p. 60). This expansion of the Leontief inverse by one row and one column had brought the induced effects of household spending into the computation of the total multiplier effects. The distribution of the total induced effect among the three industry groups is shown as follows:

Industry	Direct	Indirect	Induced	Total
Agr., Mining	0.22727	1.10935	0.06564	1.40226
Const., Mfg.	0.16364	0.24503	0.50167	0.91034
Services	0.16364	0.19047	0.76841	1.12252
Households	0.10000	0.66156	---	0.76156
Total	0.65455	2.20641	1.33572	4.19668

The induced effect here refers to the added impact of household spending on the industry groups, while the direct effect includes the added contribution of household purchases from the three input-supplying industries. The size of the induced effect is directly related to the proportion that labor is of total input purchases. The larger the value of labor inputs per $1 total purchases, the larger the induced effect. More than half of the induced effect of a $1 increase in the demand for agriculture and mining industry output is due to the purchase of services by this industry.

Both the Type I and the Type II input-output multipliers are related to changes in certain exogeneous variables, like exports and imports, which are external to the interacting industries and sectors included in Quadrant I of the interindustry transactions table. Use of the multipliers depends, therefore, on an accurate estimate or forecast of external change -- its magnitude and its relationships with the interacting industries and sectors. The internal changes are industry specific; their local impact depends on the backward or forward linkages of each industry or sector with other industries or sectors which are located in Quadrant I of the interindustry transactions table. Each of the backward and forward linkages of the external final demand, primary input, and export and import sectors with the internally interacting industries and sectors are delineated and discussed next.

Industry Sales and Purchases

Implementation of the input-output approach, based on secondary data, starts wtih the estimation of total industry sales and purchases and the use of these estimates as control totals in the determination of individual industry transactions. In this section, 1972 U.S. industry sales and purchases were derived for a 10-industry breakdown of the total U.S. economy, which was depicted earlier in the three-industry representation of the U.S. economy in Table 1.1. The presentation here differs, however, from the earlier presentation in more than the additional industry detail: Industry disbursements refer to individual commodity groups while industry purchases refer to individual industry groups. One industry may produce more than one commodity. Similarly, a given commodity may be prodeced by more than one industry.

The 10-industry breakdown cited earlier is presented in Table 3.1, to show input purchases of each of the 10 industry groups from the 10 commodity-producing groups, the three primary input sectors, and the rest-of-world

Table 3.1. Intermediate Purchases of Specified Commodities and Primary Inputs by Intermediate Demand Sectors, U.S., 1972. 1/

	Goods-Producing Industry					Services-Producing Industry					
				Manufacturing							
No. Title	Agri-culture	Mining	Con-struc-tion	Nondur-ables	Durables	Trans., Comm., Util.	Trade	Fin., Ins., Real Est.	Services	Gov. Enter., Scrap	Total Intermediate Purchases
					(million $)						
1. Agriculture	23,412	0	161	38,444	2,008	22	16	128	1,680	182	66,023
2. Mining	134	1,653	1,407	16,284	5,247	6,027	0	612	29	327	31,720
3. Construction	528	857	47	1,704	1,595	5,013	863	11,085	2,571	2,672	26,995
4. Mfg., Nondurables	10,345	810	4,256	60,741	98,976	5,582	4,340	2,763	27,072	917	187,556
5. Mfg., Durables	3,608	2,041	50,608	65,945	21,933	3,655	783	317	14,637	297	239,961
6. Tran., Comm., Util.	2,006	1,002	4,953	12,645	66,900	23,290	9,096	4,272	10,399	3,152	92,778
7. Trade	2,927	453	12,797	11,944	15,039	2,329	3,011	902	7,322	184	56,908
8. Fin., Ins., Real	5,725	3,625	2,056	6,723	6,618	5,279	11,914	33,086	17,425	716	93,167
9. Services	1,257	1,015	9,757	17,052	16,815	9,074	18,927	12,012	25,281	1,054	112,332
10. Gov. Ent; Scrap	12	41	62	901	577	549	1,167	2,070	1,606	50	7,035
11. Total	49,955	11,497	89,832	231,393	232,144	61,683	50,117	67,247	109,707	9,540	914,474
Primary Input:											
12. Emp. Comp.	4,805	6,278	60,155	76,720	127,327	51,343	91,963	39,537	109,930	12,260	580,318
13. Ind. Bus. Tax	1,554	1,401	1,218	14,341	2,770	11,659	36,716	34,088	7,183	51	110,981
14. Prop.-Type Inc.	24,070	11,207	14,734	31,809	42,155	35,510	37,424	111,430	44,154	2,350	354,843
15. Total Value Added	30,429	18,996	76,107	122,870	122,256	98,512	166,103	185,055	161,365	14,661	1,046,142
Rest-of-World:											
Noncomp. Imp.	5	7	59	1,892	640	1,410	614	166	380	381	5,103
Dummy Ind.	0	0	0	0	0	0	0	0	0	0	0
Gross Outlay 2/	80,390	30,386	165,998	356,155	405,036	161,005	216,384	252,467	269,749	24,583	1,965,719

sector (Table 3.1). In 1972, the agriculture industry, for example, accounted for $80 billion of the $1,966 billion of all industry purchases. Of this total domestic commodity purchases were $50 billion, or 62 percent. Intermediate input purchases thus were one and two-thirds times the primary input purchases. The most important intermediate purchases originated in the agriculture industry itself and in nondurable goods manufacturing. Each of the remaining 10 input-supplying industries was a source of agriculture industry inputs.

Purchases of other industry groups differed sharply from purchasing patterns of the agriculture industry group. In the U.S. economy, where very few inputs are not produced domestically (and, hence, noncomparable imports are small), the input purchases conform to the technological requirements of each industry as represented by the production function for that industry. In the input-output approach, this production function is linear and constant in its input-output relationships. For the open economy, of course, imports from rest-of-nation must be taken into account when using an industry production function to estimate or verify survey-based estimates of corresponding input purchases.

An input-output table of the Minnesota economy differs from the corresponding U.S. input-output table by the much larger purchases of intermediate inputs from industries located outside Minnesota, but in the U.S., as shown in Table 3.2. In the Minnesota table, however, imports from rest of nation include inputs which may be produced in the state, also, but which are less than total requirements. When imports exceed exports of any industry output, the net import figure is entered in the import row of the interindustry transactions table.

The Minnesota industry sales and purchases in Table 3.2 were estimated entirely from existing data sources with the use of the computer program

Table 3.2. Intermediate Purchases of Specified Industry Output and Primary Inputs by Intermediate Demand Sectors, Minnesota, 1972.1/

No.	Industry Title	Goods-Producing Industry: Agri-culture	Goods-Producing Industry: Mining	Goods-Producing Industry: Con-struction	Manufacturing: Nondur-ables	Manufacturing: Dur-ables	Services-Producing Industry: Trans. Comm., Util.	Services-Producing Industry: Trade	Services-Producing Industry: Fin., Ins., Real Est.	Services-Producing Industry: Services	Services-Producing Industry: Govern. Enter., Scrap	Total Intermediate Purchases 2/
						(thousand $)						
1.	Agriculture	918,587	0	2,989	1,626,681	37,095	3,593	16,770	8,288	11,098	2,144	2,627,245
2.	Mining	3,622	14,249	17,573	16,200	26,994	8,141	0	0	0	25	86,804
3.	Construction	22,792	16,257	840	24,622	17,771	106,843	21,119	185,823	42,991	35,773	474,531
4.	Mfg., Nondur.	239,398	12,695	81,159	1,603,544	69,799	55,777	308,650	38,077	328,821	12,608	2,750,528
5.	Mfg., Durables	43,786	47,673	638,140	205,391	1,936,974	401,897	24,491	6,160	169,221	15,872	3,125,565
6.	Tran.,Comm.,Util.	78,690	48,631	81,617	271,454	223,959	404,434	190,921	57,222	210,122	44,234	1,611,284
7.	Trade	110,058	15,471	241,845	284,714	276,717	58,316	171,506	44,077	121,039	4,588	1,328,301
8.	Fin., Ins.,Real	190,009	96,479	31,630	84,138	116,765	76,120	235,694	511,563	209,554	8,946	1,560,898
9.	Services	56,061	22,025	156,867	223,675	202,258	134,262	370,812	174,100	349,554	13,922	1,703,536
10.	Gov. Ent., Scrap	550	7,577	1,531	26,013	31,345	127,163	54,314	32,282	30,319	660	311,754
11.	Total	1,663,553	281,057	1,254,191	4,366,432	2,939,677	1,012,546	1,394,277	1,057,292	1,472,719	138,702	15,580,446
	Primary Inputs and Imports:											
12.	Value Added	1,235,761	275,516	1,359,516	1,849,193	2,718,412	1,734,623	3,776,622	3,092,059	2,479,068	242,068	18,762,838
13.	Imports	381,174	109,810	351,564	1,061,219	1,389,519	284,850	248,557	114,470	304,719	33,390	4,279,272
14.	Gross Outlay 2/	3,280,490	666,384	2,965,272	7,276,841	7,047,608	3,032,018	5,419,457	4,263,822	4,256,506	414,158	38,622,556

1/ Wilbur R. Maki, Peter L. Stenberg and Mason Chen. Economic Importance of Export Producing Industry in Minnesota. Staff Paper Series P81-3, Department of Agricultural and Applied Economics, University of Minnesota, St. Paul. 1981.

2/ Individual entries may not sum to totals because of rounding.

for the Minnesota two-region input-output model (12). Minnesota industry gross outputs and final demands were estimated, first, from a wide range of data sources. A series of input-output tables were derived subsequently which show industry output disbursements to individual industries and sectors in (1) Minnesota and (2) rest of nation. Two regional and two interregional (i.e., industry-specific exports from Minnesota to rest-of-nation and industry-specific imports from rest-of-nation to Minnesota) interindustry transactions tables were prepared with the use of the two-region computer program.

Access to an industry-specific import matrix facilitates revisions of the Minnesota interindustry transactions table when export-import balances shift from net exports to net imports. The two-region computer program also provides import and export multipliers which represent reductions in the regional multipliers due to imports from rest of nation.

Intermediate input purchases from indus-ries located in Minnesota, as a proportion of total purchases of a specific industry, will not exceed the U.S. proportion of domestic intermediate input purchases for the same industry. Any purchases of imports will reduce this internal linkage. For three Minnesota industry groups in Table 3.2, however, the internal backward linkages appear greater than for the U.S. because of industry mix. Those industries with large backward linkages were relatively more important in Minnesota than in the rest of nation. Statistical measures of their backward linkages and their relative importance are summarized as follows:

Industry	Intermediate Inputs as Prop. of Total Purchases Minn. (pct.)	U.S. (pct.)	Minnesota Total Purchases as Prop. of U.S. (pct.)
Agriculture	50.8	62.1	3.997
Mining	42.2	37.8	2.193
Construction	42.3	54.1	1.786
Mfg., Durables	41.7	57.3	2.408
Mfg., Nondurables	70.8	65.0	1.291
Transportation	33.3	38.6	2.081
Comm., Util.	33.5	37.0	1.711
Trade	25.7	23.2	2.505
Fin., Ins., Real Est.	24.8	26.6	1.689
Services	34.8	40.1	1.603
Govern. Enter.	31.6	38.8	1.576
Average	48.6	46.5	1.965

Industry mix differences in mining, nondurable goods manufacturing and trade account for the high lebels of intermediate inputs in these industries. For two ot the three industries -- mining and trade -- total purchases also were above average relative to U.S. total purchases.

The relative importance of each Minnesota industry group is indicated by the proportion of Minnesota to U.S. total purchases (in the third column above). A high proportion of total purchases, which are identical to total sales, will not also represent high proportions of employment and value added. Indeed, Minnesota mining employment is low relative to U.S. mining employment, while service employment is high.

Final Purchases and Value Added

Final purchases of commodities by each final demand sector, including rest-of-world, are listed, for the U.S. in Table 3.3. In 1972, final purchases of domestically produced commodities exceeded $1 trillion. Final purchases of primary inputs (household, and government employment and inventory adjustments) and of noncomparable imports accounted for more than $100 billion, which resulted in total final purchases of nearly $1.2 trillion in 1972.

Table 3.3. Final Purchases of Specified Commodities and Primary Inputs by Final Demand Sectors, U.S., 1972. 1/

Commodity No.	Commodity Title	Domestic Final Demand: Personal Consump. Expendi-tures	Domestic Final Demand: Government: Federal	Domestic Final Demand: Government: State & Local	Domestic Final Demand: Investment: Gross Priv. Cap. Formation	Domestic Final Demand: Investment: Chg. in Bus. Inv.	Domestic Final Demand: Total	Rest-of-World: Compe-tative Exports	Rest-of-World: Compe-tative Imports	Total Final Purchases	Total Commodity Output 2/
		(million $)									
1.	Agriculture	6,882	-1,511	267	0	2,510	8,151	4,979	-2,041	11,089	77,112
2.	Mining	3,113	11	22	252	206	627	729	-4,072	-2,716	29,075
3.	Construction	0	6,471	33,429	99,086	0	138,986	16	0	139,002	165,997
4.	Mfg., Nondur.	149,782	3,493	7,240	814	4,463	166,292	11,253	-18,464	159,081	346,623
5.	Mfg., Durables	66,773	24,070	5,676	66,566	8,892	171,977	27,255	-2,478	196,754	483,545
6.	Tran., Comm., Util.	55,094	4,253	5,427	3,380	529	68,683	5,762	-1,567	72,878	165,770
7.	Trade	140,323	1,130	1,582	10,204	1,000	154,237	4,089	2,993	161,319	218,236
8.	Fin., Ins., Real	147,700	1,144	3,821	4,432	0	157,096	2,290	-165	159,221	252,388
9.	Services	158,733	6,979	10,008	192	-164	175,749	1,335	-53	177,031	301,729
10.	Gov. Ent., Scrap	4,156	444	594	0	0	5,193	140	0	5,333	12,367
11.	Total	729,697	49,505	68,069	184,926	17,936	1,050,134	57,946	-56,835	1,051,245	1,964,290
	Primary Input:										
12.	Emp. Comp.	5,349	49,329	82,019	0	0	137,297	0	0	137,297	717,663
13.	Ind. Bus. Tax	0	0	0	0	0	0	0	0	0	110,981
14.	Prop.-Type Inc.	0	0	0	0	-7,591	-7,591	0	0	-7,591	354,112
15.	Total Value Added	5,349	49,329	82,019	0	-7,591	129,706	0	0	129,706	1,182,766
	Rest-of-World										
16.	Noncomp. Imp.	6,550	3,497	5	5	4	10,059	681	-15,843	-5,103	0
17.	Dummy Ind.	3,524	-205	0	0	0	-3,727	14,167	-3,521	6,919	6,919
18.	Gross Outlay	738,072	102,126	150,693	184,931	10,350	1,186,172	72,794	-76,199	1,182,767	3,148,485

The distribution of the U.S. final product among the five final demand sectors listed in Table 4.1 is summarized as follows:

Sector	Domestic Commodities: Total (bil.$)	Domestic Commodities: Prop. of Total (%)	All Final Purchases: Total (bil.$)	All Final Purchases: Prop. of Total (%)
Pers. Cons. Exp.	729.7	69.5	738.1	62.2
State & Local	68.1	6.5	150.7	12.7
Federal	49.5	4.7	102.1	8.6
Gr. Priv. Cap. Form.	184.9	17.6	184.9	15.6
Change in Bus. Inv.	17.9	1.7	10.4	0.9
Total	1,050.1	100.0	1,186.2	100.0

Nearly 70 percent of the final purchases of U.S. commodity output were made by the household sector, while government accounted for an additional 11 percent and investment for the remaining 19 percent. When primary input and noncomparable import purchases are included, the household and investment shares dropped to 62 percent and 17 percent, respectively, while the government share increased to 21 percent.

Domestic final product plus net exports equals domestic value added in the form,

$$DFP + (EXP - IMP) = VA,$$

or, $$1{,}186.2 + (72.8 - 76.2) = 1{,}182.8;$$

where, DFP = domestic final product in billion dollars,

EXP = total U.S. competitive exports in billion dollars,

IMP = total U.S. competitive imports in billion dollars,

VA = domestic value added in billion dollars.

Domestic value added originates from both producing industries and final demand sectors in the form of employee compensation, indirect tax receipts and property-type income. Value added is distributed between the intermediate and final demand sectors and among the three primary input sectors as follows:

Value Added	Prod. Sectors Total (bil.$)	Prod. Sectors Prop. of Total (%)	All Sectors Total (bil.$)	All Sectors Prop. of Total (%)
Employee Comp.	580.3	55.5	717.7	60.7
Indirect Bus. Taxes	111.0	10.6	111.0	9.4
Property-Type Inc.	354.8	33.9	354.1	29.9
Total	1,046.1	100.0	1,182.8	100.0

Thus, for the U.S. economy, employee compensation accounted for nearly 61 percent of total value added. In the private sector alone, however, employee componsation accounted for nearly 66 percent of total value added while property-type, including proprietorial income, was nearly 34 percent of this total.

The distribution of final purchases in Minnesota compared closely with the 1972 U.S. distribution (Table 3.4). Personal consumption expenditures accounted for nearly 70 percent of final purchases from local industry and slightly more than 62 percent of all final purchases. Government purchases were 10 percent of local industry purchases and 19 percent of all final purchases. Compensation of government employees was equivalent to 10 percent of final purchases. Private investment expenditures in Minnesota also compared closely with the U.S. pattern, accounting for over 18 percent of the Minnesota final product. The sector distribution of the 1972 Minnesota final product is summarized as follows:

Sector	Purchases From Local Industry Total (mil.$)	Purchases From Local Industry Prop. of Total (%)	All Final Purchases Total (mil.$)	All Final Purchases Prop. of Total (%)
Pers. Cons. Exp.	10,945	69.4	12,995	62.5
State & Local	1,179	7.5	2,863	13.8
Federal	371	2.4	1,105	5.3
Gr. Priv. Cap. Form.	2,836	18.2	3,475	16.7
Change in Bus. Inv.	386	2.5	343	1.7
Total	15,617	100.0	20,780	100.0

Table 3.4. Final Purchases of Specified Industry Output and Primary Inputs by Final Demand Secotrs, Minnesota, 1972. 1/

Industry No.	Industry Title	Local Final Demand: Personal Consumption Expenditures	Local Final Demand: Government: Federal	Local Final Demand: Government: State & Local	Local Final Demand: Investment: Gross Priv. Cap. Form.	Local Final Demand: Investment: Chg. in Bus. Inv.	Local Final Demand: Total	Rest-of-World: Allocated U.S.: Exports	Rest-of-World: Allocated U.S.: Imports	Rest-of-World: Exports RON	Total Final Purchases	Total Industry Output 2/
		(thousand $)										
1.	Agriculture	77,538	-9,914	4,180	0	60,634	132,438	68,487	-12,734	465,056	653,247	3,280,490
2.	Mining	83	-305	-224	731	949	1,234	3,534	-17,343	592,156	578,347	666,384
3.	Construction	0	69,352	613,498	1,777,298	0	2,460,148	267	0	30,325	2,490,740	2,965,272
4.	Mfg., Nondur.	1,471,206	44,355	73,124	224,500	93,194	1,906,379	133,931	-149,540	2,636,418	4,542,797	7,276,841
5.	Mfg., Durables	795,735	125,657	83,730	624,159	177,612	1,806,893	233,833	-234,272	2,114,711	3,922,043	7,047,608
6.	Tran.,Comm.,Util.	842,310	38,839	92,461	52,309	16,777	1,042,696	73,457	-22,582	327,163	1,420,734	3,032,018
7.	Trade	3,130,672	16,102	-3,684	191,719	33,135	3,367,944	71,083	46,454	605,676	4,091,157	5,419,457
8.	Fin.,Ins.,Real	2,445,945	10,669	63,247	62,603	0	2,582,464	30,944	-2,556	92,081	2,702,923	4,263,822
9.	Services	1,975,655	67,156	221,782	3,110	-2,958	2,264,745	19,176	-626	269,676	2,552,911	4,256,506
10.	Gov. Ent., Scrap	106,113	8,690	30,839	-100,190	6,761	52,213	17,835	-17,530	49,887	102,405	414,158
11.	Total	10,845,257	370,601	1,178,953	2,836,239	386,104	15,617,154	652,547	-410,729	7,183,139	23,042,111	38,022,556
	Primary Inputs and Imports:											
12.	Value Added	32,124	531,516	1,569,914	0	-251,451	1,882,103	245,755	-54,650	39,302	2,112,510	20,875,348
13.	Imports	2,117,245	202,843	114,583	638,417	208,180	3,281,268	352,642	-471,360	0	3,162,550	7,441,822
14.	Gross Outalys 2/	12,994,626	1,104,960	2,863,450	3,474,657	342,832	20,780,252	1,250,944	-936,739	7,222,440	28,317,170	66,939,726

1/ Wilbur R. Maki, Peter L. Stenberg and Mason Chen. Economic Importance of Export Producing Industry In Minnesota. Staff Paper Series P81-3, Department of Agricultural and Applied Economics, University of Minnesota, St. Paul. 1981.

2/ Individual entries may not sum to totals because of rounding.

Following Equation (4.1), the equality between final product and value added for the 1972 Minnesota economy is represented by the equation,

$$VA = 20{,}780 + (8{,}473 - 8{,}378) = 20{,}875 \qquad \text{Eq. (4.3)}$$

Thus, the 1972 Minnesota gross state product, as represented by total value added, was nearly $21 billion.

Exports and Imports

In 1972, U.S. competitive exports were slightly less than U.S. competitive imports, which together with noncomparable imports resulted in a negative balance of trade of $14.1 billion, as shown below:

Item	Total (bil. $)
Competitive exports	57.9
Competitive imports	-56.8
Noncomparable imports:	
Intermediate inputs	-5,1
Final purchases	-10.1
Total	-14.1

The overall balance of trade deficit was less the $14.1 billion because of intersectoral transfers (which are shown in Table 3 .3).

All U.S. foreign trade items are entered in the Minnesota interindustry transactions tables. In addition, net exports and net imports, derived from the Minnesota two-region input-output data and procedures, are included in the determination of state and regional balance of trade, as shown below:

Item	Total (mil.$)
U.S. Competitive exports	652
U.S. Competitive imports	-411
Minn. net exports	7,183
Minn. net imports (inc. noncomp.):	
Intermediate inputs	-4,279
Final purchases	-3,281
Total	-36

Thus, an apparent net balance of trade of -36 million is estimated for Minnesota in 1972. Because of intersectoral transfers with rest of nation, however, the Minnesota net balance of payments would differ from its net balance of trade. A positive overall balance of trade is indicated for Minnesota in Table 3.4 because of the inclusion of certain rest-of-nation transfers which were included, also, in the U.S. input-output table (26,27).

Derivation of export and import levels for Minnesota depends entirely upon the procedures for allocating U.S. competitive exports and imports and noncomparable imports to Minnesota, differences between total industry output and industry-specific input requirements, and the Minnesota industry output levels relative to corresponding U.S. industry output levels. The Minnesota two-region input-output data base and computer program deal with these factors simultaneously in the derivation of the external trade flows.

Employment and Earnings

Employment and earnings of the employed work force are related to industry output in deriving a variety of economic indicators, including output per worker, value added per worker, wages and salaries and other employee compensation per worker, and total hours worked. In addition, employment and income multipliers can be derived from these data as direct measures of the effects of given changes in industry employment and value added on the economic indicators cited earlier. In this section the derivation and use of employment and income multipliers are cited with reference to industry value added, as represented in Table 1.2.

The first step in the derivation of the industry value added multipliers is preparation of the value added matrix (which is discussed in the Appendix). This matrix provides a set of value added coefficients for converting the demand multipliers in Table 1.3 into value added mutlipliers. In effect, the value added conversion matrix is a series of value added coefficient ratios which account for industries in output per $1 value added -- the larger the ratio, the larger the value added impact, or, conversely, the smaller the value added coefficient, as given in Table 1.2, the larger the value added multiplier, as shown in Table 3.5. In this case, the value added multipliers in Table 3.5 vary less than the demand multipliers in Table 1.3 because of the compensating effects of the value added conversion coefficients. However, the rank order of the multipliers remains the same as a result of both a similarity in the two sets of rankings and nearly equal differences in the absolute values between the first-to-second-ranking, and second-to-third-ranking coefficients.

Interpretation of the value added multiplier is similar to the interpretation of the demand multiplier. Indeed, the value added multiplier is a form of demand multiplier, that is, it related to changes in industry value added rather than industry gross output. For example, the total value added effect of an increase in the demand for a specified industry output which is equivalent to a $1 increase in industry value added is represented by the total value added multiplier for this industry. An industry with high value added per unit of output would have a low output change relative to other industries and, hence, the value added multiplier is small and the total value added effect of a $1 increase in specified industry value added demand is also small.

Table 3.5. Illustrative Input-Output Total: Total Effect of a $1 Change in Final Demand for Specified Industry Value Added.

Sector	Goods		Services
	Agr. & Mining	Constr. & Mfg.	
Agr., Mining	1.33662	0.14587	0.05690
Constr., Mfg.	0.45731	1.70565	0.38130
Services	0.23283	0.21120	1.31282
Total	2.02676	2.06272	1.75102

EMPLOYMENT ANALYSIS

Use of detailed input-output tables in industry employment analysis is illustrated by U.S. input-output data for 1972. Both U.S. Department of Commerce and U.S. Bureau of Labor Statistics input-output data sources were consulted in the preparation of the U.S. data series presented here. Only the U.S. data series are presented in this report. Later reports in this report series will include Minnesota 1972 employment estimates which are compatible with the U.S. estimates.

Two different data series are presented -- one from the U.S. Department of Commerce, the other from the U.S. Bureau of Labor Statistics (34,35,37,38 39,40). The 80-industry breakdown from the 1972 U.S. Department of Commerce, Interindustry Economics Division input-output tables is used for both data series (Table 4.1). The Minnesota 214-industry classification system in Table 2.1 can be aggregated into the 80-industry classification in Table 4.1.

The two data series are compared in terms of (a) total employment and income and (b) per worker and per hour employment and income levels in each of the 80-industries. These comparisons are discussed, finally, with reference to state-level industry employment analysis, specifically, Minnesota.

Industry Employment and Income

Individual industry employment and income levels refer to the data base of two different input-output tables as noted earlier. Differences occur between the two models because of underlying differences in industry classification and agency orientation. The 1972 U.S. Department of Commerce input-output tables are based on the 1972 Standard Industry Classification while the U.S. Bureau of Labor Statistics input-output tables are based on the 1967 Standard Industry Classification. The 1967 U.S. Department of Commerce

Table 4.1. Relation of Minnesota 85-Industry Classification Code to U.S. and Minnesota Industry Classification Codes, 1981

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Industry No.	Industry Title	BLS 154-Ind.	Minnesota Sector 75-Ind.	Minnesota Sector 214-Ind.	Standard Industry Classification Code (1972 Edition)
1.	Live. & live. prod.	1,2	1,2	1-3	0241,pt.0191,025(exc.0254,pt.0259),21(exc.pt.0219), pt.0259,pt.0291
2.	Other agr. prod.	3-5	3,4	4-8	pt.011,pt.013,pt.014,pt.017,pt.1091,pt.0219,pt.0254
3.	For. & fish. prod.	6	5	9	081-4,091,097
4.	Agr.,for.,& fish.	7	6	10	0254,07(exc.074),085,092
5.	Iron & ferro. ores	8	7	11	101,106
6.	Nonferr. metal	9,10	8	12,13	102-5,pt.108,109
7.	Coal mining	11	9	14	1111,pt.1112,1211,pt.1213
8.	Cr. pet. & nat. gas	12	10	15	131,132,pt.138
9.	Stone & clay, quar.	13	11	16	141-5,pt.148,149
10.	Chem. & fertilizer	14	12	17	147
11.	New construction	15-20	13	18-23	pt.108,pt.1112,pt.1213,pt.138,pt.148,pt.15-17
12.	Maint. & repair	21	14	24	pt.138,pt.15-17
13.	Ord. & accessories	22,23	15	25,26	3482-4,3489,3761,3795
14.	Food & kindred	24-33	16-pt.22	27-55	20
15.	Tobacco mfgr.	34	pt.22	56	21
16.	Br. & nar. fab.	35	pt.23	57	221-4,226,228
17.	Misc. tex. goods	36,37	pt.23	58,59	227,229
18.	Apparel	38,39	pt.24	60,61	225,231-8,39996
19.	Misc..fab. text.	40	pt.24	62	239
20.	Lumber & wood	41-3	25,26,pt.27	63-74	23(exc.2441,2449)
21.	Wood containers	44	pt.27	75	2441,2449
22.	House. furniture	45	pt.28	76,77	251
23.	Other furn. & fix.	46	pt.28	78,79	252-4,259
24.	Paper & allied	47	29	80-84	26(exc.265)
25.	Paper containers	48	30	85	265
26.	Print. & publ.	49-51	31	86-8	27
27.	Chem. & related	52-4	pt.32	89-91	281(exc.28195,2865,2869),286,287,289
28.	Plas. & synthetic	55,56	pt.32	92,93	282
29.	Drugs, cleaning	57,58	pt.32	94,95	283,284
30.	Paints & allied	59	pt.32	96	285
31.	Petr. ref. & rel.	60	33	97,98	29
32.	Rubber & misc.	61-3	34	99-101	30
33.	Leather tan. & fin.	64	pt.35	102	311
34.	Footwear & other	65	pt.35	103	313-7,319
35.	Glass & gl. prod.	66	pt.36	104	321-3
36.	Stone & clay prod.	67-70	pt.36	105-112	324-9
37.	Prim. iron & steel	71,72	37,38	113-121	331,332,339,3462
38.	Prim. nonferrous	73-5	39,40.	122-5	333-6,3463
39.	Metal containers	76	pt.41	126	341
40.	Heat.,plumb. & fab.	77,78	pt.41	127,128	343,344
41.	Screw mach. prod.	79,80	pt.41	129,130	345,346
42.	Other fab. metals	81,82	pt.41	131,132	342,347,349
43.	Engines & turbines	83	pt.44	133	351
44.	Farm & gar. mach.	84	42	134	352
45.	Constr. & mining	85	pt.44	135	3531-3
46.	Mat. handling	86	pt.44	136	3534-7
47.	Metalworking	87	pt.44	137	354
48.	Special industry	88	pt.44	138	355
49.	General industry	89	pt.44	139	356
50.	Misc. machinery	90	43	140	359
51.	Off.,comp. & acc.	91,92	45	141-3	357
52.	Service industry	93	46	144	358
53.	Elec. tran. & dis.	94,95	pt.47	145,146	361,362,3825
54.	House. appliances	96	pt.47	147	363
55.	Electr. light	97	pt.47	148	364
56.	Radio,TV & comm.	98-100	pt.47	149-151	365,366
57.	Electronic comp.	101	pt.47	152-4	367
58.	Misc. electrcronic	102	pt.47	155	369
59.	Motor veh. & eq.	103	48	156	371
60.	Aircraft & parts	104	pt.49	157	372
61.	Other tran. eq.	105-8	pt.49	158-161	373-5,379
62.	Pro.,sci. & con.	109,110,113	50	162-7,171	381,382,383,387
63.	Opt.,oph. & ph.	111,112	51	168-170	383,385,386
64.	Misc. mfgr.	114-6	52	172-4	391,393,394,396
65.	Trans. & ware.	117-123	53-7	175-181	40,pt.41,42,44-7(exc.474,pt.4789)
66.	Comm., exc.ra. & TV	124	pt.58	182	48(exc.483)
67.	Radio & TV broad.	125	pt.58	183	483
68.	El., gas & water	126-8	59-61	184-6	pt.491,492,pt.493,494-7
69.	Wh. & ret. trade	129,pt.130	62,63	187,188	50,51(exc.mfgr. sales),52-7,59,7396,pt.8042
70.	Fin. & insurance	131-133	64	189-191	60,61(exc.pt.613),62-4,67
71.	Real estate & rental	134,135	65	192,193	65,66,pt.1531
72.	Hot.,pers.serv.	136-8	66	194-6	pt.70,72,762-4,pt.7699
73.	Bus. services	139-141	67	197-9	73(exc.7396),769(exc.7693),81,89(exc.8922)
74.	Eat. & dr. pl.	pt.130	68	200	58,pt.70
75.	Auto. rep. & ser.	142	69	201	75
76.	Amusements	143,144	70	202,203	78,79
77.	Med.,ed. ser.	145-9	71,72	204-9	80,82-4,86,8922,074
78.	Fed. gov. ent.	150,151	73	210,211	4311,pt.491,pt.613
79.	State & local	152,153	74	212,213	pt.41,pt.491
80.	Scrap, used & sec.	---	75	214	---

Rows:

81. Total intermediate inputs
82. Employee compensation
83. Indirect business taxes
84. Property-type income
85. Total value added, local
86. Noncomparable imports
87. Imports from RON
88. Gross outlay

Columns:

81. Total intermediate use
82. Personal consumption expenditures
83. Gross private capital formation
84. Change in business inventory
85. Fed. government purchases
86. State and local government
87. Total final use, local
88. Alloc. U.S. exports
89. Alloc. U.S. imports
90. Exports to RON
91. Gross output

input-output classification system was used in the preparation of both the historical data series and the employment projections (presented in the next report in the series). Because of orientation of the U.S. Bureau of Labor Statistics activities towards industry employment, a dichotomy of work exists between the two agencies which is not necessarily coordinated with reference to data estimate and estimation procedures.

The employment and income estimates in Table 4.2 are derived from several data sources, as indicated in the table footnotes, and adjusted to the 1972 industry employee compensation in the 1972 U.S. Department of Commerce input-output tables. Thus, all estimates in Table 4.2 are consistent with the employee compensation and, also, the value added and gross output estimates for each industry.

U.S. Bureau of Labor Statistics input-output data are presented in Table 4.3. Individual industry estimates generally differ from corresponging estimates in Table 4.2, as noted earlier. These differences are readily identified by comparing the individual industry gross output estimates (in column 1) in the two tables and, also, by comparing the two estimates of wage and salary employment (in columns 6 and 7) in Table 4.3.

Employment and Income Relationships

The 80-industry data series are reduced to the 10-industry breakdown in earlier tables for discussion purposes. In Table 4.4, the data in Table 4.2 are regrouped as in Table 3.1, starting with agriculture and ending with services, exclusive of government. Household and government workers (federal civilian, federal military, and state and local) are included in Rows 12 to 15, respectively. The summary tabulations show the gross output, employee compensation, and other value added per unit of gross output or per hour worked for each industry group.

Table 4.2. Estimated gross output, employee compensation and other value added, employment and hours worked in specified industry based on U.S. Department of Commerce, Interindustry Economics Division, input-output tables for 80 industries, U.S., 1972.

Ind. No.	Gross Output 1/	Value Added 1/ Total	Employee Compensation Total	Employee Compensation Wages & Salaries 2/	Employee Compensation Other	Ind. Bus. Taxes	Prop.-Type Income	Employment 2/ Full-Time Part-Time	Employment 2/ Full-Time Equivalent	Hours Worked 2/
	(mil.$)	(mil.$)	(mil.$)	(mil.$)	(mil.$)	(mil.$)	(mil.$)	(thou.)	(thou.)	(mil.hrs.)
1	43339.0	9563.0	1854.0	1722.0	132.0	794.0	6914.0	491.0	414.0	1014.0
2	35080.0	19600.0	2556.0	2374.0	182.0	701.0	16343.0	654.0	551.0	1351.0
3	1971.0	1267.0	395.0	358.0	37.0	59.0	813.0	30.0	24.0	61.0
4	3566.0	1733.0	1249.0	1131.0	118.0	108.0	375.0	248.0	198.0	432.0
5	1233.0	511.0	247.0	201.0	46.0	60.0	204.0	19.0	19.0	37.0
6	2270.0	1294.0	612.0	506.0	106.0	89.0	593.0	41.0	47.0	92.0
7	5442.0	3139.0	2148.0	1738.0	410.0	77.0	164.0	156.0	154.0	309.0
8	17819.0	11725.0	2126.0	1857.0	269.0	1100.0	8499.0	172.0	170.0	359.0
9	2847.0	1733.0	926.0	820.0	106.0	52.0	756.0	88.0	86.0	183.0
10	775.0	429.0	215.0	190.0	25.0	23.0	191.0	16.0	16.0	33.0
11	129561.0	54968.0	42158.0	38092.0	4066.0	1082.0	11728.0	4120.0	3384.0	6786.0
12	36417.0	21139.0	17997.0	16261.0	1736.0	136.0	3006.0	1180.0	970.0	2252.0
13	7115.0	4060.0	3600.0	2192.0	460.0	68.0	392.0	190.0	186.0	375.0
14	118301.0	32610.0	17145.0	14627.0	2518.0	5760.0	9705.0	1841.0	1757.0	3585.0
15	9243.0	4443.0	734.0	609.0	125.0	2292.0	1418.0	75.0	73.0	137.0
16	17635.0	5419.0	4449.0	3986.0	463.0	152.0	818.0	651.0	625.0	1126.0
17	5956.0	1540.0	1151.0	1031.0	120.0	33.0	356.0	153.0	147.0	260.0
18	30205.0	10195.0	9016.0	8078.0	938.0	102.0	1078.0	1409.0	1353.0	2800.0
19	4925.0	1597.0	1358.0	1217.0	141.0	22.0	218.0	164.0	157.0	334.0
20	21511.0	8155.0	4990.0	4375.0	615.0	158.0	3006.0	590.0	532.0	1204.0
21	460.0	164.0	146.0	128.0	18.0	5.0	13.0	22.0	20.0	46.0
22	7282.0	2942.0	2307.0	2127.0	180.0	29.0	606.0	361.0	352.0	565.0
23	3720.0	1662.0	1355.0	1249.0	106.0	17.0	289.0	119.0	114.0	185.0
24	19933.0	7667.0	5097.0	4437.0	660.0	270.0	2300.0	452.0	445.0	911.0
25	7975.0	2996.0	2407.0	2008.0	399.0	72.0	516.0	206.0	203.0	416.0
26	29762.0	14355.0	11405.0	10246.0	1159.0	160.0	2790.0	1102.0	1003.0	2063.0
27	24041.0	10510.0	5677.0	4920.0	757.0	264.0	4569.0	442.0	436.0	827.0
28	9664.0	3738.0	2509.0	2175.0	334.0	117.0	1112.0	189.0	186.0	353.0
29	17226.0	6904.0	3493.0	3027.0	466.0	106.0	3305.0	293.0	279.0	588.0
30	3610.0	1260.0	903.0	783.0	120.0	26.0	331.0	71.0	70.0	124.0
31	31441.0	7547.0	3225.0	2532.0	693.0	4095.0	227.0	205.0	204.0	394.0
32	20655.0	9875.0	6242.0	5283.0	959.0	862.0	2772.0	618.0	606.0	1237.0
33	1062.0	282.0	253.0	224.0	29.0	2.0	27.0	21.0	20.0	39.0
34	4529.0	1932.0	1660.0	1472.0	188.0	6.0	267.0	258.0	248.0	486.0
35	5563.0	2982.0	2064.0	1788.0	276.0	55.0	863.0	171.0	168.0	338.0
36	15266.0	7307.0	4856.0	4207.0	649.0	179.0	2271.0	470.0	461.0	932.0
37	36664.0	14924.0	11546.0	9600.0	1946.0	604.0	2774.0	754.0	745.0	1458.0
38	23926.0	5952.0	4412.0	3668.0	744.0	163.0	1378.0	461.0	455.0	891.0
39	4870.0	1501.0	1066.0	916.0	150.0	99.0	335.0	85.0	83.0	167.0
40	15347.0	6028.0	4587.0	3940.0	647.0	19.0	1422.0	491.0	482.0	973.0
41	11200.0	5220.0	4063.0	3489.0	574.0	63.0	1093.0	358.0	351.0	708.0
42	14191.0	7090.0	4927.0	4232.0	695.0	57.0	2106.0	394.0	387.0	780.0
43	5413.0	2451.0	1734.0	1505.0	229.0	14.0	703.0	97.0	96.0	191.0
44	5574.0	2420.0	1595.0	1384.0	211.0	16.0	810.0	143.0	141.0	283.0
45	7695.0	3667.0	2615.0	2269.0	346.0	27.0	1026.0	208.0	206.0	412.0
46	2815.0	1305.0	991.0	860.0	131.0	9.0	305.0	80.0	79.0	158.0
47	7155.0	4310.0	3476.0	3016.0	460.0	34.0	791.0	291.0	287.0	576.0
48	5867.0	2972.0	2243.0	1945.0	297.0	22.0	706.0	173.0	171.0	343.0
49	8149.0	4153.0	3251.0	2821.0	430.0	32.0	871.0	259.0	256.0	503.0
50	4450.0	2489.0	1931.0	1676.0	255.0	13.0	545.0	233.0	230.0	461.0
51	8036.0	3495.0	2753.0	2389.0	364.0	19.0	723.0	268.0	264.0	503.0
52	8516.0	3642.0	2367.0	2054.0	313.0	16.0	1259.0	156.0	154.0	309.0
53	10420.0	5119.0	4132.0	3630.0	502.0	27.0	960.0	378.0	373.0	713.0
54	6676.0	2802.0	1806.0	1563.0	243.0	14.0	981.0	171.0	169.0	323.0
55	5532.0	2776.0	1855.0	1605.0	249.0	9.0	931.0	198.0	196.0	374.0
56	17971.0	8323.0	7469.0	6475.0	994.0	38.0	815.0	645.0	637.0	1217.0
57	8419.0	4148.0	3657.0	3166.0	491.0	23.0	469.0	377.0	377.0	711.0
58	4292.0	2049.0	1521.0	1317.0	204.0	10.0	518.0	124.0	122.0	234.0
59	65216.0	21437.0	12474.0	10873.0	1601.0	678.0	8285.0	806.0	789.0	1596.0
60	17036.0	7860.0	7592.0	5972.0	1620.0	35.0	233.0	546.0	541.0	1025.0
61	12738.0	4700.0	4180.0	3287.0	893.0	18.0	502.0	373.0	369.0	700.0
62	6941.0	3667.0	2776.0	2449.0	327.0	25.0	867.0	255.0	253.0	487.0
63	6531.0	4018.0	2075.0	1830.0	245.0	27.0	1916.0	180.0	178.0	343.0
64	11991.0	4708.0	3631.0	3219.0	412.0	39.0	1038.0	434.0	413.0	817.0
65	74779.0	44875.0	30395.0	26691.0	3704.0	2776.0	11704.0	2531.0	2437.0	4950.0
66	31565.0	25983.0	12341.0	9505.0	2836.0	4218.0	9424.0	953.0	914.0	1759.0
67	4513.0	2374.0	1596.0	1460.0	136.0	108.0	669.0	139.0	126.0	254.0
68	50146.0	25281.0	7011.0	5939.0	1072.0	4557.0	13713.0	838.0	520.0	1027.0
69	216354.0	166103.0	91963.0	83124.0	8839.0	36716.0	37424.0	13062.0	11926.0	24819.0
70	77866.0	43970.0	31076.0	26857.0	4219.0	3455.0	9439.0	3064.0	2935.0	5447.0
71	174561.0	141038.0	8464.0	7631.0	833.0	30633.0	101941.0	1108.0	976.0	2053.0
72	30564.0	17111.0	10482.0	9538.0	944.0	1178.0	5451.0	1835.0	1586.0	3274.0
73	65716.0	47221.0	28972.0	26439.0	2533.0	607.0	17641.0	3214.0	2908.0	5711.0
74	48550.0	20948.0	15633.0	10284.0	5349.0	3341.0	1974.0	2804.0	1555.0	3258.0
75	24340.0	11429.0	6320.0	5725.0	595.0	612.0	4497.0	854.0	792.0	1611.0
76	12745.0	6674.0	4449.0	3972.0	477.0	1080.0	1145.0	674.0	512.0	1090.0
77	64906.0	57524.0	43714.0	39508.0	4206.0	365.0	13446.0	6260.0	5594.0	10130.0
78	11795.0	8244.0	9664.0	8611.0	1053.0	51.0	-1472.0	888.0	846.0	1564.0
79	12796.0	6417.0	5453.0	4869.0	584.0	0	964.0	526.0	520.0	1044.0
80 3/	142721.0	142721.0	142721.0	129392.0	13329.0	0	0	18082.0	14553.0	27237.0
81	2108355.0	1180073.0	725544.0	638043.0	85948.0	110980.0	443207.0	83058.0	73182.0	138718.0
82	5349.0	5349.0	5349.0	5283.0	66.0	0	0	2202.0	1343.0	2674.0
83	27417.0	27417.0	27417.0	24521.0	2896.0	0	0	2005.0	1934.0	3470.0
84	22643.0	22643.0	22643.0	21963.0	680.0	0	0	3410.0	2553.0	5250.0
85	87312.0	87312.0	87312.0	77625.0	9687.0	0	0	10456.0	8723.0	15843.0
86	2251166.0	1331559.0	808265.0	768040.0	49277.0	110980.0	443207.0	101131.0	87735.0	165955.0

1/ U.S. Department of Commerce, Interindustry Economics Division, Input-Output Structure of the U.S. Economy, 1972, Survey of Current Business, 59(2): 34-72. February 1979; Phil M. Ritz, Eugene P. Roberts, and Paula C. Young, Dollar-Value Tables for the 1972 Input-Output Study, Survey of Current Business, 59(4): 51-72. April 1979.

2/ Table 6.6, Wages and Salary by Industry, Table 6.7, Full-Time and Part-Time Employees by Industry, Table 6.8, Full-Time Equivalent Employees by Industry, and Table 6.10, Hours Worked by Full-Time and Part-Time Employees by Industry, Survey of Current Business, 57(7): 31-52. July 1977.

3/ Includes household and general government sectors.

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Table 4.3. Estimated gross output, wage and salary payments, proprietorial income, employment and hours worked in specified industry based on U.S. Bureau of Labor Statistics input-output tables for 80 industries, U.S., 1972.

Ind. No.	Gross Output1/	Earnings2/ Total	Earnings2/ Wage & Salary	Earnings2/ Proprie-torial	Employment Total1/	Employment Wage and Salary BLS1/	Employment Wage and Salary REIS2/	Employment Proprie-torial	Hours Worked1/
	(mil.$)	(mil.$)	(mil.$)	(mil.$)	(thou.)	(thou.)	(thou.)	(thou.)	(mil.hrs.)
1	40971.0	7295.0	1722.0	5573.0	1530.0	1398.0	491.0	1039.0	3217.0
2	57944.0	11422.0	2374.0	11422.0	4121.0	1862.0	654.0	3467.0	4286.0
3	2350.0	439.0	358.0	81.0	32.0	23.0	30.0	9.0	62.0
4	3901.0	1861.0	1131.0	730.0	222.0	207.0	243.0	15.0	398.0
5	1274.0	201.0	201.0	0	25.0	25.0	19.0	0	53.0
6	2323.0	506.0	506.0	0	63.0	63.0	47.0	0	131.0
7	5710.0	2118.0	1738.0	380.0	174.0	160.0	156.0	14.0	370.0
8	17695.0	2010.0	1857.0	153.0	161.0	143.0	172.0	18.0	340.0
9	2856.0	820.0	820.0	0	96.0	96.0	83.0	0	219.0
10	321.0	190.0	190.0	0	17.0	17.0	15.0	0	39.0
11	126746.0	48191.0	38092.0	10099.0	3691.0	3137.0	4120.0	544.0	7317.0
12	32517.0	18014.0	15261.0	2753.0	970.0	819.0	1130.0	151.0	1914.0
13	7475.0	2192.0	2192.0	0	183.0	183.0	190.0	0	385.0
14	127827.0	17598.0	15560.0	2161.0	1777.0	1739.0	1841.0	38.0	3709.0
15	10034.0	703.0	597.0	106.0	75.0	75.0	75.0	0	146.0
16	20450.0	4078.0	3986.0	92.0	592.0	590.0	651.0	2.0	1290.0
17	6871.0	1215.0	1031.0	184.0	140.0	136.0	153.0	4.0	302.0
18	35171.0	8354.0	8078.0	276.0	1473.0	1467.0	1409.0	6.0	2810.0
19	6436.0	1539.0	1217.0	322.0	182.0	175.0	164.0	7.0	366.0
20	23120.0	5363.0	4375.0	988.0	665.0	595.0	590.0	70.0	1422.0
21	520.0	123.0	123.0	0	28.0	28.0	22.0	0	57.0
22	7493.0	2271.0	2127.0	144.0	367.0	363.0	351.0	4.0	770.0
23	4492.0	1393.0	1249.0	144.0	144.0	140.0	119.0	4.0	303.0
24	21019.0	5077.0	4437.0	640.0	471.0	470.0	452.0	1.0	1039.0
25	5605.0	2006.0	2006.0	0	219.0	219.0	206.0	0	470.0
26	34045.0	11190.0	10246.0	944.0	1100.0	1084.0	1102.0	16.0	2233.0
27	29897.0	6077.0	4920.0	1157.0	456.0	450.0	442.0	6.0	973.0
28	10340.0	2175.0	2175.0	0	215.0	215.0	189.0	0	463.0
29	19353.0	3027.0	3027.0	0	273.0	273.0	263.0	0	571.0
30	4158.0	783.0	783.0	0	70.0	70.0	71.0	0	146.0
31	37008.0	3143.0	2532.0	611.0	194.0	194.0	205.0	0	415.0
32	23971.0	5982.0	5283.0	699.0	628.0	624.0	618.0	4.0	1331.0
33	1370.0	224.0	224.0	0	27.0	27.0	21.0	0	56.0
34	4748.0	1583.0	1472.0	111.0	277.0	275.0	258.0	2.0	552.0
35	6197.0	1788.0	1788.0	0	188.0	188.0	171.0	0	397.0
36	16127.0	4908.0	4207.0	701.0	481.0	471.0	470.0	10.0	1039.0
37	38127.0	9600.0	9600.0	0	865.0	865.0	254.0	0	1844.0
38	18112.0	5095.0	3068.0	2027.0	378.0	376.0	461.0	2.0	819.0
39	5203.0	916.0	906.0	10.0	84.0	84.0	65.0	0	186.0
40	10862.0	3940.0	3940.0	0	520.0	520.0	491.0	0	1088.0
41	11891.0	4492.0	3489.0	1003.0	350.0	336.0	358.0	14.0	761.0
42	17405.0	4733.0	4232.0	501.0	463.0	456.0	394.0	7.0	977.0
43	6715.0	1505.0	1505.0	0	111.0	111.0	97.0	0	234.0
44	5016.0	1384.0	1384.0	0	131.0	131.0	143.0	0	281.0
45	6459.0	2269.0	2269.0	0	204.0	204.0	208.0	0	436.0
46	3279.0	860.0	860.0	0	87.0	87.0	80.0	0	182.0
47	7955.0	3016.0	3016.0	0	289.0	289.0	291.0	0	633.0
48	6527.0	1946.0	1946.0	0	180.0	180.0	173.0	0	383.0
49	9205.0	2821.0	2821.0	0	268.0	268.0	259.0	0	572.0
50	5030.0	21614.0	1676.0	19938.0	237.0	215.0	233.0	22.0	515.0
51	6166.0	2389.0	2389.0	0	248.0	248.0	268.0	0	487.0
52	9288.0	2854.0	2854.0	0	158.0	158.0	156.0	0	332.0
53	11916.0	3620.0	3620.0	0	397.0	397.0	378.0	0	837.0
54	7869.0	1563.0	1563.0	0	186.0	186.0	171.0	0	392.0
55	6144.0	1606.0	1606.0	0	206.0	206.0	194.0	0	427.0
56	20016.0	6475.0	6475.0	0	579.0	579.0	645.0	0	1201.0
57	8495.0	3166.0	3166.0	0	355.0	355.0	377.0	0	737.0
58	4977.0	19074.0	1317.0	17757.0	130.0	124.0	124.0	6.0	278.0
59	67159.0	11916.0	10673.0	1880.0	865.0	863.0	805.0	2.0	1896.0
60	16536.0	5962.0	5962.0	0	511.0	511.0	546.0	0	1081.0
61	13979.0	11510.0	3287.0	8223.0	408.0	398.0	373.0	10.0	842.0
62	6380.0	2838.0	2449.0	389.0	293.0	290.0	255.0	3.0	548.0
63	7216.0	1830.0	1830.0	0	170.0	170.0	160.0	0	356.0
64	13236.0	3536.0	3219.0	317.0	450.0	433.0	434.0	17.0	926.0
65	75553.0	30365.0	27420.0	2945.0	2828.0	2652.0	2531.0	176.0	5997.0
66	31705.0	11962.0	9505.0	2457.0	1041.0	1009.0	953.0	32.0	2134.0
67	5290.0	1460.0	1460.0	0	143.0	143.0	139.0	0	286.0
68	59209.0	6994.0	5939.0	1055.0	728.0	713.0	533.0	15.0	1570.0
69	195804.0	110613.0	83124.0	27489.0	15089.0	13185.0	13052.0	1904.0	28671.0
70	74919.0	32733.0	26857.0	5869.0	3297.0	3119.0	3064.0	178.0	6449.0
71	174475.0	70408.0	67631.0	2777.0	975.0	824.0	1108.0	151.0	1912.0
72	26150.0	12733.0	9538.0	3191.0	2641.0	1939.0	1835.0	702.0	5125.0
73	89155.0	48199.0	26439.0	21924.0	3163.0	2680.0	3214.0	483.0	6885.0
74	43588.0	12383.0	10284.0	2099.0	3201.0	2793.0	2804.0	408.0	5816.0
75	23572.0	6892.0	5725.0	1150.0	847.0	412.0	854.0	435.0	1791.0
76	15188.0	4466.0	3972.0	492.0	800.0	714.0	674.0	86.0	1348.0
77	57790.0	5316.0	39508.0	13595.0	6906.0	6410.0	6260.0	496.0	12221.0
78	11455.0	8661.0	8661.0	0	888.0	888.0	838.0	0	1847.0
79	13929.0	4869.0	4869.0	0	526.0	526.0	526.0	0	1105.0
80 3/	142721.0	25199.0	23565.0	2181.0	18072.0	18072.0	18062.0	0	27286.0
81	2175023.0	722390.0	539567.0	179780.0	91605.0	83120.0	82234.0	10580.0	169285.0
82	5549.0	5283.0	5283.0	0	2192.0	2192.0	2202.0	0	2723.0
83	27417.0	25190.0	24521.0	669.0	2095.0	2095.0	2095.0	0	3470.0
84	22546.0	16143.0	16137.0	6.0	3410.0	3410.0	3410.0	0	5250.0
85	67312.0	54150.0	52644.0	1506.0	10465.0	10465.0	10465.0	0	15843.0
86	2317744.0	855056.0	709152.0	181941.0	109677.0	101192.0	100358.0	10580.0	196571.0

1/ U.S. Department of Labor, Bureau of Labor Statistics, Time Series Data for Input-Output Industries, Bulletin 2018, U.S. Government Printing Office, Washington, D.C. 1979.

2/ U.S. Department of Commerce, Regional Economic Measurement Division, Regional Economic Information System (REIS), Unpublished data. April 1980.

3/ Includes household and general government sectors.

Table 4.4: Estimated gross output, value added and employment ratios in specified industry based on U.S. Department of Commerce, Interindustry Economics Division, input-output tables for 10 industries, U.S., 1972.

Industry No. Title	Output per Worker: Part-time & Full-time	Output per Worker: Full-time Equiv.	Per $1,000 Value Added: Emp. Comp.	Per $1,000 Value Added: Ind. Bus. Taxes	Per $1,000 Value Added: Prop.-Type Inc.	Emp. Comp., Per $1,000 Wages & Salary	Gross Output Per Hour	Wages & Salaries Per Worker: Full-time &Part-time	Wages & Salaries Per Worker: Full-time Equiv.	Wages & Salaries Per Hour	Hours Worked Per Week: Full-time & Part-time	Hours Worked Per Week: Full-time Equiv
	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	(no.)	(no.)
1. Agr., For., Fish.	58999.	70730.	188.	52.	760.	84.	29.38	3925.	4705.	1.95	38.62	46.30
2. Mining	61760.	61760	332.	74.	551.	181.	30.00	10797.	10797.	5.24	39.60	39.60
3. Construction	31320.	38125.	790.	16.	194.	107.	18.37	10255.	12483	6.01	32.79	39.92
4. Mfg. Nondurables	43558.	45420.	633.	114.	254.	153.	22.63	8255.	8608	4.29	37.02	38.60
5. Mfg. Durables	37285.	38070.	736.	16.	249.	166.	19.38	9840.	10047.	5.12	36.99	37.77
6. Tran., Com., Util.	36092.	40281.	521.	1188.	360.	178.	20.15	9772.	10907.	5.46	34.44	38.44
7. Trade	16698.	19652.	575.	214.	211.	152.	9.44	5887.	6929.	3.33	34.03	40.05
8. Fin., Ins., Real Est.	60515.	64553.	214.	184.	1088.	146.	33.66	8267.	8818.	4.60	34.57	36.88
9. Services	17231.	19417.	669.	27.	301.	103.	10.14	6636.	7477.	3.90	32.68	36.83
10. Scrap, other 1/	8581.	10510.	1003.	0.	-3.	105.	5.61	7328.	8975.	4.79	29.44	36.05
11. Total	25384.	28810.	610.	93.	373.	135.	14.57	7689.	8727.	4.41	33.51	38.03
12. Household workers	2429.	3983.	1000.	0	0	12.	2.00	2399.	3934.	1.98	23.35	38.29
13. Federal Civilian	13674.	14176.	1000.	0	0	118.	7.90	12230.	12679.	7.07	33.28	34.50
14. Federal Military	6640.	8869.	1000.	0	0	31.	4.31	6441.	8603.	4.18	29.61	39.55
15. State & Local	8350.	10009.	1000.	0.	0	125.	5.51	7424.	8899.	4.90	29.14	34.93

1/ Household workers and general government (from rows 12 to 15).

Table 4.5 also is an aggregation of the 80 industries into 10 industries, except here the industry gross output and related employment and income relationships are based on U.S. Bureau of Labor Statistics, rather than U.S. Department of Commerce, data. Included with these data are the corresponding industry earnings estimates reported in the U.S. Department of Commerce, Regional Economic Information System. These estimates will differ from the estimates summarized in Table 4.4 because of differences in industry defintions as well as primary data sources.

Large differences in labor productivity and compensation are shown, even in the 10-industry breakdown of the U.S. data. The large variance in hours worked per worker is reduced by using output per hour,rather than output per worker, ratios. Similarly, industry-to-industry variance in employee compensation is reduced by using a per hour rather than a per worker basis.

Corresponding Minnesota data are being prepared for use with the 1972 Minnesota input-output tables. Currently, however, only 1977 base year data, and their projection to 1990, are available for use with the 1977 input-output tables. These data were prepared for the 214-industry breakdown. They are available, therefore, in much greater detail for the state, as well as the nation, starting with the 1977 base year.

Table 4.5. Estimated gross output, earnings, and employment ratios in specified industry based on U.S. Bureau of Labor Statistics input-output tables for 10 industries, U.S., 1972.

Ind. No.	Output per Worker: Total	Output per Worker: Wage & Salary	Earnings: Per $100 Gross Output	Earnings: Per Worker	Wages & Salaries per Employee	Prop. Inc. per Propri-etor	BLS Wage & Salary Emp. per 1,000 Total	REIS Wage & Salary Emp. per 1,000 Total	Per Hour: Gross Output	Per Hour: Earnings	Hours Worked per Week
	($)	($)	($)	($)	($)	($)	(no.)	(no.)	($)	($)	(no.)
1	18801.	31801.	19.	3559.	5925.	3951.	591.	241.	13.96	2.64	25.9
2	57319.	60995.	19.	10905.	10667.	16656.	940.	929.	26.67	5.07	41.3
3	34169.	40259.	42.	14204.	10255.	18492.	849.	1137.	17.25	7.17	38.1
4	49226.	49793.	19.	9238.	8579.	65151.	989.	996.	23.85	4.48	39.7
5	36355.	36959.	30.	14385.	10324.	315916.	984.	925.	16.15	6.81	40.6
6	36236.	36625.	30.	10713.	10652.	28955.	953.	878.	17.20	5.08	40.5
7	13069.	14962.	51.	6725.	5687.	12798.	874.	867.	6.94	3.57	36.3
8	58379.	63250.	41.	24143.	6267.	26280.	923.	977.	29.83	12.34	37.6
9	14755.	17423.	37.	5469.	6636.	18325.	847.	894.	7.74	2.88	36.7
10	8627.	8627.	23.	1985.	7289.	0	1000.	1001.	5.56	1.28	29.8
11	23743.	26107.	33.	7891.	7776.	16991.	907.	898.	12.85	4.27	35.5
12	2440.	2440.	99.	2410.	2599.	0	1000.	1005.	1.96	1.94	23.9
13	13674.	13674.	92.	12564.	12230.	0	1000.	1000.	7.90	7.26	33.3
14	6640.	6640.	71.	4734.	4732.	0	1000.	1000.	4.31	3.07	29.6
15	8343.	8343.	96.	8041.	7697.	0	1000.	1000.	5.51	5.31	29.1

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APPENDIX: INVERTING INPUT-OUTPUT MATRIX AND DERIVING EMPLOYMENT AND INCOME MULTIPLIERS

A. Problem: Invert three-industry table (see, p. 8) of input-output coefficients:

$$[A] = \begin{bmatrix} .22727 & .07189 & .00974 \\ .16364 & .37661 & .10173 \\ .16364 & .15129 & .20779 \end{bmatrix}$$

1. Convert (A) matrix to (I-A) matrix by subtracting (A) matrix from identity matrix (I):

$$[I-A] = \begin{bmatrix} .77273 & -.07189 & -.00974 \\ -.16364 & .62339 & -.10173 \\ -.16364 & -.15129 & .79221 \end{bmatrix}$$

2. Evaluate determinant of (I-A) matrix:

$$D=(.77273)\begin{vmatrix} .62339 & -.10173 \\ -.15129 & .79221 \end{vmatrix} -(-.07189)\begin{vmatrix} -.16364 & -.10173 \\ -.16364 & .79221 \end{vmatrix}$$

$$+(-.009744)\begin{vmatrix} -.16363 & .62339 \\ -.16364 & -.15129 \end{vmatrix}$$

$$= (.77273)\ [(.62339)(.79221)-(0.15129)(-.10173)]$$

$$-(-.07189)\ [(-.16364)(.79221)\ -(-.16364)\ (-.10173)]$$

$$+(-.009744)[(-.16364)(-.15129)-(-.16364)(.62339)]$$

$$= (.77273)9.36922)-(-.07189)(.01052) + (-.00974)(.00123)$$

$$= .35797$$

3. Identify all cofactors of determinant, D:

$$\hat{A}_{11} = \overset{(.47847)}{\begin{vmatrix} .62339 & -.10173 \\ -.16129 & .79221 \end{vmatrix}} \quad \hat{A}_{12} = \overset{-(-.14629)}{\begin{vmatrix} -.16364 & -.10173 \\ -.16364 & .79221 \end{vmatrix}} \quad \hat{A}_{13} = \overset{(.12676)}{\begin{vmatrix} -.16364 & .62339 \\ -.16364 & -.15129 \end{vmatrix}}$$

$$\hat{A}_{21} = \overset{-(-.05843)}{\begin{vmatrix} .07189 & -.00974 \\ -.15129 & .79221 \end{vmatrix}} \quad \hat{A}_{22} = \overset{(.61057)}{\begin{vmatrix} .77273 & -.00974 \\ -.16364 & .79221 \end{vmatrix}} \quad \hat{A}_{23} = \overset{-(-.12867)}{\begin{vmatrix} .77273 & -.07189 \\ -.16364 & -.15129 \end{vmatrix}}$$

$$\hat{A}_{31} = \overset{(.01339)}{\begin{vmatrix} -.017189 & -.00974 \\ -.15129 & .79221 \end{vmatrix}} \quad \hat{A}_{32} = \overset{-(-.08020)}{\begin{vmatrix} .77273 & -.00974 \\ -.16364 & .79221 \end{vmatrix}} \quad \hat{A}_{33} = \overset{(.46995)}{\begin{vmatrix} .77273 & -.07189 \\ -.16364 & .62339 \end{vmatrix}}$$

4. Derive matrix of cofactors and transposed matrix of cofactors (called the adjoint matrix):

Matrix of cofactors

$$\begin{bmatrix} .47847 & .14629 & .12676 \\ .05843 & .61057 & .12867 \\ .01339 & .08020 & .46995 \end{bmatrix}$$

Adjoint matrix

$$\begin{bmatrix} .47847 & .05843 & .01339 \\ .14639 & .61057 & .08020 \\ .12676 & .12867 & .46995 \end{bmatrix}$$

5. Divide each element in the adjoint matrix by determinant, D:

$$\begin{bmatrix} 1.33662 & .16323 & .03741 \\ .40867 & 1.70565 & .22404 \\ .35411 & .35945 & 1.31282 \end{bmatrix} = [I-A]^{-1}$$

6. Multiply original matrix [I-A] by inverse $[I-A]^{-1}$ to obtain identity matrix [I] as check on calculations:

$$[I-A] \cdot [I-A]^{-1} = [I]$$

$$= \begin{bmatrix} .77273 & -.07189 & -.00974 \\ -.16364 & .62339 & -.10173 \\ -.16364 & -.16129 & .79221 \end{bmatrix} \cdot \begin{bmatrix} 1.33662 & .16323 & .03741 \\ .40867 & 1.70565 & .22424 \\ .35411 & .35945 & 1.31252 \end{bmatrix}$$

Complete matrix multiplication as follows:

(.77273 x 1.33552)+(-.07189 x .40867)+(-.00974 x .35411) = 1.00002

(.77273 x .16323)+(=.07189 x 1.70565)+(-.00924 x .35945) = .00019

(.77273 x .03741)+(-.07189 x .22424)+(-.00974 x 1.31252) = .00010

(-.16364 x 1.33662)+(.62339 x 1.70565)+(-.10173 x .35945) = .00001

(-.16364 x .16323)+(.62339 x 1.70565)+(-.10173 x .35945) = 1.00001

(-.16364 x .03741)+(.62339 x .22424)+(-.10173 x 1.31252) = .00014

(-.16364 x 1.33662)+(-.15129 x .40867)+(.79221 x .35441) = .00022

(-.16364 x .16323)+(-.16129 x 1.70565)+(.79221 x .35934) = .00000

(-.16364 x .03741)+(-.16129 x .22424)+(.79221 x 1.31252) = .99974

Thus, derived matrix values approximate [I] values as follows:

$$\begin{bmatrix} 1.00002 & .00019 & .00010 \\ .00001 & 1.00001 & .00014 \\ .00022 & .00000 & .99974 \end{bmatrix} \cong \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B. Problem: Invert four-industry table (see p.) of input-output coefficients.

1. Convert (A) matrix to (I-A) matrix by subtracting (A) matrix from identity matrix (I):

$$[A] = \begin{bmatrix} .22727 & .07189 & .00974 & .00949 \\ .16364 & .37661 & .10173 & .29404 \\ .16363 & .15129 & .20779 & .68564 \\ .10000 & .28326 & .31926 & .00678 \end{bmatrix}$$

$$[I\text{-}A] = \begin{bmatrix} .77273 & -.07189 & -.00974 & -.00949 \\ -.16364 & .62339 & -.10173 & -.29404 \\ -.16364 & -.15129 & .79221 & -.68564 \\ -.10000 & -.28326 & -.31926 & .99322 \end{bmatrix}$$

2. Evaluate determinant of (I-A) matrix:

$$D = (e_{11}\hat{A}_{11}) - (e_{21}\hat{A}_{21}) + (e_{31}\hat{A}_{31}) - (e_{41}\hat{A}_{41})$$

$$= (.77273) \begin{bmatrix} .62339 & -.10173 & -.29404 \\ -.15129 & .79221 & -.68564 \\ -.28326 & -.31926 & .99322 \end{bmatrix} \begin{matrix} .62339 & -.10173 \\ -.15129 & .79221 \\ -.28320 & -.31926 \end{matrix}$$

$$-(-.16364) \begin{bmatrix} -.07189 & -.00974 & -.00494 \\ -.15129 & .79221 & -.68564 \\ -.28326 & -.31926 & .99322 \end{bmatrix} \begin{matrix} -.07189 & -.00974 \\ -.15129 & .79221 \\ -.28320 & -.31920 \end{matrix}$$

$$+(-.10364) \begin{bmatrix} -.07189 & -.00974 & -.00949 \\ .62339 & -.10173 & -.29404 \\ -.28326 & -.31926 & .99322 \end{bmatrix} \begin{matrix} -.07189 & -.00974 \\ .62339 & -.10173 \\ -.28326 & -.31926 \end{matrix}$$

$$-(-.10000) \begin{bmatrix} -.07189 & -.00974 & -.00949 \\ .62339 & -.10173 & -.29404 \\ -.15129 & .79221 & -.68564 \end{bmatrix} \begin{matrix} -.07189 & -.00974 \\ .62339 & -.10173 \\ -.15129 & .79221 \end{matrix}$$

To find a 3 x 3 matrix determinant, solve for determinant, D, as follows:

$$\left| \begin{matrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{matrix} \right| \left. \begin{matrix} e_{11} & e_{12} \\ e_{21} & e_{22} \\ e_{31} & e_{32} \end{matrix} \right\} = \left\{ \begin{matrix} e_{11}e_{22}e_{33} + e_{12}e_{23}e_{31} + \\ e_{13}e_{21}e_{32} - e_{31}e_{22}e_{13} + \\ e_{32}e_{23}e_{11} - e_{33}e_{21}e_{12} \end{matrix} \right.$$

Thus,

$$D = (.77273)[(.62339)(.79221)(.99322) + (-.10173)(-.68564)(-.28326)$$
$$+ (-.29404)(-.15129)(-.31926) - (-.28326)(.79221)(-.29404)$$
$$- (.31926)(-.68564)(.62339) - (.99322)(-.15129)(-.10173)]$$
$$+ (.16364)[(-.07189)(.79221)(.99322) + (-.00974)(-.68564)(-.28326)$$
$$+ (-.00949)(-.15129)(-.31926) - (-.29326)(.79221)(-.00949)$$
$$- (-.31926)(-.68564)(-.07198) - (.99322)(-.15129)(-.00974)]$$
$$- (.16364)[(-.07189)(-.10173)(.99322) + (-.00974)(-.29404)(-.29326)$$
$$- (-.00949)(.62339)(-.31926) - (0.28326)(-.10173)(-.00949)$$
$$- (-.31926)(-.29404)(-.07189) - (.99322)(.62339)(-.00974)]$$
$$+(.10000)[(-.07189)(-.19173)(-.68564) + (-.00974)(-.29404)(-.15129)$$
$$+ (-.00949)(.62339)(.79221) - (-.15129)(-.10173)(-.00949)$$
$$- (.79221)(-.29404)(-.07189) - (-.68564)(.62339)(-.00974)]$$

$$= (.77273)[.49051 - .01976 - .01420 - .06598 - .13646 - .01529]$$
$$+(.16364)[-.05656 - .00189 - .00046 - .00213 + .01576 - .00146]$$
$$-(.16364)[.00726 - .00081 + .00189 + .00027 + .00675 + .00603]$$
$$+(.10000)[-.00501 - .00043 = .00469 + .00015 - .01675 - .00416]$$

$$[I-A] = .77273(.23891) + .16364(-.04674) - .16264(.02139) + .1(=.03089)$$
$$= .184612 - .007648 - .00350 - .003089$$
$$D = .17038$$

3. Identify all cofactors of determinant, D:
(Note that above step yeilded cofactor values as follows:

$$(+)\hat{A}_{11} = .23891$$

$$(-)\hat{A}_{21} = -.04675$$

$$(+)\hat{A}_{31} = .02139$$

$$(-)\hat{A}_{41} = -.03089$$

$$-(-.15510)$$

$$\hat{A}_{12} = \left[\begin{array}{rrr} -.16364 & -.10173 & -.29404 \\ -.16364 & .79221 & -.68564 \\ -.10000 & -.31926 & .99322 \end{array}\right] \begin{array}{rr} -.16364 & -.10173 \\ -.16364 & .79221 \\ -.1000 & -.31926 \end{array}$$

$$= -.12876 - .00698 - .01536 - .02329 + .03582 - .01653$$

$$= -.15510$$

$$(.43546)$$

$$\hat{A}_{22} = \left[\begin{array}{rrr} .77273 & -.00974 & -.00949 \\ -.16364 & .79221 & -.68564 \\ -.1000 & -.31926 & .99322 \end{array}\right] \begin{array}{rr} .77273 & -.00974 \\ -.16364 & .79221 \\ -.1000 & -.31926 \end{array}$$

$$= .60801 - .00067 - .00050 - .0075 - .16915 - .00158$$

$$= .43536$$

$$-(.15204)$$

$$\hat{A}_{32} = \left[\begin{array}{rrr} .77273 & -.00974 & -.00949 \\ -.16364 & -.10173 & -.29404 \\ -.1000 & -.31926 & .99322 \end{array}\right] \begin{array}{rr} .77273 & -.00974 \\ -.16364 & -.10173 \\ -.1000 & -.31926 \end{array}$$

$$= -.078076 - .00029 - .0050 + .00095 - .07254 - .00158$$

$$= .15204$$

$$(.23591)$$

$$\hat{A}_{42} = \left[\begin{array}{rrr} .77273 & -.00974 & -.00949 \\ -.16364 & -.10173 & -.29404 \\ -.16364 & .79221 & -.68564 \end{array}\right] \begin{array}{rr} .77273 & -.00974 \\ -.16364 & -.10173 \\ -.16364 & .79221 \end{array}$$

$$= .05390 - .00047 + .00123 + .00016 + .1800 + .00109$$

$$= .23591$$

$$(.19125)$$

$$\hat{A}_{13} = \left[\begin{array}{rrr} -.16364 & .62339 & -.29404 \\ -.16364 & -.15129 & -.68564 \\ -.1000 & -.28326 & .99322 \end{array}\right] \begin{array}{rr} -.16364 & .62339 \\ -.16364 & -.15129 \\ -.1000 & -.28326 \end{array}$$

$$= .02459 + .04274 - .01363 + .00445 + .03178 + .10132$$

$$= .19125$$

$$\hat{A}_{23} = \overset{-(-.28309)}{\begin{bmatrix} .77273 & -.07189 & -.00949 \\ -.16364 & -.15129 & -.68564 \\ -.1000 & -.27326 & .99322 \end{bmatrix}} \begin{matrix} .77273 & -.07189 \\ -.16364 & -.15129 \\ -.1000 & -.28326 \end{matrix}$$

$$= -.11611 - .00493 - .00044 + .00014 - .15007 - .01168$$

$$= -.28309$$

$$\hat{A}_{33} = \overset{(.39927)}{\begin{bmatrix} .77273 & -.07189 & -.00949 \\ -.16364 & .62339 & -.29404 \\ -.1000 & -.28326 & .99322 \end{bmatrix}} \begin{matrix} .77273 & -.07189 \\ -.16364 & .62339 \\ -.1000 & -.28326 \end{matrix}$$

$$= .47845 - .00211 - .00044 - .00059 = .06436 - .01168$$

$$= .39927$$

$$\hat{A}_{43} = \overset{-(-.36124)}{\begin{bmatrix} .77273 & -.07189 & -.00949 \\ -.16364 & .62339 & -.29404 \\ -.16364 & -.15129 & -.68564 \end{bmatrix}} \begin{matrix} .77273 & -.07189 \\ -.16364 & .62339 \\ -.16364 & -.15129 \end{matrix}$$

$$= -.33028 - .00346 - .00023 - .00097 - .03437 + .00807$$

$$= -.36124$$

$$\hat{A}_{14} = \overset{-(-.12975)}{\begin{bmatrix} -.16364 & .62339 & -.10173 \\ -.16364 & -.15129 & .79221 \\ -.100 & -.28326 & -.31926 \end{bmatrix}} \begin{matrix} -.16364 & .62339 \\ -.16364 & -.15129 \\ -.100 & -.28326 \end{matrix}$$

$$= -.00790 - .04939 - .00472 + .00154 - .03672 - .03256$$

$$= -.12975$$

$$\hat{A}_{24} = \overset{(.21987)}{\begin{bmatrix} .77273 & -.07189 & -.00974 \\ -.16364 & -.15129 & .79221 \\ -.1000 & -.28326 & -.31926 \end{bmatrix}} \begin{matrix} .77273 & -.07189 \\ -.16364 & -.15129 \\ -.1000 & -.28326 \end{matrix}$$

$$= .03732 + .00570 - .00045 + .00015 + .1734 + .00376$$

$$= .21987$$

$$-(-.17409)$$

$$\hat{A}_{34} = \begin{bmatrix} .77273 & -.07189 & -.00974 \\ -.16364 & .62339 & -.10173 \\ -.100 & -.27326 & -.31926 \end{bmatrix} \begin{matrix} .77273 & -.07189 \\ -.16364 & .62339 \\ -.100 & -.28326 \end{matrix}$$

$$= -.15379 - .00073 - .00045 - .00061 - .02227 + .00376$$

$$= -.17409$$

$$(.35198)$$

$$\hat{A}_{44} = \begin{bmatrix} .77273 & -.07189 & -.00974 \\ -.16364 & .62339 & -.10173 \\ -.16364 & -.16129 & .79221 \end{bmatrix} \begin{matrix} .77273 & -.07189 \\ -.16364 & .62339 \\ -.16364 & -.15129 \end{matrix}$$

$$= .38162 = .00120 - .00024 - .00099 - .01189 - .00932$$

$$= .35198$$

4. Derive matrix of cofactors and transposed matrix of cofactors (i.e., adjoint matrix):

Matrix of Cofactors

$$\begin{vmatrix} .23891 & .15510 & .19125 & .12975 \\ .04674 & .43536 & .28309 & .21987 \\ .02139 & .15204 & .39927 & .17409 \\ .03089 & .23591 & .36124 & .35798 \end{vmatrix}$$

Adjoint Matrix

$$\begin{vmatrix} .23891 & .04674 & .02139 & .03089 \\ .15510 & .43536 & .15204 & .23591 \\ .19125 & .28309 & .39927 & .36124 \\ .12975 & .21987 & .17409 & .35798 \end{vmatrix}$$

5. Divide each element in the adjoint matrix by determinant, D:

$$\begin{vmatrix} 1.40226 & .27434 & .12555 & .18131 \\ .91034 & 2.55530 & .89238 & 1.38465 \\ 1.12252 & 1.66157 & 2.34348 & 2.12026 \\ .76156 & 1.29051 & 1.02180 & 2.10113 \end{vmatrix}$$

6. Multiply original matrix [I-A] by inverse $[I-A]^{-1}$ to obtain identity matrix [I] as check on calculations:

$[I-A] \cdot [I-A]^{-1} = [I]$

Result of matrix multiplication is as follows:

$$\begin{bmatrix} .99996 & .00002 & .00058 & .00000 \\ .00001 & .99958 & .00058 & .00000 \\ .0000. & .00000 & 1.00038 & .00001 \\ .00001 & .00000 & .000234 & .999624 \end{bmatrix} \cong \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C. Problem: Derive income multipliers, ec_j.

1. Given, income coefficient vector (see, Table 1.2):

[.10000 .28326 .33009]

where, $a_{41} = .10000$

$a_{42} = .28326$

$a_{43} = .33009$

2. Prepare matrix of income coefficients:

$[E] = [a_{4j=i}/a_{4j=j}]$

$$= \begin{bmatrix} 1.00000 & .35303 & .30295 \\ 2.83260 & 1.00000 & .85813 \\ 3.30090 & 1.16533 & 1.00000 \end{bmatrix}$$

3. Multiply Leontief Inverse, $[I-A]^{-1}$ by income matrix, [E] to obtain multiplier matrix [EC]:

$[EC] = [I-A]^{-1}[E]$

$$= \begin{bmatrix} 1.33662 & .16323 & .03741 \\ .40867 & 1.70565 & .22404 \\ .35411 & .35945 & 1.31282 \end{bmatrix} \cdot \begin{bmatrix} 1.00000 & 2.83260 & 3.30090 \\ .35303 & 1.00000 & 1.16533 \\ .30295 & .85813 & 1.00000 \end{bmatrix}$$

$$= \begin{bmatrix} 1.33662 & .05763 & .01133 \\ 1.5760 & 1.70565 & .19226 \\ 1.16888 & .41888 & 1.31282 \end{bmatrix}$$