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RELATIVE PRICES, TECHNOLOGY, AND FARM SIZE

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By all conventional measures, average farm size in the United States has grown substantially over the past half century. From 1930 to 1980 land area per farm increased over 2.5 times while real value added grew over four-fold. At the same time the number of farms decreased from 6.3 million to approximately 2.7 million.

The main purpose of this paper is to construct a theoretical framework which explains the growth in size of full-time family farms. In the last section of the paper we present empirical evidence which bears on this hypothesis.

Although the large non-family corporate farms have received much attention of late, American agriculture still consists mainly of family farms. As of 1979 only 2.4 percent of all farm and ranch land in the U.S. was cultivated by non-family corporate farms (USDA, 1979c).\(^1\) In the corn belt this figure was only 1.0 percent. A more significant trend is the growth in part-time farming. By the late 1970s income of farm families from off-farm sources exceeded net income from farming (USDA, 1979a). While part-time farming is not a part of our model, it will be clear that this phenomenon is closely related to the growth in the size of full-time family farms.

The out-migration of farm labor and the growth of farm size are two aspects of the same economic process. The rise in urban incomes serves as an incentive for people to leave agriculture. The remaining land is divided up among fewer but larger farms who are able to maintain income parity with urban people by increasing the quantity of resources at their command. While changes in the farm labor force are generally
attributed to income equilibrating forces, the growth in the size of farm is commonly attributed to economies of scale (Ball and Heady 1972; Jensen 1977; Gardner and Pope 1978; Hall and LeVeen 1978). Perhaps the difference in the explanations stems from a separation of the issues. The out-migration of farm labor is seen as part of the process of adjustment of the industry, while the farm size issue relates to the scale of the typical firm. We attempt to explain firm size but in American agriculture, these two issues cannot be separated. As will be documented later in the paper, the real quantity of labor per farm in the United States has remained relatively constant over the past half century. Therefore the factors that affected the capital/labor ratio in the industry also has affected the amount of resources per farm, i.e., farm size.

There are several analytical difficulties with the concept of economies of scale, and the way it is advanced to explain farm growth. Perhaps the most important are that it implies disequilibrium, returns to scale should vanish once farms reach optimum size, and like technological change, the concept defines, not explains, the phenomena of interest. Also, as we detail in Section 7 below, the empirical evidence does not support unequivocally the existence of the kind of scale economies that can cause persistent growth for decades.

As an alternative explanation we suggest an equilibrium theory of the size of the family farm in which three factors play the leading roles: technology, input prices and non-farm income. We concentrate on the last two since their introduction constitutes the major innovation of the paper. The analysis is comparative static in nature and dynamic adjustments are mentioned only in passing. The paper is
based on a set of restrictive assumptions formulated to capture the 
crucial features of the family farm and to highlight the major 
determinants of its size, the most specific of which is the assumption 
of exclusive employment of a fixed quantity of family labor — hired 
hands or part-time off-farm employment are not considered at this stage. 
The analysis pertains to a relatively small farm sector — urban wages 
and factor prices are determined exogenously to the sector. Other 
assumptions are formulated in the introductory sections of the analysis; 
some of them will be modified in the empirical section at the end of 
the paper.

For convenience and ease of exposition the discussion is conducted 
in terms of a field-crop farm, but, as explained in Section 7, the 
theory applies also to a livestock farm with animals (cows, hogs, 
layers) replacing land area. All symbols are redefined in Table 1 and 
the analysis is summarized in Table 2. The reader may find occasional 
reference to these tables helpful in going through the argument.

1. Scale and innovations

Simplifying, we adopt the number of acres as a measure of farm 
size (number of animal units for livestock farms). We assume that 
each farm is operated by one full-time farmer with no hired help. With 
a single operator, capital deepening — the increase of capital-labor 
ratio — takes the form of larger and more powerful machines. There 
are two sources for the increased size. Partly it reflects technological 
change, the creation of new knowledge, in the machine producing industry. 
In other cases the knowledge to build farm machinery and structures of 
various dimensions is readily available and the actual size is determined 
by economic considerations — manufactures design and produce larger
### Table 1: Glossary of Symbols

<table>
<thead>
<tr>
<th>Variables</th>
<th>Quantity per Farm</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>(L (=1))</td>
<td>(w)</td>
</tr>
<tr>
<td>Machinery</td>
<td>(K)</td>
<td>(u)</td>
</tr>
<tr>
<td>Biological inputs</td>
<td>(B)</td>
<td>(v)</td>
</tr>
<tr>
<td>Land</td>
<td>(A)</td>
<td>(\rho)</td>
</tr>
<tr>
<td>Food (agricultural product)</td>
<td>(y)</td>
<td>(P)</td>
</tr>
<tr>
<td>Rate of interest</td>
<td></td>
<td>(r)</td>
</tr>
<tr>
<td>Shadow price of mechanical services</td>
<td></td>
<td>(q_m)</td>
</tr>
<tr>
<td>Labor machinery price ratio</td>
<td></td>
<td>(\omega)</td>
</tr>
<tr>
<td>Technology parameter</td>
<td>(\lambda)</td>
<td></td>
</tr>
</tbody>
</table>

### Elasticities

| Demand for food                        | \(\eta\)          |
| Substitution in production between land and biological inputs | \(\sigma\) |
| Demand for land as a function (cross) of \(v\) | \(\varepsilon_{A/v}\) |

### Combined variables

| B/A                                    | \(b\)             |
| Machine land ratio (constant)          | \(m\)             |
| Augmented land                         | \(A^*\)           |
| (Land with its machine services)       |                   |
| Share of biological input in output   | \(s_B\)           |

### Functions

| Mechanical subprocess                  | \(f_m(\ )\)       |
| Biological subprocess                  | \(f_b(\ )\)       |
| Production as a function of A and B    | \(H(\ )\)         |
| Intensive form of \(H(\ )\)           | \(h(\ )\)         |
| Marginal products in the mechanical subprocess \((i=K,L)\) | \(f_{mi}\)       |
machines as demand arises. In fact, large tractors were pulling multi-
bottom ploughs already at the beginning of the century in the "Bonanza
farms" of North Dakota (Drache, 1964) and the same companies that produced
the modest-sized 18-50 hp farm tractors had already in the 1930's
produced earth moving equipment of very different orders of magnitude.

Thus we view innovations in machine size to be induced by factor
prices. On this point we accept the perspective of the induced innovation
hypothesis (Hayami and Ruttan, 1971) but unlike that perspective we take
technical change, associated with the creation of new knowledge on the
farm or in the input supplying industries, to be exogenous to agriculture.
The possibility that factor prices affect the direction of technical
change is, for simplicity and in order to focus on the main theme of the
analysis, not considered.

2. The Model

Only one agricultural product is produced, call it food. It is
produced in a two-process production function. The first is mechanical,
the second is biological. Both subprocesses exhibit constant returns
to scale, both are essential to production, and there is no substitution
between them. Per-farm output of food, Y, is given by equation (1).

\[ y = f(f_m(K,L), f_b(A,B)) \]

K capital
L(=1) labor
A land
B biological inputs
\[ f_m, f_b \] mechanical and biological subprocesses.
Although the mechanical and biological subprocesses occur simultaneously, it will be useful to view production as if it were conducted in successive steps. In the mechanical subprocess, $f_m$, capital and labor are combined in the usual continuous fashion to create units of mechanical services -- call these units *mechans*. Quadrant I in Figure 1 depicts isoproduct curves of *mechans*. The assumption of a fixed quantity of family labor per farm is expressed in the diagram by the vertical line at $L = 1$.

In Quadrant II we illustrate the production of *mechans* as a function of capital for the given one unit input of labor. Note particularly the assumption of diminishing returns to capital which implies increasing costs of mechanical services as the capital/labor ratio rises.

Quadrant III represents the second part of the mechanical subprocess. In this activity *mechans* and raw land are combined in fixed proportions to produce "augmented land." This is land endowed with the required amount of mechanical services, call it $A^*$. The constant *mechans* to raw land ratio is given by "$m". We assume that this ratio does not change with the level of biological inputs or with crop yields, i.e. it takes the same amount of mechanical services to spread a small or large amount of fertilizer or to harvest a field with a high or low yield.

In the biological subprocesses, augmented land ($A^*$) is combined with biological inputs such as feed, fertilizer, herbicides, water, etc. to produce food. No restriction is placed on the elasticity of substitution between $A^*$ and biological inputs.
Figure 1: The mechanical subprocess of production and the determination of farm size.
To reflect production by subprocesses, (1) can be rewritten in a hierarchical form:

\[
(1') \quad y = f(b(A, B, f_m(K, L))) = H(A^*, B)
\]

where \( H() \) is the biological subprocess. In the intensive form, food per acre is

\[
\frac{y}{A^*} = h(B/A^*)
\]

3. Determination of farm size.

Since each acre of raw land requires \( m \) units of mechanical services, the size of the full-time family farm is determined by the number of mechs produced. The number of mechs is determined in the "mechanical services department" of the farm, with the production function \( f_m() \), in the following way. Farmers consider the urban wage as the opportunity cost of their own labor. Given this wage, \( w \), and the cost of machine services, \( u \), a farmer will settle at a point of tangency of the factor price ratio, \( \omega \), and the \( M \) isoquant.

\[
(2) \quad \omega \equiv \frac{w}{u} = \frac{f_mL}{f_mK}
\]

In Figure 1, Quadrant I, \( Q_1 \) is the point of tangency for a farm operated by one family \( (L=1) \) facing a price ratio of \( \omega_1 \). In this case the farmer would hire \( K_1 \) units of machine services and produce \( M_1 \) units of mechs. At any other capital/labor ratio the family farm would not be minimizing costs while using its full complement of labor. As long as the individual farm is in business, profits will be maximized only if the family labor is fully employed. Hence \( Q_1 \) represents a profit maximizing as well as a cost minimizing point for price ratio \( \omega_1 \).
Given the constant mechs to land ratio, \( m \), the optimum size of the full time family farm (land area) is determined by the optimum number of mechs produced as shown by (3).

\[
(3) \quad A = \frac{M}{m}
\]

For example, if \( M_1 \) is the optimum number of mechs, \( A_1 \) is the optimum size of farm (Quadrant III).

Consider next an increase in nonfarm wages (exogeneous to agriculture) holding constant the price of machine services. In Figure 1 this is shown by a change in the slope of the isocost line from \( \omega_1 \) to \( \omega_2 \). At this new price ratio, \( M_1 \) would be produced by input mix \( Q_2 \) (Quadrant I). However because of the constant family labor input assumption, \( Q_3 \) is now the optimum input combination and \( M_2 \) mechs are produced. By following the broken line from \( Q_3 \) around the Quadrants in a counterclockwise fashion it becomes evident that the increase in nonfarm wages increases the optimum size of farm. The increase in farm size is accomplished by the drawing out of agriculture some farmers who take urban employment leaving the remaining land in the hands of fewer but larger farms.

In economic terms, the model tells us that as the opportunity cost of family labor increases relative to the cost of capital services, it becomes profitable for full time family farms to acquire more machinery and land. By increasing the size of their farms, farm people are able to achieve income parity with city people. Farmers that desire to remain small can substitute machines for their labor, take off-farm jobs, and become part-time farmers. In so doing they can maintain income parity.
4. **Land Values**

Because of the interrelationship between agricultural prices and land values, it will be useful at this point to consider how land prices are determined by the model. Because of the assumption of a fixed supply of agricultural land which has no alternative use, the return per acre, \( \rho \), is a residual

\[
\rho = \frac{(P_y - uK - w - vB)}{A}
\]

The price of an acre is \( \rho/r \), where \( r \) is the rate of interest.

What happens to the price of land when urban wage rates increase? Because of the higher price of labor and the reduction in the marginal productivity of capital as more capital is added per unit of labor (Quadrant II), the cost per mechan at \( Q_3 \) is higher than the cost at \( Q_1 \). Due to the assumption of a fixed number of mechan per acre, the residual return to land is lower at the new, higher wage rate. Therefore the rental value of land declines. The reduction in the implicit cost of land exactly compensates for the increased cost of producing mechan.

As long as rent is positive at the new equilibrium, \( Q_3 \), the increase in the opportunity cost of farm labor which entails a reduction in the number of farmers, an increase in capital per unit of labor, and an increase in farm size, does not effect the supply of food. The reason is that these changes do not affect the optimal level of biological inputs, and therefore yield per acre remains constant. Mechan are produced in a more capital intensive way but the same quantity of mechanical services (plowing, planting, harvesting, etc.) are performed.
Up to this point we have assumed that all land is cultivated and rent is positive. Another situation, which we term the **zero rent case**, can occur. Depending on the level of demand for food, factor costs, and productivity, the cost of production may be higher than the food price when all agricultural land is cultivated. Were such a situation to materialize, farmers will find that even with an imputed rental value of zero, their labor earnings fall short of their opportunity cost, \( w \). Equilibrium in such a case will be achieved with some land uncultivated. The equilibrium number of farms will be that number that will fix supply, and hence food price, at a level that will set \( p \) in equation (4) equal to zero.

Of course, urban wages can increase to the level that will invoke the zero rent case. In this case the increase in the opportunity cost of farm labor will reduce the number of farms, total cultivated acreage, and the supply of food. Food supply will decline until the price of food increases to the point where zero rent prevails and equilibrium is achieved.

5. **Price Changes**

   a. **Labor.** In the two preceding sections we demonstrated that an increase in the opportunity cost of farm labor (urban wage rates) will increase the optimum size of farms, reduce the rental value of farm land, and may or may not change the supply of food depending on whether the year rent case is encountered.

   In Figure 1 the capital-labor substitution phenomenon is presented in terms of the individual farm. As farms become larger they move to higher and higher isoquants, \( M_1 \) to \( M_2 \), etc. For the agricultural
sector as a whole, there is only one isoquant for the various capital-labor combinations providing all agricultural land is used. This is illustrated by Figure 2. As the price of labor increases relative to capital and the slope of the isocost line changes from $\omega_1$ to $\omega_2$, labor leaves agriculture, capital inputs increase, and the capital/labor ratio changes from $R_1$ to $R_2$. The ratios along $R_1$ and $R_2$ in Figure 2 are the same as in Figure 1. This aggregate, sector level framework is the basis for the empirical analysis of the last section of the paper.

b. **Machine services.** At first glance it may appear that a decline in the price of machine services will give rise to the same outcome as an increase in the price of labor. But this is not strictly correct. It is true that a machine price decrease will change the factor price ratio from $\omega_1$ to $\omega_2$, and the optimum point will move from $Q_1$ to $Q_3$. As in the case of a labor price increase the optimum farm size will increase, and agricultural supply will remain unchanged.

The effect on land price will, however, be different from what it was when the opportunity cost of farm labor increased. At the lower machinery services cost, the residual return to land will be higher, causing an increase in land values. Consequently farmers will enjoy a capital gain. Some of them choose to realize this gain and move to town, thus freeing land for the expansion of the farms of those who stay in agriculture. (Perhaps the old folks take capital gains to facilitate early retirement, while the young choose urban employment when nonfarm wages increase.)

c. **Biological inputs.** A reduction in the price of biological inputs -- fertilizer, pesticides, feed, etc. -- will intensify their employment, increase yields and expand the food supply. However,
Figure 2: The agricultural sector.
because of the non-substitutability assumption between chemicals and land in the production function, the decrease in the price of biological inputs will not affect the capital/labor ratio and farm size.

On the other hand, the decrease in the prices of the biological inputs will affect land values. The increased use of these inputs will increase the marginal product of land. At the same time, the increase in the supply of food causes a decrease in its price, thereby having an opposite influence on the demand for land.

To analyze the relation between biological input prices and land values, let $\sigma$ be the elasticity of substitution between $A$ and $B$ in $H(\cdot)$ of equation 1', $\eta$ the absolute value of the demand for food, and $S_B = vB/PY$, the share of biological inputs in total output. Then the elasticity of demand for land with respect to biological input prices, $E_{A/v}$, is (Ferguson, Ch. 12)

$$E_{A/v} = S_B(\sigma - n)$$

Accordingly the demand for land and land prices will increase in response to a decline in $v$ if $\sigma < n$, and decline if $\sigma > n$.

As in the case of an increase in nonfarm wages, the demand for land can decrease to the point where the zero rent case prevails and some land goes unused. Even in this case, however, the use of biological inputs, both per acre and in the aggregate, will increase; otherwise supply will not expand and food price will not decline.

d. Food. In order to more clearly see the process of adjustment, we begin at the zero rent case where some land is uncultivated. Assume the demand for food increases. In the short run the price of food will increase temporarily, resulting in a capital gain to farmers, and serving
to draw more farmers into agriculture. However, in the long run, as long as some land remains uncultivated, equilibrium food price will not change. Further increases in the demand for food eventually will be met by the land constraint. Now food price will increase in the long run as will the price of land. Because farmers enjoy a capital gain their total income, \( rW + \omega \), will rise but the opportunity cost of their labor will remain at parity with urban earnings. After the land constraint is encountered the total quantity of food supplied will increase because of the increased demand for and use of biological inputs. But farm size will increase only if the opportunity cost of farm labor (nonfarm earnings) increases relative to the price of capital services.

6. Technological Change.

Traditionally technological change has been described by shifts in a production function or the respective isoquants. However as long as the basic laws of physics, biology, chemistry etc., remain unchanged, there can be no increase in output without an increase in inputs. Therefore shifts in a production function can occur only if inputs increase in quality and their measurements do not fully reflect this quality, or if new inputs which come on the scene are omitted from the production function. However the improved inputs will not be adopted unless their real prices are lower than the inputs they replaced, or in the case of new inputs, unless their VMPs exceed their prices. Thus one can analyze technological change either in terms of shifts in a production function where inputs are not adjusted for quality or measured correctly, or in terms of lower input prices with the inputs
adjusted for quality changes. The results should come out the same.

a. Mechanical. Because neutral or non-neutral shifts in isoquants are a common way of depicting technological change, we shall run these shifts through the model and observe the consequences. Consider first a neutral technological improvement in the production of mechanical services, \( f_m(.) \) in equation (1). Graphically, the isoproduct curves in Figure 1 shift toward the origin. Then, say, the curve labelled \( M_1 \) becomes \( M_2 \) after the shift. Also the production function in Quadrant II shifts up and to the left. Since the shift is neutral and the slopes of the isoproduct curves along the rays from the origin do not change \( Q_1 \) will still be the optimum combination. Hence a neutral shift in the production function of mechanical services will increase farm size (from \( M_1/m \) to \( M_2/m \)) without increasing the measured amount of machines used.

Of course, if the machines were adjusted for quality, total machine use would increase because higher quality machines are more machines than those of lower quality. In this case the real price of machine services would have declined, providing they were adopted, and the same result obtained as in section 5b above.

The conventional case of machine biased technological change in \( f_m \) is depicted in Figure 3. Again this assumes that the quantity and price of the machines have not been adjusted for quality changes. The isoquant \( M_1 \) becomes \( \hat{M}_1 \) and the price ratio \( \omega_1 \) which was tangent to \( M_1 \) at \( Q_1 \) is now tangent to \( \hat{M}_1 \) at \( Q_2 \). Equilibrium along the \( L=1 \) line will now be at \( Q_3 \) and optimum farm size will increase. Unlike the previous case of neutral technological change in \( f_m \), the measured amount of machines now increases. In constant quality units the increase in machine use would be greater here than in the previous case because of
Figure 3: Biased technological change in the mechanical service sector.
the increased MPP of machines vis-a-vis labor. In both the neutral and non-neutral shifts, the level of biological inputs is not affected leaving food supply and food prices unchanged. Because of the assumptions underlying the model, the lower production costs (reduction in the real price of machine services) are reflected in larger residual returns to land and higher land values.

b. Biological. Consider a neutral technical change in \( f_b \), that is, in \( H(\cdot) \), with a technology parameter, \( \lambda \),

\[
y = \lambda H(A, B)
\]

An increase in \( \lambda \) will increase yield and supply of food. It will also reduce the price of food. To analyze the demand for \( B \) and the returns to land, note that value of marginal product of an input changes with technology according to

\[
\frac{\partial}{\partial \lambda} (PAH_i) = PH_i(1 - 1/\eta) \quad i = A, B
\]

Therefore, if the demand for food is inelastic, neutral technological change in the biological sub-process will call forth a decrease in the amounts of the factors of production; that is, it will entail a decrease in the use of biological inputs, measured without equality adjustments, and lower land values. The opposite will be true if demand is elastic.

If the agricultural sector were to acquire both land services and the factor \( B \) on the free market at fixed prices, then (since technological change is assumed neutral), the use of both factors would have contracted or expanded proportionally. With land in fixed supply and being the residual rent receiving factor, the quantitative change in the employment of the biological factors depends on the elasticity of substitution in \( H(\cdot) \).
Table 2. Summary of Comparative Statics

<table>
<thead>
<tr>
<th>Demand</th>
<th>Farm Size</th>
<th>Number of Farms</th>
<th>Product Supply</th>
<th>Product Price</th>
<th>Land Rent</th>
<th>Machine Use</th>
<th>Biological Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>food (up)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prices</th>
<th>Farm Size</th>
<th>Number of Farms</th>
<th>Product Supply</th>
<th>Product Price</th>
<th>Land Rent</th>
<th>Machine Use</th>
<th>Biological Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor (up)</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>machine (down)</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>biological</td>
<td>c &gt; n</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>inputs (down)</td>
<td>c &lt; n</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technological Change</th>
<th>Farm Size</th>
<th>Number of Farms</th>
<th>Product Supply</th>
<th>Product Price</th>
<th>Land Rent</th>
<th>Machine Use</th>
<th>Biological Inputs</th>
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<tbody>
<tr>
<td>mechanical neutral</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>machine biased</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
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<tr>
<td>biological</td>
<td>n &gt; 1</td>
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<tr>
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<td>n &lt; 1</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes:

a. Zero rent case not included;

b. Directions of changes are those observed most often in modern agriculture.

c. Mechanical and biological inputs are not adjusted for quality in cases of technological change.
Biological technological change will not change farm size. It may, though, tilt the economy over to the zero rent case with idle land, zero rent and a smaller number of farms than the one prevailing before the change.

7. Other Issues
   a. Livestock. Livestock share with field crops the same general features of the production process: mechanical services are proportional to the size of the enterprise and virtually unaffected by yield (milk per cow, eggs per layer, feed conversion ratios in meat animals and poultry, etc.). Once the appropriate mechanical services have been provided, the yield per unit of livestock is determined by biological factors including inputs such as feed, medicine, and genetic material. The dual process production function of equation (1) can be applied without modification, where A now represents number of livestock units and B is mainly feed. The previous analysis of the size of farms and intensity of utilization of biological inputs can be applied equally to livestock farms and field crop operations.

   There is however one difference between the two types of farms. Because no factor is in fixed supply in the livestock case, there is no residual receiver of rent. Therefore the zero rent case applies, except in the short run situation of a growing livestock industry where the rate of growth is constrained by the size of the breeding herd. In this case rent to breeding stock capital could be positive.

   The optimal size of the livestock farm also is determined by the process depicted by Figure 1. In equilibrium the number of farms will be such that the supply forthcoming together with demand will set a
price of the product that will equate costs and returns in the industry. In other words, one can view labor as the residual income receiver, and the number of livestock farms will be such that on-farm labor earnings will be equal to the urban wage rate.

A significant share of livestock production in the U.S. is on diversified farms, typically on farms combining a field crop and a livestock enterprise. Virtually all hogs and dairy cows in the Midwest and in the East and a substantial share of beef fattening are done on farms that produce their own feed. However, poultry almost everywhere and dairy farms in the Southwest and West are usually completely specialized. In the past, diversification was more pronounced than today and was probably practiced mainly in order to reduce risk and to assure a home supply of livestock products. In a modern, market-oriented agriculture, the main advantage of on-farm production of feed lies in the elimination of cost of handling, marketing and transportation between the field crop and the livestock farmers. For diversified farms, machinery such as grain elevators, silo unloaders, milking machines, and manure disposal equipment play a similar role in livestock production to the one played by tractors and equipment in field crop operations. For the discussion of farm size one can, therefore, assume that field crops and livestock enterprises grow proportionally. The question of the determinants of optimal enterprise combination lies outside the scope of this paper.

b. **Economies of Scale.** The analysis in this paper has demonstrated that growth in farm size can be explained without reference to the catch-all
"economies of scale". However, such economies are also not so clearly supported by the empirical record, at least as we read it. True, some of the ordinary least square estimates of Cobb-Douglas production functions have yielded high scale coefficients. Griliches (1964), for example, reported sums of elasticities for an aggregate, state level production function in the range of 1.2 to 1.3. However, other, particularly farm level estimates, indicated decreasing returns (Kislev 1966; Hoch 1976). Moreover OLS estimates can be criticized as being biased upward due to the omission of a management factor which is positively correlated with size of operation (Mundlak 1961). Indeed, covariance analysis with a firm effect accounting for management, yields, in most cases, decreasing returns even in samples which OLS estimates indicate strong economies of scale (Hoch 1976).

A possible explanation for the difference in results is the age distribution phenomenon. The inclusion in the same sample of young (and old) farmers having comparatively small operations and inefficient asset combinations together with well established farmers in their prime age and balanced asset structure, will result in upward biased OLS estimates of returns to scale. Such bias is eliminated with covariance analysis.

Other evidence, coming from synthetic "engineering" firm analysis, have indicated economies of scale that are sharply reduced once the size of a one or two-man operation is reached (Madden and Partenheimer 1972; Hall and LeVeen 1978). The significant aspect of those findings for the explanation of the historical growth in farm-size is not the mere existence of scale economies at the family-farm range of operation, but rather the persistent upward shifts of optimum size often found in
cost analyses and attributed to the market appearance of new machines (Rodewald and Folwell 1977). There is, however, an identification problem here. Undoubtedly, new machines reflect, in part, new exogenous technological developments which reduce the real cost of mechanical services and induce growth in farm size. Partly, however, they also reflect endogenous changes in equipment needed to adopt production to a rising opportunity cost of farm labor. Attributing all growth in farm size to exogenous technological developments clearly overstates the effect of this factor.

c. Structure. The structural assumptions on which this paper rests are of two kinds: (a) the formulation of the theory and its empirical application in terms of a single product and uniform production conditions implies that the same economic factors are assumed to similarly affect growth of farms of different types and regions, (b) the relevancy of the study depends on the stability and the resiliency of the family farm. However, the analysis not only relies on the assumption of this institutional viability, it can also shed explanatory light on the forces that maintained the industrial organization of American agriculture.

The U.S. farm sector owes its structure to the historical patterns of settlement and homesteading. Its stability is fostered by strong economic forces whose operation is best illustrated with the few episodes of large scale farming in the mid-west. For example, some of the Bonanza Farms of the Red River Valley cultivated in the last third of the 19th century more than 20,000 (one even 55,000) acres (Briggs, 1932; Drache, 1964) and similar sizes were also reported for Illinois and Iowa (Gates, 1932). These huge enterprises, that depended on a large
hired labor force and substantial investment in power and equipment ceased to operate and were sold, subdivided into family units, at the beginning of this century.

The dominance of the family unit in agriculture and of the large corporation in the nonfarm sector testify to the lack of significant economies of scale in farming and to their existence in other industries. Owners of large amounts of wealth will, therefore, invest their capital in the nonfarm sector. If, in addition, large farm enterprises run into scale diseconomies, such large holdings will be subdivided into family units. Thus, the American family farm was preserved by ample non-farm investment opportunities.

In the less developed countries, on the other hand, where investment opportunities outside agriculture are less attractive, the rich have to accumulate their capital in land even if large holdings are comparatively inefficient (Berry and Cline 1979). This suggests an hypothesis on the connection between agrarian structure and urban development. The empirical test of such an hypothesis is, of course, outside the scope of the present study.

6. Empirical Evidence

Changes in factor prices change the capital/labor ratio according to the elasticity of substitution. This explains the growth of mechanical services and farm size. Biological inputs are not considered at this stage.

a. Salient features. The major magnitudes associated with farm growth are presented in Table 3. The empirical analysis of farm size reported below is limited to the period 1930-1970. Long run
adjustments to the large price fluctuations of the mid 1970s have yet to run their course, although data for 1976 are included in the table to provide an indication of future trends.

Sources and definitions of data are detailed in the Appendix. The salient features to note in Table 3 are: (1) the substantial growth in farm size as indicated by the near quadrupling of value added and more than doubling of acres per farm; (2) the relative constancy of family and hired labor per farm; (3) the large growth of power and machinery; (4) the increase in the price of labor relative to machinery.

The labor variable has been adjusted for quality changes due to the increased educational level of the rural farm population over the period. This quality adjustment increased the per farm family labor input about 20 percent between 1930 and 1970 which serves to offset the 18 percent reduction in the unadjusted family labor input due to the decline in the average size of family. Thus the assumption of a constant (quality adjusted) family labor input appears justified. Real wages in manufacturing and agriculture, both adjusted for schooling, increase substantially over the period with the latter closing the gap in more recent years.

Because of the difficulty of measuring quality changes in machinery, several measures of machine inputs, prices, and costs are presented. We would argue that custom rates provide the most accurate measure of the real cost of machine services. This market determined figure should reflect reductions in the real cost of machine services not only because of improvements in quality but also due to the preferential tax treatment of capital, mainly investment credit and accelerated depreciation. Note that the real cost of fuel, a major part of machine costs, moves parallel to custom rates.
Table 3. U.S. Agricultural Data—Per Farm Output, Inputs, and Prices.

<table>
<thead>
<tr>
<th>Measure</th>
<th>1930</th>
<th>1950</th>
<th>1970</th>
<th>1976</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value added dollars</td>
<td>$2972</td>
<td>$4745</td>
<td>$11,992</td>
<td>$12,819</td>
</tr>
<tr>
<td>Land area acres</td>
<td>157</td>
<td>215</td>
<td>374</td>
<td>394</td>
</tr>
<tr>
<td>Labor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>family man/years</td>
<td>1.42</td>
<td>1.43</td>
<td>1.36</td>
<td>1.34</td>
</tr>
<tr>
<td>hired man/years</td>
<td>.49</td>
<td>.43</td>
<td>.48</td>
<td>.62</td>
</tr>
<tr>
<td>Wage rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing Index</td>
<td>59</td>
<td>100</td>
<td>124</td>
<td>130</td>
</tr>
<tr>
<td>Agriculture Index</td>
<td>62</td>
<td>100</td>
<td>138</td>
<td>148</td>
</tr>
<tr>
<td>Power and machinery</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USDA Index</td>
<td>--</td>
<td>100</td>
<td>240</td>
<td>289</td>
</tr>
<tr>
<td>15-year one-hoss-shay Index</td>
<td>45</td>
<td>100</td>
<td>298</td>
<td>419</td>
</tr>
<tr>
<td>10 percent depreciation Index</td>
<td>43</td>
<td>100</td>
<td>328</td>
<td>446</td>
</tr>
<tr>
<td>Machinery prices and costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USDA Index</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>157</td>
</tr>
<tr>
<td>Bureau of Labor Statistics Index</td>
<td>93</td>
<td>100</td>
<td>108</td>
<td>119</td>
</tr>
<tr>
<td>Hayami and Ruttan Index</td>
<td>104</td>
<td>100</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Custom rates Index</td>
<td>111</td>
<td>100</td>
<td>86</td>
<td>157</td>
</tr>
<tr>
<td>Gasoline Index</td>
<td>132</td>
<td>100</td>
<td>83</td>
<td>88</td>
</tr>
<tr>
<td>Diesel fuel Index</td>
<td>--</td>
<td>100</td>
<td>75</td>
<td>113</td>
</tr>
</tbody>
</table>

Notes: all monetary figures are in constant 1969 dollars. Value added is deflated by the Index of Prices Received by Farmers while wage rates and machinery prices are deflated by the Consumer Price Index. See appendix for further details on data sources and construction.
The official USDA price index has been criticized for neglecting to take quality into account (Griliches 1960, and Fettig 1963). As a result the actual quantity of power and machinery is likely to have grown more rapidly than the official USDA index indicates. Moreover the rapid write-off of machinery by farmers makes it appear that the stock of machinery is smaller than it really is. Our indexes of machine stocks which were built up from value of shipments data using alternative 15-year one-hoss-shay and constant 10 percent depreciation assumptions allow for longer life machines. Consequently they show a substantially larger increase in machinery stocks than the USDA index.

b. Elasticity of substitution. An empirical test of the model requires an unbiased estimate of the elasticity of substitution between capital and labor as well as constant quality measures of inputs and their respective prices. To incorporate simultaneity in the labor market, the elasticity of substitution between capital and labor is estimated in the following two equation model, expressed in logarithms:

\begin{align}
(7) \quad y &= \alpha_1 + \delta w + e \quad \text{demand} \\
(8) \quad w &= \alpha_2 + \delta \bar{w} + \gamma y + \varepsilon \quad \text{supply}
\end{align}

- \(w\) = farm wage rate;
- \(y\) = value added per unit of labor;
- \(\bar{w}\) = manufacturing wage rate;
- \(e, \varepsilon\) = independently distributed random disturbance terms.

The demand equation is derived from the CES production function (Arrow, Chenery, Minhas and Solow 1961; Minasian 1961). The supply of farm labor is influenced by manufacturing wages, \(\bar{w}\), a proxy for nonfarm earnings opportunities, and farm earnings, \(y\).
The demand equation can be identified and estimated with manufacturing wages as an Instrumental Variable. In Table 4 we report the results of these estimates along with OLS estimates of equation (7). The estimates are cross section for each of the six census years shown, using states as the unit of observation (n=48, see the Appendix for details of data construction). To account for differences in size of states, weighted regressions also are estimated using number of farms in each state as weights.

In general the IV estimates are higher than the OLS figures. All the IV estimates in the weighted regressions are not significantly different from 1.8, therefore we use this value for c in the calculations to follow.

c. Other estimates. Both Griliches (1964) and Binswanger (1974) reported elasticity estimates that were not significantly different from 1 which is substantially smaller than our figures. The differences in the estimates are evidently due to differences in specification, in particular, Griliches and Binswanger disregarded simultaneity of urban-rural labor income, grouped several states to single observations, and did not use weighted regressions. Also Binswanger used a multi-factor formulation. Low elasticity values are hard to reconcile with historical developments. By our calculation, the ratio of wage to machine costs increased almost 3 times over the 40 year, 1930-1970 period and the ratio of machine to labor inputs increased over the same period by a factor of more than 6.5. These magnitudes suggest either an elasticity of substitution which is close to 2 or substantial changes in demand. Demand for machinery may have increased as a result of the introduction
Table 4: Elasticity of Substitution

<table>
<thead>
<tr>
<th></th>
<th>Unweighted Regressions</th>
<th></th>
<th>Weighted Regressions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>IV</td>
<td>OLS</td>
<td>IV</td>
</tr>
<tr>
<td>1949</td>
<td>1.48</td>
<td>1.81</td>
<td>1.54</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>(7.71)</td>
<td>(6.81)</td>
<td>(11.00)</td>
<td>(8.03)</td>
</tr>
<tr>
<td>1954</td>
<td>1.83</td>
<td>1.96</td>
<td>1.84</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>(11.16)</td>
<td>(8.31)</td>
<td>(13.53)</td>
<td>(9.52)</td>
</tr>
<tr>
<td>1959</td>
<td>1.23</td>
<td>1.65</td>
<td>1.33</td>
<td>1.47</td>
</tr>
<tr>
<td></td>
<td>(6.12)</td>
<td>(5.48)</td>
<td>(7.73)</td>
<td>(5.72)</td>
</tr>
<tr>
<td>1964</td>
<td>1.13</td>
<td>1.48</td>
<td>1.32</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>(4.67)</td>
<td>(3.47)</td>
<td>(6.35)</td>
<td>(3.82)</td>
</tr>
<tr>
<td>1969</td>
<td>1.07</td>
<td>1.78</td>
<td>1.22</td>
<td>1.69</td>
</tr>
<tr>
<td></td>
<td>(2.30)</td>
<td>(1.71)</td>
<td>(2.90)</td>
<td>(1.96)</td>
</tr>
<tr>
<td>1974</td>
<td>1.35</td>
<td>2.10</td>
<td>1.23</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>(2.97)</td>
<td>(1.69)</td>
<td>(2.96)</td>
<td>(1.69)</td>
</tr>
</tbody>
</table>

Notes:  
a. The estimated equation is (7), number of observations 48.

b. The figures in parenthesis are t ratios. The t values for IV regressions are calculated according to Maddalla, 1977, p. 239.

c. Weighting is by the number of farms in each state for each year.
of seasonally sensitive varieties for example, but it is hard to see how such changes may have doubled the demand for machinery on the farm, which they should have done to explain the quoted quantitative relations.

Binswanger used in his study USDA time series data in which the ratio of machinery service to labor cost rose after the Second World War. He, therefore, concluded that biased technical change must have been important in the intensive process of agricultural mechanization. This conclusion, which runs counter to our theoretical assumptions and interpretation of the data, is, however, unwarranted because Binswanger did not adjust machinery prices for quality. The quality unadjusted prices which registered increases are economically meaningless; farmers react to the effective cost of machinery, in standard efficiency units. Effective cost fell relative to labor cost and mechanization can therefore be explained as a straightforward reaction to price signals without the need to suggest biasedness in technical change.

d. Explanation of growth in farm size. The average annual rate of change over the 40-year-period (1930-70) in the ratio of labor to machine cost was 2.5 percent, while the capital/labor ratio grew at the rate of 4.9 percent per annum (Table 5). Price changes explain, therefore, about half the input change if the often quoted value of $\sigma = 1$ is accepted, and 92 percent of the change if $\sigma = 1.8$ is adopted. If one accepts 1.8 for the elasticity of substitution, then the "over-explanation" in the first period (1.381 in line 3b) might be attributed to the unusual circumstances associated with the Great Depression and the WW II years. The 1950-70 period can be looked upon as a time of "catching up" to the adjustments that would have occurred in the preceding two decades.
Table 5: Changes in Prices and Inputs and Explanation of Size
(Average Annual Percentage Changes)

<table>
<thead>
<tr>
<th></th>
<th>1930-50</th>
<th>1950-70</th>
<th>1930-70</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Wage to machine cost</td>
<td>.032</td>
<td>.018</td>
<td>.025</td>
</tr>
<tr>
<td>2. Capital-labor ratio</td>
<td>.042</td>
<td>.057</td>
<td>.049</td>
</tr>
<tr>
<td>3. Percentage explained</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a.   θ = 1</td>
<td>.762</td>
<td>.316</td>
<td>.510</td>
</tr>
<tr>
<td>b.   θ = 1.8</td>
<td>1.381</td>
<td>.561</td>
<td>.918</td>
</tr>
<tr>
<td>4. Farm size</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Value added</td>
<td>.022</td>
<td>.047</td>
<td>.034</td>
</tr>
<tr>
<td>b. Land</td>
<td>.016</td>
<td>.028</td>
<td>.022</td>
</tr>
<tr>
<td>5. Production of methods</td>
<td>.013</td>
<td>.018</td>
<td>.016</td>
</tr>
<tr>
<td>6. Percentage explained</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Of value added</td>
<td>.591</td>
<td>.383</td>
<td>.471</td>
</tr>
<tr>
<td>b. Of land area</td>
<td>.813</td>
<td>.643</td>
<td>.727</td>
</tr>
</tbody>
</table>

Notes:
1. Manufacturing wage rate and custom rates;
2. Family and hired labor and machine one-hoss-shay stock;
3. a. line 1/line 2; b. 2(line 1)/line 2;
4. line 2 times 0.319 (see text);
5. line 5/lines 4.
under more normal circumstances, as well as a time of adjusting to changing relative price ratios.

Given the stock of capital, farm size is determined by the quantity of mechanical services (mechans) produced. To gauge changes in this variable we adopt the production function estimate of Griliches (1964). His coefficients for labor and machinery were .426 and .200 respectively. Assume that these coefficients represent the relative elasticities of labor and machines in the production of mechans and that this subprocess is subject to constant returns to scale. Then in a linear homogeneous mechans production function the machine coefficient will be $\frac{.200}{.426 + .200} = .319$. Applying this figure to the growth in power and machinery we obtain an annual rate of growth, over the 1930-70 period, of the production of mechanical services of 1.6 percent (Table 5). This growth explains 72.7 percent of growth in land per farm and 47.1 percent of the growth in value added per farm. The difference is probably due to the fact that growth in value added reflects quality improvements of biological factors: chemicals, seeds, medicine, etc.

8. Epilogue

Over the 20 year 1950-70 period, the ratio of manufacturing wages to machine costs (custom rates) increased at a rate of 1.5 per cent per annum while land per farm increased at an annual rate of 2.8 per cent (Table 3). However the trend in the factor price ratio was reversed during the 1970-76 period when machine costs rose faster than wages at an annual average rate of about 10 percent.

The model predicts that the reversal of the capital/labor price trend should eventually stop the growth in farm size, and perhaps even
reverse it. It is too early to tell whether this trend will persist, although it is interesting to note that the annual growth in land area per farm declined to almost one-third of its previous rate during the 1970-76 period. If the relative growth in energy costs persists, one should not be surprised to see a decline in farm size. Of course, if the relatively high inflation rates of the 1970s continue, land will continue to be an attractive investment, and farm size may grow for this reason alone. Our model which assumes zero inflation is not designed to handle the inflation phenomenon. Many interesting questions remain including the impact of inflation on farm size, the reasons for the increase in use of biological inputs, the increased popularity of part-time farming, as well as the explanation for changes in the size distribution of farms.

9. **Summary and Conclusions.**

In this paper we have developed an equilibrium theory of optimum farm size. According to our model, the growth in the size of farms in the United States has occurred because of the increase in the opportunity cost of family labor relative to the cost of machine services. As nonfarm incomes have increased, farm people have attempted to achieve income parity by increasing the size of their farms. The growth in farm size was made possible by farmers who left agriculture to take advantage of higher earnings in the nonfarm sector. The land which they released has been incorporated into fewer but larger farms. Moreover, we have explained the growth in farm size without relying on the "catch-all" phrases of economies of scale and technological change.
Footnotes

* Hebrew University, Rehovot, Israel and The University of Minnesota, St. Paul, Minnesota. We are indebted to Zvi Griliches, Hans Binswanger and members of seminars at Minnesota, Yale and Rehovot for comments and suggestions. Most of the work on this study was done in Minnesota when Yoav Kislev was visiting the Department of Agricultural and Applied Economics. The project was supported in Israel by the United States-Israel Agricultural Research and Development Fund - BARD.

1/ Of course many family farms have incorporated for tax purposes and to facilitate transfer between generations. "Farming corporations, like partnerships, tend to be closely held by a few family shareholders. Eighty percent of privately held farming corporations had five or fewer shareholders, and 79 percent were family owned, with family members directly involved in daily operations. Ninety percent of the closely held corporations had most of their management provided by shareholders. Farming is the primary, and often the only, business for these corporations. Corporations are frequently operated similar to partnerships and often formed to preserve the family farming operation by facilitating the transfer of assets between generations." (USDA, 1979b, p. 8).

2/ While we believe that the opinion in the text is shared by many economists, we could find no written statement with reference to American agriculture. For an elaboration in the context of developing economies, see Ranis and Fei (1961).
3/ Linear homogeneity of the mechanical subprocess is not essential; a homothetic production process will yield identical analytic results.

4/ We are indebted to John Fei for suggesting the four-quadrant configuration.
References


Berry, R. Albert and William R. Cline, 1979, Agrarian Structure and Productivity in Developing Countries, Baltimore and London: Johns Hopkins University Press.


Appendix

Value added: Gross farm income minus expenditures on feed, livestock, seed, fertilizer, and miscellaneous items divided by number of farms. The time series data for Table 3 are from Agricultural Statistics, corresponding years, and the cross section data for the regressions shown in Table 4 are from the Census of Agriculture, 1949 through 1974.

Land area: Acres per farm from Agricultural Statistics.

Labor: Includes family workers and hired workers as defined in Agricultural Statistics for the time series data in Table 3. For the cross section data used in the regressions, labor is years of farm operator labor minus days per year of off farm work plus labor-years of hired labor. The latter figures are obtained by dividing expenditures on hired labor by hourly wage rates without room and board in agriculture. Farm operator labor was obtained by assuming one operator per farm. The cross section data on hired labor and number of farms are from the Census of Agriculture. Both the time series and cross section labor variables are adjusted by the same procedure used by Griliches (1963). For the time series data the labor quality adjustment coefficients are extrapolated between Census years.

Power and machinery: The USDA index is from Agricultural Statistics. The 15-year one-hoss-shay index is computed by cumulating the value of shipments of machinery to farmers over the 15-year period preceding the year in question; the 10 percent depreciation figure is derived by depreciating the value of shipments of machinery by 10 percent per year starting in 1910 and cumulating the figures forward to the year in
question. All figures are deflated by the CPI, 1969 = 100, before computing the depreciated values. The 1930 and 1950 figures also included the value of horses and mules on farms at those points in time. All data are from Agricultural Statistics, respective years, except 1914 and 1919 values of farm machinery shipments from Statistical Abstract, 1928.

Wage rates: The time series and cross section figures on average gross weekly earnings in manufacturing are from Statistical Abstract, corresponding years. The time series figures on agricultural wages are from Agricultural Statistics. The cross section wage rates are from Farm Labor, corresponding years, (the per hour series without room and board). Both the cross section and time series wage rates are adjusted for educational differences (except when agricultural wages are divided into expenditures on hired labor to determine labor years). As mentioned, the labor years are adjusted separately.