Staff Paper Series

P72-20 JULY 1972

A Programming Approach to Urban Transit Planning

by
Terry Roe
and
Mathew Shane

Department of Agricultural and Applied Economics

University of Minnesota
Institute of Agriculture
St. Paul, Minnesota 55101

Staff Paper P72-20 July 1972

A PROGRAMMING APPROACH TO URBAN TRANSIT PLANNING

Terry Roe and Mathew Shane

Staff Papers are published without formal review within the Department of Agricultural and Applied Economics

A PROGRAMMING APPROACH TO URBAN TRANSIT PLANNING[a]

Terry Roe and Mathew Shane[1]

I. INTRODUCTION

Determination of the optimal transit mix in an urban environment requires the interfacing of many separate factors. For one thing, with social investments, one must consider social as well as private benefits. Taken from the perspective of a unified metropolitan transit commission,[2] it will be assumed that the objective is to maximize net social benefits.[3] This is accomplished by investing and setting prices for various transit services. In this study, these will be assumed to be limited to three types of systems: (1) automobile, (2) bus and (3) personalized rapid transit.[4] Of particular interest to this study is to investigate the affects of introducing a PRT system into an

---

[a]Presented at the ASCE National Transportation Engineering Meeting, Milwaukee, Wisconsin, July 17, 1972.

[1]The authors are Assistant Professors, Economic Development Center, University of Minnesota.

[2]Although the Twin Cities has a Metropolitan Transit Commission as do other major cities across the country, this is not a unified transit commission in the sense used above. The MTC is only responsible for public transit excluding highways.

[3]In [11] they consider a similar problem although not from the perspective of a unified transit commission and not with an explicit objective function.

[4]Personalized rapid transit as defined at the National Conference of Personalized Rapid Transit is "the class of fixed-guideway transit systems for which the stations are off the main line, the vehicles operate individually under automatic control and are auto sized, and trips are usually non-stop from origin to destination."

environment where there is currently only auto and bus operating.

The objective of the study is to demonstrate the usefulness of an explicit quantification of metropolitan transportation problem using a quadratic programming approach. Because of the availability of data, the peak hour traffic flow for Minneapolis, Minnesota as of 1970 will be chosen as a basic modeling device. In a previous paper [8], the authors set forward a strategy for making optimal transit decisions. This paper applies the framework previously presented to an explicit transit environment.

Many of the social cost and benefit factors involve difficult measurement problems. To avoid these problems, social factors will be dealt with indirectly rather than directly. What will be presented is the direct costs associated with changing the transit mix. It is then up to the policy decision maker or society in general to decide whether the net social benefits such as reduced air pollution from automobiles is worth that cost. Thus we avoid the issue of subjectively evaluating external benefits and costs, but still provide a basis for evaluating these.

In the next section, the explicit model will be introduced along with a discussion of the Minneapolis transit problem and the date derivations. That will be followed by a consideration of the outcome of the following simulation analysis: (1) shifts in demand, (2) changes in the modal service characteristics and (3) the introduction of PRT into the transit environment. The first involve investigating the impact of dynamic behavior on the modal split while the second involves the implications of changing technology or investments in the transit environment. Lastly, the comparison of the 1970 actual solution with the base simulated run provides some estimate of the short run impact of introducing an alternative public transit mode into an existing auto-bus system.

One further clarifying remark needs to be made on the Minneapolis transportation problem. Several factors in the Twin Cities makes it extremely difficult for public transit systems to compete with the private automobile. For one thing, the cities have perhaps the lowest average population density of any conparably sized metropolitan area in America--densities of 5,000 per square mile compared with the 60,000 per square mile densities in New York City and the 20,000 per square mile densities in Washington, D.C.. In addition, because of dispersal of economic activity and the excellant highway system, the time differential of the current public bus system and private automobile for most trips--and only 9 percent of the daily trips are to the CBD's--is extremely important. Thus whereas the average time differential in Washington, D.C. is ten minutes [2] and the differential in New York City during peak hour might be negative, the time differential in favor of auto may run from 20 to 30 minutes on an average trip and could result in a differential of up to one hour in those areas where public service is poor or where the trip involves several transfers.[5/] A third bias in favor of the automobile is the extreme weather conditions of the Twin Cities which mitigates against the inconvenience of public systems. The average temperature in January is 8° and it is not uncommon for it to be below zero for up to one month.

[5/] One of the authors trip from home to the office takes 15 to 20 minutes by car in the peak hour and one hour to one hour and fifteen minutes by bus.

Given this distinct bias away from public transit, the results of this analysis become ever more significant. If a PRT system can compete effectively in this environment, then the case for PRT is ever stronger in less extreme cases.

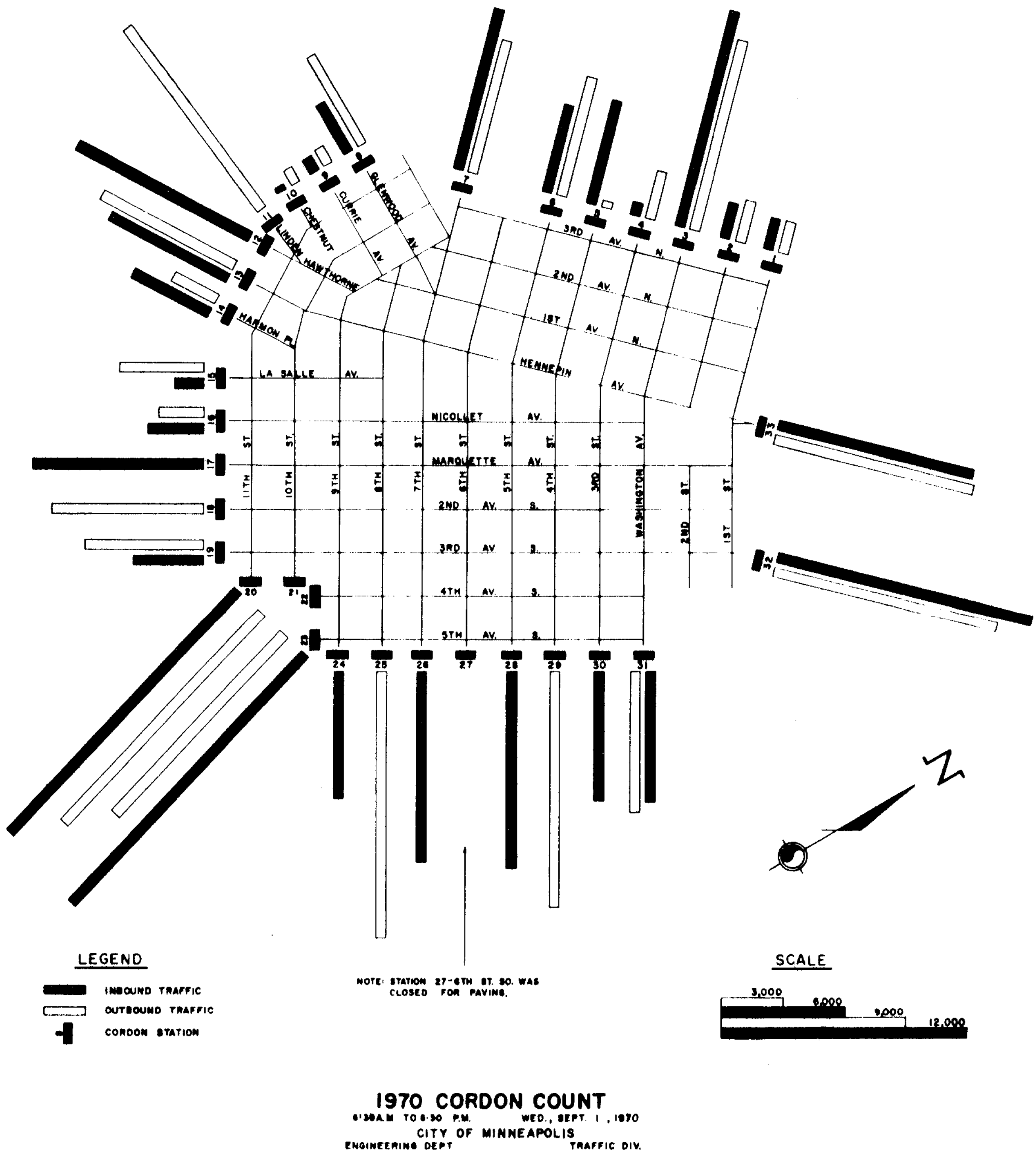
II. TRANSIT MODEL

The model presented here is based on a generalized transportation objective subject to various linear constraints. Although the framework is general, this paper will investigate the explicit problem of transportation into Minneapolis based on the 1970 Minneapolis Cordon Count. Chart I summarizes the traffic flows reported during that count. As can be seen, there were 32 routes in and out of Minneapolis at that time of which approximately one-third were major routes. Since approximately 14 percent of public transit demand occurs during the peak hour flow, it was the problem of peak hour flow rather than the more general one of daily flow presented in Chart I which was chosen for investigation.

Considering the current trips into the CBD, the problem is first to determine impacts on automobile and bus use of introducing a hypothetical PRT system. By comparing the actual auto-bus use with the augmented tri-modal system, a measure of this impact will be obtained.

Several basic assumptions will be made to define the problem. (1) An aggregate linear demand function is assumed to exist for each of the modes over all routes. This demand function is assumed to be a function of either the explicit or implicit price and the convenience and on line

CHART 1: CORDON STATION AND GRAPHICAL 12 HR. TOTALS OF VEHICLES ENTERING AND LEAVING AT EACH STATION.



time[6] given the basic transit parameters of per capita income, population density and employment. Analytically this can be written as

(1) $$T_k^d = -a_k P_k + \sum_{i \neq k} a_i P_i - b_k T_k + \sum_{i \neq k} b_i T_i + \beta_{1k} Y + \beta_{2k} D + \beta_{3k} E$$

where T_k^d is demand for trips into the CBD on the k^{th} system, P_k is the price of trip on the k^{th} system, P_i is the price on the substitute systems, T_k and T_i are the total time involved with the trip on the k^{th} and i^{th} systems and Y, D and E are the income, population density and employment parameters.

The following explicit demand functions were assumed:[7]

(1a) $$T_A^d = -6{,}666\, P_A + 2{,}166\, P_B + 2{,}333\, P_{PRT} - 6{,}666\, T_A + 1{,}667\, T_B + 1{,}667\, T_{PRT} + 29{,}087$$

(1b) $$T_B^d = +2{,}166\, P_A - 6{,}666\, P_B + 2{,}500\, P_{PRT} + 2{,}666\, T_A - 6{,}666\, T_B + 2{,}666\, T_{PRT} + 13{,}959$$

(1c) $$T_{PRT}^d = +2{,}333\, P_A + 2{,}800\, P_B - 6{,}666\, P_{PRT} + 2{,}666\, T_A + 2{,}666\, T_B - 6{,}666\, T_{PRT} + 21{,}331$$

The time and price coefficients were chosen so as to fit the Cordon data given reasonable estimates of price and time. The intercept terms relate

---

[6] The convenience time is involved in getting to and from the systems and waiting time. In this paper convenience time is weighted twice on line time.

[7] In [5] and [7] gross price elasticities are derived from empirical evidence. These indicate that the responsiveness of ridership to price changes are relatively small. These are consistent with what has been assumed here.

to the income, population density and employment while the price and time variables determine overall transit demand.

The next element of the problem which needs to be specified is the transit supply functions. This is assumed to have two components--a fixed investment component which is independent of the number of trips and a linear and quadratic variable component directly related to the number of vehicle trips. Stated analytically, this is:

(2) $$TC_k = FC_k + c_{1k}N_k + c_{2k}N_k^2$$

where TC_k is the total costs of the k^{th} systems, FC_k is the fixed investment for costs of the k^{th} system, N_k is the vehicle trips and c_{1k} and c_{2k} are the cost coefficients.

The following explicit cost functions were utilized in this study:

(2a) $$TC_A = 15{,}000 + .50\ N_A + .00005\ N_A^2$$

(2b) $$TC_B = 2{,}020 + 2.10\ N_B + .003\ N_B^2$$

(2c) $$TC_{PRT} = 15{,}000 + .25\ N_{PRT} + .00004\ N_{PRT}^2$$

The intercepts or fixed cost components are based on the discounted overhead cost of one hour's use. The linear term was derived from estimates of average costs per vehicle for a ten mile trip. In the case of auto, this includes a variable cost of 50 cents for the trip and a quadratic term which reflects social costs including parking, pollution and excess energy use. The bus cost function was computed from data given the authors by the Twin Cities Metropolitan Transit Commission and includes labor, maintenance and gas and oil expense for a ten mile modal trip. The estimates for PRT are based on imputed technological requirements for labor, maintenance and energy. The estimates of PRT utilize the assumptions of a 30 horsepower electric engine, computer controlled vehicle and minimum structure.

The demand and cost functions provide the basis for computing the objective function. The objective is to maximize imputed net social profit which in this case is imputed total revenue, the sum of the demand functions time explicit and implicit price per trip, minus total cost functions. Restated mathematically, the problem is to find the vector $\{P_k, T_k, N_k; FC_k\}$ which maximizes

$$(3) \qquad Z = \sum_k (TR_k - TC_k).$$

The objective function (3) will possess an unconstrained maximum if the revenue and cost functions have the usual neoclassical shape. This is guaranteed in this problem by the assumption of linear demand and marginal cost functions and hence quadratic revenue and total cost functions. However, several restraints will be placed on (3) to restrict the solution space.

The first set of restraints relate the number of vehicles positively with the time on the trip. This can be thought of as a production function restraint where time and a stock of vehicles are inputed to result in an output of trips. In explicit form,[8/] these are:

[8/] These time constraints are consistent with what is presented in [3] and [6]. Unfortunately given the limitations of the model--that the constraints must be linear--the representation is linear. A more realistic restraint would be quadratic with conjestion reaching a critical stopage point.

(i) $T_A = .00005\ N_A + .25$

(ii) $T_B = 1.5\ T_A$

(iii) $T_{PRT} = .00004\ N_{PRT} + .11$

The next major set of restraints involve price time relatives. The following price time relatives were utilized in the base run:

(iv) $P_A = 1.75\ T_A$

(v) $P_{PRT} = .75\ T_{PRT}$

(vi) $P_B = .27\ T_B$

Finally, the last set of restraints involve converting the demand function set in terms of passenger trips into demand for vehicles. This was done using the assumptions that there was an average 1.36 person per car, 41 persons per bus and 1.79 persons per PRT. The bus and auto figures were taken from the averages derived from the 1970 Cordon Count while the PRT figure was derived based on an assumption of higher use than auto in peak periods, but not as great as potential capacity. Most recent proposals have considered PRT vehicles which hold between 2 and 6 passengers. Stating this analytically we get:

(A.1) $T^d_A/N_A = 1.36$

(A.2) $T^d_B/N_B = 41$

(A.3) $T^d_{PRT}/N_{PRT} = 1.79$

With these assumptions, the following vehicle restraints were imposed:

$$\text{(vii)} \qquad N_A = -\ 4{,}902\ P_A + 1{,}593\ P_B + 1{,}716\ P_{PRT} - 4{,}902\ T_A + 1{,}225\ T_B + 1{,}225\ T_{PRT} + 21{,}334$$

$$\text{(viii)} \qquad N_B = 52\ P_A - 256\ P_B + 60\ P_{PRT} + 64\ T_A - 256\ T_B + 60\ T_{PRT} + 340$$

$$\text{(ix)} \qquad N_{PRT} = 1{,}333\ P_A + 1{,}429\ P_B - 3{,}944\ P_{PRT} + 1{,}530\ T_A + 1{,}530\ T_B - 3{,}944\ T_{PRT} + 17{,}334$$

Thus the problem investigated in this paper is to maximize objective function (3) subject to constraints (i) through (ix).

III. SIMULATION ANALYSIS

In this section, the simulated results will be presented for the model system of the Twin Cities presented in the last section in three basic parts. First a solution is obtained to the model presented in the previous section based on current estimates of peak hour trip demand, costs and constraints on traffic flows. This solution is referred to as the base solution. The remaining two parts of this section are simulations involving changes in trip demand and modal service characteristics.

1. Base Solution

The base solution is presented in Table 1. Personalized rapid transit is the preferred mode in this case, followed by auto and bus respectively. The direct cost or price of the auto trip exceeded the price of the bus trip by a factor of 4.3 and the total trip cost (Col. 1 Plus Col. 3) is $3.10 for auto, $2.15 for bus and $1.94 for PRT. The lower preference for bus relative to auto in spite of the lower user costs suggests that there are demand attributes of this mode which have been excluded from the model. These on net would raise the relative cost of bus. Convenience, flexibility in addition to alternative uses are just a few of these factors. Although they are not explicitly in the model, their average effect is reflected in the demand intercepts of equations (1a), (1b) and (1c).

Comparing the actual 1970 solution (Table 1.b) with the base solution (Table 1.a) indicates that the most significant effect of the introduction of PRT is a reduction in auto use of about 44 percent and a reduction in bus use of about 74 percent. In other words, PRT substituted more for buses than autos. However, the reduction in auto use with its concommitment implications on highway building needs, air and noise pollution is still highly significant.

The base solution presented in this section is used as a means of evaluating the changes in modal demand and service characteristics presented in the following sections.

TABLE I.A BASE SOLUTION

Mode	Direct Cost ($)	Trip Duration (Hrs.)	Time[a] Cost ($)	Vehicle Flows[b]
Auto	1.97	.452	1.13	11,711
Bus	.46	.676	1.69	83
PRT	.83	.444	1.11	14,919

TABLE I.B ACTUAL 1970 SOLUTION

Mode	Direct Cost ($)	Trip Duration (Hrs.)	Time[a] Cost ($)	Vehicle Flows[b]
Auto	2.00[c]	.45	1.13	21,208[d]
Bus	.30	.75	1.88	313[d]
PRT	--	--	--	--

---

[a] An opportunity time cost of $2.50 is assumed. See previous section.

[b] It is assumed that the average number of persons per vehicle are 1.36, 41 and 1.79 for auto, bus and PRT, respectively. See previous section.

[c] Based on an estimate of .50 for travel costs and 1.50 parking costs.

[d] Taken from the 1970 Minneapolis Cordon Count.

2. Changes in Modal Demand

There are several reasons for investigating the affects of changes in modal demand. Public and private investment in transit services involves the construction of facilities and purchases of equipment having an employable life of many years. The economic utilization of these facilities and equipment requires that during their employable life they yield a return sufficient to cover the variable and fixed costs of their operation and maintenance. However, during this time substantial changes in socio-economic conditions will occur. This would include such things as changes in family income distribution, adjustments in personal tastes and preferences for transit services and changes in the spatial distribution of economic activity. These can be expected to lead to significant shifts in relative modal demand. Consequently, it is not unlikely that these adjustments will make some systems more profitable while making others unprofitable. Thus a situation where one transit mode is over utilized contributing to inefficiency and conjestion may occur while another less demanded system is underutilized.

Finally, although it is difficult to estimate the separate effects of many of the variables endogenous to the mode by parametrically changing demand intercepts, the effects of aggregate changes in these variables on the endogenous variables of the model can be simulated.[9/]

[9/]Simulating the effects of variation in the endogenous variable coefficients of (1a), (1b) and (1c) yields information on the consequences of changes in user tastes and preferences associated with the three modes considered here. However, because of limited space, this analysis is not reported here.

Changes in Total Trip Demand. The simulation of the effects of changes in total demand is accomplished by uniformally changing the intercepts of equations (1a), (1b) and (1c) from -20 per cent to +20 per cent. A zero per cent change is equal to the base solution in the previous section.

As might be expected, the affects of increasing demand is an upward movement in all variables of the model (Figure I). However, the relative rates change are significantly different. A ±20 per cent change in the intercepts of equations (1a), (1b) and (1c) changed the number of PRT trips by ± 4,060, auto trips by ± 4,677 and bus trips by ± 984 for a total change of ± 9,721 trips. This implies that a 20 per cent overall change in the demand intercepts leads to a 22 per cent increase in trips.

This increase in trips implied an increased trip duration of ± .196 hours for the auto and PRT and .296 hours in the case of the bus. Direct auto cost per trip increased at a more rapid rate than that for PRT while bus cost per trip changed at the slowest rate.

The results of this simulation suggest that changes in aggregate demand have a substantial effect on modal trips, costs, duration and use. For a given uniform increase in demand, it appears that the number of PRT trips increase at a somewhat faster rate than the bus and auto modes. This occurs in the case of the bus mode because the increase in bus duration time represents a time cost to the bus user which is not sufficiently offset by the rather slow increase in bus trip cost. This induces a substitution of PRT for bus trips. The relatively rapid increase in auto trip cost relative to the increase in PRT trip cost also induces a substitution of PRT trips for auto trips.

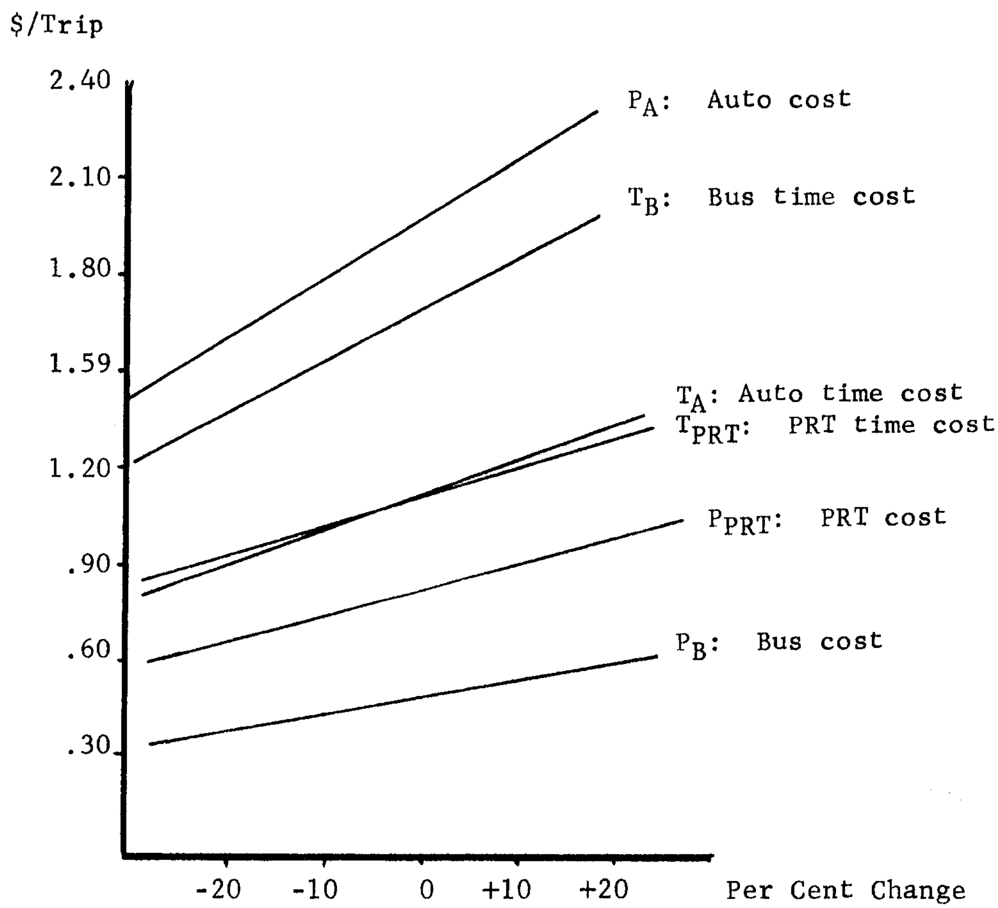


FIGURE I. Simulation Results from Uniform Shifts in Modal Demand

SOURCE: Appendix Table A.1.

Changes in Auto Trip Demand. Information on the effects of changes in auto trip demand is accomplished by changing only the intercept of equation (1a). The effect of a ± 20 per cent variation in that intercept resulted in a ± 3,276 change in auto trips and a ± 1,359 change in PRT trips from the base solution. However, the number of bus trips changed in the opposite direction by ∓ 2,255 trips. This results in a total change in trips of ± 6,875.

The changes in modal direct and time costs are presented in Figure II. The increase in direct auto trip cost reflects the increased variable costs of auto operation and parking as a consequence of a larger number of autos. Similarly, the increase in time cost reflects longer trip duration time as a consequence of increased traffic conjestion.

The increase in these variables induces auto users to substitute PRT trips for auto trips. This is reflected in the increase in PRT trips. This substitution effect is "dampened" somewhat however since the increase in PRT demand induces an increase in PRT trip cost and time cost.

Another factor explaining the increase in PRT demand and the resulting increases in PRT trip cost and duration is the decrease in bus trip demand induced by the increase in auto trip demand. Bus trip demand decreased because of the increase in auto traffic which resulted in longer bus trip duration times and an increase in bus operating costs. These changes induced some former bus users to substitute PRT trips for bus trips, thereby causing an increase in PRT trip demand.

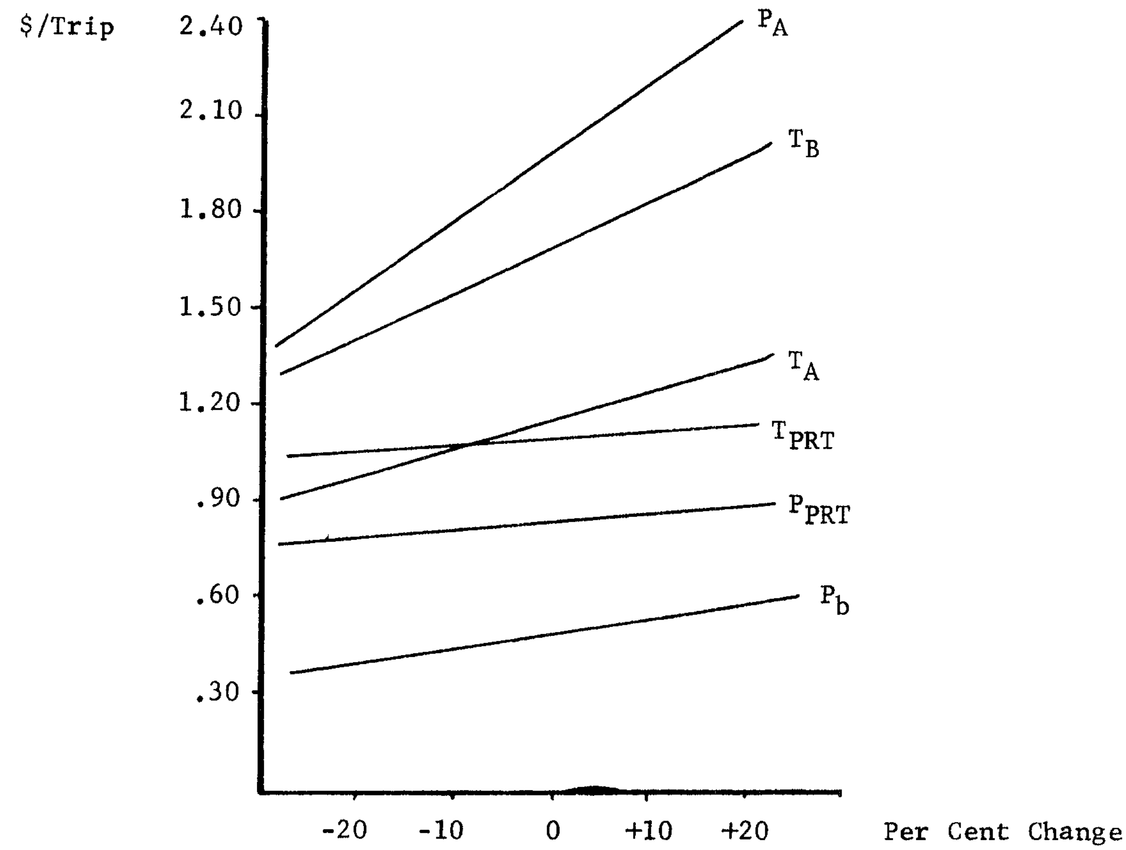


FIGURE II. Simulation Results from Shifts in Auto Demand

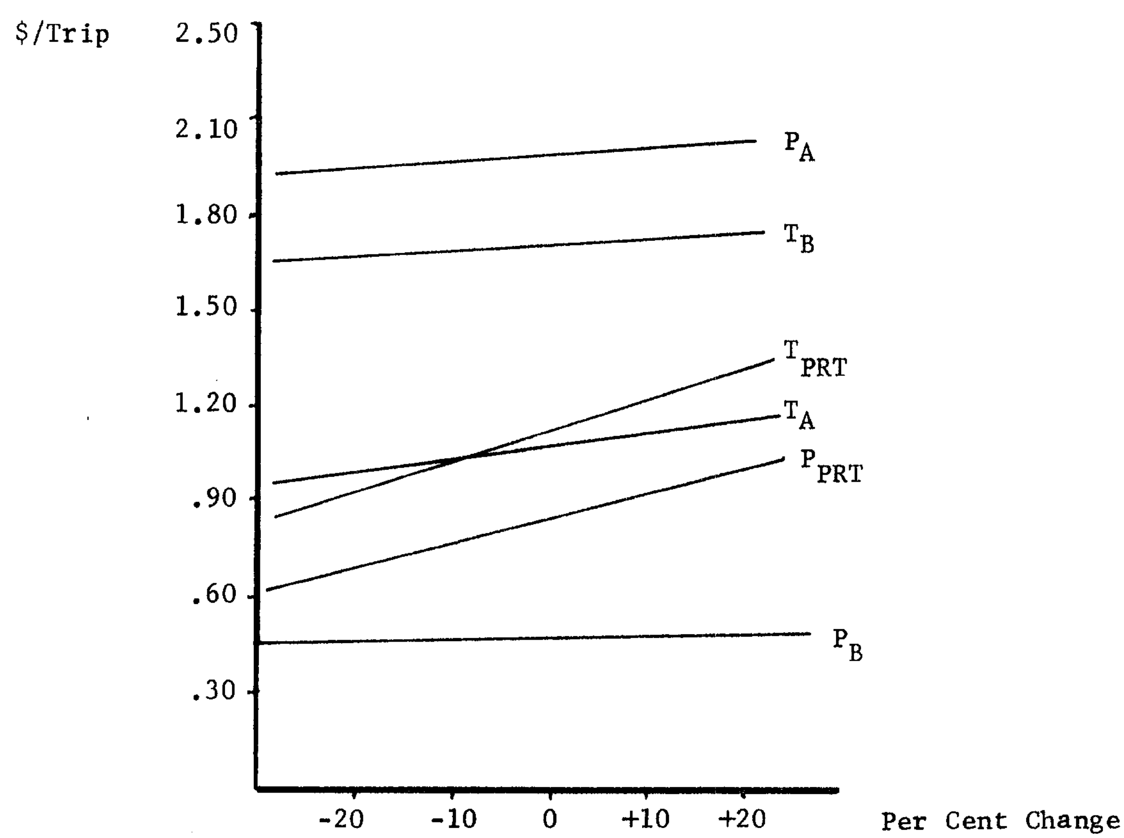


FIGURE III. Simulation Results from Shifts in PRT Demand

<u>Changes in PRT Trip Demand.</u> A change of ± 20 per cent in PRT demand resulted in a change of ± 3,986 PRT trips, a change of ± 279 auto trips and a change of ± 451 bus trips for a total change of ± 4,730 trips. The direct trip costs and trip time costs associated with these changes are presented in Figure III.

The primary effects of an increase in PRT demand are the changes in PRT trip and time costs with relatively minor influences being reflected in the auto and bus modes. This occurs because PRT mode conjestion constraint is not related to auto and bus use. On the other hand, since auto and bus utilize the same routes, an increase in auto use is directly reflected in an increase in bus time.

A slight indirect impact on auto and bus conjestion occurs however because increased trip and time costs induces some PRT trip users to substitute auto and bus trips for PRT. This increase in auto and bus trips in turn induces slight auto and bus conjection. It can be concluded therefore that the effects of changes in variables directly affecting PRT flows is not likely to have a substantial effect on other transit mode flows and associated costs.

3. Changes in System Service Characteristics

Changes in system service characteristics result from advances in technology and are assumed to be changes which alter trip duration time including changes in both on-line and inconvenience time. In the case of autos and buses this may include changes in trip time resulting from redesigned intersections, traffic conjestion points, and express lanes for peak hour traffic demands. In the case of PRT this may include more efficient scheduling and higher average speeds.

The costs and benefits of developing and adopting this new technology is not considered in this paper. The cost side would present a formidable problem. However, the benefits could be computed by comparing the consumer surplus[10/] generated by the base solution and the altered solutions. The total benefits received would provide a limit on the total costs that could be incurred to achieve the change. Any costs exceeding that limit would imply a net loss to the investment.

Affects of Changes in Auto Service Characteristics. The simulation of changes in auto service characteristics is accomplished by altering the coefficients in the constraint matrix associated with auto trip times from minus 20 to plus 20 percent.

Linear changes in these coefficient resulted in non-linear changes in the solution values of the model variables. A ± 20 per cent change from the base solution caused a change in auto flows of - 3,014 and 2,402 trips, a change in bus flows of - 2,665 and a 2,132 trips and an inverse change in PRT flows of + 161 and - 1,289 trips. The changes in modal trip and time cost is presented in Figure IV.

[10/] Consumer surplus is defined to be equal to the difference between what consumers are willing to pay and what they actually pay. In this case this is equal to $\int_0^{T_k^{d\prime}} T_k^d \, dT_k^d - T_k^{d\prime} P_k{}'$ and is a measure of the benefits derived from changing the amount consumed.

As expected, a decline in the factors leading to auto conjestion decreased auto trip time and direct auto trip costs even though auto trip flows increased as a result. The change in auto trip time is also reflected in bus trip time because both use the same routes. Therefore, bus trip time and direct bus cost decreased.

The reduction in auto and bus trip direct costs and duration induced users of PRT to substitute auto and bus trips for PRT trips. With a decline in PRT trips, PRT direct trip cost and trip duration decreased, i.e., a movement down the PRT cost function occurred.

It can be concluded from this simulation that an advancement in technology which decreases auto conjestion, increases flows of both autos and buses while decreasing PRT flows and lowers all modal direct costs and trip costs. These increased flows result from PRT users switching to auto and bus modes as well as encouraging trips from previously non-system users.

Affects of Changes in Bus Service Characteristics. The analysis of changes in bus service characteristics is accomplished by altering the coefficients in the constraint matrix associated the efficiency of bus trips.

It was found that the number of bus vehicle trips is very sensitive to changes in trip duration time. For changes in the efficiency coefficients exceeding + 13 per cent, the bus mode did not appear in the solution of the model, i.e., it appeared at zero levels.

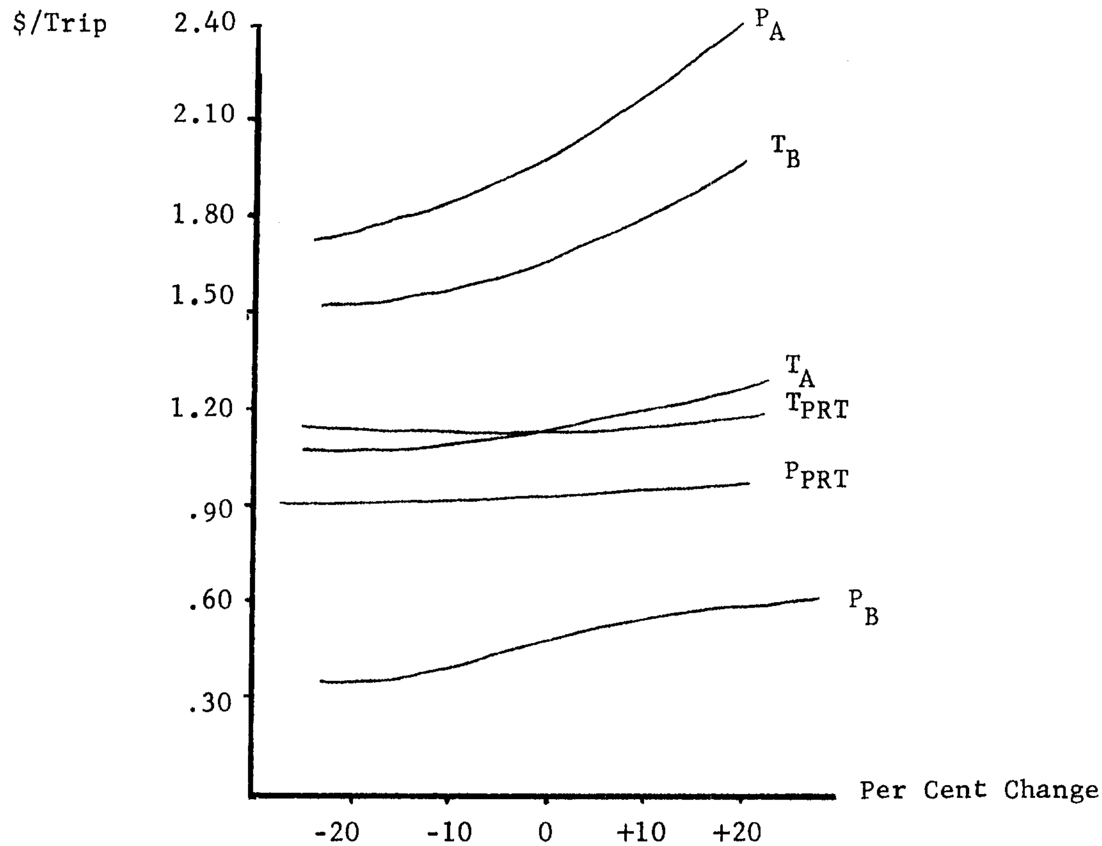


FIGURE IV. Simulated Results of Changes in Auto Trip Efficiency

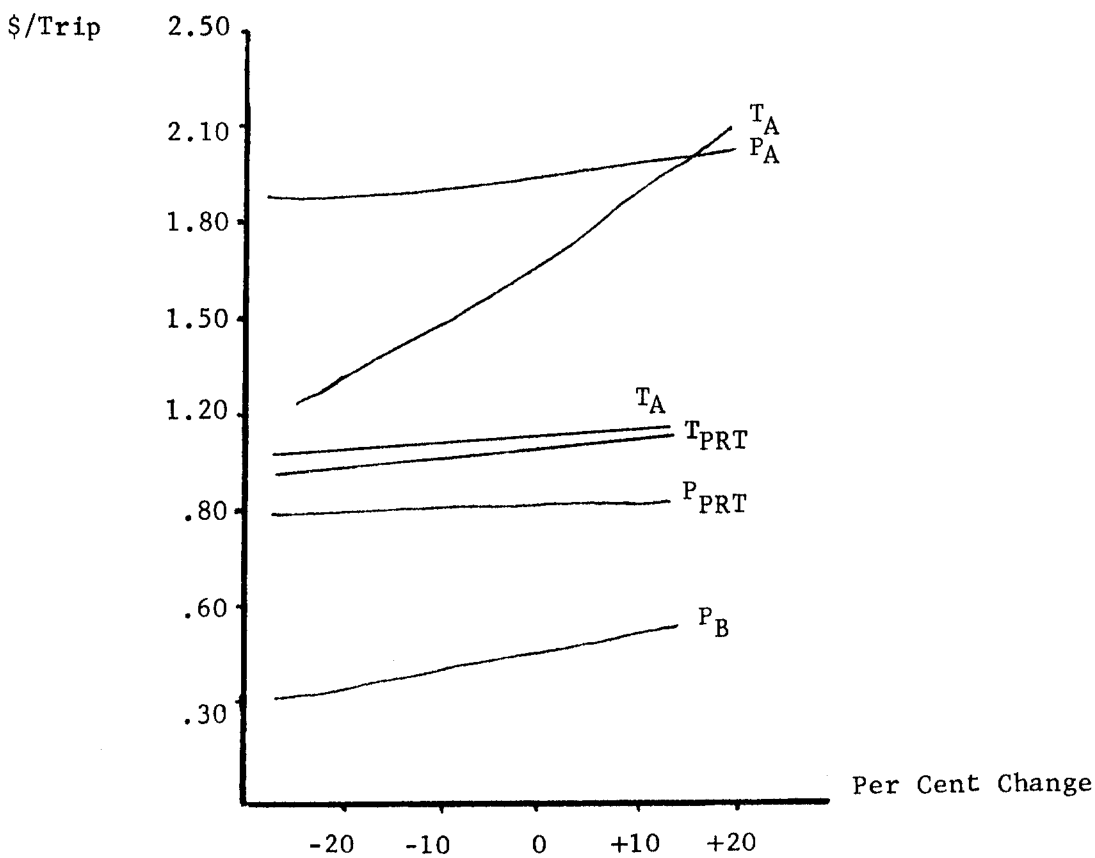


FIGURE V: Simulated Results of Changes in Bus Trip Efficiency

Changes in the efficiency coefficients of a ± 10 per cent resulted in a change in auto trips from the base solution of -472 and +245 respectively. Changes in bus trips are from 4,674 to -2,378 trips while changes in PRT is from -902 to 467 trips. The corresponding changes in trip and time costs appear in Figure V.

Changes in bus efficiency coefficients produced the expected decline in bus trip duration time and cost thereby increasing the number of bus trips. This decline induced former PRT users to substitute bus trips for PRT trips leading to a decline in both PRT trip cost and duration time.

A similar auto user substitution occurred leading to proportionally smaller changes in auto trips and cost. Auto trip duration time did not change significantly since the simulated change in bus trip efficiency did not directly alter auto trip efficiency and because the total change in flows over auto and bus routes does not change appreciably.

Affects of Changes in PRT Service Characteristics. The simulated effects of changes in modal flows and costs from changes in PRT service characteristics yielded relatively larger changes in PRT flows and costs and smaller changes in auto and bus flows and costs than the previous analysis. This again implies the significant sensitivity of the public system to service as compared to direct cost changes.

A ± 20 per cent change in PRT service characteristics resulted in nonlinear changes in auto flows of from -304 to 235 trips and changes in bus flows from -492 to 369 trips.

The changes in auto and bus trip and time costs are also relatively small (Figure VI) while the changes in PRT flows (from +1,881 to -2,438 trips) and costs are relatively large.

Comparing the changes in PRT usage with those of auto-bus lends to the inevitable conclusion that an increase (decrease) in the service characteristics implies an increase (decrease) in total transit demand. In our system this could occur from two forces: (1) an increase in the number of trips by previous transit consumers and (2) new consumers are induced to take trips who previously did not use any system. This inducement effect is very significant and occurs whenever service characteristics are altered. This is consistent with the basic transit problem of reducing spatial costs.

These results substantiate the concusion drawn in the previous demand simulation--that changes in variables directly affecting PRT produce small indirect effects on other modes.

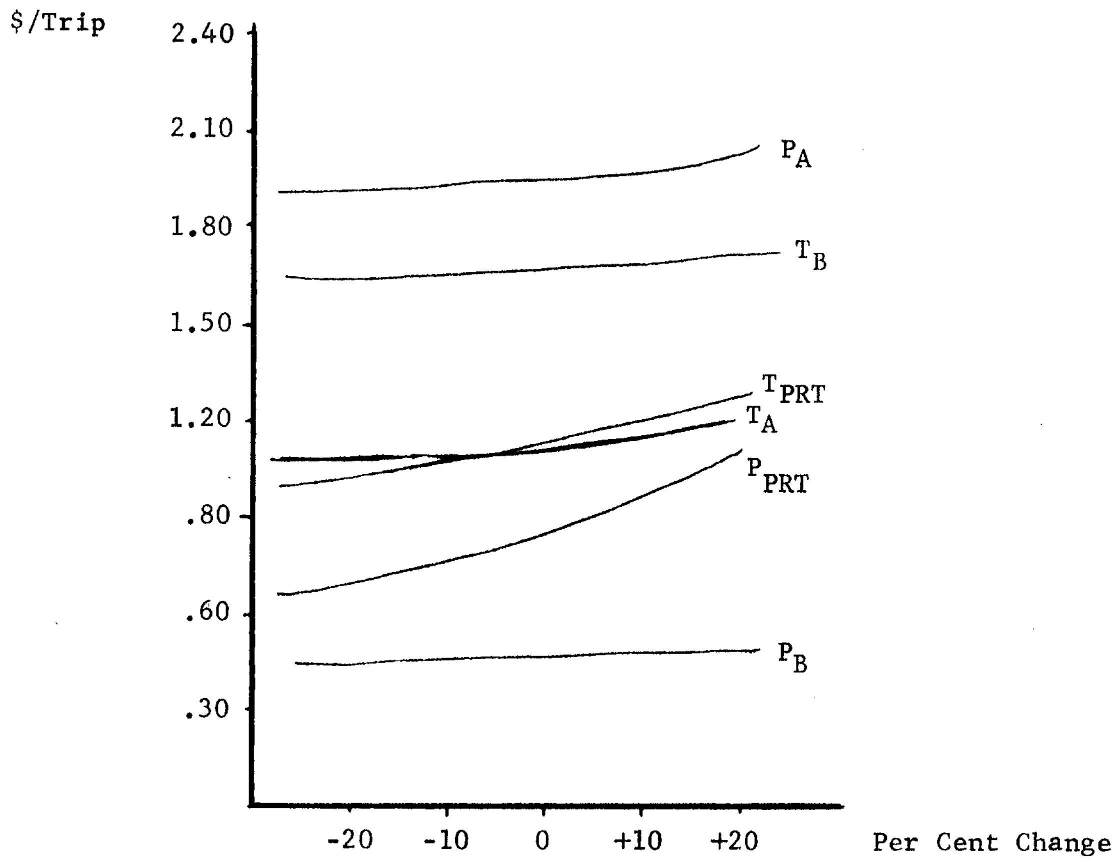


FIGURE IV. Simulated Results of Changes in PRT Trip Efficiency

IV. SUMMARY AND CONCLUSIONS

Based on 1970 Minneapolis Cordon Count, a mathematical programming model was developed and investigated. It is suggested as a way of quantifying the direct and indirect impacts of various policy changes on the transit environment. The results of the analysis of changes in the transit environment caused by demand alteration and technological innovations are reported. The primary conclusions drawn from this analysis are:

(1) the introduction of a PRT system is likely to produce a greater relative reduction in bus use than in auto use;

(2) during periods of increases in aggregate transit demand, PRT use will likely increase at a faster rate than either the increases in auto and bus as a consequence of a higher rate of conjestion associated with these other modes;

(3) increases in auto trip demand will tend to increase PRT demand at a somewhat faster rate than bus demand because of the conjestion resulting from increased auto flows;

(4) changes in PRT trip demand and service characteristics will have small indirect affects on auto and bus since the PRT does not compete directly with the auto and bus in the area of conjestion; and

(5) changes in bus service characteristics is likely to have small effects on both auto or PRT use.

The most significant overall conclusion is that the introduction of a PRT system leads to significant (more than 40 per cent) reduction in the private automobile. This occurs because the PRT in contra-distinction to other types of public transit such as the bus is successful in competing with the service characteristics of the auto. The PRT because of its flexibility and non-stop origin to destination ride can substantially reduce the time cost of public service system. In fact, the basic concept behind PRT is to imitate the flexibility and convenience of the auto by having small structure and vehicles and utilizing the off-line station while minimizing some of the major social cost items such as air, noise and esthetic pollution.

The success of PRT under the Minneapolis simulated conditions implies even more promise to these types of systems in other major metropolitan areas since it could be argued that with the increased conjestion present, the PRT would have a time cost advantage over the auto rather than just a relative equality estimated for Minneapolis.

BIBLIOGRAPHY

1. J.E. Anderson, J.L. Dais, W.L. Garrard, and A.L. Kornhausore. Personal Rapid Transit, University of Minnesota, 1972.

2. W. Avery, "The Potential Role of Mechanically Linked Systems in Urban Transportation," in [1].

3. L. Breiman, "Space-Time Relationships in One-Way Traffic Flow," Transportation Research, Volumn 3 Number 3, September 1969.

4. R.L. Carstens and L.H. Csanyi, "A Model for Estimating Transit Usage in Cities in Iowa," Highway Research Record Number 213, Washington, D.C., 1968.

5. J. Curtin, "Effects of Fares on Transit Riding," Highway Research Record Number 213, Washington, D.C., 1968.

6. J. Gani and R.C. Srivastava, "Some Models for Traffic Conjestion," Transportation Research, Vol. 3 Number 3, September 1969.

7. W. Lassow, "Effect of the Fare Increase of July 1966 on the Number of Passengers Carried on the New York City Transit System," Highway Research Record Number 213, Washington, D.C., 1968.

8. T. Roe and M. Shane, "Optimal Cost-Benefit Analysis of Urban Trans-Portation Systems: Its Use In Policy Implementation," in [1].

9. T. Roe and M. Shane, "A Fixed-Charge Quadratic Algorithm," submitted to The Journal of Management Science, December 1971.

10. Traffic Division, Public Works Department, Minneapolis, Minnesota. City of Minneapolis Central Business District 1970 Cordon Count, Minneapolis, 1970.

11. H. Wold and B. Martin, "Evaluation of Mutually Exclusive Design Projects," Highway Research Board, National Research Council, Special Report #92, 1967.

APPENDIX TABLES

SIMULATED DATA

TABLE A.1: RESULTS OF DEMAND SIMULATIONS

PRICE PER TRIP			TIME COST PER TRIP*			NUMBER OF VEHICLES			
Auto	Bus	PRT	Auto	Bus	PRT	Auto	Bus	PRT	Solution
1.97	.46	.83	1.13	1.69	1.11	11711	83	14919	Base
1.63	.38	.68	.93	1.40	.91	9098	59	11394	Lowered all demand intercepts by 20%
1.80	.42	.75	1.03	1.55	1.01	10404	71	13427	Lowered all demand intercepts by 10%
2.15	.50	.90	1.23	1.84	1.21	13017	89	16411	Raised all demand intercepts by 10%
2.32	.54	.98	1.38	1.99	1.30	14323	107	17904	Raised all demand intercepts by 20%
1.66	.39	.79	.95	1.42	1.06	9329	138	14161	Lowered auto demand intercepts by 20%
1.82	.42	.81	1.04	1.56	1.08	10506	111	14540	Lowered auto demand intercepts by 10%
2.13	.50	.85	1.22	1.83	1.13	12915	56	15298	Raised auto demand intercepts by 10%
2.29	.54	.87	1.31	1.96	1.16	14119	29	15676	Raised auto demand intercepts by 20%
1.95	.45	.72	1.11	1.67	.96	11506	72	12692	Lowered PRT demand intercepts by 20%
1.96	.46	.77	1.12	1.68	1.03	11608	78	13805	Lowered PRT demand intercepts by 10%
1.99	.46	.89	1.14	1.70	1.18	11813	89	16032	Raised PRT demand intercepts by 10%
2.00	.47	.94	1.14	1.72	1.25	11915	95	17146	Raised PRT demand intercepts by 20%

*Based on the assumption of a time cost of $2.50 per hour.

TABLE A.2: RESULTS OF SYSTEM SERVICE SIMULATIONS

PRICE PER TRIP($)			TIME COST PER TRIP($)			NUMBER OF VEHICLES			
Auto	Bus	PRT	Auto	Bus	PRT	Auto	Bus	PRT	Solution
1.77	.41	.81	1.02	1.52	1.07	12888	118	14439	Alter auto trip efficiency by -20%
1.87	.44	.82	1.07	1.60	1.09	12331	102	14666	Alter auto trip efficiency by -10%
2.09	.49	.84	1.20	1.79	1.12	11016	63	15202	Alter auto trip efficiency by +10%
2.23	.52	.86	1.27	1.91	1.15	10234	40	15572	Alter auto trip efficiency by +20%
1.91	.36	.80	1.10	1.32	1.07	11364	197	14415	Alter bus trip efficiency by -20%
1.95	.41	.82	1.12	1.51	1.09	11535	140	14664	Alter bus trip efficiency by -10%
2.00	.51	.84	1.14	1.88	1.12	11896	25	15180	Alter bus trip efficiency by +10%
1.87	.52	.84	1.08	1.92	1.11	12929	0	15047	Alter bus trip efficiency by +20%
1.95	.46	.73	1.12	1.67	.98	11538	74	23958	Alter PRT trip efficiency by -20%
1.96	.46	.78	1.12	1.68	1.04	11619	79	23217	Alter PRT trip efficiency by -10%
1.99	.46	.89	1.14	1.70	1.18	11815	89	21430	Alter PRT trip efficiency by +10%
2.00	.47	.95	1.15	1.72	1.27	11935	95	20338	Alter PRT trip efficiency by +20%

*Based on the assumption of a time cost of $2.50 per hour.