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Optimal Cost-Benefit Analysis of Urban Transportation Systems: Its Use in Policy Implementation

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Introduction

Determination of the benefits and costs associated with the introduction of a personalized rapid transit system into a metropolitan area is a complicated question which requires the integration of many separate pieces of analysis. Our objective is to present the relevant issues along with a preliminary framework for integrating these issues into a unified analysis. To simplify this, we will first consider the issues from the points-of-view of the user, nonuser and transit authority separately, and then see how they act jointly to determine the optimal transit configuration.

Several simplifying assumptions will be made. We will consider only three possible transit modes: (1) automobile, (2) bus, and (3) personalized rapid transit. The benefits and costs derived will be for a specific route which consists of two activity centers and two internal entry and exit points. Further, we will assume that the relevant

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objective of transit implementation is to maximize net social product, a concept which will be clarified further in the paper. The use of this objective function and the assumption of linear supply and demand functions for transportation will lead to the use of a quadratic integer programming framework.

We will now move into a discussion of the user, nonuser and transit authority problems followed by the derivation of supply and demand functions for the three transit modes.

The User Problem

In terms of the economic feasibility of introducing a new transit innovation, it is extremely important to know that the system's use will generate adequate revenue. As Sommers states:

> If an innovation fulfills no real need and satisfies no predicted latent demand, it is unlikely to generate profitable volumes if introduced, and certainly offers no benefits to a society already overburdened with the irrelevant. Given a transportation system, it is essential to predict its acceptance as part of the design evaluation process. [13, p.2]

For the user, the demand for alternative transit modes is considered to be a function of the following system characteristics:[1/] (1) time, (2) convenience, (3) cost per ride, (4) comfort, (5) safety, (6) weather reliability, (7) mechanical reliability, and (8) noise.[2/] When choosing

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[1/] Lancaster [6] followed by Quandt and Baumol [8] first introduce the concept of evaluating transit systems in terms of their characteristics compared to institutional arrangements.

[2/] Sommers [13] utilized these categories as a means of defining transit service. His problem, however, is quite different from the one we are considering, that of intercity transits compared with intra-city transit.

a particular transit alternative, each individual will subjectively quantify these characteristics and choose that system which for that occasion results in the lowest cost. It is obvious, for instance, that the choice of transit mode might be quite different on a clear spring day than on a snowy winter one.1/ In this case, the weight placed on the weather reliability factor would change dramatically. It is also clear that the evaluation of alternative systems will depend on the income of the individual and the nature of the trip. For example, the cost per ride might be considered less important if the success of a business trip depends on reducing the travel time to a minimum or if the company as distinct from the individual pays the fare.2/ On the other hand, a low income individual using the system on his own time might consider the fare as the overriding factor in determining transit mode.

The problem is quantifying these different factors. The most easily quantifiable are time, convenience and cost, while the most difficult are the demand implications of comfort, safety, weather, mechanical reliability, and noise. Time, convenience and cost relate most directly to the problem of traveling between two points, while reliability and safety relate to the probability of completing the trip, i.e., risk factors. Given the characteristics of traveling between two points, one would choose that

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1/ Ibid., p. 7.

2/ In the same paper by Sommers [13, p.5] businessmen rank the relative importance of these characteristics on the trips between Washington to New York and Washington to Philadelphia. The fare ranks sixth in order of importance behind time, convenience, comfort, safety, and weather reliability. Only noise and mechanical reliability were ranked lower.

system where the probability of arriving unimpeded is greatest. Although we can qualitatively determine the effects of increasing the risk factors, the quantitative results are much more difficult to measure. While using the direct cost factor to determine the optimal transit mix, we will consider the nature of the bias introduced by the risk factors. In a similar manner we will consider noise and comfort characteristics.

Although the duration of the trip and the amount of the fare are obviously quantifiable, the convenience factor is not so easily defined. However, the specification of the components of the convenience variable will assist in this definition. The main factors of convenience are the distance of the station from the origin and destination of the trip, the frequency of service and the number of transfers involved. While these characteristics can all be partially reduced to a time variable, this does not take into account such additional factors as discomfort in winter. However, such factors could be included by weighting the convenience time more heavily than the time spent on the system. Later in the paper, a model which handles the quantification of convenience in this way will be introduced.

The Nonuser

We will now turn to a discussion of the factors involved in determining the cost and benefits to the nonuser. These costs and benefits are generated by the external effects (referred to as externalities) of introducing an additional transit mode into the economic and physical environment.[1/] They can be broken down into three different classes: pollution, economic development and induced transit effects. These externalities provide some with a basis for arguing in support of public subsidization of rapid transit.[2/] In the following pages we will discuss the three types of social benefits and costs and the issue of transit subsidization.

Perhaps the least desirable side effect of the current transit mix is pollution. The automobile contributes approximately 50 percent of the air pollution in major metropolitan areas [1] and there is considerable public pressure to reduce this source of pollution. This can be accomplished by reducing the pollution content of auto emissions or by reducing the relative importance of the automobile on the urban transit scene. Given the gravity of the air pollution factor, along with the ever-increasing use of land for highways and the equally increasing congestion of major urban auto routes, one readily understands the urgent need for public transit.

Associated with excessive land use are the problems of noise pollution and aesthetic pollution. In addition to the loss in private housing, highway expansion programs frequently encroach further upon public park lands, the destruction of which dehumanizes the urban environment. Although

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[1/] In a paper by Manheim [7], it is argued that the basic problem with economic analysis is the exclusion of these external effects. See particularly, p. 8-9. In our analysis we try to overcome this criticism by explicitly including these external effects.

[2/] See for instance "Technical Report No. 6, Financial Plan," [16] prepared for the Twin Cities Metropolitan Transit Commission.

landowners might find parking lots and ramps financially rewarding alternative uses for this land would be desirable if the result were not increased parking rates.

All transit systems create some environmental effects. However, in view of the excessive environmental effects caused by the automobile, any shift to other transit modes should lead to improved environment. If the economic mechanism is working correctly, the value of land adjacent to a transit mode should reflect the effects which a particular transit system has on its environment. This mechanism provides one way of quantifying the aggregate subjective evaluation of environmental effects confined to a limited area, such as noise pollution. Air pollution, however, is distributed across the entire community and would obviously not be reflected in this measure.

Whereas the environmental effects are the major social cost items associated with introducing a new system or expanding an existing system, the economic effects are the major social benefits. The primary purpose of transportation is to reduce the cost of space and thereby reduce the cost of moving goods and services between different points. If a new transit system is sufficiently successful in reducing spatial costs it induces additional economic activity by increasing the size of a given market. Thus a major concern in the introduction of a new system is the economic development effects of the areas involved. The problem is that it will not effect everyone equally. Suppose a particular PRT route is built between downtown and a remote shopping area. The net effect will be to increase the access to these areas, thus increasing the level of economic activity. The downtown and remote area become more valuable business property while the housing along the route is made less desirable. Let us assume that in this case the increase in value of the business is greater than the loss in value of the housing along the route, that is, the social benefit to cost ratio is greater than one. However, the incidence of impact is also unequal. A few businesses reap a very large benefit while many home owners pay a relatively small cost. Economists argue that in cases such as this, it should be possible to tax those who gain and to redistribute their excess gains among those who lost. If we could determine with some accuracy who benefits and loses and by what amounts, this redistribution process would be relatively easy. Unfortunately, we rarely achieve that degree of accuracy. Consequently, some parties will benefit at the expense of others. Thus to make a new transportation alternative politically feasible, it is necessary to compare the incidence of impact on various groups. An aggregate cost-benefit ratio is thus an inadequate measure of the social desirability of a particular transit investment.

The third class of social externalities is the induced benefits to other transit modes caused by increased expenditures on public transit. If successful, the introduction of a new transit mode will cause a redistribution of transportation usage away from existing modes in favor of the new mode. This will benefit not only those who make direct use of the new mode, but also those who continue to use the now less crowded existing modes. This is particularly true of the automobile. If a new PRT route to the downtown area reduces the peak load of auto traffic, then the efficiency of the auto mode is increased. This benefit will subsequently be analyzed in our model.

We have now arrived at the question of public subsidies for rapid transit. It is often argued that public transit must be subsidized in one form or another. For example, in a study by Aerospace Corporation [4], it is assumed that three-fourths of the capital cost of a new system will be paid by Federal funds. In a study for the Twin Cities Metropolitan Transit Commission [16], a similar assumption was made. The argument for such subsidization is that the social benefits exceed the private benefits and that therefore everyone should pay some of the cost. Although this may in fact be true, it should be possible to isolate those groups which benefit most from new metropolitan transit and to tax them in order to cover part of the operating cost.

Further, the idea that public transit should be subsidized implies that existing transit modes are incapable of generating adequate demand. Systems which require subsidization are not designed to provide competitive alternatives to private enterprise. Thus one test of the economic viability of any new transit system is its ability to attract ridership adequate to cover all expenses. If it is then felt that certain groups of individuals should be encouraged to use the system, such encouragement should be given directly to the consumer in the form of discriminant fares and not through a general subsidy of the system. The losses incurred by existing public transit systems are just another symptom that such systems are no longer viable alternatives.

The Transit Commission

The objective of the transit commission is to minimize the cost of operation and investment for any level of transit service. This involves minimizing variable costs for any level of service in the developed system, but more importantly for our purposes, it requires choosing a system which will provide maximum service at minimum cost. Let us now consider what is involved in this decision.

It must first be decided what service characteristics the system should possess. In the sense that the auto mode is the main competitive alternative, the system should contain as many of the service features of the auto mode as possible, while remaining under public control and minimizing its major disadvantages. These disadvantages include the environmental factors discussed above. The service characteristics are tho-e noted under the user section, that is, time, convenience and cost. However, a competitive level of the risk factors snould also be maintained.

With these in mind, all potential alternatives should be evaluated in terms of cost, both private directly related to the system and other net social costs. The following direct cost factors should be included: (1) initial equipment and construction costs, (2) land usage, both direct purchase and indirect tax loss, (3) operating labor costs and (4) maintenance and repair costs.

Different individuals acting in different circumstances will evaluate a service differently. Similarly, different urban environments will require different evaluation of cost characteristics. For the downtown area land usage may be the critical variable, while construction costs may be more important in a suburban area. Thus it is conceivable that more than one system could and should be developed even in the same metropolitan area.

It is obvious that based on this analysis one would reject any system which provides equivalent or inferior service at increased cost. Unfortunately, very little of the current debate on urban transit revolves around economic considerations. Technical feasibility and political concerns have dominated the debate. Studies show that rapid rail or other on-line station systems compete poorly with either the auto or PRT. PRT estimates indicate that construction and equipment will cost from 1/3 to 1/5 that of a comparable rapid rail system.[1/] In terms of land use, the ratio is approximately four to one. In terms of the number of stations, a similar ratio is derived. Labor costs would not be significantly different, since both would be computer-run. Thus based on this superficial an analysis, rail-type systems appear not to be a viable

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[1/] The Bart system is costing 17 million/mile while the Washington, D.C. system is estimated at 33 million/mile [16]. In a system designed for Las Vegas by Aerial Transit Systems of Nevada Inc., it is estimated that 16 miles of one-way PRT will cost 50 to 60 million dollars or 3.1 to 3.75 million per one-way mile. This is 1/5 of the cost of the Washington, D.C. system on a two-way mile basis. Although this result overstates the difference because of fund evaluation and tunnelling, the result is predictable just from the relative structure sizes. Estimate obtained directly from Aerial Transit Systems.

economic alternative. The real issue for PRT systems is not whether they can compete with rail, which they obviously can, but whether or not they are able to compete with the auto. Here we return to our initial point: the only viable alternative to the auto is a system which can generate sufficient usage to justify the investment of public funds.

Net Social Product

The user, nonuser and transit autority problems can now be integrated into a framework of net social product.[1/] Involved is finding those transit demands, supplies and externalities which represent an optimism in relation to the private and social costs of supplying transit service. The net social product function contains three elements: the total social product induced by transit supply and demand over transit modes and routes, plus the product from external benefits such as increased sales of goods and services, minus the external costs such as decreased land values or increased environmental contamination. Net social product is defined as the sum of the products for each route traveled by each mode, i.e., the area "under" the excess demand function associated with each route and mode.[2/]

In the classic case developed by Samuelson [12] NSP (net social product) is maximized through the competitive forces of the market. In our

[1/] The concept of net social product utilized here is similar to that defined by Samuelson [12].

[2/] The excess demand function is defined to be the difference between demand and supply, i.e., positive if demand exceeds supply and negative if supply exceeds demand.

model these forces do not lead to its maximization since the transit market induces externalities. However, the framework used here includes these externalities in the maximization process. This is similar to including shipment costs in the transfer of goods and services between regions as illustrated by Samuelson [12] and Judge and Takayama [5]. It differs from their framework in that the external costs induced by transit implementation are compensated either by transit authority profits or external gains. This will become clear in later sections of the paper.

The Model

The model presented here is based on a hypothetical transportation problem. Although the framework is simplified, the strategy developed can be directly transferable to an urban transit problem. Thus this framework provides a basis for transit policy formulation and public decision making.

The model can be categorized into four parts: (1) the demand for transit services, (2) the supply of transit services, (3) the determination of modal splits, and (4) the inducement of social externalities.

Transit Route Example. To demonstrate this framework, the following route is assumed (Figure 1).

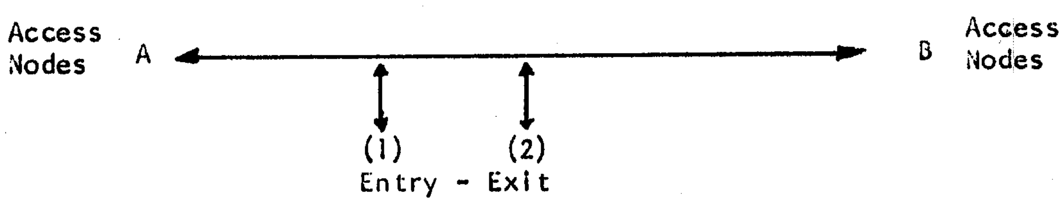

Figure 1: Depiction of Transit Routes Between Growth Centers A and B With External Access Nodes and Internal Entry-Exit Routes

Two growth centers exist (denoted as A and B) each having external access nodes and internal entry-exit routes. These generate possible route configurations (depicted in Figure 2), where a star (*) indicates an

	A	B	(1)	(2)
A	0	*	*	*
B	*	0	*	*
(1)	*	*	0	0
(2)	*	*	0	0

Figure 2: Depiction of Possible Route Configurations

acceptable route while a zero (0) indicates a non-acceptable route.[1/] In addition, suppose that the only current form of transit over the routes indicated in Figure 1 is private automobiles. Thus, the problem is two fold:

> (1) To determine the current level of auto transit efficiency and major restraints and costs to increase efficiency ignoring alternative transit technologies.
>
> (2) To determine the selection of the "best" transit system mix (automobile, bus and PRT) from a given set of transit technologies and route applications (such as number of stops, capacity).

The following assumptions define the nature of the problem. A potential transit demand is assumed to be associated with each of the possible routes and modes. This is a potential demand since it need not

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[1/] Thus it is clear that for purposes of simplification, we are not allowing transit between internal stops.

be satisfied. For example, while it is assumed that a demand exists for express bus transit between A and B, it is not assumed that the demand must be met.[1/] Two types of transit technology are considered as partial substitutes for the private auto: personalize rapid transit and bus. Bus transit is of two types: (1) express from A to B and B to A with no stops, and (2) from A to B and B to A with stops at entry-exit points (1) and (2). A PRT system is also assumed with identical routing to that of the bus.

Interaction between automobile, bus and PRT modes takes place as they compete for the transit users. It is possible, for instance, that a decrease in fare or travel time by bus or PRT may induce less auto traffic and congestion and thus might make the auto mode more competitive. Similarly, there will be induced responses from any change in the transit mix.

Finally, four types of externalities are assumed. Increased traffic at either growth center A or B is assumed to increase the sales of goods and services of the respective center. A decrease in traffic is assumed to decrease these sales. It is also assumed that the expansion of existing auto and bus routes or the construction of a route for a PRT system causes a loss in tax base and future income tax stream from the property utilized for the transit route. Lastly, it is also assumed that the value of property paralleling the transit routes suffer a depreciation in

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[1/] Obviously, the demand might not be sufficient to warrant express bus service.

value with an increase in traffic flows.[1/]

Mathematical Formulation. The following notation is used to define the specific mathematical programming problem.

$TR_{kij} = c^{*}_{kij}\ T^{d}_{kij}$ denotes the total revenue from passenger trips demanded (T^{d}_{kij}) at price c^{*}_{kij} per trip over routes i, j per unit of time on transit mode k where,

i, j = A, B, 1, 2

$k = b_1, b_2, c, r_1, r_2$ and where

b_1, b_2 denote an express bus and a bus with two stops respectively,

c denotes transit by private auto,

r_1, r_2 denote a PRT system with no stops and two stops respectively,

$TC_{kij} = c_{kij}\ T^{s}_{kij}$ denotes the total cost from supplying passenger trips (T^{s}_{kij}) at unit cost c_{kij} per trip over routes i, j per unit of time on transit mode k,[2/]

H_{kij} denotes the weighted average of time in transit and inconvenience time per trip to go from i to j on transit mode k,

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[1/] If land values appreciate in value as potential industrial sights the character of the framework considered here does not change.

[2/] In the case of the auto, this cost includes variable auto costs plus the variable costs of roadway maintenance.

F_k denotes the fixed cost of land, equipment and support facilities incurred in supplying the k-th mode of transit equipment plus the loss in tax base of private property condemned and utilized for the k-th transit system,

L_{A1}, L_{12}, L_{2B} denotes the change in value of private property paralleling the transit routes between A and (1), (1) and (2) and (2) and B,

M_A^R, M_B^R denote the increase in sales of goods and services of growth centers A and B respectively,

M_A^c, M_B^c denotes the decrease in sales of goods and services at growth centers A and B respectively,

N_A, N_B are assumed constant in this problem and denote the level of traffic flow from nodes servicing centers A and B.

Passenter trip demand is assumed to be a linear function of C^*_{kij}, H_{kij}, N_A, N_B and some base level of economic activity (M_A, M_B) at centers A and B and user income (Y) and population density (D). These base levels of activity are augmented by M_A^R, M_B^R, M_A^c, M_B^c. Thus, for example, the demand for trips on the k-th mode (say b_2) from A to B is the linear function[1/]

[1/] This demand function is expressed in "disaggregated" form since these variables are expressed in the objective function (1). The aggregate form of the function could be derived from the estimation of a function containing the variables suggested by Quandt and Baumol [8].

$$
\begin{aligned}
T^{d}_{b_2AB} = {} & \gamma + \beta_1 H_{b_2AB} + \beta_2 C^{*}_{b_2AB} + \beta_3 H_{b_1AB} + \beta_4 C^{*}_{b_1AB} + \beta_5 H_{r_1AB} \\
& + \beta_6 C^{*}_{r_1AB} + \beta_7 H_{r_2AB} + \beta_8 C^{*}_{r_2AB} + \beta_9 H_{cAB} + \beta_{10} C^{*}_{cAB} \\
& + \beta_{11} M^{R}_{A} + \beta_{12} M^{C}_{A} + \beta_{13} M^{R}_{B} + \beta_{14} M^{C}_{B} + \beta_{15} N_A + \beta_{16} N_B + \beta_{17} M_A \\
& + \beta_{18} M_B + \beta_{19} Y + \beta_{20} D
\end{aligned}
$$

The total revenue function ($c_{kij}\, T^{d}_{kij}$) for each k,i,j is therefore a quadratic.

The total cost function for each transit mode k on routes i, j can be expressed as

$$C_{kij}\, T^{s}_{kij} = g_{kij}\,(C_{kij}, H_{kij}; F_k).$$

The derivation of this cost function may have been obtained through engineering or simulation studies, or actual observation. It should be understood that it represents the supply of trips T^{s}_{kij} such that unit cost, time and fixed costs are a minimum. For the mathematical programming problem considered here, this total cost function for any k, i, j can be linear and/or quadratic. In this paper, it is assumed to be quadratic.

Finally, we assume that the number of trips (T_{kij}) actually taken is obtained when

$$T_{kij} = T^{s}_{kij} = T^{d}_{kij}.$$

The mathematical programming problem that is consistent with the maximization of NSP of the transit problem can now be stated. The problem is to find the number of trips for all k, i, j and therefore transit mode technology, and the level of externalities to maximize the total net return of transit over all k, i, j and the corresponding external economies and diseconomies. That is, find the values of the vector $\{C^*_{kij}, H_{kij}, L_{A1}, L_{A2}, L_{2B}, M^R_A, M^c_A, M^R_B, M^c_B\}$ which maximizes[1/]

$$(1) \quad Z = \sum_k \sum_i \sum_j (TR_{kij} - TC_{kij}) - d_1L_{A1} - d_2L_{12} - d_3L_{2B} + d_4M^R_A - d_5M^c_A$$
$$d_6M^R_B - d_7M^c_B.$$

Restated in matrix notation, this is:

$$Z = \underline{F}\ \underline{\delta}' + \underline{a}\ \underline{C}' + \underline{C,H}\ \mathbf{B}\ \underline{C,H}' + \underline{d}\ \underline{L,M}'$$ [2/]

where the bar denotes a vector, ' denotes transpose, and bold face letters denote a matrix. The elements of $\underline{a}$ and the quadratic form B are constants and are obtained from the subtraction of the total revenue and total cost functions corresponding to like modes and routes. The elements of $\underline{d}$ are also constants. The maximization of (1) is subject to the following restraints:

(i) m = 1,2,...,34 restraints (Appendix B, Table B-1) which state that the number of trips (T_{kij}) transacted from i to j on mode k is

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[1/]This objective function is specified such that the solution values are prices C^*_{kij} and time H_{kij}. The tableau specified below also includes trips T_{kij}. However, the coefficients associated with each T_{kij} is zero.

[2/]See Appendix A for a solution procedure to this type of integer quadratic programming problem.

dependent on the time (H_{kij}) and price C^*_{kij} of the i j route and on corresponding times and prices of other alternative transit modes over the same route(s), i.e., the number of trips from A to B (B to A),

$$(1.1) \quad T_{kij} = b^*_{kij} + \sum_k Q_{kij} H_{kij} + \sum_k R_{kij} C^*_{kij} + \sum_j D_j M^R_j + \sum_j D^*_j M^C_j$$

for k = b_1, b_2, c, r_1, r_2, i = A,B and j = A,B, and the number of trips from A to 1, A to 2, B to 1 and B to 2,

$$(1.2) \quad T_{kij} = b^*_{kij} + \sum_k Q_{kij} H_{kij} + \sum_k R_{kij} C^*_{kij} + \sum_e D_{ej} M^R_{ej} + \sum_e D^*_{ej} M^C_{ej}$$

where k = b_2, c, r_2, i = A,B, c = A,b, j = 1,2, e = A,B, and the number of trips from 1 to A, 1 to B, 2 to A and 2 to B,

$$(1.3) \quad T_{kij} = b^*_{kij} + \sum_R \sum_j Q_{kij} H_{kij} + \sum_R \sum_j R_{kij} C^*_{kij} + \sum_e D_e M^R_e + \sum_e D^*_e M^C_e$$

where i = 1,2, j = A,B and R = b_2, c, r_2.

(ii) m = 35,36,...,68 restraints (Appendix B, Table B-II) stating that an inverse relationship exists between transit time and cost for any mode k and route i j,

$$(1.4) \quad C_{kij} = b^*_{kij} - Q_{kij} H_{kij}.$$

(iii) m = 69,70,...,90 restraints (Appendix B, Table B-III) relating to traffic congestion and stating that the time associated with the i,j-th route for auto and bus modes is a positive linear function of the number of trips on these modes, e.g., from A to B, (1.5) $$H_{kAB} = \sum_{e} eAB\ T_{eAB} + \sum_{m} [\sum_{i} mAi\ T_{mAi} + \sum_{j} mjB\ T_{mjB}]$$

where $e = b_1, b_2, c$, $m = b_2, c$, $i = 1,2$ and $j = 1,2$.

(iv) m = 91,92,...,98 restraints (Appendix B, Table B-IV) stating that the time on mode b_2 (bus with two stops) or r_2 (PRT with two stops) on route A,B (B,A) is the sum of times on routes A to 1 and 1 to B (B to 2 and 2 to A) plus stopage time,

(1.6)
$$H_{kAB} = H_{kA1} + H_{k1B} + b_k^*,$$
$$H_{kBA} = H_{kB2} + H_{k2A} + b_k^*,\ k = b_2, r_2$$

where b_k is the mean stopage time.

(v) m = 99,100...,102 restraints (Appendix B, Table B-V) stating that the sales of goods and services at A and B is dependent on modal traffic flows,

(1.7)
$$M_i^R = \sum_{k} \sum_{j} \delta_{kji}\ T_{kji},\ \text{for } i = A,B,\ \ddagger\ j = A,B,1,2 \text{ and}$$
$$M_i^C = \sum_{k} \sum_{j} \delta_{kij}\ T_{kij},\ \text{for } i = A,B\ \ddagger\ j = 1,2,A,B.$$

(vi) m = 103,104,105 restraints (Appendix B, Table B-V) stating that the change in value of private property paralleling the transit routes is inversely related to the traffic on the route,

$$L_{A1} = \sum_k [\sum_j \delta_{kAj} T_{kAj} + \sum_i \beta_{kiA} T_{kiA}] \ \forall k \text{ and } j = 1,2,B \neq i = 1,2,B$$

(1.8)

$$L_{12} = \sum_k [\sum_j \delta_{kAj} T_{kAj} + \sum_i \beta_{kiB} T_{kiB} + \sum_e \gamma_{kBe} T_{kBe} + \sum_n \rho_{knA} T_{knA}]$$

where $i = A,1$, $j = 2,B$, $e = 1,A$ and $n = 2,B$, and where the expression for L_{2B} is identical to L_{A1}.

(vii) m = 106,107,...,113 restraints (Appendix B, Table B-VI) state that the number of passenger trips per u-it of time on the express bus mode over the i,j-th route must not exceed its capacity

(1.9)

$$\delta_{b_1} T_{b_1AB} \leq b^*_{b_1},$$

$$\delta_{b_1} T_{b_1BA} \leq b^*_{b_1},$$

and the capacity restrictions on the bus making multiple stops between A and (1)

(1.10) $$\sum_i \delta_{b_2Ai} T_{b_2Ai} \leq b^*_{b_2}, \ i = 1,2,B$$

between (1) and (2)

(1.11) $$\sum_i \delta_{b_2Ai} T_{b_2Ai} + \beta_{b_21B} T_{b_21B} \leq b^*_{b_2}, \ i = 2,B$$

and between (2) and B

(1.12) $$\sum_j \delta_{b_2jB} T_{b_2jB} \leq b^*_{b_2}, \ j = A,1,2.$$

Similar expressions exist between routes B to A and likewise for PRT (restraints m = 120,121,...,127).

(vii) m = 114,115,...,119 restraints (Appendix B, Table B-VI) stating that auto and bus trips must not exceed road capacity between A and (1)

(1.13) $$\sum_k \sum_i \delta_{kAi} \; T_{kAi} \leq b_k^*, \; k = b_1, b_2, c \text{ and } i = 1,2,B$$

between (1) and (2)

(1.14) $$\sum_k \left[\sum_i \delta_{kAi} \; T_{kAi} + \beta_{k1B} \; T_{k1B}\right] \leq b_k^*, \; k = b_1, b_2, c \text{ and } i = 2,B$$

and between (2) and B

(1.15) $$\sum_k \sum_i \delta_{kiB} \; T_{kiB} \leq b_k^*, \; k = b_1, b_2, c \text{ and } i = A,1,2$$

where similar expressions exist between B to A.

(viii) All variables are equal to or greater than zero and

(1.16)
$$\delta_k = 1 \text{ if } C_{kij}^*, H_{kij} > 0,$$
$$\delta_k = 0 \text{ if } C_{kij}^*, H_{kij} = 0, \forall, k, i \text{ and } j.$$

The tableau containing the above restraints is summarized in Table 1.

TABLE 1: TABLEAU OF RESTRAINT MATRIX CORRESPONDING TO RESTRAINTS (1.1) TO (1.15)

a/ A_{11}	A_{12}	A_{13}	=	b_1'
b/ A_{21}	A_{22}	A_{23}	=	b_2'
c/ A_{31}	A_{32}	A_{33}	=	$0'$ g/
d/ A_{41}	A_{42}	A_{43}	=	b_4'
e/ A_{51}	A_{52}	A_{53}	=	$0'$
f/ A_{61}	A_{62}	A_{63}	$\leq$	b_6 g/

a/Appendix Table B-I

b/Appendix Table B-II

c/Appendix Table B-III

d/Appendix Table B-IV

e/Appendix Table B-V

f/Appendix Table B-VI

g/Null vector

Analysis

Step 1. There are at least two approaches that can be used in arriving at the "best" transit mix. First, the programming model suggested above can be specified such that auto traffic flows are "forced" int the optimal solution at levels actually observed.

The approach suggested here however, is to specify the model such that all bus and PRT modes are excluded from appearing in the optimal solution, i.e., bounded from consideration. The model is then solved to determine the optimal auto traffic flow pattern.

This analysis will yield three categories of information. First, the solution will yield information on the isomorphic characteristics of the programming model and the "real world" problem it depicts. Second, parametric analysis can be performed to determine the level of sensitivity in the optimal solution to the estimated variance levels of the parameter estimates of demand, resource restrictions and transformation coefficients.

Finally, this analysis will yield information on the degree of efficiency of current auto-traffic flows, where bottlenecks (resource restrictions) exist, the benefits and cost of relaxing these restrictions, as well as providing insight into the means of improving over-all efficiency of auto traffic flows at minimum cost.

Step II. Given that in the first step of the analysis above the model is judged satisfactory, the next step is to consider the costs and benefits from the introduction of additional transit technology. In terms of the programming model presented above, this is accomplished by specifying the model such that bus and PRT modes can be included in the optimal solution, i.e., by relaxing the bounds that prevented their consideration in Step I.

The solution of (1) subject to the restraints (1.1) through (1.15) yields values of

$$\{T_{kij},\ C^{*}_{kij},\ H_{kij},\ L_{A1},\ L_{A2},\ L_{2B},\ M^{R}_{A},\ M^{c}_{A},\ M^{R}_{B},\ M^{c}_{B}\} \tag{2}$$

for all k, i and j from which modal splits, route configurations, corresponding time and fare costs, and associated externalities are obtained.

From the dual of the solution to (1), insight is obtained into those resources (such as bus capacity) that are binding or limiting, the cost of these limitations or the benefit occurring from their relaxation. In addition, the sensitivity of the solution to prices and time (H_{kij}) can be obtained.

Benefits and Costs

Computation. Solution (2) provides all of the essential information relating to costs and benefits required for each mode of transit. However, this information should be disaggregated in order to draw insights into the magnitude of the "gainers and losers" of our hypothetical situation (Figure 1).

Benefits accuring to the k-th transit mode over the i,j-th route is the total revenue TR_{kij} evaluated at the corresponding k,i,j solution values (2). The total costs TC_{kij} are also obtained from these corresponding solution values. Thus, for each mode and route a cost to benefit ratio can be obtained. The benefits accruing to the change in land values in this hypothetical situation is obviously zero and the costs are positive while the benefits accuring at the growth centers depends on the net change in business activity or $d_4\, M_A^R / d_5\, M_A^c$ where these values are obtained from (2).

Finally, it should be noted that from the dual of the solution to (1), values (referred to as weights or multipliers) are obtained which relate to the restrictions on restraints (1.1) through (1.15). The multipliers provide insight into the extent to which benefits and/or costs will change with a relaxation of constraint restrictions.

Use in Policy Decision Making. To demonstrate the use of this framework as a tool in policy decision making, we advance two suppositions on solution (2). First, suppose that a transit mode mix of auto, bus and PRT are among the basic variables of this solution. Also, suppose that the benefit-cost ratio for each mode is slightly greater than one. Finally, suppose that the benefit-cost ratio computed for the growth centers A, B is substantially greater than one.

Now, given a welfare condition that essentially states: welfare is increased if a combination of goods and services can be produced and consumed such that if a set of producers or consumers are made better off no producers or consumers are made worse-off,[1/] i.e., we must compensate the "losers" (benefit-cost ratio less than one) of our hypothetical situation by taxing the gainers (benefit-cost ratio greater than one) and distributing this tax revenue to the losers either directly or indirectly, say by lowering the "losers" taxes.

In terms of our hypothetical situation, this implies taxing A and B either directly or by requiring that they pay a portion of the transit costs and compensate the land owners paralleling the routes from A to (1), (1) to (2) and (2) to B (Figure 1).[2/] If this type of reallocation is not

[1/] This welfare criteria is referred to as Pareto Optimality.

[2/] It can be shown that this reallocation of revenue does not change the value of Z in (1).

politically feasible, then considerable doubt must be cast on implementing the type of transit system mix suggested by (2).[1/]

For the second supposition, suppose that the solution (2) is unchanged from the solution in step 1 where the "optimal" auto traffic flow pattern was derived. This implies that the introduction of a bus or PRT system is not consistent with our overall welfare criteria.[2/] The question then becomes: to what extent must either the bus or PRT system be subsidized to induce its use? There are at least two alternative ways to analyze this question. One being the subsidization of fares; the other the subsidization of fixed costs (F_k).

We shall only consider the latter. In this case a parametric analysis may be undertaken where the fixed costs associated with bus and/or PRT are reduced until these modes of transit appear in solution at positive trip levels. The amount by which the fixed costs of these modes is reduced becomes the amount of subsidization that must be secured from "outside" sources.

Within the framework of the model present here, this subsidization cannot be justified on an equitable basis. In order to justify it on an

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[1/] It may be well to note here that the benefit to cost ratio of say the bus and PRT can be less than one and the solution (2) unchanged. If this is the case, then the revenue from the increase in sales and services at growth center A and/or B is sufficiently large to overcome the "loss" suffered by the transit authority. In this case, A and/or B should be taxed to overcome this loss.

[2/] This leads us back to the subsidization issue and it is obvious that at least in terms of the variables included in the model, public transit is not a viable alternative.

equitable basis, it should be argued that the use of these systems generates beneficial externalities that are not considered in our framework. It may also be argued that we are concerned with the depreciable life of a transit system which may be greater than 20-50 years in case of PRT. Therefore we are concerned with demand, business activity and externalities over this entire period. However, it is extremely difficult to derive meaningful estimates of these variables 20-50 years in the future, although the directional change in these variables may be argued on a "heuristic" basis. If these directional changes appear to induce future demand, business activity, etc., then an "external" subsidization may be feasible.

Finally, the following rather short-run type of question may be considered within this framework. The solution (2) derives different fares (C^*_{kij}) for each route i,j. A policy question may be: is the operator savings of charging for all i,j an average fare (based on the weighted average $\sum_i \sum_j C^*_{r2,ij} T_{r2,ij} / \sum_i \sum_j T_{r2,ij}$) worth its cost? That is, to use an average fare simplifies the mechanics of fare collection and reduces associated costs. However, in this case, the short trip is subsidizing the longer trip. This may induce a decrease in "short" trip demand and an increase in "long" trip demand. This can be analyzed by not permitting the appropriate fare ($C^*_{r2,ij}$) to vary and deriving a solution to (1) with this restriction. This solution can then be compared to the former where all C^*_{kij} are variables.

This same type of analysis may be used to differentiate between demanders. For example, it may be socially desirable to provide lower fares to ghetto residents or the elderly. The effect of this decision could be analyzed in a manner similar to the above case.

While various other types of situations could be considered, the situations presented above should provide insight into the flexibility of this approach.

Other Methodological Considerations

It was pointed out that the introduction of fixed costs into (1) complicates the derivation of (2). In constructing a model of this type, consideration must be given to the trade-offs between the isomorphic properties of a model and the precision aspects of the model. If the level of error induced by approximating true quadratic functions by a linear function is "small", the isomorphic sacrifice may be small and the gain in precision large. However, this depends on the judgement of the "model builders" and the particular problem under consideration.

The problems of peak loads can also be partially considered by replicating the model presented here for say morning, mid-day and evening conditions. These three replications could then be "attached" by a series of row equations where dependence between the replications existed. A second method would be to estimate variance and co-variance of daily demands and incorporate this into quadratic form (8). This analysis would then proceed similarly by solving the model for various levels of accepted variance.

Finally, it is apparent from Table 1 that while the consideration of additional corridors, entry-exit routes and alternative transit mixes is possible, the model readily expands such that a point is soon reached where it can no longer be handled by any computer available. If in this case, and a model of the form presented here is still appealing, then the approach to consider is simulation. The simulation would be conducted on (1) subject to the specified restraints. The objective would be to derive values of (2) such that these values are feasible (do not violate any constraints) and are "in the direction" of maximizing (1).

APPENDIX A

Problem (1) is of the following form:

(A.1) $$Z = \underline{a}\,\underline{x}' + \underline{x}\,\underline{B}\,\underline{x}' + \underline{f}\,\underline{\delta}' \quad \text{a max.}$$

subject to

(A.1.1) $$A\,\underline{x} \underline{\leq} \underline{b}',$$

$$\underline{x} \underline{\geq} 0,$$

where

$\underline{a}$ is a n component row vector of constants

$\underline{x}'$ is a n component column vector of variables

B is a nxn symmetric definite or semi-definite matrix of constants

$\underline{f}$ is a n component row vector of constants

$\underline{b}'$ is a n component column vector of constants

A is a mxn matrix of constants

$\underline{\delta}'$ is a n component column vector such that

$$\delta = \begin{matrix} 0 \text{ if } X_i = 0 \\ 1 \text{ if } X_i > 0. \end{matrix}$$

The fixed charges $\underline{f}$ introduce discontinuities at the origin thus violating the convexity assumptions of quadratic programming even though B is a definite or semi-definite quadratic form. This problem is related to the linear integer fixed charge problem for which several computationally efficient approximate solution methods [2,10,14] and less computationally efficient though exact solution methods [15] exist.

A solution procedure to the integer quadratic problem stated above is being developed and will be available shortly. The procedure suggested here utilizes an efficient quadratic programming algorithm. The first step of this solution procedure is stated below.

The true optimal value of the objective function (Z) can be bounded after one solution to the above problem (A.1) (with a slight modification) by a traditional quadratic algorithm. This is accomplished by defining the new problem

(A.2.0) $$Z_L = \underline{f}^* \underline{X}' + \underline{X} B \underline{X}'$$

subject to

(A.2.1) $$A\underline{X}' \leq b,$$

$$\underline{X}' \geq 0,$$

where the vector of constants of $\underline{f}^*$ are:

$$f_i^* = \frac{f_i}{b_j} + b_j,$$

and where b_j is the upper bound (capacity restraint) associated with the X_i component of $\underline{X}$.

It can be shown that the solution to (A.2.0) will yield a value of Z_U such that

(A.3.0) $$Z_U \geq Z.$$

Now, let $\underline{X}^o$ denote the optimal solution to (A.2.0). The derivation of the upper bound to (A.1.0) is then obtained by substituting the values $\underline{X}^o$ into (A.1.0) and computing the resulting value of the objective function. Denoting this value as Z_L, it can be shown that

$$Z_L \leq Z. \tag{A.4.0}$$

Condition (A.3.0) and (A.4.0) bound the true optimal value of (A.1.0) thus permitting the maximum error of this approximate solution procedure to be determined.

APPENDIX B

The matrix tableau of linear restraints corresponding to (1.1) through (1.15) is depicted in Table B-I through Table B-VI. The tableau contains a total of 127 row equations and 109 column equations where each column variable appears in the objective function (1). The tableau is subdivided into six submatrices, A_{op}, o = 1,2,...,6 and p = 1,2,3 where the sumatrices $A_{o,3}$ also contains the right hand side restrictions ($\underline{b}'$) for each row equation. The constants (coefficients) in the tableau are denoted by "*" where a negative coefficient is denoted by "$\underline{*}$".

To demonstrate the correspondence between the tableau and the restraints (1.1) through (1.14) consider restraint (1.4) for k = b_1 and m = 69. The equation for this restraint is found in Table B-III, row No. 69. The stars * associated with T_{b_1AB}, T_{b_2AB}, T_{cAB} corresponds to the coefficients $\sum_e \delta_{eAB}$. The stars associated with T_{b_2A1}, T_{b_2A2}, T_{cA1}, T_{cA2} correspond to the coefficients $\sum_i \beta_{mAi}$ and the stars associated with T_{b_21B}, T_{b_22B}, T_{c1B}, T_{c2B} correspond to the coefficients $\sum_j \beta_{mjB}$. The star $\underline{*}$ associated with H_{b_1AB} corresponds to the coefficient of H_{kAB} which, when moved to the right of the equal sign, is a negative one. In this case, the corresponding right hand side element (b_{39}) of submatrix A_{33} is zero.

TABLE B-1a: SUBMATRIX A_{11} CORRESPONDING TO RESTRAINTS (1.1) TO (1.3) AND COLUMNS 1 THROUGH 34

	T_{b1AB}	T_{b2AB}	T_{b1BA}	T_{b2BA}	T_{b2A1}	T_{b2A2}	T_{b21A}	T_{b22A}	T_{b2B1}	T_{b2B2}	T_{b21B}	T_{b22B}	T_{cAB}	T_{cBA}	T_{cA1}	T_{cA2}	T_{c1A}	T_{c2A}	T_{cB1}	T_{cB2}	T_{c1B}	T_{c2B}	T_{r1AB}	T_{r2AB}	T_{r1BA}	T_{r2BA}	T_{r2A1}	T_{r2A2}	T_{r21A}	T_{r22A}	T_{r2B1}	T_{r2B2}	T_{r21B}	T_{r22B}
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TABLE B-1b: SUBMATRIX A_{12} CORRESPONDING TO RESTRAINTS (1.1) TO (1.3) AND COLUMNS 35 THROUGH 78

	H_{b_1AB}	H_{b_2AB}	H_{b_1BA}	H_{b_2BA}	H_{b_21A}	H_{b_22A}	H_{b_2A1}	H_{b_2A2}	H_{b_21B}	H_{b_22B}	H_{b_2B1}	H_{b_2B2}	C_{b_1AB}	C_{b_2AB}	C_{b_1BA}	C_{b_2BA}	C_{b_21A}	C_{b_22A}	C_{b_2A1}	C_{b_2A2}	C_{b_21B}	C_{b_22B}	C_{b_2B1}	C_{b_2B2}	H_{cAB}	H_{cBA}	H_{c1A}	H_{c2A}	H_{cA1}	H_{cA2}	H_{c1B}	H_{c2B}	H_{cB1}	H_{cB2}	C_{cAB}	C_{cBA}	C_{c1A}	C_{c2A}	C_{cA1}	C_{cA2}	C_{c1B}	C_{c2B}	C_{cB1}	C_{cB2}
1	$\underline{*}$	*											$\underline{*}$	*											*										*									
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TABLE B-1c: SUBMATRIX A_{13} CORRESPONDING TO RESTRAINTS (1.1) TO (1.3) AND COLUMNS 79 THROUGH 109 AND RHS

•	H_{r1AB}	H_{r2AB}	H_{r1BA}	H_{r2BA}	H_{r21A}	H_{r22A}	H_{r2A1}	H_{r2A2}	H_{r21B}	H_{r22B}	H_{r2B1}	H_{r2B2}	C_{r1AB}	C_{r2AB}	C_{r1BA}	C_{r2BA}	C_{r21A}	C_{r22A}	C_{r2A1}	C_{r2A2}	C_{r21B}	C_{r22B}	C_{r2B1}	C_{r2B2}	M_R	M_C	M_A	M_B	L_{A1}	L_{12}	L_{2B}		b*(RHS)
1	*	*											*	*											*	*	*	*				=	*
2	*	*											*	*											*	*	*	*				=	*
3			*	*											*	*									*	*	*	*				=	*
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TABLE B-11a: SUBMATRIX A_{22} CORRESPONDING TO RESTRAINTS (1.4) AND COLUMNS 35 THROUGH 7[illegible]*

	H_{b1Ab}	H_{b2Ab}	H_{b1bA}	H_{b2bA}	H_{b21A}	H_{b22A}	H_{b2A1}	H_{b2A2}	H_{b21B}	H_{b22B}	H_{b2B1}	H_{b2B2}	C_{b1Ab}	C_{b2Ab}	C_{b1bA}	C_{b2bA}	C_{b21A}	C_{b22A}	C_{b2A1}	C_{b2A2}	C_{b21B}	C_{b22B}	C_{b2B1}	C_{b2B2}	H_{cAb}	H_{cbA}	H_{c1A}	H_{c2A}	H_{cA1}	H_{cA2}	H_{c1B}	H_{c2B}	H_{cB1}	H_{cB2}	C_{cAb}	C_{cbA}	C_{c1A}	C_{c2A}	C_{cA1}	C_{cA2}	C_{c1B}	C_{c2B}	C_{cB1}	C_{cB2}
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68																																												

TABLE B-11b: SUBMATRIX A_{23} CORRESPONDING TO RESTRAINTS (1.4) AND COLUMNS 79 THROUGH 109 AND RHS

	H_{r1AB}	H_{r2AB}	H_{r1BA}	H_{r2BA}	H_{r21A}	H_{r22A}	H_{r21B}	H_{r22B}	H_{r2B1}	H_{r2B2}	C_{r1AB}	C_{r2AB}	C_{r1BA}	C_{r2BA}	C_{r21A}	C_{r22A}	C_{r2A1}	C_{r2A2}	C_{r21B}	C_{r22B}	C_{r2B1}	C_{r2B2}	M_R	M_C	M_B	M_3	L_{A1}	L_{12}	L_{2B}		b*(RHS)
35																														=	*
36																														=	*
37																														=	*
38																														=	*
39																														=	*
40																														=	*
41																														=	*
42																														=	*
43																														=	*
44																														=	*
45																														=	*
46																														=	*
47																														=	*
48																														=	*
49																														=	*
50																														=	*
51																														=	*
52																														=	*
53																														=	*
54																														=	*
55																														=	*
56																														=	*
57	*										*																			=	*
58		*										*																		=	*
59			*										*																	=	*
60				*										*																=	*
61					*										*															=	*
62						*										*														=	*
63							*										*													=	*
64								*										*												=	*
65									*										*											=	*
66										*										*										=	*
67											*										*									=	*
68												*										*								=	*

TABLE B-IIIa: SUBMATRIX A_{31} CORRESPONDING TO RESTRAINTS (1.5) AND COLUMNS 1 THROUGH 34

.	T_{b1AB}	T_{b2AB}	T_{b1BA}	T_{b2BA}	T_{b2A1}	T_{b2A2}	T_{b21A}	T_{b22A}	T_{b2B1}	T_{b2B2}	T_{b21B}	T_{b22B}	T_{cAB}	T_{cBA}	T_{cA1}	T_{cA2}	T_{c1A}	T_{c2A}	T_{cB1}	T_{cB2}	T_{c1B}	T_{c2B}	T_{r1AB}	T_{r2AB}	T_{r1BA}	T_{r2BA}	T_{r2A1}	T_{r2A2}	T_{r21A}	T_{r22A}	T_{r2B1}	T_{r2B2}	T_{r21B}	T_{r22B}
69	*	*			*	*					*	*	*		*	*					*	*												
70	*	*			*	*					*	*	*		*	*					*	*												
71			*	*			*	*	*	*				*			*	*	*	*														
72			*	*			*	*	*	*				*			*	*	*	*														
73			*	*			*	*						*			*	*																
74			*	*			*	*	*					*			*	*	*															
75	*	*			*	*							*		*	*																		
76	*	*			*	*					*		*		*	*					*													
77	*	*				*					*	*	*			*					*	*												
78	*	*									*	*	*								*	*												
79			*	*				*	*	*				*				*	*	*														
80			*	*					*	*				*					*	*														
81	*	*			*	*					*	*	*		*	*					*	*												
82			*	*			*	*	*	*				*			*	*	*	*														
83			*	*			*	*						*			*	*																
84			*	*			*	*	*					*			*	*	*															
85	*	*			*	*							*		*	*																		
86	*	*			*	*					*		*		*	*					*													
87	*	*				*					*	*	*			*					*	*												
88	*	*									*	*	*								*	*												
89			*	*				*	*	*				*				*	*	*														

TABLE B-IIIb: SUBMATRIX A_{32} CORRESPONDING TO RESTRAINTS (1.5) AND COLUMNS 35 THROUGH 78*

.	H_{b_1AB}	H_{b_2AB}	H_{b_1BA}	H_{b_2BA}	H_{b_21A}	H_{b_22A}	H_{b_2A1}	H_{b_2A2}	H_{b_21B}	H_{b_22B}	H_{b_2B1}	H_{b_2B2}	C_{b_1AB}	C_{b_2AB}	C_{b_1BA}	C_{b_2BA}	C_{b_21A}	C_{b_22A}	C_{b_2A1}	C_{b_2A2}	C_{b_21B}	C_{b_22B}	C_{b_2B1}	C_{b_2B2}	H_{cAB}	H_{cBA}	H_{c1A}	H_{c2A}	H_{cA1}	H_{cA2}	H_{c1B}	H_{c2B}	H_{cB1}	H_{cB2}	C_{cAB}	C_{cBA}	C_{c1A}	C_{c2A}	C_{cA1}	C_{cA2}	C_{c1B}	C_{c2B}	C_{cB1}	C_{cB2}
69	*																																											
70		*																																										
71			*																																									
72				*																																								
73					*																																							
74						*																																						
75							*																																					
76								*																																				
77									*																																			
78										*																																		
79											*																																	
80												*																																
81																									*																			
82																										*																		
83																											*																	
84																												*																
85																													*															
86																														*														
87																															*													
88																																*												
89																																	*											
90																																		*										

*Submatrix A_{33} is a null matrix-

TABLE B-IVa: SUBMATRIX A_{42} CORRESPONDING TO RESTRAINTS (1.6) AND COLUMNS 35 THROUGH 73*

	H_{b_1AB}	H_{b_2AB}	H_{b_1BA}	H_{b_2BA}	H_{b_21A}	H_{b_22A}	H_{b_2A1}	H_{b_2A2}	H_{b_21B}	H_{b_22B}	H_{b_2B1}	H_{b_2B2}	C_{b_1AB}	C_{b_2AB}	C_{b_1BA}	C_{b_2BA}	C_{b_21A}	C_{b_22A}	C_{b_2A1}	C_{b_2A2}	C_{b_21B}	C_{b_22B}	C_{b_2B1}	C_{b_2B2}	H_{cAB}	H_{cBA}	H_{c1A}	H_{c2A}	H_{cA1}	H_{cA2}	H_{c1B}	H_{c2B}	H_{cB1}	H_{cB2}	C_{cAB}	C_{cBA}	C_{c1A}	C_{c2A}	C_{cA1}	C_{cA2}	C_{c1B}	C_{c2B}	C_{cB1}	C_{cB2}
91		*					*		*																																			
92		$\overline{\underline{*}}$						*		*																																		
93				*	*						*																																	
94				$\overline{\underline{*}}$		*						*																																
95																																												
96																																												
97																																												
98																																												

*Matrix A_{41} is a null matrix.

TABLE B-IVb: SUBMATRIX A_{43} CORRESPONDING TO RESTRAINTS (1.6) AND COLUMNS 79 THROUGH 109 AND RHS

	H_{r1AB}	H_{r2AB}	H_{r1BA}	H_{r2BA}	H_{r21A}	H_{r22A}	H_{r2A1}	H_{r2A2}	H_{r21B}	H_{r22B}	H_{r2B1}	H_{r2B2}	C_{r1AB}	C_{r2AB}	C_{r1BA}	C_{r2BA}	C_{r21A}	C_{r22A}	C_{r2A1}	C_{r2A2}	C_{r21B}	C_{r22B}	C_{r2B1}	C_{r2B2}	M_R	M	M	M	L_{A1}	L_{12}	L_{2B}		b
91																																=	*
92																																=	*
93																																=	*
94																																=	*
95		*					*		*																							=	*
96		*						*		*																						=	*
97				*	*						*																					=	*
98				*		*						*																				=	*

TABLE B-Va: SUBMATRIX A_{51} CORRESPONDING TO RESTRAINTS (1.7) AND (1.8)
COLUMNS (1) THROUGH (34)

	T_{b1AB}	T_{b2AB}	T_{b1BA}	T_{b2BA}	T_{b2A1}	T_{b2A2}	T_{b21A}	T_{b22A}	T_{b2B1}	T_{b2B2}	T_{b21B}	T_{b22B}	T_{cAB}	T_{cBA}	T_{cA1}	T_{cA2}	T_{c1A}	T_{c2A}	T_{cB1}	T_{cB2}	T_{c1B}	T_{c2B}	T_{r1AB}	T_{r2AB}	T_{r1BA}	T_{r2BA}	T_{r2A1}	T_{r2A2}	T_{r21A}	T_{r22A}	T_{r2B1}	T_{r2B2}	T_{r21B}	T_{r22B}
99			*	*			*	*						*			*	*							*	*			*	*				
100	*	*			*	*							*		*	*							*	*			*	*						
101	*	*									*	*	*								*	*	*	*									*	*
102			*	*					*	*				*					*	*					*	*					*	*		
103	*	*	*	*	*	*	*	*					*	*	*	*	*	*					*	*	*	*	*	*						
104	*	*	*	*		*		*	*		*		*	*		*		*	*		*		*	*	*	*		*		*	*		*	
105	*	*	*	*					*	*	*	*	*	*					*	*	*	*	*	*	*	*					*	*	*	*

TABLE B-Vb: SUBMATRIX A_{53} CORRESPONDING TO RESTRAINTS (1.7) AND (1.8)
COLUMNS 79 THROUGH 109 AND RHS*

	H_{r1AB}	H_{r2AB}	H_{r1BA}	H_{r2BA}	H_{r21A}	H_{r22A}	H_{r2A1}	H_{r2A2}	H_{r21B}	H_{r22B}	H_{r2B1}	H_{r2B2}	C_{r1AB}	C_{r2AB}	C_{r1BA}	C_{r2BA}	C_{r21A}	C_{r22A}	C_{r2A1}	C_{r2A2}	C_{r21B}	C_{r22B}	C_{r2B1}	C_{r2B2}	[illegible]	[illegible]	[illegible]	[illegible]	L_{A1}	L_{12}	L_{2B}		b
99																									*							=	*
100																										*						=	*
101																											*					=	*
102																												*				=	*
103																													*			=	*
104																														*		=	*
105																															*	=	*

*Matrix A_{52} is a null matrix.

TABLE B-VIa: SUBMATRIX A_{61} CORRESPONDING TO RESTRAINTS (1.9) TO (1.15) AND COLUMNS 1 THROUGH 34

	T_{b_1AB}	T_{b_2AB}	T_{b_1BA}	T_{b_2BA}	T_{b_2A1}	T_{b_2A2}	T_{b_21A}	T_{b_22A}	T_{b_2B1}	T_{b_2B2}	T_{b_21B}	T_{b_22B}	T_{cAB}	T_{cBA}	T_{cA1}	T_{cA2}	T_{c1A}	T_{c2A}	T_{cB1}	T_{cB2}	T_{c1B}	T_{c2B}	T_{r_1AB}	T_{r_2AB}	T_{r_1BA}	T_{r_2BA}	T_{r_2A1}	T_{r_2A2}	T_{r_21A}	T_{r_22A}	T_{r_2B1}	T_{r_2B2}	T_{r_21B}	T_{r_22B}
106	*																																	
107			*																															
108		*			*	*																												
109		*				*					*																							
110		*									*	*																						
111				*					*	*																								
112				*				*	*																									
113				*			*	*																										
114	*	*			*	*							*		*	*																		
115	*	*				*					*		*			*					*													
116	*	*									*	*	*								*	*												
117			*	*					*	*				*					*	*														
118			*	*				*	*					*				*	*															
119			*	*			*	*						*			*	*																
120																							*											
121																									*									
122																								*			*	*						
123																								*				*					*	
124																								*									*	*
125																										*					*	*		
126																										*				*	*			
127																										*			*	*				

TABLE B-VIb: SUBMATRIX A_{63} CORRESPONDING TO RESTRAINTS (1.9) TO (1.15) AND COLUMNS 79 THROUGH 109 AND RHS*

	H_{r1AB}	H_{r2AB}	H_{r1BA}	H_{r2BA}	H_{r21A}	H_{r22A}	H_{r2A1}	H_{r2A2}	H_{r21B}	H_{r22B}	H_{r2B1}	H_{r2B2}	C_{r1AB}	C_{r2AB}	C_{r1BA}	C_{r2BA}	C_{r21A}	C_{r22A}	C_{r2A1}	C_{r2A2}	C_{r21B}	C_{r22B}	C_{r2B1}	C_{r2B2}	M_R	M_A	M_B	M_D	L_{A1}	L_{12}	L_{2B}		b
106																																≤	*
107																																≤	*
108																																≤	*
109																																≤	*
110																																≤	*
111																																≤	*
112																																≤	*
113																																≤	*
114																																≤	*
115																																≤	*
116																																≤	*
117																																≤	*
118																																≤	*
119																																≤	*
120																																≤	*
121																																≤	*
122																																≤	*
123																																≤	*
124																																≤	*
125																																≤	*
126																																≤	*
127																																≤	*

*Matrix A_{62} is a null matrix.

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