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## A TIME-SEQUENCED APPROACH TO THE ANALYSIS OF OPTION VALUE

by

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## 1. Introduction

Burton Weisbrod's 1964 seminal article on option value spawned a large literature which addresses the following question: will an individual who is uncertain about his or her future demand for a good be willing to pay a premium, in excess of the expected value of use, for the right to retain the option of future use? This difference between maximum sure willingness-to-pay for the option of future use (option price) and the expected value of future use (the mathematical expectation of Hicksian consumer surplus) is option value.

It generally is conceded that when preferences are uncertain, option value can be positive or negative (Smith, 1983, and Bishop, 1982, provide overviews of this literature). These results are of dubious theoretical interest, but of some practical importance.

They are of dubious theoretical interest because, given current institutions, the option price is the correct ex-ante measure of welfare change under uncertainty (Anderson, 1979). If compensation for a change in regime could be exacted ex-post, after uncertainty was resolved, then the expectation of Hicksian equivalent variation would be an appropriate ex-ante measure of welfare change. Alternatively, if contingent claims markets exist, then the expected value of equivalent variation again is appropriate (Graham, 1981). However, neither contingent claims markets nor the ability to determine ex-post compensation exist. Therefore, it may be concluded that option price is relevant to measuring welfare changes under uncertainty and expected consumer surplus is irrelevant. Why then should we study option value?

The answer to this good question is that the sign and size of option value is of considerable practical importance in project analysis. Individual option prices may be assessed (perhaps) by contingent valuation techniques, but these analyses are quite expensive to undertake. One-way tests for project acceptance based on expected surplus would be available if the sign of option value is determinate. For, if a project passed (failed) a benefit-cost test which uses expected surplus measures and it is known that option value always is positive (negative), then the project could be accepted (rejected).

Naturally, this approach leaves a zone of indeterminancy, which may be filled only if the magnitude of option value is known. As well, if the issue is the optimal size of a project, then the magnitude of option value, and not just its sign, must be known. Of course, this is equivalent to saying that you need to know option price. This has led some investigators (Freeman, 1984, and Smith, 1984) to seek a bound for option value. Unfortunately, useful analytical results along these lines have been difficult to obtain.

Most of the option value literature has dealt with Weisbrod's original notion of demand uncertainty. The difficulty that arises in establishing a sign for option value is the need to compare the marginal utility of income across states: with different utility functions in each state, nothing definite can be said in this regard. This realization led Bishop (1982) to consider supply-side options. That is, if demand for a resource is certain but its supply is uncertain, then the problem of state-dependent marginal utility of income is eliminated and the sign of option value can be established. Freeman (1985) has pointed out that Bishop only studied one case of supply-side uncertainty and concluded that in the other cases, option value again is indeterminate.

The assumption of the supply-side analyses that demand is certain, but supply is not, seems relevant to many current resource policy issues. As well, based on the positive analytical results obtained by Bishop (1982), more work along these lines appears warranted. In this paper, supply-side option value is investigated.

In the option value literature, analyses most often have been based on static models and have used the common postulate that individual preferences satisfy the von Neumann-Morganstern axioms and, hence, have an expected utility representation. In these analyses, little attention has been paid to underlying choices and constraints. This is natural, given the well-known foundations of expected utility analysis. However, it is argued in this paper that this possibly has led to a misrepresentation of actual choice situations of interest in policy discussions.

In particular, it seems that inadequate attention has been paid to temporal aspects of the risky choices at issue, and the timing of possible solutions of uncertainty relative to the time when choices must be made. Consideration of temporal risk (in the sense of Dreze and Modigliani, 1972) undermines the expected utility foundation on which previous research has been based. Since most, if not all, actual choices involve temporal risk, this appears to be a serious problem.

The issue of time sequencing has been raised in the option value literature in the guise of quasi-option value (Arrow and Fisher, 1974). Here, the central issue is the timing of choices relative to the timing of resolution of uncertainty. Specifically, Arrow and Fisher and others (see Henry, 1974; Epstein, 1980; Hanemann, 1983; and Graham-Tomasi, 1983) seek to determine if the prospect of learning reduces the benefits of implementing irreversible investments relative to the case when learning is ignored. The general result is that, even under risk neutrality, there is a benefit to maintaining flexibility (a quasi-option value of not undertaking irreversible investment-) due to expected learning possibilities. In fact, Conrad (1981) suggests that quasi-option value is equal to the expected value of information. Here, I very briefly address quasi-option value (QOV)(Smith, 1983 calls this time-sequenced option value) and its relationship to the time sequenced approach taken here.

The paper is organized as follows. In the next two sections, a certainty model is used to establish what one wishes to measure in the stochastic case and how these measurements can be used to select a project. Section 2 addresses individual welfare change measures, while Section 3 provides a review of how a planner could use information on individual welfare change to choose a project. In Section 4, possible sources of uncertainty are discussed. Section 5 contains an analysis of supply-side option value in a setting where there is no temporal risk and individuals have standard von Neumann-Morganstern utility functions. I provide an alternative approach to that used by Bishop (1982) and Freeman (1985) and am able to obtain some positive results. In the sixth section, the problems introduced by a move to temporal risk is studied and several results from this literature are derived in terms of supply-side uncertainty. The results here are quite negative: temporal risk greatly complicates the study of option value. The next section shows in the case of uncertainty how the planner could use individual welfare change measures to select a project. This section also addresses quasi-option value. The final section provides a discussion and points out some empirical implications.

It should be stressed at the outset that this paper is exploratory in nature. It represents an attempt to draw inferences from the general economic literature on temporal risk for the modeling of option prices and option values in the analysis of projects with uncertain environmental consequences. There remains a great deal of work to be done. I seek here to illustrate the kinds of difficult questions that arise when time is composed with uncertainty in the study of welfare change and project appraisal.

## 2. A Certainty Model: The Individual

In this section, a simple model of a project in a dynamic setting and measures of welfare change are set out. This will serve as a foundation for the stochastic models to be analyzed in the sequel. It also has some important implications for project analysis which carry through to the stochastic case and, therefore, to the study of dynamic option prices.

The individual has preferences over alternative sequences of goods consumed and environmental quality. Let $c_t \varepsilon E^n$ (Euclidean n-space) be a vector of consumption goods at date t. Included in $c_t$ are labor supplies (measured as negative) as well as visits to recreation areas. Let $c = (c_1, \ldots, c_T)$ be a sequence of such consumptions; the individual's time horizon is date T. Prices of consumption goods are given by the spot price vector $p_t \varepsilon E^n$. This includes the prices of visits to recreation areas.

The level of environmental quality at various locations at date t is given by a vector $q_t \varepsilon E^m$. This vector is exogenous to the individual. However, as the individual has preferences over alternative quality vectors, these have components measured in an "individual payoff-relevant" fashion. The vector $q_t$ will depend on the "output" of a "project" that is being anticipated. A project is represented by a sequence of points on the real line $v = (v_1, \ldots v_\tau)$ which may be thought of a "project size." Of course, the project may outlive the individual; generally $\tau \neq T$. Often, a project is represented in the literature by

$$v_t = \begin{cases} 0 \text{ if the project is not implemented} \\ 1 \text{ if the project is implemented} \end{cases} \quad \text{all } t.$$

But this is not necessary and the more general approach allows alternative "phasings" of projects, which may be important under uncertainty.

The project affects payoff-relevant environmental quality variables via a biological/physical process function. Thus, a project may affect fish populations of relevance to recreationists by controlling amounts of a pollutant which is detrimental to ecosystem functioning more generally. In a dynamic model, the history of outputs of a project, as well as the history of environmental quality will affect current enviornmental quality. This can be captured by specifying a difference equation which governs the time path of environmental quality which does not have a Markovian structure. Let

$$\hat{v}_t = (v_1, v_2, \ldots, v_{t-1})$$
$$\hat{q}_t = (q_1, q_2, \ldots, q_{t-1}).$$

Then

$$q_{t+1} = f(\hat{v}_t, v_t, \hat{q}_t, q_t). \quad (1)$$

Regarding individual preerences, it is assumed that all individuals are finite-lived. Let $z_t = (c_t, q_t)$ be a consumption goods/environmental quality bundle at date t ($z_t \varepsilon E^n \times E^m$) and let $z^T = (z_1, \ldots, z_T)$. For notational convenience, let $Z = E^{nT} \times E^{mT}$. The following axioms concerning individual behavior are posited to hold.

Axiom 1.1: Each individual's choices from Z are represented by a binary relation R on Z where R is a weak order and R is monotonic.

Axiom 1.2: Let $\xi$ be the usual topology on Z. Then $\{z: z \varepsilon Z, z R y\} \varepsilon \xi$ and $\{z: z \varepsilon Z, y R z\} \varepsilon \xi$ for every z, y $\varepsilon$ Z.

The following representation theorem is well known.

Theorem 1.1: If individual preference orderings satisfy axioms 1.1-1.2, then there exists a real-valued utility function U(z), continuous in the usual topology on Z, such that z R y iff U(z) > U(y).

Proof: Fishburn (1970) theorems 3.1, 3.5, and Lemma 5.1.

Let $\alpha \epsilon (0,1)$ be the (constant for convenience) one-period, market rate of interest at which individuals can borrow and lend. The individual has an exogenous sequence on non-wage incomes $\{w\}_t^T$. Then the budget constraint may be written

$$B(p^T,w,\alpha) = \{C \epsilon E^{nT} : \sum_{t=1}^{t=T} \alpha^{t-1} p_t^T \cdot c_t \leq \sum_{t=1}^{t=T} \alpha^{t-1} w_t ; c \epsilon C\}$$

where $p^T = (p_1, \ldots, p_T)$, $w = (\sum_t \alpha^{t-1} w_t)$ and C $E^{nT}$ is the set of feasible consumptions, assumed closed and bounded below.

It is natural to impose the following assumptions:

A1: B(.) is non-empty.

A2: $\{w_t\}$ is bounded.

It is clear that B(.) is compact in $E^{nT}$.

Let

$$V(p^T,q^T,w,\alpha) \equiv \sup_c \{U(z^T) : c \epsilon B(p^T,w,\alpha)\},$$

where $q^T = (q_1, \ldots, q_T)$. Since U is continuous and B is compact and non-empty, the supremum is attained.

In a world of certainty, we can define measures of welfare change using this intertemporal indirect utility function. Let $(p_o^T,q_o^T)$ be the initial situation and let $(p_v^T,q_v^T)$ be the situation subsequent to implementation of a project v. The compensating variation (cv) and equivalent variation (ev) are defined implicitly by (suppressing $\alpha$)

$$V(p_o^T, q_o^T, w) = V(p_v^T, q_v^T, w-cv) \qquad (1)$$

$$V(p_v^T, q_v^T, w) = V(p_o^T, q_o^T, w+ev). \qquad (2)$$

An important special case of this arises when the utility function U is separable. Here, I impose more structure on preferences by means of the following axiom.

> Axiom 1.3: $(z : z \in Z, z \, R \, y)$ and $(z : z \in Z, y \, R \, z)$ both are open in the usual topology on Z (continuity) and are convex.

To discuss separability and the existence of instantaneous utility functions, reconsider the sequence z. Recall $z_t \in E^n \times E^m$; z is constructed by considering the T-fold Cartesian product of $E^{n+m}$ with itself and with $z^T$ an element of this space. Now, consider preferences on each $z_t$ individually. Thus, let $Z = \Pi_{t=1}^{T} Z_t$, where $(Z_t, \xi_t)$ is a topological space for each t. Let $\xi = \Pi_t \xi_t$ be the product topology for Z. It is well known that if each $(Z_t, \xi_t)$ is a connected and separable space, then $(Z, \xi)$ is connected and separable in the product topology. Therefore, it makes sense to discuss properties of the instantaneous utility functions which are similar to those of the overall utility function discussed above.

Let $z_{-t} = (z_1, \ldots, z_{t-1}, z_{t+1}, \ldots, z_T)$ be the consumption/quality bundle at all dates other than date t. For fixed $z_{-t} = z^o_{-t}$, the preference ordering R induces a preference ordering on $Z_t$ given by $x_t \, R_{x_{-t}} \, x'_t$ if and only if $(x_{-t}, x_t) \, R \, (x_{-t}, x')$ for any $x_t$, $x'_t$ in $Z_t$.

Axiom 1.4: For each $t \varepsilon (1, \ldots, T)$ $x_t \; R_{x_{-t}} \; x'_t$

implies $x_t \; R_{x_{-t}} \; x'_t$ for all $x_{-t} \varepsilon \prod_{i \neq t} Z_i$.

The following theorem provides a utility function representation for separable preferences.

Theorem 1.2: The preference ordering posited in Axioms 1.1 to 1.4 may be represented by a continuous, quasi-concave function $U: \prod_t Z_t \to E$ which may be written

$$U(z^T) = \hat{U}(u_1(z_1), \ldots, u_T(z_T))$$

where $u_t : Z_t \to E$ and $\hat{U} : E^T \to E$, and

$\hat{U}$ as well as each $u_t$ is increasing, continuous (in the product topology and usual topologies respectively) and quasi-concave.

Proof: The existence of a continuous utility function taking the separable form is proved by Katzner (1970). That the component functions $\hat{U}$ and $u_t$ are quasi-concave if U is (which follows from axiom 1.3), is shown by Blackorby, et al., (1978), Theorem 4.1.

Let $y_t$ be income allocated to consumption at date t, and let $B_t(p_t, y_t) = (c_t : p_t \cdot c_t \leq y_t, \; c_t \; \varepsilon \; C)$. Define $V_t(p_t, q_t, y_t) = \max_{c_t} (U(c_t) : c_t \; \varepsilon \; B_t(\cdot))$.

Then

$$V(p_t, q, w) = \max_{\{w_t\}} \; \{\hat{U}(\{V(p_t, q_t, w_t)\}_{t=1}^{t=T}): \sum_{t=1}^{t=T} \alpha^{t-1} y_t \leq w\}.$$

The instantaneous indirect utility functions can be used to define instantaneouc measures of welfare change, i.e.,

$$V_t(p_t^o, q_t^o, w_t^o) = V_t(p_t^v, q_t^v - cv_t, w_t^v)$$

$$V_t(p_t^v, q_t^v, w_t^v) = V_t(p_t^o, q_t^o + ev_t, w_t^o), \text{ for } t \varepsilon [0,T].$$

Here, when the project is implemented, the consumer may respond by reallocating income through time as well. This point is crucial, for it creates the following inequality:

$$V(p_v^T, q_v^T - cv, w_v^T) = \hat{U}(\{V_t(p_t^v, q_t^v - cv_t, w_t^v)\}_{t=0}^{t=T}) \leq$$

$$V(p_v^T, q_v^T, \sum_{t=1}^{t=T} \alpha^{t-1}(w_t - cv_t) = V(p_v^T, q_v^T, w - \sum_{t=1}^{t=T} \alpha^{t-1} cv_t).$$

This implies, since V is increasing in its second arguement, that

$$cv \geq \sum_t \alpha^{t-1} cv_t.$$

Thus, if the present value of consumer surplus is non-negative, so is the present value welfare change measure cv.

Similarly,

$$\sum_t \alpha^{t-1} ev_t \geq ev,$$

whence if the present value of equivalent variations is negative, so is the true welfare charge measure. These give two one-way tests, but leaves a zone of indeterminancy. Moreover, we have the following theorem.

Theorem 1.3: There is no U with U, $\hat{U}$ and $\{u_t\}$ continuous, increasing, and quasi-concave, such that the present value of instantaneous $cv_t$ or $ev_t$ is an exact index of welfare change for all projects.

Proof: Blackorby, Donaldson and Moloney (1984).

Before turning to an assessment of how the equivalent variation measure of welfare change for individuals can be used in making choices among projects by a social planner, I introduce the intertemporal expenditure function and discuss briefly the money metric measure of welfare change.

Dual to the lifetime indirect utility function introduced above is the lifetime expenditure function defined by

$$V(p^T, q^T, w) = V^o \iff E(p^T, q^T, V^o) = w.$$

The money metric (see McKenzie and Pearce, 1982) is defined by

$$Y(v) \equiv E(p^T(0), q^T(0), V(p^T(v), q^T(v), w)).$$

The definition of the expenditure function shows that

$$ev(v) = Y(v) - w. \tag{2}$$

The money metric gives minimum the cost of achieving the level of the utility with the project, when the project has not been implemented. Since Y is a monotonic increasing function of an indirect utility function, it is itself an indirect utility function. Importantly, both the ev and the money metric are invariant to increasing monotonic transformations of the underlying ordinal utility function.

The money metric and equivalent variations possess an important property that the compensating variation does not have. The cv is not an exact measure of welfare change in that it may not correctly rank several projects relative to a base project, although it will correctly make pairwise comparisons (Hause, 1975; Chipman and Moore, 1980).

To sum up the results of this section, the equivalent variation and money metric are useful measures of individual welfare change due to the implementation of a project. In a dynamic setting, these should be defined relative to the lifetime indirect utility or expenditure functions. This would seem to underscore the usefulness of survey techniques in eliciting willingness-to-pay since lifetime compensation measures (or their annualized equivalent) can be directly assessed. However, the lifetime approach does create a few difficulties for the definition of an appropriate criterion for selection of a project by the planning authority. These are addressed, at least partially, in the next section.

3. Project Selection under Certainty

The difficulties of moving from individual to social valuations of projects are of two kinds. The first is the much discussed possibility of providing an axiomatic foundation for a social preference ordering or welfare function which is based on individual orderings. This issue is not addressed here; the existence of a preference ordering for the planning authority which has certain properties is merely asserted. The second difficulty derives from the focus on lifetime indirect utility functions in Section II. In particular, if it is asserted that the planner has preferences over indirect utilities, and it is not assumed that each "generation" consists of a single individual (see, e.g., Ferejohn and Page, 1978), then some work is required to establish a benefit-cost foundation for social choices.

The individual theory above used the sequences $p^T$ and $q^T$, which are sequences with terminal date corresponding to the individual's planning horizon. These are subsequences of $p^{\bar{\tau}} = (p_1,\ldots,p^{\bar{\tau}})$ and $q^{\bar{\tau}} = (q_1,\ldots,q^{\bar{\tau}})$, where $\bar{\tau}$ is the horizon relevant to the planning authority. These price and environmental quality sequences depend on the project that is implemented. The environmental quality sequence depends on the project as represented by equation (1). In the sequel $q^{\bar{\tau}}(v)$ is used to denote this dependence. Being purposely vague, I write $p^{\bar{\tau}}(v)$ as well. It is assumed that both of these functions are unique without specifying conditions under which this will be true. For $t \in (\tau, \bar{\tau})$,

$$q_{\tau+s+1} = f((\hat{v}_\tau, 0^s), 0, \hat{q}_{t+s}, q_{t+s}),$$

where $0^s$ is the zero vector in $E^s$. Similarly, let $p_t = p_t(0)$ for $t \in (\tau, \bar{\tau})$.

The set of possible projects is given by $\Delta \subset E^{\tau}$, $\Delta = \{v \varepsilon E^{\tau} : v$ is feasible$\}$. An individual is said to care about a project if his/her lifetime indirect utility varies with changes in v. Formally, agent i cares about the project set $\Delta$ if $V(p^T(v), q^T(v), w) \neq V(p^T(v'), q^T(v'), w)$ when $v \neq v'$ for some $v, v', \varepsilon \Delta$.

There are several ways in which an individual might not care about a project. If the individual is not alive, then (presumably) $V^i(.) = 0$ for all $v \varepsilon \Delta$. As well, some prices might not depend on the project and an individual might not consume any of the goods (including recreation) with project-sensitive prices. If $V^i$ is independent of changes in environmental quality when consumption of recreation is zero and the individual does not care about price changes for goods (s)he does not consume, then (s)he will not care about the project. This is the case of "weak complimentarity" discussed in the valuation literature (Bradford and Hildebrandt, 1977).

Let $M^i = \{t : i$ cares about $\Delta$ at $t\}$, and let $t(i) = \inf \{t : t \varepsilon M^i\}$. To avoid mathematical complexities which are not of concern in this paper, the following assumptions are imposed.

A3.1: The number of agents at each date t is finite.

A3.2: $\bar{\tau} < \infty$.

A3.3: $t(i) \leq \bar{\tau} - T^i$ for all i.

Let ${}^tI = \{i : t(i) = t\}$ and denote the power of ${}^tI$ by $I^t$. Individual i's planning horizon is given by $T^i$; purely to ease notational burden, I assume that $T^i = T$ for all i.

The vector of lifetime indirect utilities is a vector in $E^I$, where $I = \sum_t^{\bar{\tau}} I^t$. By A3.1 and A3.2, this is a finite-dimensional space. The planning authority is presumed to have preferences on $E^I$ as given in the following axioms.

Axiom 3.1: The planner's preferences are represented by a binary relation P $E^I \times E^I$ where P is a weak order, which is monotonic and continuous in the usual topology.

Under axiom 3.1, P can be represented by a real valued social welfare function.

Theorem 3.1: If the planner's preferences satisfy axiom 3.1, then there exists $W : E^I \to E$, with W continuous and such that $W(V^1,\ldots,V^I) > W(\overline{V}^1,\ldots\overline{V}^I)$ if and only if $(V^1,\ldots, V^I)\ P(\overline{V}^1,\ldots, \overline{V}^I)$.

Proof: Fishburn (1970).

Theorem 3.1 establishes a social welfare function defined on sequences of lifetime indirect utility profiles. However, a problem arises in this approach. The arguments of W are individual utilities, which can be subjected to an arbitrary monotonic transformation with affecting underlying behavior. Undertaking such a transformation may drastically change the social rankings involved. Clearly, this is an undesirable characteristic for a social welfare function to have. Rather than dealing carefully with specification of W, it is more convenient to measure the arguments of W such that they are invariant to such monotonic transformations. The money metric described in the previous section is an obvious candidate.

Furthermore, one is interested in deriving social rankings of alternative projects induced from this ranking of utilities. That is, one seeks a ranking P* defined by v P* v' if and only if g(v)P g(v'), where $g: \Delta \to E^I$ given individual lifetime utility vectors as a function of projects. An important special case for which this is straightforward and which will be useful when uncertainty is introduced is where the social welfare function is linear. Thus, I impose

$$\text{A3.4:} \quad W(V^1,\ldots, V^I) = \sum_{i=1}^{i=I} b_i V^i.$$

The implementation of a project entails a cost and, therefore, the central planner must devise some method of financing the effort. Assume that lump-sum financing is possible. Let the spot expenditures required to implement a project v be given by

$$k(v) = (k_1(v), \ldots, k_\tau(v)).$$

The planner has several options for financing project v. A financing scheme is a vector of payments $s(v) = (s^1(v), \ldots, s^I(v))$ which specifies $s^i(v)$, the payment by agent i to finance project v. The set of feasible financing schemes is given by

$$S(v) = \left\{ s(v) : \sum_{t=1}^{t=\bar{\tau}} \alpha^{t-1} \sum_{i=1}^{i=I^t} s^i(v) \geq \sum_{t=1}^{t=\tau} \alpha^{t-1} k_t(v) \right\}$$

The central authority will choose a feasible project/financing scheme pair so as to maximize social welfare. That is, it will solve

$$\max_{v \varepsilon \Delta} \sum i \; b_i \; Y^{*i}(v),$$

where

$$Y^{*i}(v) = Y^i(v, s^{*i}(v)) = E^i(p^T(0), q^T(0), w^i, V^i(p^T(v), q^\tau(v), w^i - s^{*i}(v))$$

for

$$s^*(v) \; \varepsilon \; \underset{s(v)\varepsilon S(v)}{\text{argmax}} \sum_i b_i Y^i(v, s^i(v)).$$

It is interesting to point out that the following theorem governs a relationship between choices of v and choices of lifetime indirect utility vectors.

Theorem 3.2: v P* v' if and only if

$$\sum_i b_i \left[ ev^i(v) - ev^i(v') \right] > 0.$$

Proof: By theorem 3.1, $\vec{V}$ P $\vec{V}'$ iff $W(\vec{V})>W(\vec{V}')$, where $\vec{V}=(V^1, \ldots, V^I)$.

Whence by A3.4, $\vec{V}$ P $\vec{V}'$ iff

$$\sum_i b_i \, [V^i(v) - V^i(0) - (V^i(v') - V^i(0))] > 0.$$

Since $Y^i$ is a utility indicator and $Y^i(0) = w^i$,

$\vec{V}$ P $\vec{V}'$ iff $\sum_i b^i[Y^i(v) - w^i - (Y^i(v') - w^i)] > 0$,

By definition of P* and by (2), the result follows.

The magnitude of $ev^i(v)$ will depend on the financing scheme used. It is not possible to separate these decisions. McKenzie (1983, chapter 8) shows how the ordinal properties of W can be used to determine losses due to use of non-optimal financing schemes.

4. Uncertainty

I now turn to possibilities for generalizing the framework developed in the previous section to the case of uncertainty. As discussed in the introduction, it is critical that when uncertainty is addressed, that it is clear what it is that uncertainty surrounds, who faces the uncertainty, and what that agent can do about it. There are several ways that uncertainty can enter the model developed in Section II of the paper. Here several are identified that seem relevant in the option value literature:

(i) Ecological uncertainty. Given a $\nu \in \Delta$ it is not known what level of environmental quality will obtain. This may be represented by making (1) a random function. There are two ways to capture this, each representing a different source of uncertainty.

First, one could think of the function f itself as being unknown. That is, one may not know how ecosystem function maps projects into environmental quality. Second, even if the true f is known, the sequence of quality outcomes might be stochastic. In fact, both of these are operating to make uncertainty relevant. If the former operated without the latter, a simple experiment at date zero would resolve all of this type of uncertainty. If the latter operated without the former, then learning about ecosystem function would not be possible unless it is interpreted as trying to discover the probability law driving the stochastic process; clearly biological investigation seeks more than this.

(ii) Economic uncertainty. It seems plausible to assume that future prices and incomes are risky.

(iii) Preference uncertainty. The majority of the literature on option value has investigated the implications of state-dependent preferences (demand uncertainty) where individual preference orderings are uncertain.

(iv) Political/Regulatory uncertainty. The project itself may be risky. The project may entail some enforcement which may be applied at various levels in the future or may no yield compliance.

(v) Social uncertainty. When confronted with a project which can be implemented at alternative levels and where aggregate willingness to contribute to funding the project is involved, individuals may hold uncertainty about the contributions of other agents. This often is discussed in terms of strategic bias in contingent valuation assessments of willingness-to-pay where the presumption is free-riding behavior, but this is a special case of more general problems of social interdependence in provision of public goods.

(vi) Planning uncertainty. Even if agents know their own preferences, the planning authority may not know them. Thus, the planning authority may have uncertainty about preferences even if individuals do not.

The theoretical option value literature has focused on uncertainty of types (i), (ii), and (iii) above, though one analysis of time-sequenced option value has examined uncertainty of type (iv) (Graham-Tomasi, 1985). The ecological uncertainty has taken a particular form in the literature on supply-side option value (Bishop, Freeman, 1985), in which quality either is good enough to allow a particular recreational activity or quality deteriorates to the point where the activity no longer is available. Thus, just two states are possible. It is common in this literature to see this uncertainty represented as price uncertainty, with the entry fee for activity at the rate equal to some finite price of the activity is available and an infinite price if it is not. A generalization of this approach is presented below. Usually, though not always (Hartman and Plummer, Freeman, 1984) it is assumed that prices of other goods and income are non-stochastic.

5. Individual Uncertainty with an Expected Utility Representation

The majority of analyses of option value employ a static model and use an expected utility representation of individual preferences. In this section I take a similar approach to modeling preferences and investigate extensions of the material developed above to the case here. The focus is on ecological uncertainty, that is, on supply-side uncertainty. Given a project, there is a probability structure on environmental quality induced by the probability structure on ecosystem functioning. To gain an expected utility representation, a static problem is analyzed. In section 7 I consider a two-period problem.

Let $(\Omega, \tau, \mu^{\omega})$ be a probability space. The function f defined in (1) can be turned into a random function representing the two sources of ecological uncertainty in the following fashion. Let

$$\Phi = \{f: E^m \times E \times \Omega \rightarrow E^m: \text{ f if continuous, is Borel measurable relative to } \tau \text{ for all fixed } (g,\nu) \in E^m \times E, \text{ and } \mu^{\omega} - \text{integrable}\}.$$

Assume

$$\text{A5.1:} \quad f \in (f^1, \ldots, f^F) \quad f^i \in \Phi \text{ for all } i, \text{ with } \pi^i = \text{Prob } [f = f^i].$$

Then, the induced probability measure on environmental quality, conditional on the project $\nu$ and initial (non-random) enviornmental qualtiy $q_o$ is

$$\mu^{\nu}(Q_1) = \sum_i \mu\{\omega \in \Omega: \ f^i(q_o, \nu, \omega) \in Q_1\} \ \pi^i$$

$$\text{for } Q_1 \in \beta(E^m), \text{ the Borel sets of } E^m.$$

In this section it is supposed that the individual has preferences on the space of probability measures on $(Q_1)$ which satisfy the non-Newmann and Morganstern axioms. Formally, let L be the space of lotteries on environmental qualtiy, i.e.,

$$L = \{\mu^{\nu}(Q_1): \ \nu \in \Delta\}.$$

Axiom 5.1: The individual's choices from L are representable by a binary relation R which satisfies

(i) is a weak order

(ii) $(\mu^1 \ R \ \mu^2) \Rightarrow \alpha\mu^1 + (1-\alpha)\mu^3 \ R \ \alpha\mu^2 + (1-\alpha)\mu^3$ for $\mu^1, \mu^2, \mu^3 \in L$ and $\alpha \in (0,1)$

(iii) $(\mu^1 \ R \ \mu^2)$ and $(\mu^2 \ R \ \mu^3) \Rightarrow \alpha\mu^1 + (1-\alpha)\mu^3 \ R \ \mu^2$ and $\mu^2 \ R \ \beta\mu^1 + (1-\beta)\mu^3$ for some $\alpha, \beta \in (0,1)$ and $\mu^1, \mu^2, \mu^3 \in L$.

Then one can show

Theorem 5.1: For all $\mu^{\nu}$, $\nu \in \Delta$, let the sets $\{\mu^{\nu} \in L: \mu^{\nu} \ R \ \mu^{o}\}$ and $\{\mu \in L: \ \mu^{o} \ R \ \mu\}$ be open in the weak topology and strictly convex. Then there exists a continuous function $V: \ E^{3n + 2m+1} \rightarrow E$ such that

$$\mu \ R \ \mu^{o} \Leftrightarrow \int_{\Omega} V(\cdot)d\mu > \int_{\Omega} V(\cdot)d\mu^{o},$$

where

$$V(p_0, p_1, w, c_0, q_0, q_1) = \sup_{C_1 \in B(p_0, p_1, w, c_0)} U(c_0, q_0, c_1, q_1) \text{ for } B(\cdot) = \{c_1: \ \alpha P_1 C_1 \leq w_1 + \alpha \ w_2\}.$$

Moreover, this supremum is attained, and $C_1^* \in \arg\max_{C_1 \in B(\cdot)} U(\cdot)$ is continuous.

Proof: The existence of the functions U and V follow from Axiom 5.1 and Fishburn (1970), Theorem 8.4. Continuity of V follows from openness of the upper and lower contour sets (Varian, 1978). That the supremum is attained derives from the Weierstrauss Theorem, the continuity of U and the compactness of $B(\cdot)$. Upper semi-continuity of $c_1^*$ follows from the maximum principle of Berge (1963); but $c_1^*$ is unique due to the strict convexity of the upper contour sets of $\mu$, and therefore $c_1^*$ is continuous.

To define welfare measures for changes in the measure $\mu^{\nu}$ due to choices of $\nu \in \Delta$, let $\mu^{o}$ be the measure induced by project $0 \in \Delta$. For further reference, let $F^{o}(q_1)$ and $F^{\nu}(q_1)$ be the probability distribution functions for $\mu^o$ and $\mu^\nu$. There is a one-to-one correspondence between $\mu$ and F (Ash, 1970). The compensating option price (COP) and equivalent price (EOP) are defined implicitly by

$$\int_\Omega V(p,\ q,\ w - COP(\nu))d\mu^\nu = \int_\Omega V(p,\ q,\ w)d\mu^o.$$

$$\int_\Omega V(p,\ q,\ w)d\mu^\nu = \int_\Omega V(p,\ q,\ w + EOP(\nu))d\mu.$$

These, of course, are natural analogs of the cv and ev measures of welfare change defined in Section II. In most of the option value literature, the COP measure is called the option price (e.g. Smith, 1983; Freeman, 1985). As discussed in the introduction, considerable attention has focused in this literature on the relationship between COP and the expected value of consumer surplus. The motivation for this concern is two-fold. First, in the absence of contingent claims markets, or the ability to extract ex-post compensation from agents, it is thought that COP is the proper measure of ex-ante WTP for the project. Second, since consumer surplus measures are used to determine project choice (as in Section 3 of this paper), investigators are interested in whether use of consumer surplus over or under estimates true ex-ante WTP.

One difficulty with this discussion is that the COP measure only is an appropriate index of welfare when binary choices among projects are being made. This is for the same reason that the cv measure is inappropriate. This is stated formally in the following theorem.

Theorem 5.2: The COP($\nu$) measure is not a valid measure of welfare change.

Proof: Define certainty equivalent environmental quality levels CEQ(p, w, $c_o$, $\mu$) by

$$\int V(p,\ q,\ w,\ c_o,\ q)d\mu = V(p,\ CEQ(\cdot),\ w,\ c_o)).$$

Then by definition,

$$V(p,\ CEQ(p,\ w,\ c_o,\ \mu^\nu)\ w - COP(\nu), c_o)) =$$

$$V(p,\ CEQ(p,\ w,\ C_o,\ \mu^o),\ w,\ C_o)).$$

> But, by arguments in Chipman and Moore (1980), COP($\nu$) only is a valid index for binary choices. If there is more than one $\nu \in \Delta$ other than $\nu = 0$, COP may not rank these correctly.

In their analyses of option value, Schmalensee (1972) and Bishop (1982) uses the EOP. Of course, whether EOP or COP is used will not matter if there are only two possible projects.

As discussed above, much of the option value literature is concerned with the relationship between an ex-ante measure such as COP or EOP and the expected value of ex-post measures. Freeman (1985) has pointed out that the supply-side of many of these analyses is a special case of the more general case of a change in distribution that he (and I) consider. In particular, these analyses presume that only type (iii) uncertainty, demand uncertainty exists, substitute two degenerate measures $\mu^o$ and $\mu^\nu$ on the supply side, and let m = 1.

Briefly, the formulation is as follows. Let V be the individual's indirect utility function, a Borel measurable function of $\omega \in \Omega$, and let $\mu^D$ be a probability measure on the $\sigma$-algebra on $\Omega$. On the supply side, assure that $\mu^\nu$ and $\mu^o$ both are degenerate, assigning probability one to outcomes $q^\nu$ and $q^o$ respectively. Then, in state $\beta$, the equivalent variation ev($\beta$) is

$$V^\beta(p, q^\nu, w) = V^\beta(p, q^o, w + ev(\beta)),$$

and the expected equivalent variation is

$$\int ev(p)dF^D(\beta).$$

The following result has been much-discussed.

> Theorem 5.3: With $\mu^o$ and $\mu^\nu$ degenerate and $\mu^D$ non-degenerate, EOP can be greater or less than expected equivalent variation.
>
> Proof: The proof follows that of Bishop (1982), where our definition of ev is substituted for his.

Note that in the formulation in Bishop and elsewhere (e.g. Andersen, 1981) it is assumed that under $0 \epsilon \Delta$, $q_1^o < q_{1min}$, where $q_{min}$ is the minimum quality such that the site is not available. This is formalized as $q_1^o \Rightarrow c_{1j} = 0$ where $c_{1j}$ is visits to the site and is accomplished by a pricing function $p(\nu)$ with $p_{1j}(0) = \infty$; $p_{1j}(\nu) = p_{1j} < \infty$.

The literature which addresses ecological uncertainty in the absence of preference uncertainty is somewhat confusing regarding definitions of equivalent and compensating option price. In the definitions above, equivalent option price (EOP) uses the situation without the project as a base adn asks how much money must be given to the individual to forego the benefits induced by the project. The compensating option price (COP) uses the situation with the project as a base and asks how much can be taken away from the individual to return him or her to the pre-project level of utility.

In the analyses by Bishop (1982) and Freeman (1985) of ecological uncertainty, only two situations are compared; thus, the difficulty of ranking projects by the COP measure may not arise. However, it is important to note that the proof of Theorem 5.2 used a certainty equivalent approach. When one defines a welfare change measure for each state, then which measure is appropriate may depend on whether the before-project or after-project probability measure is degenerate.

Both Bishop and Freeman study a model with only two possible outcomes, one of which corresponds to a level of quality such that use of the site is zero. They then define the ex-post compensation measure in the state in which the resource is available by income change that equates indirect utility with and without the resource. This is the natural approach. Here, I consider a model with many possible states. Thus, the ex-post measure for each state is defined relative to with and without project realizations of quality. That is, if

$q^o \varepsilon Q^o$ is the realization without the project and $q^\nu \varepsilon Q^\nu$ is the realization with project $\nu \varepsilon \Delta$, then $ev(q^o, q^\nu)$ and $cv(q^\nu, q^o)$ are defined implicitly by (suppressing p)

$$V(q^o, w - ev(q^o,q^\nu)) = V(q^\nu, w)$$

$$V(q^o, w) = V(q^\nu, w - ev(q^\nu, q^o))$$

In the most general situation in which there is risk about environmental quality both with and without the project. Then expected values of ex-post welfare measures are given by

$$\int_{Q^o} \int_{Q^\nu} ev(q^o,q^\nu)dF^\nu(q^\nu)dF^o(q^o) = \int_{Q^\nu} \int_{Q^o} cv(q^\nu,q^o)dF^o(q^o)dF^\nu(q^\nu)$$

Having chosen a base outcome given by the first argument of the ev(.,.) and cv(.,.) function (e.g., ev $(q^o, q^\nu)$ gives the ev of a move from outcome $q^o$ without the project to outcome $q^\nu$ with the project), both of these will correctly compute the welfare change in each state. That is, conditional on outcome $q^o$, the L.H.S. measure will assign the same welfare measure to two indifferent with-project outcomes $q^\nu$. The same is true for the R.H.S. where the conditioning base event is the with-project event $q^\nu$.

Returning to the analyses of Bishop (1982) and Freeman (1985), consider two special cases. In the first, the situation without the project is risky, while the project provides a desirable sure outcome, and in the second, environmental quality without the project is given by a sure undesirable outcome, while the project provides a risky quality. These correspond to Case B and Case C in Freeman (1985), respectively; he notes that Bishop studies Case B.

Consider first Case B. Here, since the situation with the project is fixed, it makes some sense to use the cv measure in each state. Then, a fixed base is used for comparison to each of the risky outcomes without the project. It is easy to show that the COP is greater than the expected value of the ex-post cv measures, at least for a finite number of states.

Theorem 5.4: (Bishop, 1982) Let $F^o(q)$ be non-degenerate with probability mass $\Pi^o = (\Pi^o, \ldots, \Pi^o_Q)$ and let $F^\nu(q)$ be degenerate, with Prob $[q = q^\nu] = 1$. Then, if $V(\cdot)$ is strictly concave and increasing in income, the COP is greater than the expectation of cv.

Proof: The cv measure in state i is defined implicitly by

$$V(q^\nu, w - cv^i) = V(q^i, w).$$

Compensating option value is defined by

$$\sum_i \Pi^o_i V(q^i, w) = V(q^\nu, w - COP).$$

By concavity of $V(w,q)$ in w,

$$V(q^\nu, w - cv^i) < V(q^\nu, w - COP) + (COP - cv^i)V_w(q^\nu, w-COP).$$

Since the LHS is equal to $V(w,q^i)$ by definition, multiplication by $\Pi^o_i$ gives

$$\Pi^o_i V(q^i, w) < \Pi^o_i V(q^\nu, w-COP) + \Pi^o_i(COP - cv^i)V_w(q^\nu, w-COP).$$

Summing over i yields

$$\sum_i \Pi^o_i V(q^i, w) < V(q^\nu, w-COP) + V_w(q^\nu, w-COP)[COP - \textstyle\sum_i^o \Pi_i cv^i].$$

By the definition of COP,

$$0 < V_w(q^\nu, w-COP)[COP - \textstyle\sum_i \Pi^o_i cv^i],$$

which provides the result.

Actually, with many possible states, the use of the cv as the ex-post compensation measure and COP as the option price, and the definition of cv in each state allows a simpler proof than that used by Bishop in the two-state world.

Next, consider Case C. Freeman (1985) uses a cv measure and proves that the sign of option value (the difference between COP and the expectation of ev) is ambiguous. Here is presented a similar result, and also it is shown that with an equivalent option price approach and use of ev in each state, the sign of option value can be determined.

Theorem 5.5: Let $F(q)$ be non-degenerate with probability mass $\Pi^{\nu} = (\Pi_1^{\nu}, \ldots, \Pi_Q^{\nu})$, and let $F^o(q)$ be degenerate with Prop $[q=q^o] = 1$. Then, with V increasing and strictly concave in income, the relationship between COP and expected cv is not determinate. A sufficient condition for COP - E(ev) to be positive is that the marginal utility income is the same for each state.

Proof: The cv in each state is defined by

$$V(q^i, w-cv^i) = V(q,w)$$

and COP is defined by

$$\sum_i \Pi_i^{\nu} V(q^i, w-COP) = V(q^o, w).$$

By strict concavity of V in w,

$$V(q^i, w-cv^i) < V(q^i, w-COP) + [COP-cv^i] V_w (q^i, w-COP).$$

$$\Leftrightarrow V(q^o, w) < V(q^i, w-COP) + [COP-cv^i] V_w (q^i, w-COP)$$

$$\Leftrightarrow \Pi_i^{\nu} V(q^o, w) < \Pi_i^{\nu} V(q^i, w-COP) + \Pi_i^{\nu} [COP-cv^i] V_w (q^i, w-COP).$$

This holds for each i, whence by definition of COP,

$$0 < \sum_i \Pi_i^{\nu} [COP-cv^i] V_w (q^i, w-COP).$$

The difficulty in establishing a sign for option value is presented by the marginal utility of income. If this is the same at (w-COP) for each $q^i$, then this term can be factored out to yield

$$0 < COP - \sum_i \Pi_i^{\nu} cv^i.$$

The value of an equivalent option price approach is that the marginal utility of income term appears only with a fixed state. Thus, equivalent option value is positive.

Theorem 5.6: Assume the conditions of Theorem 5.5. Then EOP is greater than E(ev).

Proof: The proof is exactly the same as for the proof of Theorem 5.4 using EOP and $ev^i$ defined by

$$V(q^o, w-ev^i) = V(q_i^{\nu}, w)$$

$$\sum_i V(q_i^{\nu}, w) = V(q^o, w-EOP).$$

The discussion of the relationship between the ex-ante measures of COP and EOP and the expected value of ex-post measure cv and ev is due to a desire to determine if use of cv and ev in project evaluation systematically over or under estimates true ex-ante WTP. However, two points may be made. First and most obviously, knowing that expected ev underestimates EOP is not particularly useful if you don't know by how much. Thus, Smith (1984) tries to find a bound for the size of the discrepancy. Unfortunately, Smith's approach requires a fairly strong restriction on preferences and only works for two possible states. Second, most analyses of projects do not use the expected ev or cv measure. Rather, they ignore uncertainty altogether and presume that the expected outcome is the true outcome. Thus, they calculate the Hicksian welfare measure at the expected value. Formally, let

$$ev(\overline{q}^{\nu}) = ev(\int q dF^{\nu}(g))$$

$$ev(\overline{q}^{o}) = ev(\int q dF^{o}(q)).$$

If $ev(\overline{q}^{\nu}) > ev(\overline{q}^{o})$, then the project is said to make the individual better off and the analysis proceeds as in Section 3. It may be possible to derive an approximation to EOP based on readily observable variables and the deterministic ev using expected values. The author will present such an approximation in a future paper.

## 6. Individual Uncertainty: Generalized Expected Utility

The model of the previous section, which predominates the option value literature, is static; this was captured in the previous section by assuming that $C_0$ is fixed and concentrating on the relationship between $C_1$ and $q_1$. As well, it was assumed that $C_1$ could be chosen after observing $q_1$. When this assumption is dropped and the model becomes dynamic, there are two difficulties that arise.

First, atemporal von Neumann-Morganstern (vN-M) utility theory applied in a dynamic setting requires that preferences on income (or here, environmental quality) be defined solely on income vectors. In the language of dynamic programming, a plan for choosing actions given states induces a probability distribution on the vector of payoffs. As optimal plan (if one exists) is one that maximizes the expectation of vN-M utility function on such vectors. As pointed out by Kreps and Porteus (1978), this rules out the possibility that an individual may prefer earlier to later resolution of uncertainty. They illustrate this by the following example. Suppose the payoff vector is (5,10) with probability 1/2 and (5,0) with probability 1/2. Then under the vN-M axioms, since 5 is the first-period payoff for sure, the individual should be indifferent between a flip of a fair coin at $t = 0$ and a flip of the coin at $t = 1$ to determine which vector obtains. In fact, individuals may prefer earlier resolution of uncertainty.

Kreps and Porteus (1978, 1979) derived a generalization of atemporal vN-M theory, which they called temporal von Neumann-Morganstern utility theory. In their theory, uncertainty is dated by the time of its resolution. These entities are called temporal lotteries. They present axioms for preferences defined as these temporal lotteries which allow a temporal vN-M representation. Below, their framework is applied to our problem concerning environmental quality.

The second problem that arises concerns induced preferences when a choice must be made before uncertainty resolves. Even if all uncertainty resolves at a single date and the underlying preferences on consumption have an expected utility representation, induced preferences will, in general, not satisfy the independence axiom and will be "non-linear in the probabilities." This has been observed by Markowitz (1959), Mossin (1969), Spence and Zeckhauser (1972), and Dreze and Modigliani (1972). Kreps and Porteus (1979) derive necessary and sufficient conditions for induced preferences in the temporal case to take the temporal vN-M form. These are quite strong. Machina (1982, 1984) has proposed an approximation approach called generalized expected utility theory, which copes with this difficulty without sacrificing the basic foundation of expected utility theory. In this section, these results are developed in terms of a model of ecological uncertainty.

Uncertainty is represented in same way as in the previous section. We assume that the space $\Omega$ of possible realizations of the "experiment" giving rise to environmental uncertainty is compact. Let $D_t$ be the space of Borel probability measures on $Q_t$.

Lemma 6.1: $D_t$ is a compact metric space.

Proof: By assumption, f is continuous function onto $Q_t$ for fixed $q_{t-1}$. By Theorem 3.5 in Kolmogorov and Fomin (1970), $Q_t$ is compact: $Q_t \varepsilon E^m$ so it is a metric space. The result follows from Parthasarathy (1967), Theorem 6.4.

Endow $D_t$ with the weak topology. If $g(q)$ is continuous, then the weak topology is the weakest topology for which the functional $\int g(x)d\mu(x)$ is continuous for $\mu \in D$. Alternatively, one could give $D_t$ the Prohorov metric, since convergence in the Prohorov distance of a sequence of measures on a Polish space is equivalent to weak convergence of this sequence (Lukacs, 1975, p. 74).

Clearly, the probability measure on $Q_1$ is conditioned on the realization of $q_0$ due to the structure of the function f. Thus, define $D_0$ as the space of all Borel probability measures on $Q_0 \times D_1$. Elements of $D_0$ are called temporal lotteries. I introduce the following axioms on individual preferences regarding probability measures.

Axiom 6.1: The relation R is asymmetric and negatively transitive.

Axiom 6.2: The sets $[\mu_0 \in D_0 : \mu_0 R\, \mu_0']$ and $[\mu_0 \in D_0 : \mu_0' \, R\, \mu_0]$ are both open in the weak topology.

Axiom 6.3: If $\mu_0 \, R\, \mu_0'$ and $\alpha \in (0,1)$, then $[\alpha\mu_0 + (1-\alpha)\,\mu_0''] \, R\, [\alpha\mu_0' + (1-\alpha)\,\mu_0'']$.

Axiom 6.4: Let $\mu_{i0}$ be degenerate with outcome $(q_0,\mu_1)$. If $(q_0,\mu_1)\, R\, (q_0,\mu_1')$ and $\alpha \in (0,1)$, then $[q_0,\ \alpha\mu_1 + (1-\alpha)\,\mu_1''] \, R\, [q_0,\ \alpha\mu_1' + (1-\alpha)\,\mu_1'']$.

Axiom 6.1 and 6.2 are obvious analogs of Axiom 5.1 and the condition of Theorem 5.1 regarding continuity. Axiom 6.3 is a substitution axiom similar to Axiom 5.2 for time zero; Axiom 6.4 is a substitution axiom for time 1. The following restates Theorem 2 in Kreps and Porteus (1978).

Theorem 6.1: Axioms 6.1 to 6.4 are necessary and sufficient for these to exist continuous functions $V_1 : Q_0 \times D_1 \to E$ and $U_0 : Q_0 \times E \to E$ with $U_0$ increasing in its second argument such that if $V_0 : Q_0 \times D_1 \to E$ is given by

$$V_0(q_0,\mu_1) = U_0(q_0, \int V_1(q_0,q_1)\, d\mu_1),$$

then $\mu_0 \; R \; \mu_0'$ if and only if

$$\int V_0(q_0,\mu_1)\, d\mu_0 \geq \int V_0(q_0,\mu_1)\, d\mu_0'.$$

Proof: Kreps and Porteus, 1978, Theorem 2.

The relationship between temporal vN-M theory as given by Theorem 6.1 and the atemporal theory studied the previous section is given by the following result.

Theorem 6.2: If $U_0(q_0,r)$ is affine in $r$, then the temporal respresentation collapses to the atemporal vN-M utility. This is the case if and only if, in addition to Axioms 6.1 to 6.4,

$$(q_0,\alpha\mu_1 + (1-\alpha)\mu_1') \; I \; \alpha(q_0,\mu_1) + (1-\alpha)(q_0,\mu_1'),$$

where I is the equivalence derived from R in the usual way.

Proof: Kreps and Porteus, 1978, Theorem 3 and its corollary.

Thus, the kinds of analyses usually undertaken in the literature of option value, where atemporal vN-M utility is assumed, can be extended without modification if preferences satisfy the substitution axioms and are neutral to the resolution of uncertainty. However, it seems unlikely that individuals are neutral with respect to the resolution of uncertainty.

Consider now the induced preference problem and the relationship between the timing of choices of $C_0$ and the timing of the resolution of uncertainty. As mentioned above, induced preference generally will not have an expected utility representation. In fact, it generally will not have a temporal vN-M representation. Kreps and Porteus (1979) derive necessary and sufficient conditions for the former to take on the latter form.

Note that in the above formulation, the first-period consumption decision was not explicitly introduced. At date zero, after observing the outcome of the temporal lottery $\mu_0$, the agent chooses $C_0$ from $B(\cdot)$, the budget set. Note that it is possible to have uncertainty enter the budget set (via income or price uncertainty), so that the constraint set for time zero decisions depends on the realization of the date zero lottery, as long as it does so continuously.

In the previous section, the conditions of Theorem 5.1 were stated assuming $q_0$ fixed. Alternatively, it could be assumed that the individual chooses $(C_0, C_1)$ after observing the outcome of $(q_0, q_1)$. I now uncouple these. Continue to assume preferences representable by the expectation of the continuous vN-M function $V: Q_0 \times Q_1 \times B \to E$, just as in Section V. Here, however, after observing $q_0$, the agent chooses $C_0$ to maximize

$$\int_{Q_1(\Omega)} V(q_0, q_1, C_0, C_1^*)\, d\,\mu_1.$$

The following states standard properties of value functions.

Lemma 6.2: $V^* : Q_0 \times D_1 \to E$ defined by

$$V^*(q_0, \mu_1) = \sup_{C_0 \in B(\cdot)} \int_{Q_1(\Omega)} V(q_0, q_1, C_0, C_1^*)\, d\,\mu_1$$

is continuous, the supremum is attained, and $C^*: Q_0 \times D_1 \to B$ is continuous.

Proof: The proof is a fairly tedious restatement of results from the dynamic programming literature (see Kreps and Porteus (1979a)) and not reproduced here.

Induced preference can now be edfined on $D_0$ by

$$\mu_0 \; R_0 \; \mu_1' \text{ if } \int_{Q_1(\Omega)} V^*(q_0,\mu_1) \, d\mu_0 > \int_{Q_1(\Omega)} > V^* \, (q_0,\mu_1') \, d\,\mu_0.$$

Lemma 6.3: $R_0$ is asymmetric, negatively transitive, continuous, and satisfies the substitution axiom for $t = 0$.

Proof: Kreps and Porteus (1979) Proposition 2.

Thus, induced preference satisfies axioms 6.1 to 6.3, and by Theorem 6.1, induced preference is temporal vN-M if axiom 6.4 holds, i.e., if the substitution axiom holds for $t = 1$. The following results follow from Kreps and Porteus (1979).

Theorem 6.3: Induced preference is <u>atemporal</u> vN- if and only if, for all $\mu_1$ and $\mu_1'$, $C_0^*(\mu_1) = C_1^*(\mu_i')$.

Proof: By Kreps and Porteus (1979) Lemma 1, the $C^* : Q_0 \times D_1 \to B$ given by

$$C^*(q_0,\mu_1) = \underset{C_0 \,\varepsilon\, B(\cdot)}{\arg\max} \; V(q_0,q_1,W_0,W_1,C_0,C_1^*) d\mu_1,$$

is an upper-semicontinuous correspondence. By Proposition 2 and Corollary 1, induced preference is atemporal vN-M if and only if $C^*(q_0,\mu_1) \cap C^*(q_0,\mu_1') = 0$. Theorem 6.3 follows from this result and the fact that $C^*(q_0, \mu_1)$ is singleton-valued under the assumption of that upper and lower contour sets on $D_0$ under R are strictly convex sets.

Theorem 6.4: Induced preference is temporal vN-M if and only if

(i) $(q_0,\mu_1) \; I \; (q_0,\mu_1')$ implies $C^*(q_0,\mu_1) = C^*(q_0, \mu_1')$

(ii) $(q_0,\mu_1) \; R \; (q_0,\mu_1')$ implies $(q_0, \alpha\mu_1 + (1 - \alpha)\mu_1') \; R \; (q_0,\mu_1)$ for all $\alpha \; \varepsilon \; (0,1)$.

Proof: Kreps and Porteus (1979), Proposition 4 provide a statement for non-singleton C*. The result is immediate.

These results are quite strong and not easily checked. Sufficient conditions take the form of a restriction on the form of the utility function. The following result generalizes one in Kreps and Porteus (1979).

Theorem 6.5: Suppose that

$V(q_o, q_1, w_o, w, c_o, c_1^*) = \phi_1(q_o, c_o) + \phi_2(q_o, c_o)\phi_3(q_o, q_1, c_1^*)$, let $U_1(q_o, q_1, c_o^*) \equiv \phi_3$ and let $u_o(q_o, \beta)$

$$\equiv \max_{c_o \varepsilon B} \phi_1 + \phi_2(\cdot)\beta \text{ for } \beta \varepsilon \tau(q_o), \text{ where}$$

$$\tau(q_o) = \{\beta \varepsilon E: \ \beta = \int_{Q_1(\Omega)} \phi_3(\cdot)d\mu_1 \text{ for } \mu_1 \varepsilon D_1\}.$$

Then if $\mu_o$ is strictly increasing in $\beta$, induced preference is temporal vN-M with $U_1$ and $\mu_o$ representing induced preference.

Proof: It suffices to verify the substitution axiom for t =1; the result then follows from Theorem 6.1. This is obvious from the fact that V is linear and increasing in $\beta$ and $\beta$ is linear in $\mu_1$. By hypothesis,

$$\max(\phi_1 + \phi_2\, \beta(\mu_1)) - \max\, (\phi_1 + \phi_2\beta(\mu_i))>0. \text{ But,}$$

$$\max\, [\phi_1+\phi_2\beta(\alpha\mu_1+(1-\alpha)\mu_i''))]-[\max(\phi_1+\phi_2\beta(\mu_1'+(1-\alpha)\mu_1'')]$$

$$= \max(\phi_1 + \phi_2\, \alpha\beta(\mu_1) + \phi_2(1-\alpha)\beta(\mu_1'')$$

$$- \max(\phi_1 + \phi_2\, \alpha\beta(\mu_1') - \phi_2(1 - \alpha\beta(\mu_1'')$$

$$= \max(\phi_1 + \phi_2\, \alpha\beta(\mu_1)) - \max(\phi_1 + \phi_2\, \alpha\beta(\mu_1') > 0.$$

While this condition is straightforward, it is restrictive. Kreps and Porteus (1979) develop an approximation to induced preference which is temporal vN-M, but do not claim that theirs is a "best" approximation in any sense. Machina (1982, 1984) makes use of "generalized expected utility theory," which does make use of a best approximation under the assumption that induced preferences are Frechet differential.

Before embarking on this approximation procedure, let us summarize what the issues are. The agent is assumed to have a vN-M utility function defined on $(q_0, C_0, q_1, C_1)$. When $C_1$ is chosen, everything else is known. Maximizing out $C_1$ provides the function $V(q_0, q_1, C_0)$. Given some $q_0$, the distribution on $q_1$ is known, based on the function f. First period consumption $C_0$ is chosen after $q_0$ is observed, but before $q_1$ is. Thus, one can use $C_0^*(q_0, F_1(q_1 \mid q_0))$ as this optimal choice and define

$$\hat{V}(q_0) = \int V(q_0, q_1, C^*(q_0, F_1(q_1 \mid q_0)))dF_1(q_1 \mid q_0).$$

Overall rankings of temporal lotteries $F_0$ on $Q_0 \times \hat{D}_1$ are made on the basis of $J(F_0) = \int \hat{V}(q_0)\, dF^0(q_0)$.

Now, it is clear that preferences on temporal lotteries are linear in the probabilities given by $F_0$. However, the induced preferences on $F_1$ are not linear in the probabilities; Kreps and Porteus show that they are convex. Machina's (1982, 1984) insight uses intuition from ordinary calculus: a differential of a non-linear function is the best linear approximation to that function at that point. Thus, the best linear approximation to the non-linear preference functional is provided by differentiation provided it is smooth. The appropriate concept of differentiation here is Frechet differentiation.

I begin the application of Machina's analysis to the option value problem by converting the above analysis to the use of distribution functions. For each $\mu_j^i \in D_j$ there is a unique distribution function $F_j^i$ in the space $\hat{D}_j$ of distribution function on $Q(\Omega)$. Endow the space $\hat{D}_j$ with the weak topology, as with the space $D_j$. Machina uses the notion of the Frechet derivative of the value functional. This requires that one define a norm on the space

$$\Delta\hat{D}_j = \{\lambda(F^* - F) \mid F, F^* \in \hat{D}_j, \lambda \in E\}.$$

Then the following result holds.

Lemma 6.3: The topology of weak convergence on $\hat{D}_j$ is induced by the $L^1$ metric

$$d(F,F^*) = \int \left| F^*(q) - F(q) \right| \; dq.$$

Proof: Machina (1982), Lemma 1.

The norm on $\Delta\hat{D}_j$ is then $\|\lambda(F^* - F)\| \equiv |\lambda| \, d\,(F,F^*)$. One can now discuss the differentiability of the induced preference functional

$$J(F_1) = \int_{Q_1(\Omega \mid q_o)} V(q_o,\ q_1,\ \overset{*}{c}_o(q_o, F_1(q_1 \mid q_o))\ dF_1(q_1 \mid q_o)$$

Assume that $J(F_1)$ is once Frechet differentiable. That is, assume that there exists a continuous linear functional $\psi\,(\cdot, F_1)$ on $\Delta\hat{D}_1$ such that

$$\lim_{\|F^* - F\| \to 0} \quad \frac{\left| J(\overset{*}{F_1}) - J(F_1) - \psi(\overset{*}{F_1} - F_1;\ F_1) \right|}{\|F^* - F\|} = 0$$

Machina (1982) shows that existence of $\psi(\cdot, \cdot)$ implies the existence of a local utility function $\theta(\cdot,\ F_1^o)$ such that for any $F_1^o$ and $F_1^1 \in \hat{D}_1$

$$J(F_1^1) - J(F_1^o) = \int \theta(q_1;\ F_1^o)\ [dF_1^1(g_1 \mid q_o) - dF_1^o(q_1 \mid q_o)] + O(\|F_1^1 - F_1^o\|).$$

where $O(\cdot)$ is a function of higher order than its argument. Thus, the difference in preference the functional consists of a linear term plus a higher order term. The linear term is the difference in the expectation of $\theta(\cdot)$ with respect to the two distributions. Thus, the induced preference ordering takes on a local expected utility representation.

These local utility functions can be used to analyze "large" changes in distributions by use of a path integral. Let $\beta \in [0,1]$; we define the path $\{F(\cdot\,;\beta) \mid \beta \in [0,1]\}$ from $F^o$ to $F^\nu$ by $F(\cdot\,;\beta) = \beta F^\nu + (1-\beta)F^o$. Then $V(F^\nu) - V(F^o)$ is given by the integral of $dV(\cdot\,;\beta) \mid d\beta$ as $\beta$ runs from 0 to 1.

Machina (1982; 1984) provides a number of useful results regarding the local utility functions $\theta(\cdot,\cdot)$ and the overall rankings of temporal prospects based on the funcation $J(\cdot)$. For example if all local utility functions are concave, then overall choices will exhibit risk aversion. Thus, one would expect results that rely solely on risk aversion to carry over to the generalized case. Unfortunately, this is not so for Bishop's proof of the non-negativity of supply-side option value. The reason is familiar: establishing the sign of option value for supply-side uncertainty requires a singly utility function. Here, the utility function corresponding to $F^o$ is different than the utility function corresponding to $F^\nu$ if $F^o$ and $F^\nu$ are sufficiently different. Thus, for projects which significantly will affect environmental quality, the assumption of one utility function cannot be used when there is temporal risk. Formally, I state this as

> Theorem 6.6: Under temporal ecological risk, the sign of supply-side option value is indeterminate, if $F^o$ and $F^\nu$ differ "significantly."

The main result of this section, Theorem 6.6, is a negative one. The sign of supply-side option value is indeterminate when risk is temporal under conditions that allow its determination when risk is timeless. However, Machina (1984) derives a numberof useful results concerning monotonicity and concavity of the induced utility function $V(q_o,q_1\ C^*(\cdot))$ and distributions that are ordered by stochastic dominance or differ by increases in risk. I will not repeat these here; the results generally are not surprising given that most propositions in the timeless setting relying on risk aversion carry over to the temporal setting if all of the local utility functions exhibit risk aversion. While many of Machina's results could rule out from consideration certain projects in $\Delta$, it is apparent that a total ordering on $\Delta$ generally would not be forthcoming based on these results. For example, if a project induces a distribution which differs by a mean preserving increase in risk from the distribution induced by $v^o$, then $\hat{\nu}\ P^*\ \nu^o$ never would hold if individual utility functions are concave in $q_1$. But certainly most projects of interest will give rise to changes in mean as well as increases or decreases in risk.

Of course, this does not mean that welfare evaluations cannot proceed when individual's face temporal risk. As with the static option price, one knows what one wishes to measure and one has techniques available, i.e., contingent valuation methods, to obtain it. The relevant measure is EOP defined by

$$J(F_0^{\nu}(q), w) = J(F_0^{0}(q), w-EOP(F_0^{\nu}, F_0^{0}, w)),$$

where $J(F_0^{\nu}, \cdot)$ is defined as above an alternative temporal lotteries, where $F_0^{\nu}$ is the temporal lottery induced by project $\nu \varepsilon \Delta$ and $0 \varepsilon \Delta$ is the "project" which is defined by the status-quo. What one is unable to obtain in this framework is the sign of option value. This seems to be an elusive quest.

## 7. Project Choice Under Uncertainty

As in the case of certainty, it is up to the central planner to select a project from $\Delta$, based on individual willingness-to-pay for them. Three issues arise here. First, suppose that there is no planning uncertainty. That is, the planner is able to obtain EOP $(F^o, F^\nu, w)$ for each individual and for each $\nu \varepsilon \Delta$. The analysis proceeds exactly as in Section 3; based on the weights $b_i$ of the social welfare function, the planner selects $\nu \varepsilon \Delta$ such that the weighted EOP is maximal, after incorporating a feasible financing scheme for the project.

The second question that arises concerns the possibility that the planner's preferences can be formulated over projects such that the planner's preferences satisfy the von Neumann-Morganstern axioms. Clearly this will only be the case if individual utilities satisfy these axioms. Thus, in this section I consider a static model. The answer to this question, based on Wilson's (1968) analysis of the theory of syndicates, demonstrates the appeal of the linear welfare function. This is undertaken below.

The third question concerns the assumption, maintained throughout the paper so far, that uncertainty is exogenous. As Bishop (1982) points out, there is a connection between supply-side option value and the literature on quasi-option value (Arrow and Fisher, 1974), in which learning may take place.

Regarding the question of project selection, I now incorporate into the risky choice problem the financing decision, and determine a relationship between group and individual payoffs as functions of the project and outcomes of the random event.

Suppressing dependence of a previous quality, if project $\nu \varepsilon \Delta$ is implemented and event $\omega \varepsilon \Omega$ obtains, realized environment quality is $f(\nu,\omega)$.

Individual i's assumed von Newmann-Morganstern utility function is $V^i(q, \omega) = V^i(f(\nu,\omega)w)$ and equivalent variation is defined by

$$V^i(f(0,\omega)w^i - ev^i(\nu,\omega)) = V^i(f(\nu,\omega),w).$$

As in Section 3, under financing scheme $s(\nu) \varepsilon S(\nu)$, i pays $s^i(\nu)$. The payoff to person i from implementation of project $\nu$ is $m^i(\nu,\omega) \equiv ev^i(\nu,\omega) - s^i(\nu)$.

Since environmental quality is a public good, the group payoff from implementing project $\nu$ is

$$g(\nu,\omega) \equiv \sum_i m^i(\nu,\omega) = \sum_i ev^i(\nu,\omega) - k(\nu).$$

To develop a tie to the linear welfare function of section 3, begin by supposing that the planner seeks to implement a financing scheme that is Pareto efficient.

Denote the expected utility of the ith agent under project $\nu$ by

$$J^i(\nu,s^i(\nu), F^i) \equiv \int_\Omega V^i(f(\nu,\omega), w - s^i(\nu))dF^i(\omega).$$

The standard proofs of the following lemmata are omitted.

Lemma 7.1: The set $\tau(\nu)$ defined by $\tau(\nu) = \{J^i(\nu,s^i,F^i): \; s^i \varepsilon S(\nu)\}$ is convex.

Lemma 7.2: If $s(\nu)$ is Pareto efficient then there is a set of weights $\{b^i(\nu),\ i = 1, \ldots, I\}$ with $b^i(\nu) \geq 0$ such that $s(\nu)$ solves

$$\max_{s(\nu)\varepsilon S(\nu)} \sum_i b^i(\nu) J(\nu,s^i(\nu),F^i(\omega)).$$

The following result is stated by Wilson (1968).

Theorem 7.1: $s(\nu)$ is Pareto efficient if and only if there exist non-negative weights $\{b^i(\nu)\}$ and a function $\lambda(\nu,w)$ such that

(i) $s(\nu) \varepsilon S(\nu)$

(ii) $b^i \nu \frac{\partial V}{\partial w} (\cdot) h^i(w) = \lambda(\nu,w) \quad \lambda = 1, \ldots, 1$

for almost all $w \varepsilon \Omega$ for which $b^i(\nu)h^i(\omega) > 0$, where $h^i = F^{i'}(\cdot)$, i.e., $h^i$ is the density corresponding to i's subject probability measure on $\omega$.

"Proof": By Lemma 7.2 the planner wishes to solve a constrained minimization problem, with weights defined by the tangent hyperplane to $\tau(\nu)$. This hyperplane exists by Lemma 7.1. The function $\lambda(\nu,\omega)$ can be thought of as the Lagrange multiplier in the constrained maximization problem, where the constraint is given by (i). Thus, $s(\nu)$ and $\lambda(\nu,\omega)$ can be found as by finding (pointwise) a paddle-point of the Lagrangean, i.e., by solving

$$\sup_{s} \inf_{\lambda} L(b^i, J^i, h^i, k)$$

where

$$L(\cdot) = \int \{\sum_i b^i(\nu)V(f(\nu,\omega),w-s^i(\nu))h^i(\omega) - s^i(\nu)\lambda(\nu,\omega)\}$$

This theorem concerns the choice of a Pareto efficient financing scheme. The central question of this analysis concerns the overall problem faced by the planner, which includes the choice of a feasible project. The central question is whether there exists some overall utility function such that, in choosing a Pareto efficient project, the planner will maximize the expectation of this function. The answer to this question is stated in the next proposition.

Theorem 7.2: There exists a group utility function $V^o(q,w)$ such that the choice of a Pareto efficient project involves solving

$$\max_{\nu \varepsilon \Delta} \int V^o(f(\nu,\omega),w)d\omega.$$

if $b^i(\nu)$ are independent of $\nu$.

Proof: Given Theorem 7.1, the overall problem is to solve

$$\sup_{\nu\varepsilon\Delta} \int \inf_{\lambda} \{\lambda k + \sum_i \sup_{s\varepsilon S} [b^i V^i h^i - \lambda s^i]\} d\omega.$$

Define the "rent" measure

$$\psi^i(d_i) \equiv \sup_x [V^i(q,x) - d_i x].$$

Then the above problem can be simplified to read

$$\sup_{\nu\varepsilon\Delta} \int \inf_{\lambda} \{\sum_i [b^i h^i \psi^i(\frac{\lambda}{b^i h^i})] - \lambda k\} d\omega.$$

Define

$$V^o(f(\nu,\omega),w,\nu) \equiv \inf_{\lambda} \{\sum_i [b^i(\nu)h^i \psi^i(\frac{\lambda}{b^i h^i}) - \lambda k\}.$$

Then the preferred project solves

$$\sup_{\nu\varepsilon\Delta} \int V^o(f(\nu,\omega),w,\nu)d\omega.$$

This $V^o$ will depend on $\nu$ only through the transition equation on environmental quality if the weights $b^i(\nu)$ are independent of $\nu$.

The theory of syndicates, applied here to the analysis of provision of a public good, concerns the relationship between individual preference representations and group preference representations. The key result is that if the social welfare function is linear (as in Section 3), then there is a "utility function" for the planner such that choice of efficient projects amounts to maximization of the expected value of this function.

It is important to note that the only source of uncertainty in the model is ecological uncertainty. There is no planning uncertainty (in the language of Section 4) since the planner is assumed to know the individual vN-M utility functions and the individual probability density functions. With planning uncertainty, the planner does not know these individual preferences.

The case of pure planning uncertainty raises a number of interesting problems of analysis. The first concerns the form of the planner's objective function. Anderson (1979) has proposed that planner's preferences in this situation be assumed to take an expected utility form. This approach might be considered to be controversial. Second, since uncertainty gives rise to possibilities for learning, there is a possibility that the planner can devise a mechanism to discover the true preferences of individuals. This issue is the topic of the large literature on incentives. That is, can a principle (in this case the planner) design an incentive scheme which induces an agent (individuals in society who care about the project) to act in accord wtih the principle's goals (reveal their preferences for a public good). The theory of incentives has been reviewed recently by Laffont and Maskin (1982). They study particularly simple forms of individual utility functions (quasi-linear) and planner choice rules which are similar to those posited here where the individual "weights" are the same for all individuals. While it appears that the literature abounds with impossibility theorems, these are often seeking incentive schemes with quite strong properties. It would seem possible for the planner to learn something of individual preferences which will be of use.

The third issue is that raised by the literature on quasi-option value. Until now, all of the timing of resolution of uncertainty relative to the timing of choices in projects and consumption has been assumed exogenous. The QOV literature seeks to deduce the effect of possibilities for learning on willingness to undertake projects which are irreversible.

In terms of the current model, let $\Delta = \Delta_o \times \Delta_1$, where $\Delta_t = [0,1]$. Suppose that Int $\Delta_t = \phi$ for $t = 0,1$, and that projects are irreversible in that $\nu_o = 1 \Rightarrow \nu_1 = 1$, while $\nu_o = 0$ is consistent with $\nu_1 = 0$ or $\nu_1 = 1$. The QOV literature then compares two decision frameworks. In one framework it is assumed that no new information will become available. Thus, the planner chooses immediately from $\Delta$ one of (0,0), (0,1) or (1,1). In the other decision scheme a sequential decision is possible, i.e., conditional on $\nu_o$, and the outcome of an experiment $y \varepsilon Y$ that provides information, the planner chooses $\nu_1^* (\nu_o,y)$. Clearly, if $\nu_o = 1$, then $\nu_1^* = 1$ irrespective of the outcome of the experiment. However, if $\nu_o = 0$, then $\nu_1^* (0,y)$ is undertaken. Using a backward induction approach, the optimal choise of $\nu_o$ can be determined based on a likelihood function for the experiment. Provided the information service Y has value (increases expected payoff) the central result of the QOV literature is that, if $\nu^* = (1,1)$ is optimal in the non-sequential decision framework, it may be that $\nu_o^* = 0$ is optimal in the sequential decision framework. The difference in expected payoff with $\nu_o = 1$ in the non-sequential and sequential cases is QOV.

This result is intiutively plesing and corresponds to Machina's (1984) observation that an individual never will prefer a temporal prospect to an identically distributed timeless one. In the context of the current model, it appears that merely observing the outcome $q_o$ constitutes learning since the probability distribution on $q_1$ is conditioned on the outcome $q_o$ due to the nature of the transition equation f. Thus, learning here can be passive and involves no cost. Of more interest, since this surely will be recognized by a planner and built into the sequential decision framework, is the possibility of actively learning about which $f \varepsilon \Phi$ is the true ecological process function. An experiment which involves this additional source of learning would be sufficient for passive observation of $q_o$. This would give rise to an additional source of QOV.

Much of the analysis in the QOV literature assumes that Int $\Delta_t = \phi$, as above. Graham-Tomasi (1983) presents a model of pure planning uncertainty for the case $\Delta_t = [0,1]$ in which the concept of "quasi-option tax" (QOT) is presented. Although his model is very different than that considered here and so the details of the analysis are not relevant, QOT is an adjustment to initial development benefits in the learning case that would lead to the same level of initial development as in the non-sequential case. Moreover, QOT is a potentially estimable number, given bythe expected present value of the second period loss if an irreversible decision is implemented at the myopically profitable level, where the loss is averaged over the possible states of nature under which the decision-maker would reverse the decision if he/she could.

8. Discussion

In this paper, I have attempted to explore the foundations of supply-side option value and project appraisal under uncertainty. The key result is the following: when temporal risk is present, the analysis of option prices and option values significantly is complicated. Since almost all situations discussed in the option value literature involve temporal risk, the analyses of this literature seriously are called into question. However, this is not really a significant insight since most of the analyses of option value have a negative result: option value is not determinate in sign. The key insight for the analytical option value literature is the following: existing studies in which positive results have been obtained, e.g., Bishop's (1982) result on supply-side option value and our own Theorem 5.6 in the same area, no not hold in an obvious way under temporal risk. As well, Freeman's (1984) and Smith's (1984) bounds on option value would need to be reexamined under temporal risk using an extension of Machina's (1984) generalized expected utility analysis to the case of state dependent preferences. An alternative is the use of the restriction of Kreps and Porteus (1979) to obtain temporal von Neumann-Morgenstern utility representations. The use of atemporal vN-M representations undoubtedly is too strong.

Another alternative to all of these machinations is to explicitly model the intervening choices, as in Dreze and Modigliani (1972). This is the approach taken in the QOV literature. While a complete analysis along these lines is likely to result in too much deatil so that analytical tractability is lost, for some decisions (or under separability assumptions) this may prove useful.

Regarding empirical studies, it is clear that the use of contingent valuation techniques to measure option price holds the key to correct project appraisal under uncertainty. It may turn out that empirical regularities exist. My own feeling is that this will not be the case, and such an approach is similar to the search for a single discount rate for use in the analysis of public projects. It is likely that decisions will differ sufficiently that regularities will not exist.

Regarding the conduct of these empirical studies to determine option prices, two important points emerge. When setting the context of the questions in the survey, it is curcial that respondents understand the temporal aspects of the choices being made. Inadequate attention has been given to this issue in existing studies. Can individuals change their minds? Will a reassessment be made as learning takes place? Need payments be equal annual payments, or can WTP lump-sum payments be allocated through time in any fashion?

A second point concerns the existence of local utility functions. The utility functions depend on initial probabilities and on all probabilities in a global analysis. This may prove to be important in teh assessment procedure, particularly regarding specification bias in regressions explaining willingness-to-pay.

While the overall results of this paper seem quite negative, this is not the acutal intent of the analysis. Rather, it is to suggest that much work remains to be done. But, this is not surprising given the difficulty of analyses involving both time and risk.

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