REAL OPTIONS AND AGRICULTURAL LAND VALUES

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Real Options and Agricultural Land Values

Researchers have been unable to show a time series correspondence between cashflow from agriculture and land values. Featherstone and Baker used vector autoregressive techniques and found that time series in cash rents, discounted at the prevailing t-bill rate did not explain land prices. Baker and Featherstone (1988) speculated that from time to time land values behave like a speculative bubble. Falk (1991) examined Iowa farmland prices and discovered that while cash rents and land prices did move together, the volatility in price movements are more volatile than the volatility in land prices over the long run. Clark, Fulton and Scott using Indiana data from Featherstone and Baker (1987) reject the hypothesis that cash rents and land prices move together. Hanson and Myers rejected the hypothesis that the relationship between rents and land value was due to time varying discount rates. The hypothesis that government payments affect land prices was examined by Weersink et al. In that study, which used a VAR structure that allowed for endogenously determined time-varying discount rates, it was found that the land market (at least in Ontario) viewed income from production with greater uncertainty than government support payments and thus more heavily discounted the cash flow stream from operations.

Falk and Lee comprehensively investigate the decomposition of shocks as they relate to long run land values. They break land price variability into fundamental and nonfundamental components. Fundamental shocks relate to economic conditions, which appear to be of a permanent nature, but there are also temporary fundamental shocks that give rise to volatility in the short run. These temporary fundamental shocks may be attributable more to fads than the speculative bubbles hypothesized by Featherstone and Baker (1988). The third category of shocks is nonfundamental shocks. This category of shock affects land price but does not affect other market fundamentals such as prices and interest rates. Importantly, Falk finds that temporary fundamental shocks and nonfundamental shocks explain nearly 96% of the year-to-year fluctuation in land prices while permanent fundamental shocks explain only 4%. In the longer run, markets rationally adjust to these shocks such that their contribution falls to less than 25% of the variability in long-run land prices. Falk and Lee conclude that while the land capitalization model appears to correlate with economic information in the long run, it appears to overreact in the shortrun and in a manner inconsistent with the land capitalization model.

The studies referenced above use statistical models to test for the validity of the capitalization model. The notion that land prices are not correlated with fundamental economic information at least in the short-run gives rise to the possibility that an alternative model that challenges the present value model needs to be considered.

There is an emerging literature called real options theory which blends conventional corporate finance and capital budgeting with the theory of rational options pricing to explain behaviour which justifies economically the acceptance of projects which have a negative net present value or the rejection of projects with a positive net present value. This new theory on management, as described in theory by Dixit and Pindyck and practice by Amran and Kulatilaka, uses the argument that capital investments are like a call option on the future flow of risky cash flows. This call option provides every decision-maker with the right but not the obligation to accept a capital investment at some future date. The action taken today is contingent more on the timing at which the intrinsic value of the option comes into the money and the risks of the project than the discount rate or sequential cash flows normally considered in capital budgeting. Indeed because the call option provides the right but not the obligation to undertake a project at a future date it implies that a decision does not have to be made immediately. If the real option has a positive value then it may be rational to forgo the opportunity today and postpone the decision until more information becomes available and uncertainty or ambiguity about project flows diminishes.

The purpose of this paper is to investigate theoretical conditions under which farmland prices would deviate from the conventional capitalization model. In this paper I review the classical capitalization model and then develop the real options valuation formula and illustrate its use in understanding farmland prices.

This paper departs from the conventional approach to real options valuation. Dixit and Pindyck and Amran and Kulatilaka, for example, focus on capital investments for which the option is owned entirely by the purchaser. The importance of this paper is that it posits a new theory whereby the buyer has a want for the asset but the real option value really rests with the seller. This theory is the result of equilibrium conditions that force the value of land to equal its zero NPV bid price at any moment in time. In this instance the buyer can postpone a purchase in the hopes that commodity prices or cashflow will rise but when the time comes the bid price will have increased. Consequently there is no advantage for a buyer to postpone a purchase, but the seller can postpone the sale in the hopes that bid price increasing economic conditions will arise from the uncertainties in the market place. Referring to a game introduced by Zhao and Kling the natural outcome would be for the buyer to compensate the seller, in other words buy a part of the real option value from the seller, while the seller can remove the uncertainty of waiting by selling the land for the bid price plus a portion of the real option value. Importantly this theory admits that equilibrium land prices will not necessarily be consistent with the capitalization model at each moment in time. There can be departures and as will be shown the departures should increase with increasing risk, increasing returns, or both.

Land Bid prices and the Net Present Value Rule

The conventional approach to the pricing of farmland is to estimate the future cash flows from revenue generating activities, synergistic benefits, capital acquisition, and salvage values etc. The present value of these cash flows dictates the maximum bid price for a tract of land. The seller will attempt to extract more than this bid price and the buyer will attempt to pay less and the closing price will often lie somewhere in between. Conventional bid price models include taking the present value of a perpetual stream of operating cashflows; or unequal cashflows such as the model described in Ellinger, Barry, Baker, and Salint; a perpetual model with growth taxes and an infinite number of homogenous buyers and sellers as discussed in Baker, Ketchabaw and Turvey.

Under the NPV rule the economics is straightforward. If the selling price is less than the bid price then the land purchase will have a positive net present value and wealth will have been created. In contrast if the sale price is in excess of the bid price then purchasing the land will result in a negative NPV and wealth will have been lost. Since a buyer would not purchase land at a price greater than its present value, and an equally aware seller would not sell for less than the present value of future cashflow foregone, then the conventional wisdom is that bid-ask prices would be equal at the present value and the transaction would take place at that point. In other words, land is priced to extract all future wealth from the buyer and this is to the benefit of the seller (at least when the selling price exceeds the seller's original purchase price).

To illustrate how options values can emerge from the NPV framework, consider the following land capitalization model,

(1) $$v = \frac{\pi(1+g)}{i-g}$$

where A is current period cashflow from the land, i is the risk-adjusted discount rate, and g is the growth rate. This equation, bounded to the positive domain, is exponential in growth with the limit of v approaching infinite as g approaches the discount rate i.. In addition, buyers and sellers can do no more than estimate what future values of growth and cashflow should be. These would probably be based upon recent patterns and perhaps some economic forecasts, but most surely under conditions of risk they will not be constant. The more important variable is growth since this provides the intertemporal dynamic to the observable static cashflow estimate Π. To gain insight into how farmland investments can be better managed under uncertainty it is useful to partition equation (1) in terms of its static component, Π/i which we will refer to as the Marshallian bid price, from the growth component.

(2) $$v = \frac{\pi}{i} + \frac{\pi\, g(l+i)}{i(i-g)}$$

The second term is the growth component and it is this component which econometric models have found difficult to measure. The problem is illustrated in Figure 1 which illustrates the equation (1) bid prices assuming π=$100/acre per year and for discount rates from 5% to 15% and growth rates from 0% (the Marshallian bid price) to 10%. The vertical axis has farmland values ranging from $666 to over $10,000 depending on what discount rate and growth rate combination is assumed. If π and i are known with reasonable, the stochastic component to land prices will be determined by the expected value and variance of g. Consequently, the range of land values on the vertical axis represents feasible points within the domain of the probability distribution describing land values

Consider the curve for i=10%. Suppose that the buyer observing a 5% growth in cashflows in recent years bids $2,100 for the land. If in time actual growth drops to 0%, the land value drops to $1,000 and if actual growth turns out to be 7% the land value increases to $3,567. In one instance the farmer has suffered significant erosion of equity and in the other a significant gain. The problem now becomes one of not only identifying the risks associated with farmland ownership but also in understanding and managing these risks.

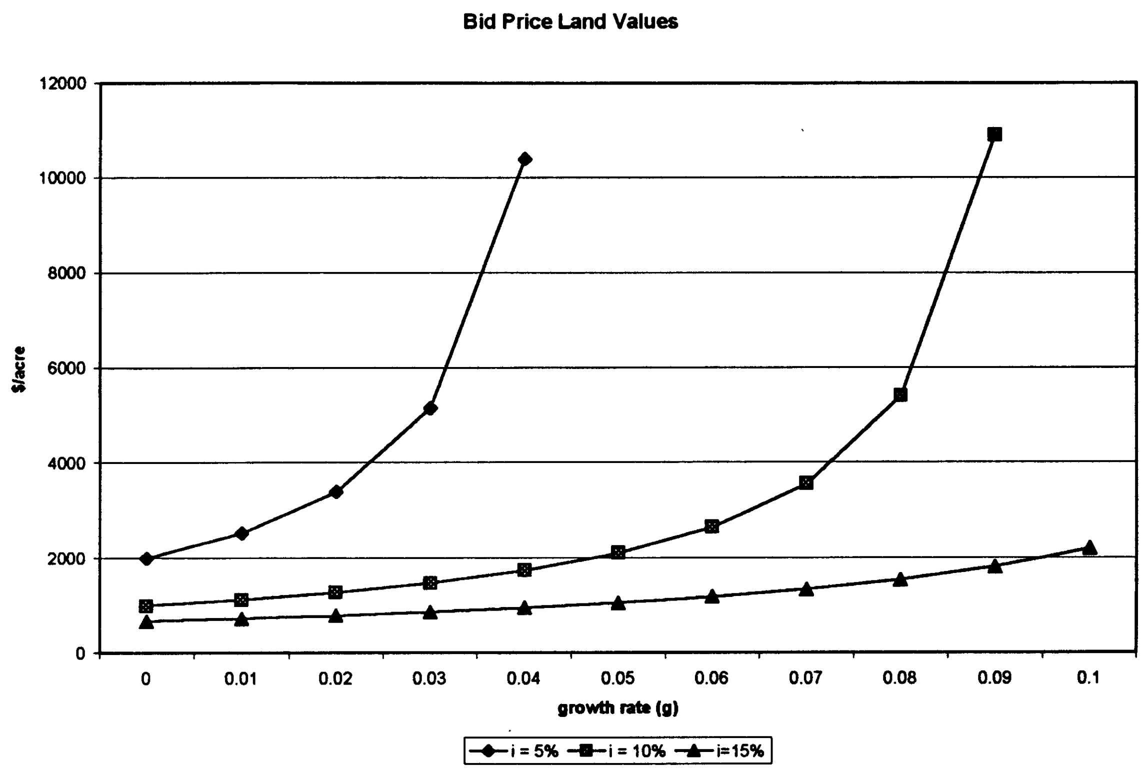


Figure 1: Bid Price Land Values for Different Discount and Growth Rates and Π=$100/acre/year

The Real Option Value of Farmland

In a real options framework the value of land will equal its fundamental bid price value plus a call option on future land price increases resulting from increases in cashflow.

(3) $F(\pi, t) = V(\pi,r,t) + c(\pi,t)$

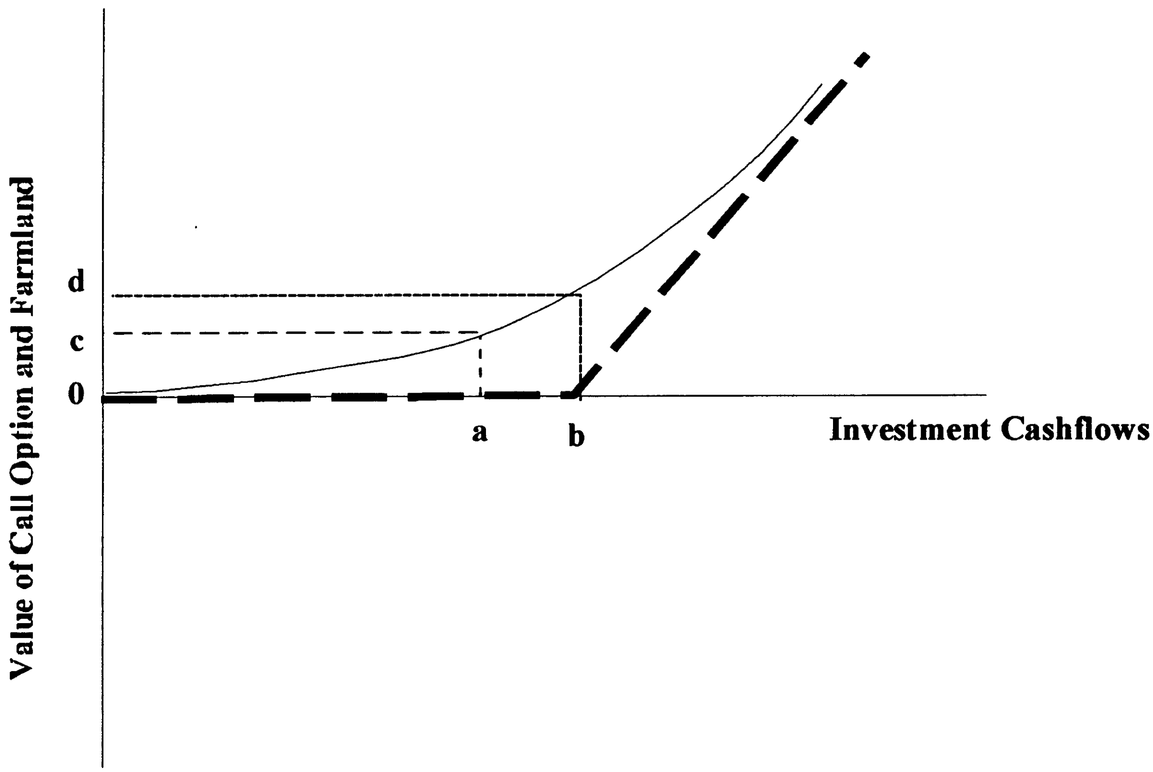
Where F() is the economic value, V() is the bid price as determined by conventional present value rules (e.g. equation 1) , and c() is the real call option. Assume for now that V() once determined remains constant (i.e. I = V()) for the buyer and the seller over the life of the option. (This assumption will be relaxed later on and is used now only for exposition). If F() – V() = 0 then the bid-ask contract price results in a zero NPV for the buyer. In this case c()=0 and the timing to sell is optimal for the vendor; if F() – V() = c() then the land price was sold at a price higher than the present value of the cash flows and the vendor successfully extracted the value of this option from the buyer; the buyer has paid a fair price for the land but has also purchased the call option which provides the right to sell the land at a future date if prices improve. If F() – V()

< c() then the vendor has not fully captured the option value and has likely sold too soon, whereas the buyer has purchased the land for the fair price of discounted cash flows plus the call option at a discount. Finally if F() – V() > c() then the vendor has been able to extract more than the fundamental value of future cash flows and the option value, whereas the buyer has purchased too soon and could have benefited from postponing the purchase if and until the implicit call option decreased in value. Figure 2 illustrates the value of farmland in a real options framework.

In Figure 2 the vertical axis represents the net present value of cashflows from the investment, and the vertical axis represents the value of the investment. Under traditional bid price modeling the rule is to purchase the land as long as the present value of cash flow exceeds the price of the land. Otherwise don't invest. In Figure 2 the present value bid price would be at point b. If future cashflows exceed this then there is a positive net present value and the thick hatched payoff line represents this to the right of 'b'. If the present value of cashflows is less than the asking price, say at point a, then there would be a negative NPV and the horizontal investment would not be made. Hence the thick hatched line to the left of point b.

The curved line in Figure 2 represents the option value. In the Dixit-Pindyck framework, at point 'b' there is a value to postponing the investment until the present value of cashflows improves substantially. The rationale behind waiting is that because cashflows are governed by some underlying stochastic dynamic process (which will be discussed further below) there is a positive probability that cashflow will increase and the NPV of the investment will rise, but there is also a probability that the cashflows will decrease and the NPV will decline. The option to waiting provides the right but not the obligation to buy the land at a price equal to 'b' at some future date. The positive value to waiting does not occur at point 'b' alone. At point 'a' there is also a positive probability that cashflows will increase sufficiently to convert a negative NPV project into a positive one at some future date, but because the probability of this is more remote the call option value is lower. In either case the land purchase is viewed as value maximizing because the right to make an irreversible investment is delayed from the present to the future, and because there is no obligation associated with this right there is additional value in flexibility.

The Real Option Value of Farmland



If markets are efficient however, and the vendor is not myopic then $V(\pi_t) = I_t$ at each instance in time: That is $dI_t = dV(\pi_t)$ with the cost of the investment changing dynamically. As cash profits change, so too will the real option values of postponing the decision. Indeed, as will be shown later the real option price will increase or decrease as π increases or decreases. In either circumstance there is no incentive to postpone the investment, since no matter when and at what π, the NPV will always be zero.

However, the vendor has every incentive to postpone selling the property. To the vendor, the initial investment is already sunk so the objective to maximize capital gains would be to postpone the decision to sell if Prob $(\pi_t > \pi_o) > 0$.

In a real options framework, uncertainty causes behaviours that are inconsistent with traditional NPV rules. In the traditional model any asset will be sold if $I \geq V(\pi)$ and purchased if $I \leq V(\pi)$. The common solution which describes equilibrium is to transact at $I=V(\pi)$. However, uncertainty causes a wedge to form between the vendor and the purchaser. In order for a transaction to take place the purchaser must compensate the vendor for exercising the option to postpone the sale before it is optimal to do so.

Suppose that the option to postpone the sale is equal to $F(\pi)>0$. $F(\pi)$ represents the expected increase in capital gain due to rising π. In order to induce a sale today the price to the purchaser would have to be $I+\delta F(\pi) > V(\pi)$. That is since $F(\pi)$ reflects an uncertain future outcome the vendor may be willing to give up $(1-\delta)\ F(\pi)$ of this expected option value to eliminate the risk of π falling, while the purchaser would be willing to pay $\delta F(\pi)$ of the real option value today in order to receive possibility of a $(1-\delta)\ F(\pi)$ gain in the future.

In such a game both vendor and purchaser are better off since the expected NPV will be positive to the purchaser, while some portion of future capital gains is provided to the vendor. A model for pricing these options is developed in the next section.

Market Prices, Bid Prices and Real Options Values

In this section we develop the real options pricing model along the lines of Pindyck and Dixit. The procedure assumes that farmland values evolve over time according to geometric Brownian motion. There are two objectives to providing the derivation. First, it is hypothesized that farmland transactions implicitly include the call option value of future growth opportunities and this is why econometric models often fail to support the land capitalization model. Second, by deriving the call option value a new opportunity for investment presents itself. Instead of purchasing the land immediately, the buyer can simply purchase the option itself. By purchasing the option the vendor agrees to sell the farmland at a specified price in the future. The buyer owns this right but is not obliged to exercise it.

The stochastic process is an Ito process which when solved with certain boundary conditions provides what Dixit refers to as the smooth pasting condition. The smooth pasting condition is critical because it shows that the optimal time to invest is when the value of the real option on the land equals the net present value of the land, and the marginal change of the real option equals the marginal change in the value of land. This is referred to as the investment trigger and is shown in Figure 2 as the point at which the current options valuation line becomes tangent to the NPV profile line. This smooth pasting condition is critical because it suggests that the value of the real option when it is exercised is equal to the value of the investment in land.

That is the real option has intrinsic value. The second critical observation of the smooth pasting condition is that sometimes the net present value at which land is transacted is actually less than the optimal trigger. This is important because it provides an economic rationale into why so many econometric studies tend to reject the present value model of land pricing. Could it be that the market accurately reflects the real option value of land and that this option includes the fundamental value of land based upon certainty conditions plus a call option on the value of land based upon its intrinsic value under conditions of uncertainty?

The procedure for pricing a real option value for farmland is developed along the lines of Dixit and Pindyck (Chapter 6). In their chapter they provide a solution based on market arbitrage and contingent claims analysis as well as a solution using dynamic programming. The dynamic programming approach is developed in this paper because it does not require the rather restrictive conditions required by the contingent claims approach. In particular, the contingent claims approach requires the existence of a perfectly correlated asset that spans the volatility of farmland prices.

The dynamic programming approach provides a similar solution to the contingent claims approach except that in the former the natural discount rate is exogenously determined, whereas with spanning the creation of a riskless arbitrage portfolio through continuous delta hedging requires that the appropriate discount rate be the risk free rate.

There are two steps involved in the solution to the real options valuation. The first step is to determine a formula that describes the value of land as a function of the stochastic variables. The value of land is denoted by $V(\pi)$ where π is the stochastic variable that determines value. Once the basic land price formula is determined, the second step determines the real option value. It should be noted that the real option value is denoted in terms of waiting for optimal wealth maximizing conditions to occur. This is markedly different from financial options that are exercised on or before a particular date if a particular condition (e.g. the option is in-the-money) occurs.

It is assumed that the variable Π evolves over time as a random walk and can thus be described by Brownian Motion and the following Ito process:

(4) $d\pi = \alpha\pi dt + \sigma\pi dz$

where α is the expected instantaneous change in π per unit of time (the growth rate), σ is the standard deviation of the percentage change (natural logarithm) in π, and dz is the increment to a standard Wiener process with $dz = \varepsilon_t \sqrt{dt}$, dt is the incremental change in time between successive observations in π, and ε_t is a random error term with an expected value equal to zero and standard deviation equal to 1.

Equation (4) is a very general description of π. In practice π should represent the residual after-tax cash flow for the investment. A rather simple illustrative representation of what π might represent can be described by the following equations where P is price, Y is output and W are (deterministic) costs.

(5) $\pi = PY - W$

(6) $dP = \alpha_p Pdt + \sigma_p P\, dz_p,$

(7) $dY = \alpha_y Ydt + \sigma_y Y\, dz_y$

Applying Ito's lemma to (5) yields

(8) $d\pi(P,Y,W,t) = [PY(\alpha_p + \alpha_y + \sigma_{py})\, dt + P\sigma_p dz_p + Y\sigma_y\, dz_y]T.$

Hence, under stochastic conditions the change in profits is lognormally distributed with a mean of $PY(\alpha_p + \alpha_y + \sigma_{py})$ T and variance $P^2\sigma_p{}^2 + Y^2\sigma_y{}^2 + 2PY\sigma_{py}$.

Equation (8) is a simplified representation of the stochastic events that will affect cash flow and consequently farmland prices[1]. For convenience and simplification in exposition we will assume that equation (4) sufficiently represents the intertemporal risks that impact farmland values, $v(\pi)$; that is we will use $d\pi = \alpha\pi dt + \sigma\pi dz$, but we will keep in mind that the true dynamic is best measured in an empirical sense by equation (8).

The term α in (4) measures the rate at which π is expected to change at each instance in time. In terms of the pricing of capital assets α represents the growth rate. Thus, we can write the equation for farmland pricing in terms of the perpetual growth model.

(9) $$V(\pi) = \frac{\pi}{r - \alpha}$$

Applying Ito's lemma to $v(\pi)$ the instantaneous change in land price is determined by

[1] As a further illustration suppose that input costs w also followed $dw = \alpha w\, wdt + \sigma_w\, Wdz_w$ then the change in cash becomes $d\pi = [PY(\alpha_y + \alpha_p + \sigma_{py}) + \alpha w\, W]dt + P\sigma_p\, dz_p + Y\sigma_y\, dz_y + w\, \sigma_w\, dz_w$ which is lognormally distributed with mean $PY(\alpha_y + \alpha_p + \sigma_{py})T$ and variance $[P^2\sigma_p{}^2 + Y^2\sigma_y{}^2 + w^2\sigma_w{}^2 + 2\,[PY\sigma_{py} + PW\sigma_{pw} + YW\sigma_{yw}]]T$.

(10) $dV(\pi) = \frac{\alpha\pi}{r-\alpha} + \frac{\sigma}{r-\alpha}\pi dz$

which is normally distributed with mean $\frac{\alpha\pi}{r-\alpha}T$ and variance $\left(\frac{\sigma\pi}{r-\alpha}\right)^2 T$.

Given variability in π, farmers face a decision with options like quality. With σ>0 there is a probability that π will increase in the future. With increasing π the present value of cash flow will increase. In the simplest of models suppose that the current economic relationship is given by $I \leq V(\pi)$ so that investing now will provide a positive or non-negative net present value.

The Dynamic Programming Solution to Real Options Valuation

The Dixit-Pindyck solution is based on the notion that the real option value fluctuates with time and risk. Given that V(π) describes the Marhsallian value of land given π, α and r, the option value F(π)is given by

(11) $F(\pi) = E[F(\pi) + d\,F(\pi)]e^{-rdt}$

In (11) the current options price is given by the expected value of the option price plus the change in its value over time. This is then discounted to the present. Applying Ito's lemma to (11), using dπ, and using the fact that (1-rdt) is equivalent to e^{-rdt},

$$F(\pi) = F(\pi) + [1/2\,\sigma^2\pi^2F'' + \alpha\pi F'] - r\,Fdt,$$

or

(12) $\frac{1}{2}\sigma^2\pi^2F'' + \alpha\pi F'' - r\,F = 0.$

Equation (12) is the stochastic differential equation that provides the solution for F(π). To obtain this solution we add three boundary conditions. These are

(13) $F(0) = 0$

(14) $F(\pi^*) = V(\pi^*) - I$

and

(15) $F'(\pi^*) - V'(\pi^*)$.

Condition (13) says that if cash flows are zero the option will be zero. Condition (14) is the value matching condition. It says that at some level of cash flow π^*, the value of the option must equal the present value of the investment. It is equation (14) that gives rise to the notion that the option value to the buyer is zero if land is priced to market. In this instance π^* is observed, and $NPV = -I + V(\pi^*)$. That is, the buyer is always paying the present value of future cash flow and this will always result in a zero NPV. This, in turn, implies that the buyer has no incentive to postpone the decision since the NPV will be zero for any π including π^*. The buyer will only have a positive option value if the market is myopic and $I<V(\pi^*)$. However, to the seller who observes I at t=0, the option to sell is clearly driven by the potential capital gain in moving from $I=V(\pi)$ to $I=V(\pi^*)$.

Condition (15) is the smooth pasting condition. It says that the optimal time to sell occurs for some π^* such that the incremental gain in options value exactly equals the incremental gain in net present value. The smooth pasting condition ensures that at some point the option value will become tangent to the options payoff curve (as in Figure 2).

The solution to (12) is obtained by assuming that

(16) $F(\pi) = A\pi^{\beta}$

where A and β are parameters to be determined. Applying the appropriate calculus to (16) and substituting into (12) yields the quadratic equation.

(17) $\frac{1}{2}\sigma^2 \beta(\beta-1) + \beta\alpha - r = 0$

Solving for positive root of β gives

(18) $$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left((\frac{\alpha}{2} - 1/2)^2 + \frac{2r}{\sigma^2} \right)^{1/2}$$

Solving the smooth pasting condition (15) in terms of A, and substituting A into the value matching condition (14) yields

(19) $$\pi^* = \left[\frac{\beta}{\beta - 1}\right] I\,(r - \propto).$$

Equation (17) gives the optimal level of cash flow (π^*) at which the option should be exercised. Note that π^* is independent of time. The decision rule is to postpone the investment or disinvestment condition until π^* occurs. Finally

(20) $A = \frac{\pi^*}{(r-\alpha)\beta\pi^{*\beta}}$, and

the value of the option can be solved by

(21) $F(\pi) = A\pi^{*\beta}$.

Illustrated Examples of Real Options Valuation

In this section real options values for farmland will be calculated under hypothetical and simulated conditions. The examples will explicitly rely on equations 9 and 18-21. Under each scenario cashflow is represented over a fixed range while growth, discount rates, and volatility are allowed to vary. The key variables of interest are the value of the investment, $F(\pi) = V(\pi) + c(\pi)$ and its components. $V(\pi)$ represents the present value bid price at any π, and $c(\pi)$ the real option value to postponing the investment. $F(\pi)$ therefore represents the true value of the investment if the option is exercised optimally at π^*.

In addition recall that in the context of capital assets $V(\pi) = \pi/(i - \alpha)$ at any observation of π (including π^*) at any moment of time. This means that to the buyer the value matching condition (13) is always equal to zero and it is never optimal to postpone the investment. Setting I equal to the current (t=0) value of $V(\pi)$ the real option is owned by the seller who can optimize wealth, as a first-best solution, by postponing the sale until $\pi=\pi^*$. This means that the owner of the land owns an asset equal to I in fundamental value plus an option with value $c(\pi^*)$, exercisable at π^*.

The Value of Uncertainty

In a real options framework uncertainty creates value because with increasing uncertainty comes an increasing probability that asset values will increase further. As shown below the trigger level of cashflow, π^* will increase with volatility. Note once again that this says nothing about how long the sale will be postponed for but only the conditions under which selling the land will be optimal.

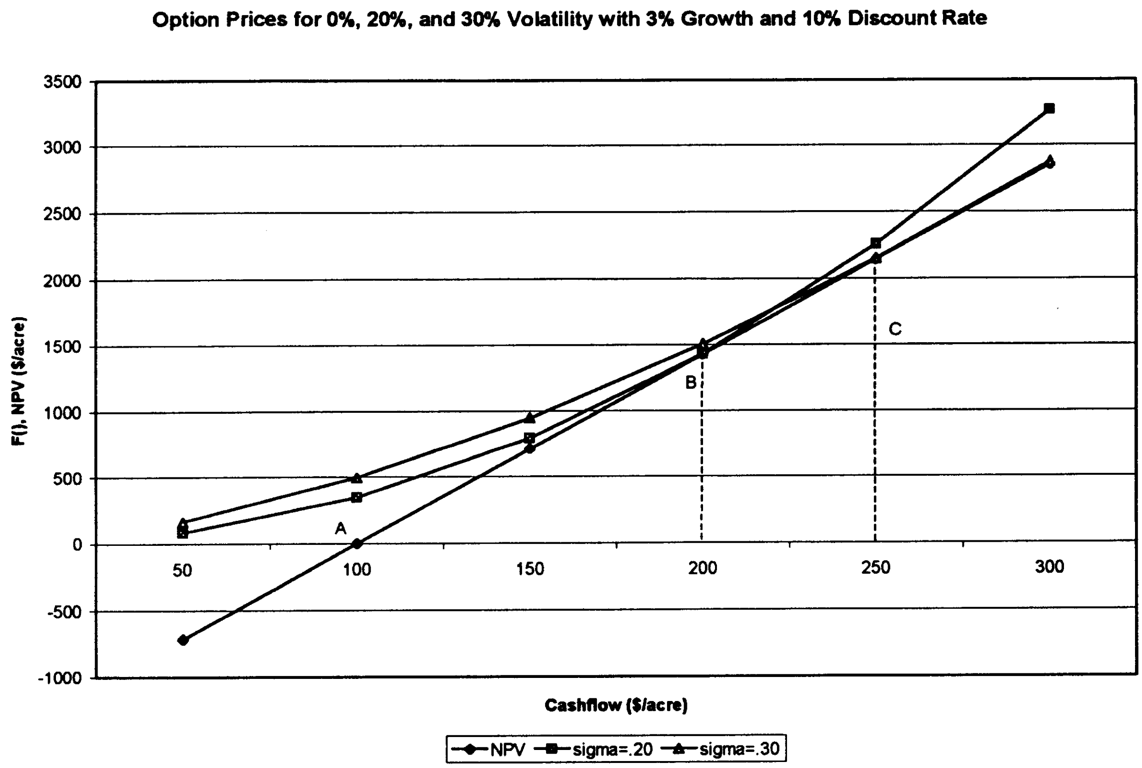
Table 1. illustrates the real options valuation framework. The base assumptions in Table 1 are that the discount rate i=10% , growth (α) is 3%/year, and current cashflow expectations are

\$100/acre. Based on these assumptions the bid-price value of land is \$1,428. For σ=.20, π^*= \$196/acre and for σ=.30, π^*= \$267/acre. The term π^* represents the trigger value at which the option should be exercised and the farmland sold. This means that with current cashflows of \$100/acre and a current bid price of \$1,428 it would be optimal for the seller to postpone the sale until cashflow (under the 20% volatility scenario) increases to \$196/acre and the bid price increases to \$2,800/acre. Beyond \$196 the option value increases but this has little significance since it is diverging from the relevant increases in cash flow.

Table 1. Real Options Valuation Using 10% Discount Rate for Volatility of 20% and 30%

Cash Flow (\$/acre)	50.00	100.00	150.00	200.00	250.00	300.00
Bid Price Value V()	714.29	1,428.57	2,142.86	2,857.14	3,571.43	4,285.71
Current Bid Price	1,428.57	1,428.57	1,428.57	1,428.57	1,428.57	1,428.57
Real Option Value , sigma=.20	84.35	347.21	794.41	1,429.17	2,253.74	3,269.91
Trigger Cashflow, sigma=.20	196.03	196.03	196.03	196.03	196.03	196.03
V(p*) , sigma=.20	2,800.50	2,800.50	2,800.50	2,800.50	2,800.50	2,800.50
Real Option Value , sigma=.30	163.41	495.49	948.09	1,502.45	2,147.31	2,874.85
Trigger Cashflow, sigma=.30	266.56	266.56	266.56	266.56	266.56	266.56
V(p*) , sigma=.30	3,807.99	3,807.99	3,807.99	3,807.99	3,807.99	3,807.99

The real option prices are illustrated in Figure 3 for the cashflow possibilities in each column of Table 1. It can be seen from the figure that the smooth pasting conditions indeed occur at the point of tangency, and that even if cashflow falls below 100 there is still a positive option value given historical volatility.

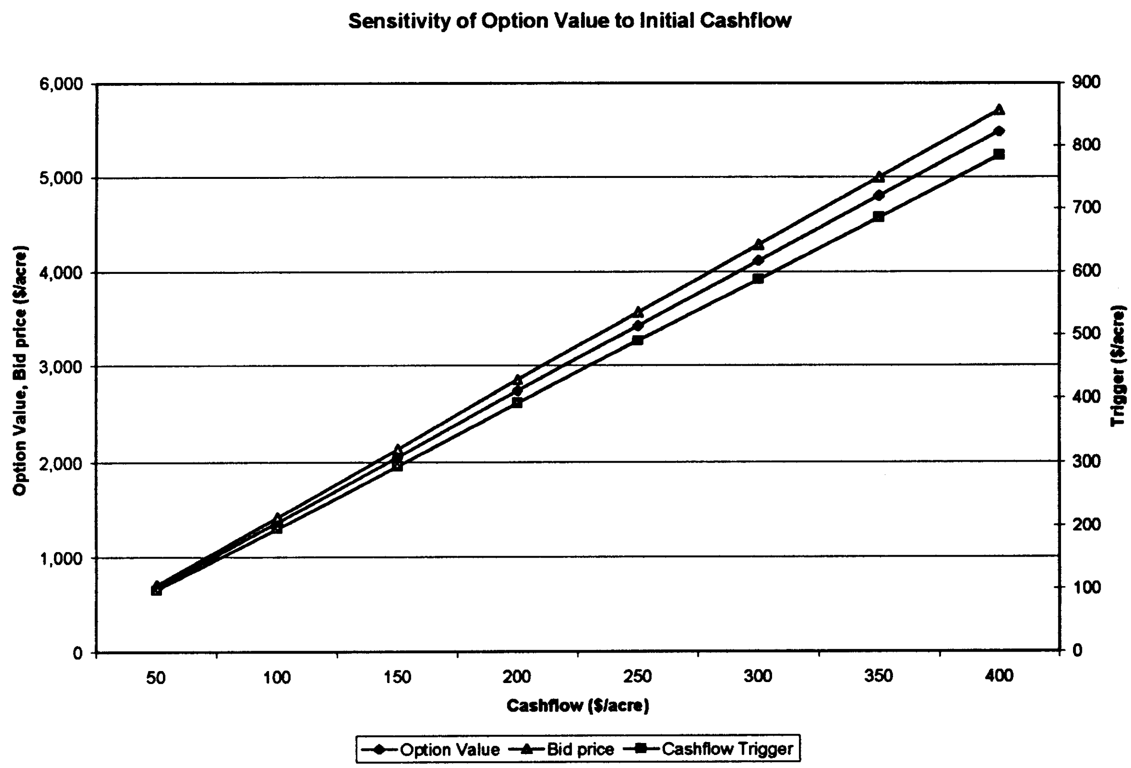


Sensitivity of Option Value to Cashflow

The sensitivity of the option value to the initial cashflow conditions is presented in this section. The conventional bid price model shows that the bid price increases linearly with changes in A.[2] This is shown in Table 2 and figure 3 where the bid price increases from $714/acre to $5,714/acre as cashflow increases from $50/acre to $400/acre with volatility of 20%, growth of 3% and a hurdle rate of 10%. Typical cashflow for most crops will range between $100 and $200/acre. At $100/acre the trigger, π^*, is $196 which means that the seller should wait until the price of her crops increases from $100 to $196/acre. At that price the value of land will have increased from $1,429 to $2,372 for an increase in wealth of $1,372 or the value of the real option. As cashflow increases so does the option value. For initial cashflow of $200/acre the option value increases to $2,744 while cashflow at $400/acre indicates a real option value of $5,488. In Table 2 the strike trigger is always 1.96 times the base cashflows. Consequently the land values at π^* are also 1.96 times that of the bid price values. The linear relationship can be seen in figure 3.

Table 6: Sensitivity of Option Valuation to Cashflow with Sigma=.20, Hurdle Rate=10%, and Growth=3%

Cashflow ($/acre)	50	100	150	200	250	300	350	400
Bid Price ($/acre)	714	1,429	2,143	2,857	3,571	4,286	5,000	5,714
Strike Trigger ($/acre)	98	196	294	392	490	588	686	784
Farmland Value at Trigger ($/acre)	1,400	2,800	4,201	5,601	7,001	8,401	9,802	11,202
Value of Option ($/acre)	686	1,372	2,058	2,744	3,430	4,116	4,802	5,488



[2] The calculus applied to the bid price formula is $dV = (1+g)/(i-g)\ dA > 0$.

Sensitivity of Option Values to Varying Hurdle Rates

The value of the real options is contingent on all of the economic conditions that would normally impact the bid price. In the deterministic bid price model the present value of cash flows will decrease with an increasing discount rate. For example in table 2 the bid price is \$3,333 for a hurdle rate of 7.5% but only \$882 if the hurdle rate increases to 20%.[3,4]

Table 2: Sensitivity of Option Valuation to Hurdle Rate with Sigma=.20, Growth=3%, Cashflow=\$150/acre

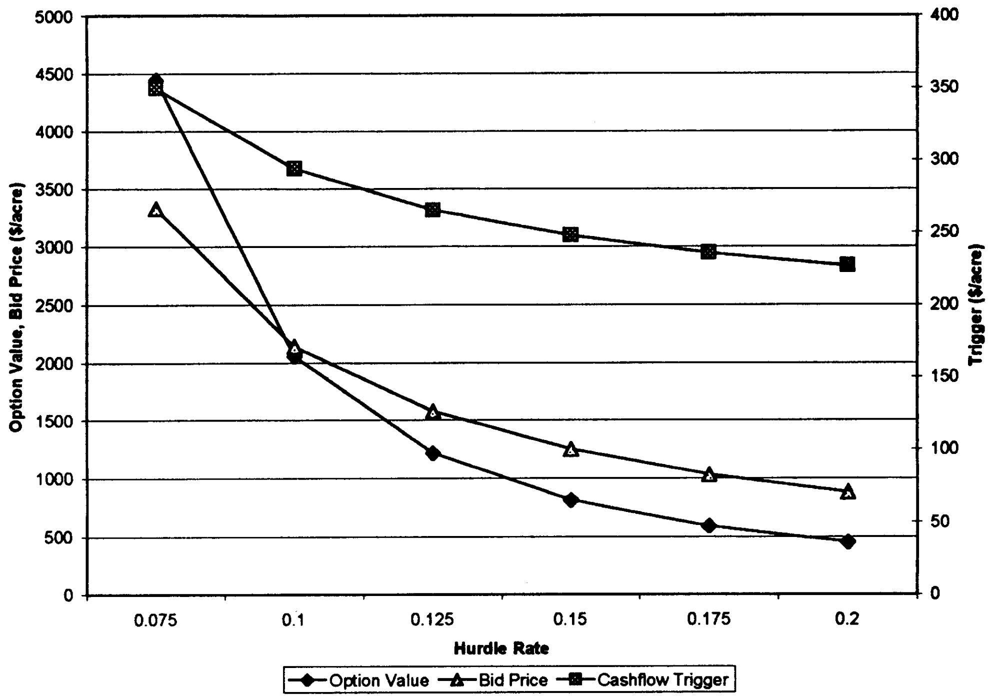
Hurdle Rate (%)	0.075	0.1	0.125	0.15	0.175	0.2
Bid Price (\$/acre)	3,333	2,143	1,579	1,250	1,034	882
Strike Trigger (\$/acre) π^*	350	294	265	248	236	227
Farmland Value at Trigger (\$/acre) $V(\pi^*)$	7,778	4,201	2,794	2,065	1,626	1,334
Value of Option (\$/acre) $F(\pi^*)$	4,444	2,058	1,215	815	591	452

The impact of the discount rate on the trigger value , π^*, is negative as the present value of the future cash possibilities decreases. At 7.5%, π^* = \$350/acr but at 20% it is \$227/acre. Consequently the value of land at the trigger also diminishes. At 7.5% it is \$7,778 when evaluated at \$350/acre and \$1,334 when evaluated at 20%. Consequently the real option value $F(\pi^*) = V(\pi^*) - I$ also diminishes from \$4,444 to only \$452. This is illustrated in Figure 4 with π^* recorded on the right hand Y axis and bid prices and options values recorded on the left. Figure 4 shows that as the discount rate increases the value of the option decreases at a rate greater than the bid price itself. Up to approximately 10% the value of the option is actually greater than the current bid price value itself, but to the right of the intersection point the option value is less than the current bid price.

[3] The calculus of this relationship is $dV = -A(1+g)/(i-g)^2\, di < 0$

[4] The terms hurdle rate and discount rate are used interchangeably. In the purest of interpretations the discount rate should equal the weighted average cost of capital with the return to equity appropriately adjusted for risk. This is the minimum discount rate. The hurdle rate is often used to refer to discount rates that exceed the risk adjusted discount rate.

Sensitivity of Option Values to Hurdle Rates



Sensitivity of Option Value to Growth Rate

The value of land will increase with the growth rate and as shown in Table 3 and figure 5 so will the real option value[5]. At 0 growth the bid price is $1,500. Even in the absence of growth uncertainty still creates an option value. The trigger is $243/acre, meaning that the seller should wait until cashflow rises from $150/acre to $243 before selling the land. The value of land at that point will be $2,425/acre for a real option value of $925/acre. In agriculture growth rates are generally small so the option values would probably be no more than the $2,058 calculated for 3% growth. However, from time to time agriculture does witness some extraordinary growth in certain sectors of the economy that could be described by the high option values illustrated for 7% growth in Table 3. These have often been attributed to speculative bubbles the most famous of which would have been the tulip bubble of the 17th century. One can only imagine how Dutch farmland would have increased exponentially in value as the value of tulip bulbs increased.

Likewise the real option value of that farmland would have increased at an ever-increasing rate. A more recent example would be the real option value of ratites that so the value of breeding pairs of emus rise to $40,000 in the early 1990's only to fall back to several hundred dollars or less by the late 1990's. Importantly it is the real option wedge between the current asset value and the real option value that creates the appearance of a bubble. While one might argue the wisdom of optimistic growth forecasts the resulting asset valuations are nonetheless rational within a real options framework. Thus when, for example, Featherstone and Baker find evidence of a speculative bubble or Falk and Lee find evidence of a fad, a real options framework provides a structured approach to understanding and comprehending the underlying economic principles involved.

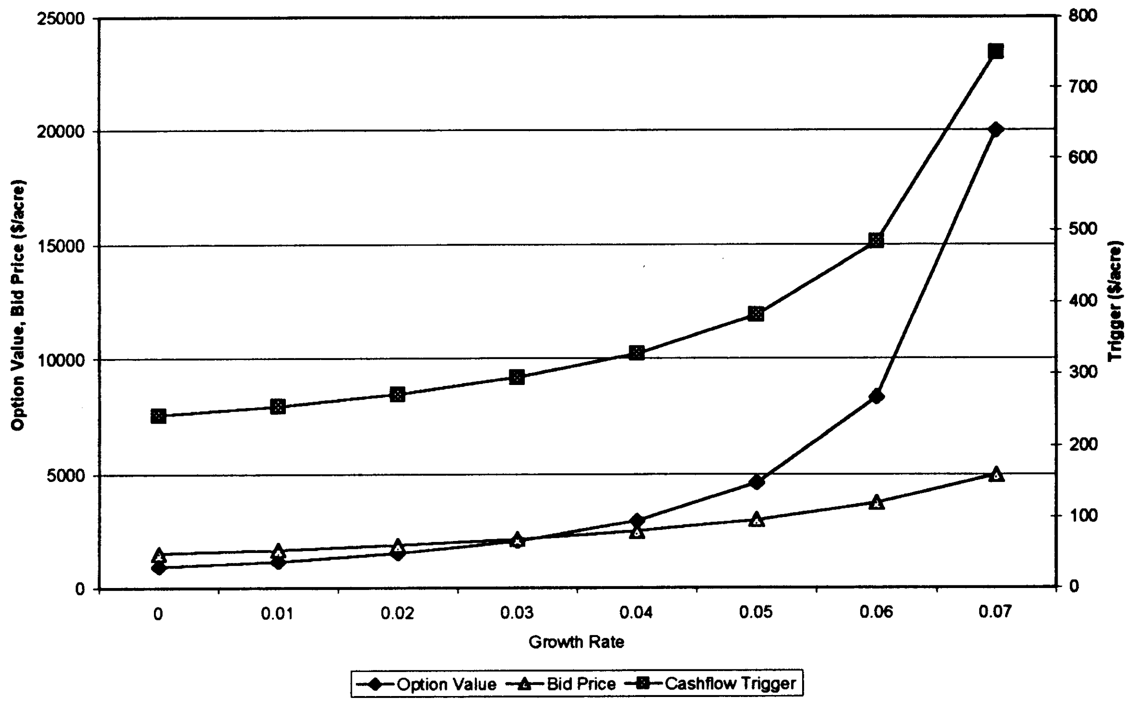
Table 3: Sensitivity of Option Valuation to Hurdle Rate with Sigma=.20, Hurdle Rate=10%, Cashflow=$150/acre

Growth Rate (%/year)	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07
Bid Price ($/acre)	1,500	1,667	1,875	2,143	2,500	3,000	3,750	5,000
Strike Trigger ($/acre)	243	255	271	294	327	382	484	750
Farmland Value at Trigger ($/acre)	2,425	2,833	3,392	4,201	5,458	7,630	12,093	25,000
Value of Option ($/acre)	925	1,166	1,517	2,058	2,958	4,630	8,343	20,000

Figure 6 illustrates the effect of growth on farmland values again with π^* on the right axis. The figure shows that the real option value of farmland is less than the current bid price when growth is low, but at about 3% the real option value exceed the value of the bid price itself. This option value increase coincides with the exponential compounding of growth as g approaches i.

[5] The calculus on the bid price with respect to growth is given by $dV = A(1+i)/(i-g)^2\ dg > 0$

Sensitivity of Option Values to Growth Rates



Sensitivity of Options Values to Volatility

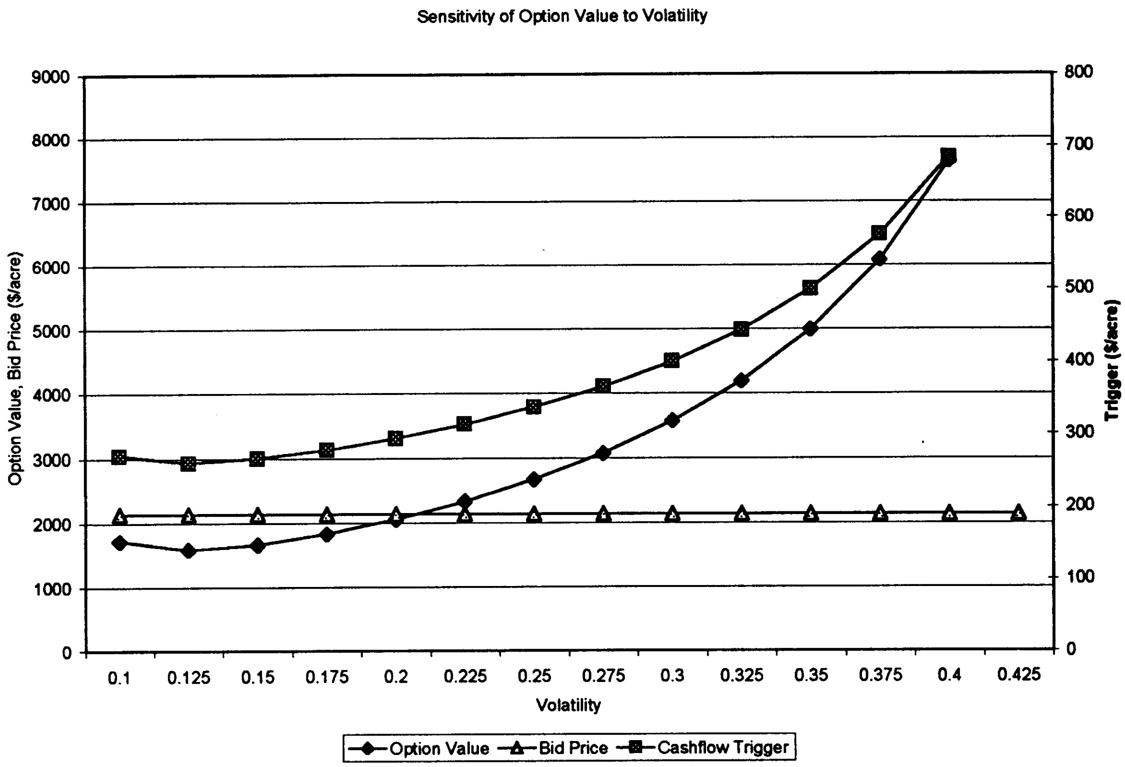
The existence of real options arises from uncertainty. Without uncertain the probability that there will be an unanticipated increase in cashflows at some future date is zero. In general the annual volatility in agricultural prices is between 15% and 30%. This means that given a current observation on prices, for example, there is a 66% chance that the price could increase or decrease by 15% to 30% depending on the commodity. Likewise, uncertainty in cashflows will also adjust dynamically in an uncertain way that gives rise to the possibility of future increases. The sensitivity of real options values to volatility is shown in Table 5 and illustrated in Figure 7 for values of σ ranging from 10%/year to 40%/year. Since volatility does not enter into the conventional bid price model its value remains constant at $2,143 under the given conditions. At 10% volatility it is optimal for the seller to postpone the investment until π^* is $271/acre. At the trigger the value of land would be $3,866/acre giving rise to an option value of $1,723/acre. The more volatile the cashflows from the land the greater the real option value of land. At 20% volatility the real option value is $2,058 and at 40% the value is $7,633.

Table 5: Sensitivity of Option Valuation to Volatility with Hurdle Rate=10%, Growth=3%, and Cashflow=$150/acre

Volatility (%/year)	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Bid Price ($/acre)	2,143	2,143	2,143	2,143	2,143	2,143	2,143
Strike Trigger ($/acre)	271	267	294	337	400	500	684
Farmland Value at Trigger ($/acre)	3,866	3,811	4,201	4,811	5,712	7,140	9,776
Value of Option ($/acre)	1,723	1,668	2,058	2,668	3,569	4,997	7,633

The relationship between risk and value is important. It suggests that with increasing risk the seller of land will hold onto the land in hopes of receiving extraordinary future gains. As indicated earlier the buyer does not have this option. If the game described in Zhao and Kling is correct then in order for the buyer to acquire the land she will have to purchase at least part of the future option value from seller. Consider for example the 25% volatility values in Table 5. The current bid price is $2,143/acre using conventional present value methods. But under conditions of risk the seller has this option to postpone the sale and this is valued at $2,668. The option itself is negotiable. Suppose that a 50-50 agreement is reached. The buyer will pay $2,143 for the fundamental value of land plus 50% of the option value for $1,334. As a result an asset that is valued at $2,143 by conventional bid price modeling would actually transact at $3,477. The transaction price will also increase with increasing uncertainty. This theory is actually Pareto efficient. As indicated earlier the buyer has purchased an asset that still has a $1,334 option attached to it while the seller has been compensated with the certainty equivalent value of the option.

It is useful at this point to return to the recent findings of Falk and Lee. In their study they found that there were short run departures from the basic capitalization model. In their figure 2 (page 704) the relationship between fundamental and non-fundamental shocks is evident for Iowa farmland prices from 1924 to 1994. The greatest nonfundamental shocks occurred in the 1970's at precisely the same time that Baker and Featherstone suggest a speculative bubble. In correlating the finding of this paper to that (and

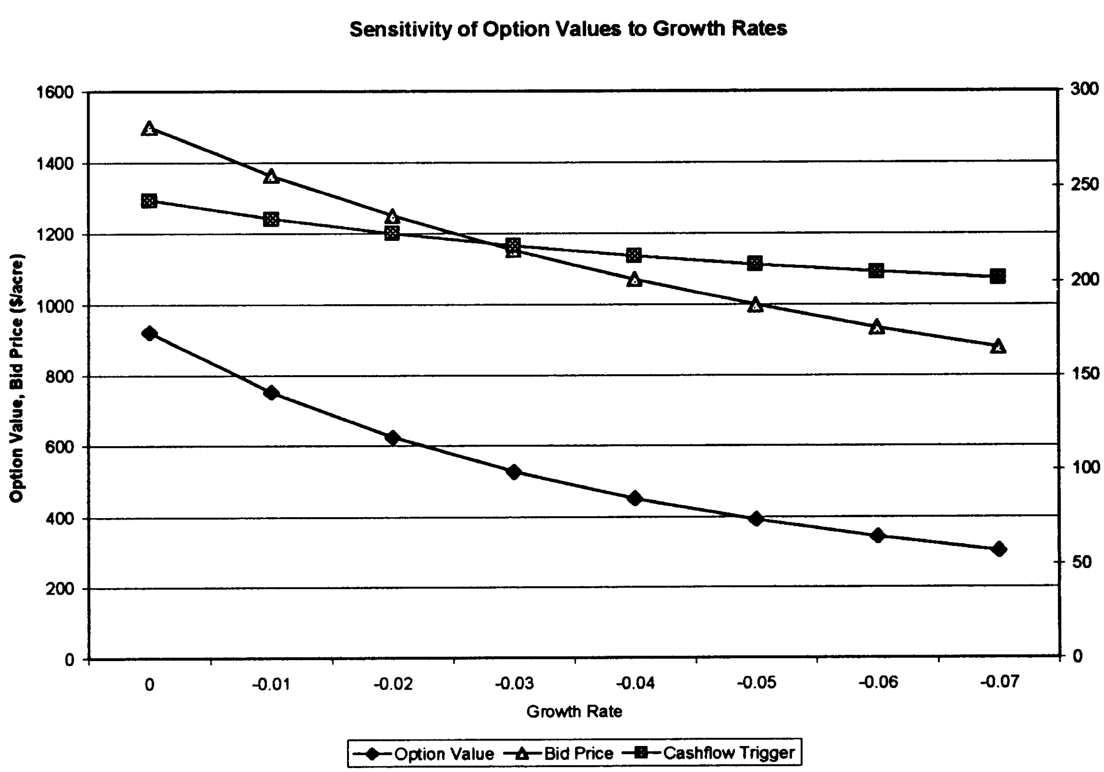


other) studies, the theory posed here is that the sudden increase in land values was due to transactions in real options brought on by increasing volatility or a combination of volatility and growth expectations etc.. The departure from the basic capitalization model should be expected in short-run land price movements as value is created from uncertainty and the resulting option value becomes part of the transacted price. In this theory the failure of econometric studies to find evidence of the capitalization model should be expected. While the current study does not empirically estimate the cointegrated values of cashflow and land values the obvious null hypothesis would be that there is no significant difference between actual land prices and the predicted behaviour from the land capitalization model. Failure to reject this hypothesis would provide evidence that real-options values do not exist. However, upon rejecting the hypothesis, as so many studies have done, this would indeed provide evidence that real options play a rational role in the determination of land prices. It would appear therefore that variance decomposition studies such as Falk and Lee could prove to be very fruitful in testing such hypothesis.

Discussion and Conclusions

The emerging theories and literature on real options provides a rich and reasonable approach to explaining many of the discrepancies that agricultural economists have discovered when positive theories such as the pricing of farm land do not hold under rigorous empirical testing. The theory posited in this paper is that divergent findings on farmland pricing found by Featherstone and Baker, Weersink et al, Falk and Falk and Lee may well be explained by the positivist economics that underscores real options pricing. Real options emerge when investors have the option to act or postpone making an investment decision when the investment has uncertain outcomes in unto itself, or is derived from an underlying stochastic process. This real option emerges because the decision maker always has an option to do nothing, invest now or postpone the investment. For market determined assets such as land this means that the observed land price does not always match the price predicted from the simple discounting of future cashflows. It is argued that uncertainty drives a wedge between the conventional bid price and the real option value associated with wanting to sell the land. This option does not occur for buyers, at least when growth is positive since the market will always charge at least the present value of future cashflows, i.e. the bid price at any level of cashflow.

This is not to say that the buyer never has an option. Figure 9 shows the option value to



the seller when the growth is increasingly negative. As growth decreases land bid prices decrease as does the trigger and the option value. A put-call parity will exist in a real options framework as it does in financial options. As the value of the call option to the seller decreases the value of a put option to the buyer will increase in tandem. The put option gives the buyer the right but not the obligation to postpone the purchase until cashflow, and hence the value of land, decreases further. Under this theory the seller would be more willing to sell the land immediately at higher cash values than later when cash is expected to decrease. The option to postpone the purchase would be the difference between the current bid price and the lower bid price if cashflows actually decrease further.

In this framework whether agriculture is facing a bull market or bear market is important. The bull market will give rise to positive and increasing call option values for the seller, whereas a bear market defined by increasingly negative growth provides a positive and increasing call option value to the buyer. In both scenarios the holder of the option will negotiate with the other to acquire at least a portion of the option. In a bull market the seller will extract some of the option value from the buyer, and in a bear market the buyer will attempt to extract some of the option value from the seller. In both cases the actual transaction price will be either higher or lower than the traditional capitalization model would expect. This is a testable hypothesis that should be rigorously examined because if it is true then much of our previous understanding of land-price dynamics would need to be reconsidered.

Another area of important research deals with the relationship between asset valuation and public policy. The conclusions reached in a real options framework is that the option value of farmland will increase with increased cashflow, growth, and risk, and decreasing discount rates. Agricultural policies are for the most part directed towards reducing risk and one would think that this would lead to a decrease in option values and hence land prices. Weersink et al found a different result. In their study they found that the rate of discount was higher for operating cashflow and lower for government-originated cashflow. This means that a marginal increase in a dollar of cashflow from operating income would lead to less of an increase in land values than a dollar of subsidy from government. This result might at first appear to refute the hypothesis that government intervention leads to a reduction in option values. However it must be recognized that for the most part the purpose of government intervention is to truncate the probability distribution below the support level while keeping the distribution of outcomes above the support level in tact. This is not easily remedied in the framework presented here because no allowance is made for truncated volatility. Nonetheless, given our understanding of the real option pricing process it is reasonable to suggest that the call option to the seller in a bull market would at least remain the same if current conditions are above the support level, and would likely increase if the current conditions are below the support level.

The real impact of government policy is its impact on the put option available to buyers in a bear market. As economic conditions worsen, support policies reduce or eliminate entirely the downside probabilities. Since the probability of lower cashflow is eliminated there is a lower bound on the value of farmland. The imposition of this lower bound will inevitably reduce the value of the real put option available to the buyer. In fact, the put option value would equal zero at the support level.

Government intervention then plays a very important role in the pricing of farmland. The above reasoning suggests that in a bull market the rate at which farmland prices increase will be greater than the rate at which farmland prices decrease in a bear market. Weersink et al hint at this in their study noting that falling farm incomes are offset by increasing farm subsidies that have lead to an overall increasing trend in land prices. They did not however differentiate the effects in an increasingly profitable economy versus an unprofitable one. This again is a testable hypothesis that can be examined in two stages. The first stage would identify the existence of a wedge between observed land prices and the capitalization model that can be attributed to real options values. The second phase, which is only valid if evidence is found in the first phase, would specifically test for land price responses to government programs. Perhaps a model framework combining Falk and Lee and Weersink et al with specific dummy or trend variables for bear and bull markets could be used.

The notion of real options as proposed by Dixit and Pindyck provides a rich set of logic to understanding the valuation of capital asset pricing under conditions of risk. It provides a unifying framework that ties our thinking on conventional investments (such as farmland) together with modern options pricing. In this paper their model was used to investigate the pricing of capital assets under conditions of risk. The resulting framework can be used to explain much of the recent literature that shows time and again that the standard land capitalization model or bid price framework does not hold up to empirical testing. Refutation of the bid price model at this point is probably foolhardy, because as shown in this paper the basic model may be ignoring a very important element, uncertainty. Uncertainty gives rise to real options. This real option value in turn provides a wedge between the bid price estimated under certainty and the market valuation under uncertainty. It is suggested that it is this wedge that has been discovered by many of the econometric studies that refute the capitalization model of farmland pricing.

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