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Calum G. Turvey and Alfons Weersink

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Department of Agricultural, Resource and Managerial Economics
College of Agricultural and Life Sciences
Cornell University, Ithaca, New York 14853

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THE DEMAND FOR AGRICULTURAL LOANS AND THE LENDER-BORROWER RELATIONSHIP

Calum G. Turvey and *Alfons Weersink*[1]

Much of the economic literature on loan demand views it in terms of the lender-borrower relationship. Early work in this field, namely Freimer and Gordon; and Jaffee and Modigliani, viewed loan demand and credit risk as being passive to the borrower but active to the lender; that is the loan offer curve made explicit use of credit risk, whereas the borrower's decision did not. The Jaffee-Modigliani model of credit rationing has received wide attention and encouraged substantive debate as to exactly what the borrower-lender relationship should be.

The question of whether or not borrowers actively incorporate credit risk in their demand for loans is an important one. In terms of pure conjecture it seems unreasonable to characterize borrowers as making risky investment decisions using financial leverage independently of the incremental increase in financial risk associated with debt financing. Azzi and Cox argue that if borrowers offer collateral or equity to secure debt, lenders would be able to recover at least some portion of financial obligation below the default rate of return, and as they argue borrowers must satisfy the collateral/equity needs in order to convert a desire for loan into a demand.

In related work Smith argues that borrower's equity acts as an 'external economy' to the lender which implies an intrinsic stochastic dependency between the borrower and lender which allows for an increasing supply of debt as borrower equity increases. Baltsenberger commenting on Smith, and Hansen and Thatcher, note a certain independence between the contractual rate of interest charged the borrower and loan demand. Baltsenberger; and Hansen and Thatcher argue that loan demand must be viewed by both borrower and lender in terms of loan quality, where quality refers to the riskiness of the loan measured in terms of debt relative to equity. Thus the borrower's interest payment must be related to the amount of equity provided and the probability of default. In Baltsenberger's formulation parieto efficient loan contracts consistent with a competitive equilibrium in the loan markets exists, whereas in Smith's model parieto efficient contracts cannot exist if equity is an external economy to the lender and firms have limited liability. Rather, Smith argues that parieto efficient contracts emerge only through negotiated contracts wherein lenders constrain borrower's equity capital to a minimum while borrowers constrain debt capital to a maxima.

A related class of problems occurs when there is more than one class of borrower distinguished in a perfect information economy by investments with differing expected returns and/or risk. In such an economy parieto efficient outcomes can be established through lenders' offerings of multiple contracts with each being unique to the risk class of individuals. To avoid credit rationing of any sort borrowers must be willing to accept interest rates above the prevailing market price while lenders must be allowed to offer differential interest rates to different classes of borrowers. In equilibrium, credit rationing would not occur.

However, in reality information is not perfect, and obtaining perfect information comes at a cost to the lender. Asymmetric information may be sufficient to drive equilibrium solutions which are characterized by credit rationing (Stiglitz and Weiss; Jaffee and Russel). For example, it is plausible that a lender may assess multiple loan applicants investing in projects of equal expected returns, but different risks. Since probability information is far more difficult to garnish than

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[1] The authors are associate professors in the Department of Agricultural Economics and Business, University of Guelph, Guelph, Ontario.

expected returns there would be sufficient ambiguity in establishing which borrowers are high risk and which borrowers are low risk. Lenders can then employ screening devices such as credit scoring models to objectively sort risk classes and thus eliminate demand rationing (Bester). However credit scoring models are in themselves imperfect because they may reject or ration loans which would otherwise be acceptable or accept loans which would otherwise be rejected. Alternatively, lenders may charge all borrowers the same rate of interest based upon the pooled risk, but this is again inefficient since loans would be a bargain, and therefore attract, high risk types, while being to dear, and therefore rationing low risk types.

Pooled interest rates can also act as a signalling device, which can lead to adverse incentive effects which actually increase the riskiness of loan portfolios. Recall again that pooled interest rates are sufficient to attract high risk types to the loan market, but they may also encourage low risk types to increase their desired loan amount and encourage adoption of higher risk projects (Stiglitz and Weiss; Jaffee and Stiglitz).

While arguments such as those presented above focus on the borrower-lender relationship they have not been used to establish characteristics which can be used to empirically estimate loan demand curves. Perhaps this is because a unified, generally agreed upon notion of what constitutes demand or the lender-borrower relationship has yet to surface. Yet there is much in these theories which dictate what a candidate loan contract curve could look like, while explaining some potentially observable characteristics of the lender-borrower relationship.

While in theory opinions on loan demand and supply are diverse, in practice loan demand is probably related to all of the factors considered above. First, because of the borrower-lender relationship the desired demand is unlikely to be observed. In terms of data, what is observed are contracts whereby borrowers and lenders have negotiated a loan amount at a specified price. From the theory we can suppose that factors such as probability of default, debt-equity structure, limited liability, and security/collateral, are all factors to be considered. But it is not observed, nor could it possibly be observed, how individual market participants negotiate, say equity structure and collateral, in order to satisfy the lender-borrower relationship.

What can be observed are ex post characteristics of this relationship and the probability of default given various performance measures. These are the same characteristics which are used to obtain objective *ex ante* probabilities from credit scoring models which are used to screen loan applicants. Thus if we can use quantitative and qualitative variables to compute loan default probabilities from credit scoring models then we should be able to determine the 'demand for loans' from credit scoring models as well. However, 'demand for loans' in this context must be viewed as a hybrid 'loan contract' function of the borrower/lender relationship along a locus of feasible loan outcomes.

The purpose of this paper is to empirically estimate the demand for credit on loans made to Canadian farmers through the Farm Credit Corporation. In the next section we develop a general model of the borrower-lender relationship assuming both are profit maximizers and note that the implicit demand for debt is a function of the probability of default and the implicit supply of debt is a function of this same probability. The result implies that when information is perfect and costless all loans lie on a concave (backward-bending) contract curve. The importance of the result is that it refutes the notion that loan demand and loan supply can generally be viewed in isolation, which thus implies that empirical estimation using observed loan contracts may be justified. In addition we show that downward sloping loan demand curves are guaranteed to exist if loan default probabilities are constant along their slope and lenders are profit maximizers. The converse possibility that distinct loan supply curves are downward sloping holds too if lenders apply credit scoring or other screening devices which restrict risk in a safety-first context, while borrowers are profit maximizers. These concepts are then expanded to show how ambiguous probability information about multiple classes of borrowers, can lead to credit rationing. From analyzing the basic structure of the lender-borrower relationship we use empirical credit scoring models to derive loan 'demand' functions for different risk classes of agricultural loans. The advantage of this theoretical model guiding the empirical model is that consideration of hedonic pricing such as that proposed by Baltsenberger, as well as collateral/equity consideration as proposed by Smith, Azzi and Cox; and Stiglitz and Weiss, can be evaluated within the basic structure of the Jaffee-Modigliani model.

The Borrower-Lender Relationship

The Borrower's Problem

This section develops, along the lines of Smith, a general model of the borrower-lender relationship assuming a profit maximization objective. The firm has a fixed amount of wealth, W_o, which can be invested in a riskless asset or as equity in a leveraged risky investment. The investment horizon and loan payback period is for one year. If θ is the proportion of wealth invested in the risky asset, and $(1-\theta)$ in the riskless asset then end-of-period wealth, defined by initial wealth plus profits, is:

$$\bar{\pi} = (1-\theta)W_oR_f + (\theta W_o+D)\bar{R} - ID \tag{1}$$

where R_f equals 1 plus the riskless rate of return, $\bar{R}$ equals 1 plus the expected rate of return on risky investment, and I is 1 plus the interest rate on debt, D. We assume limited liability which protects personal holdings of the riskless investment. Hence the loan is in default if $r < r^*$, where

$$r^* = ID / (\theta W_o+D) \tag{2}$$

is the critical or breakeven (1 plus the) rate of return. Since $\partial r^*/\partial I$, and $\partial r^*/\partial D$ are positive, increases in interest rates or debt increase the chance of default, while $\partial r^*/\partial \theta$ and $\partial r^*/\partial W$ indicate that increased equity or initial wealth decreases the chance of default. Let $f(r)$ be the probability density function about the random return on the risky investment. Then with limited liability and loan default risk expected terminal wealth is given by

$$\begin{aligned} Max_{\theta,D}\, \bar{\pi} = {} & (1-\theta)W_oR_f + (\theta W_o+D)\bar{R} - ID \\ & - \int^{r^*}[(\theta W_o+D)R-ID]\, f(r)dr\,. \end{aligned} \tag{3}$$

Integrating the last term by parts and substituting in r^* yields

$$Max_{\theta,D}\, \bar{\pi} = (1-\theta)W_oR_f + (\theta W_o+D)\bar{R}-ID+(\theta W_o+D)\int^{r^*}F(r)dr \tag{4}$$

The firm's choice is to maximize terminal wealth by choosing the optimal equity investment in risky to riskless assets, θ, and how much of the risky investment is to be financed with debt. Assuming that all second order conditions are satisfied, the first order conditions for an interior maximum are

$$\frac{\partial \pi}{\partial \theta} = W_o(\bar{R}-R_f) + (\theta W_o+D)\ F(r^*)r^*_\theta + W_o\int^{r^*} F(r)dr = 0 \tag{5}$$

and

$$\frac{\partial \pi}{\partial D} = \bar{R}-I + (\theta W_o+D)\ F(r^*)r^*_D + \int^{r^*} F(r)dr = 0 \tag{6}$$

where $r^*_\theta = \partial r^*/\partial\theta = -W_oID/(\theta W_o+D)^2$ and $r_D^* = \partial r^*/\partial D = \theta W_oI/(\theta W_o+D)^2$. Substituting (5) into (6) yields the optimal condition

$$\partial\bar{\pi}/\partial D = R_f - (1-F(r^*))\ I = 0\ , \tag{7'}$$

or

$$F(r^*) = \frac{I - R_f}{I}\ , \tag{7}$$

the implicit solution of which provides the loan demand curve $D(\theta,W_o,R_f,I,F'(r^*))$. The condition implies that debt will be used in place of equity until the probability of default $F(r^*)$ equals the loan risk premium as a percent of the interest cost. As the riskless rate increases the investor places relatively more resources into risk investment and borrows less for risky investment so loan default risk decreases. As interest rates increase the probability of default increases as fixed financial obligations increase.

The Lender's Problem

The lender's problem is to maximize expected end-of-period wealth $\bar{Z}$. Here the lender must choose the optimal amount of debt, B, given the borrower's equity position, and facing the possibility of bankruptcy. If $r < r^*$ then the borrower under limited liability foregoes $(\theta W_o+B)R < IB$. The lender takes a loss on the loan but the loss may not be total. It is also assumed that the lender can acquire all of B at the opportunity cost rate δ. The lender's objective function is

$$MAX_{\theta B}\ \bar{Z} = \int^{r*} [\theta W_o+B]R\ f(r)dr + IB\ {}_{r*}\!\int f(r)dr - \delta B \tag{8'}$$

or

$$MAX_{\theta B}\ \bar{Z} = (I-\delta)B-(\theta W_o+B)\ \int^{r*} F(r)dr \tag{8}$$

First order conditions are given by $\partial\bar{Z}/\partial B$ and $\partial\bar{Z}/\partial\theta$. Here the derivative $\partial\bar{Z}/\partial\theta$ refers to Smiths view that the proportion of equity a borrower puts into risky investment is an external economy to the lender. Differentiation yields the following first order conditions.

$$\partial\bar{Z}/\partial\theta = -W_o\int^{r*} F(r)dr - (\theta W_o+B)\ F(r^*)r^*_\theta = 0 \tag{9}$$

and

$$(10)\quad \partial\bar{Z}/\partial B = (I-\delta) - \int^{r*} F(r)dr - (\theta W_o+B)\ F(r^*)\ r^*_B = 0\ . \tag{10}$$

Here r^*_θ and r^*_B are the same as for the borrower except B is substituted for D. Substituting (9) into (10) gives

$$\partial\bar{Z}/\partial B = I(1-F(r^*))-\delta = 0 \tag{11'}$$

or,

$$F(r^*) = \frac{I-\delta}{I}\ . \tag{11}$$

In (11), $I-\delta$ is the risk premium required by the lender in order to supply the amount of debt implied by $F(r^*)$. This is the same solution obtained by Jaffee and Modigliani and thūs takes on the general backward-bending shape of the Jaffee-Modigliani loan supply curve.

Equilibrium in the Borrower/Lender Relationship

Equations (7) and (11) provide the optimal conditions for loan demand and supply. If a parieto efficient contract is to be made then clearly there must be agreement between the borrower and lender on not only the probability of default, $F(r^*)$, but also the amount of equity contributed to investment, and the interest rate charged. Importantly a necessary condition for optimality is that $\delta = R_f$; that is unless the lender's opportunity cost of capital (passbook account charges, GIC's, and operating/monitoring costs) equals the risk free rate, competitive equilibrium parieto efficient loan contracts cannot occur[2]. However, under the assumption that $R_f = \delta$ there is a further point of importance. If equity is treated as an external economy to the lender then there does not exist, in the usual sense of the terms, distinct loan demand curves or loan supply curves. All parieto efficient demand and supply combinations are satisfied along this curve.

The implicit solution to the loan contract problem is given by equations (7) and (11) which are equivalent for contracts D=B and $R_f=\delta$. Implicit differentiation yields

$$\frac{dD}{dI} = \frac{1-F(r^*) - IF'(r^*)r_I^*}{IF'(r^*)r_D^*} \tag{12}$$

Since the denominator is always positive the slope of the loan contract curve is determined by the numerator. Using $r_I^* = D/(\theta W_o+D)$, the slope of the contract curve is determined by

$$1 \gtrless r^* \frac{F'(r^*)}{1-F(r^*)} \tag{13}$$

The term $F'(r^*)/(1-F(\theta^*))$ in (13) is the conditional probability of default (Smith). Since $\partial r^*/\partial I > 0$, $\partial r^*/\partial D > 0$ and $\partial^2 r^*/\partial I\partial D > 0$, r^* increases along the demand curve until $dD/dI = 0$, and then decreases as D decreases. The conditional probability of default increases at all points along the contract curve. Loan contract curve with these features is illustrated in Figure 1, where the vertical axis is measured in terms of the optimal debt-equity ratio, and the horizontal axis, the marginal interest rate. The main results of the lender-borrower relationship are summarized in the following paragraphs.

Proposition I: Under perfectly competitive conditions, loan demand and loan supply curves are identical for both the borrower and lender.

The result implies that along a single locus of loan contracts expressed in terms of debt, equity and interest rates marginal expected profits of the borrower equal marginal expected profits of the lender. The proof is obtained by noting that $\partial\bar{Z}/\partial B = \partial\bar{\pi}/\partial D = 0$ for D = B at an optimum. That both demand and supply lie along a single contract curve, as discussed above, is proven by setting $\delta = R_f$ and noting that equation (7) and equation (9) are identical for all I.

Proposition II: For both borrower and lender marginal profits are constant along the contract curve as I increases.

Proof: Assume that D=B and $R_f=\delta$, so that (7') and (11') are equivalent. Integrating both sides of (7') over outcomes below r^* so that $\int^{r^*}F(r)dr=\int^{r^*}(I-r_f)/I\,dr = (I-R_f)r^*/I$. Substituting this into (4) and (8)

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2 To see this note that $\partial F(r^*)/\partial\delta < 0$ in (11. If $\delta > Rf$, $(I-\delta)/I > (I-Rf)/I$ the acceptable risk to the borrower for a given interest rate will always be greater than that acceptable to the lender, and the borrower will be forever rationed.

gives conditional profit functions π^* and Z^*. Differentiating with respect to I, yields $\partial\bar{\pi}^*/\partial I=0$ and $\partial\bar{Z}^*/\partial I=0$. The result implies that optimal contracts along the contract curve for increasing I represent parieto optimal contracts, because profits are neither increasing or decreasing for borrower or lender. The result contrasts with Jaffee and Modigliani and Kalay and Rabinovitch, who find that marginal profits are increasing for lenders along the contract curve.

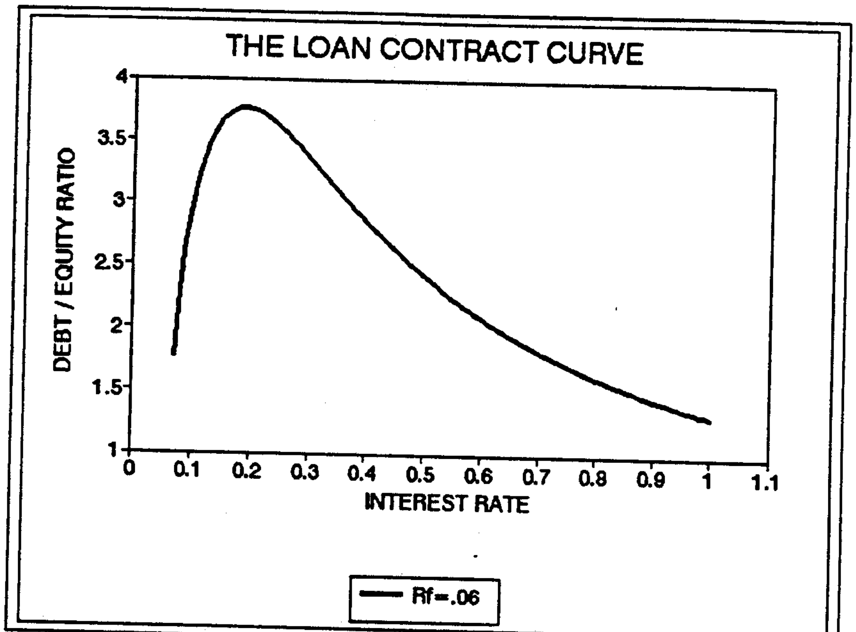

Figure 1

***Corollary I:* Given perfect information about $F(\bullet)$, and $R_f=\delta$, parieto optimal loan contracts are not characterized by any credit rationing equilibria: that is given propositions I and II both borrowers and lenders can have their respective loan demands and supplies satisfied at the prevailing market rate of interest. Furthermore since these parieto optimal outcomes are also competitive equilibria,lenders would never have to act as discriminating monopolists, as described in Jaffee and Modigliani.**

***Proposition III:* For any given I, borrower profits increase as debt is varied any direction from the optimum while lender profits decrease.**

Proof: From (7) it is found that $\partial^2\bar{\pi}/\partial D^2=IF'(r^2)r^*_D > 0$ for the borrower, and from (11'), $\partial^2\bar{Z}/\partial D^2=-IF'(r^*)r^*_D<0$. The results imply that borrower's profits are convex in (θ,D) while lenders' profits are concave in (θ,B).

***Proposition IV:* Debt and equity are gross substitutes for borrowers and gross compliments for lenders.**

Proof: First from (7'), $\partial^2\pi/\partial D\partial\theta=IF'(r^*)r^*_\theta < 0$, for $r^*_\theta < 0$. Second from (11'), $\partial^2\bar{Z}/\partial B\partial\theta=-IF'(r^*)r^*_\theta > 0$. The proposition is an interesting one for it provides the key motivation for borrower behavior. When the contract curve is upward sloping borrowers increase debt for each dollar of equity, thereby transferring equity into risk-free personal holdings which are bankruptcy protected. The preference to the borrower is to leverage increased financial risk, and risk of default, through increased personal holdings in riskless assets. As interest rates increase there is a point where financial risk is so high, that in order to maintain profits, equity is transferred from personal holdings to the risky investment so that the proportion of debt relative to equity in risky investments decrease.

In contrast, lenders would prefer to see an increase in borrower provided equity, since this would reduce expected losses due to default. Lenders would be willing to increase the amount of debt available if the lender were to provide more equity. Azzi and Cox; Smith; Baltsenberger; Stiglitz and Weiss; and Jaffee all provide collateral-based arguments supporting this proposition.

However, the result is indicative of conflicting objectives in the borrower-lender relationship. This conflict arises from the concavity-convexity conditions outlined in Proposition III. Should the lender demand more equity for a given amount of debt the borrower would, all other things being equal, anticipate a marginal decrease in profits and would thus respond by decreasing debt until marginal profits are zero. Likewise, lenders facing a demand for debt which is disproportionate to equity contributed, would anticipate a marginal decrease in expected profits, and respond by increasing the equity requirement until marginal profits are zero. Through this mechanism a single, zero marginal profit contract curve emerges. Indeed, competitive equilibria, must be described by this process if loan contract outcomes are to be parieto optimal. This notion of equilibrium is consistent with Smith.

Proposition V: All other things held constant, as R_f increases relative to δ, lenders would be rationed for all l, and as δ increases relative to R_f borrowers would face credit rationing for all l.

Proof: As defined by Jaffee and Modigliani, credit rationing occurs to borrowers if they cannot obtain all that is demanded at the prevailing market price, and in Jaffee, lenders are rationed if they cannot supply the optimum amount of debt at the prevailing market price.

Given (7) and (11) and Corollary I, parieto optimal, competitive equilibrium solutions occur along a single contract curve for both borrower and lender, if $R_f=\delta$. However, there are many frictions in the market which may cause δ to differ from R_f. For example, variations in deposit rates, excessive costs of servicing and monitoring may all rise, at least in the shortrun, to affect equilibrium. When a parieto optimal equilibrium does not exist then supply and demand can be identified along uniquely identifiable curves. That is for $R_f\neq\delta$ the risk premiums $(l-R_f)\neq(l-\delta)$.

The effect on optimal leverage can be seen by noting that $\partial F(\bullet)/\partial R_f=\partial F(\bullet)/\partial\delta=-1/l<0$ sos that the optimal probability of default must decrease. Thus, as illustrated in Figure 2, the demand curve will lie everywhere above the supply curve is $R_f>\delta$, and everywhere below for $R_f<\delta$. The characteristics of this problem prohibit parieto efficient-competitive equilibrium solutions from occurring, and depending on Sign$(R_f-\delta)$ either borrowers or lenders would be rationed for all l.

Proposition VI: Parieto optimal loan contracts require complete agreement between borrower and lender as to the exact probability distribution which characterizes $\bar{R}$.

Proof: The proof is by example. Suppose that borrowers and lenders are in fundamental agreement about $\bar{R}$ and its variance σ^2_R, but do not subjectively agree on the nature of the underlying probability distribution. Assume that the borrower perceives risk to follow a two-parameter logistic distribution of the form $F(r^*)=1/(1+e^{(r^*-\bar{R})/\sigma})$, while the lender assumes a more even dispersement of probabilities as suggested by the uniform distribution $F(r^*)=(r^*-a)/(b-a)$ where $a=\bar{R}-3\sigma$ and $b=\bar{R}+3\sigma$.

For $R_f=\delta=.06$, $\sigma=.15$, and $\bar{R}=1.15$, the two contract curves are depicted in Figure 3. For the borrower, the lender's perception of risk would be sufficient to ration credit t all points to the left of the intersecting curves, while the lender would be rationed at all points to the right. Likewise, if it were the lender perceiving the logistic distribution and the borrower, the uniform, rationing criteria would be reversed.

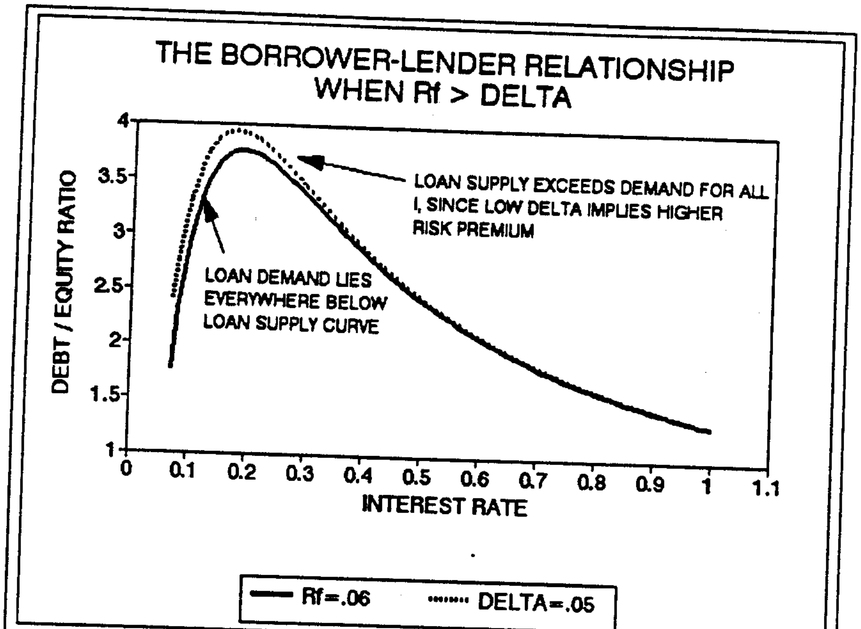

Figure 2

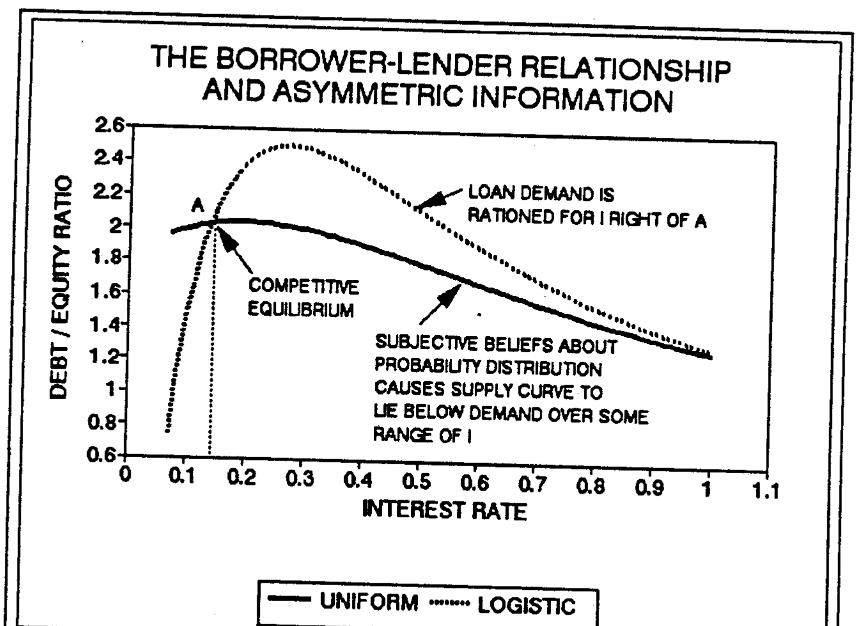

Figure 3

That differences in risk perception, or asymmetric information about outcome probabilities, can lead to different contract curves provides the following corollaries.

***Corollary II:* When lenders and borrowers differ in their perception of risk distinct loan demand and supply curves can be defined, and**

***Corollary III:* Differences in risk perception between the borrower and lender can lead to outcomes for which either the borrower or lender are rationed over some range of interest rates.**

Proposition VII: An exogenous increase (decrease) in downside-risk holding I constant, leads to a corresponding decrease (increase) in the contracted loan amount.

Proof: Assume a mean preserving transformation which either increases downside risk through increased variance, or increases downside risk through allocating probabilities from the upper to lower end of the probability distribution as in Rothschild and Stiglitz or Menezes et al. Then the transformed density function $G(r^*)$ must be greater than the original distribution. Let $dF(r^*)=G(r^*)-F(r^*)>0$. Then from (7') or (11'), $dD/dF(\bullet)=-1/F'(r^*)r^*_D<0$. The result implies that loan contract curves shift down (up) as downside risk increases (decreases).

Proposition VIII: An exogenous increase (decrease) in downside-risk, holding debt constant, leads to an increase (decrease) in the contracted loan rate.

Proof: Using the same assumptions as Proposition VII, then from (7') or (11') $dI/dF(\bullet)=I^2/R_f=I^2/\delta>0$. Propositions VII and VIII are depicted in Figure 4, for a logistic distribution with $\bar{R}=1.10$, and $\sigma=.10$, .15 and .20. By changing the variance, contract curves shift down and to the right, thus confirming the propositions. We have also the following corollaries:

Corollary IV: As downside-risk increases (decreases) the slope along the loan contract curve becomes flatter (steeper).

Proof: Define the conditional probability of default by $\lambda(r^*)=F'(r^*)/(1-F(r^*))$, then divide the numerator and denominator of equation (12) by $(1-F(r^*))$ to get $dD/dI=(1-r^*\lambda(r^*))/I\lambda(r^*)r^*_D$. Then $d^2D/dId\lambda(\bullet)=-\lambda(r^*)^{-2}<0$. Thus, high-risk contracts are likely to be less responsive to increasing interest rates than low-risk contracts. Evidence of this is provided in Figure 4, as risk increases the slope of the contract curves become flatter.

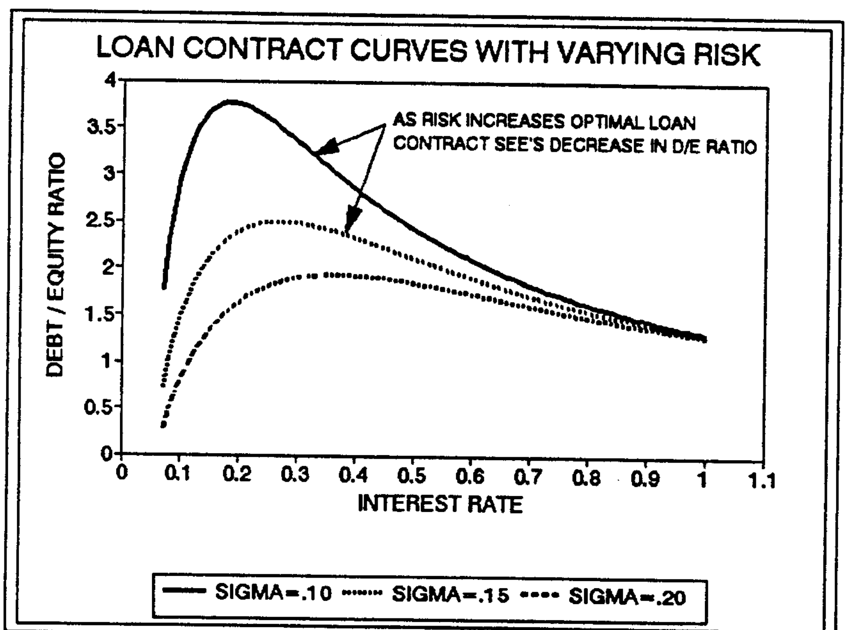

Figure 4

Corollary V: Mutually identifiable groups of borrowers with distinct, measurable, and heterogenous investment risk profiles can be separated into different loan classifications by lenders.

The corollary follows proportions VI and VII and Corollary IV. If we assume, as do Stiglitz and Weiss and Bester, that single lenders with multiple borrowers cannot due to asymmetric information possibly identify all default risks *a priori* then they face a problem of adverse selection. Then lenders will use screening or signalling devices in order to identify borrower' risks and create risk pools. Formal credit evaluation techniques such as credit scoring can be used to create such pools, and can be used to reduce the number of adversely selected borrowers.

Credit scoring is one such screening device. Since credit scoring models are used to estimate $F(r^*)$, the probability default, they can be used, according to (7) to identify diverse groups for which $F_1(I)>F_2(I)>F_3(I)$, where the subscripts represent customer types and the probabilities are measured relative to a given interest rate. This too is illustrated in Figure 4 where the three curves can be viewed as contract curves for three different risk groups. Through screening devices, such as credit scoring, lenders can offer different loan contracts. If properly identified, these contracts will be parieto optimal (Bester). However, if either type I error (i.e., a good loan is rejected), or Type II error (i.e., a bad loan is accepted), then credit rationing outcomes, consistent with adversely selected groupings can occur.

Proposition IX: For profit maximizing lenders and borrowers with self-imposed (or safety-first) credit risk constraints, unique demand and supply curves, can be defined with the loan demand curves being everywhere downward sloping.

Proof: Set the right-hand side of equation (7) equal to probability $\bar{F}$, such that $F(r^*)\leq\bar{F}$. Total differentiation yields $dD/dI=-r^*_I/r^*_R=-D(\theta W_o+D)/I\theta W_o<0$ and $d^2D/dI^2>0$.

This situation is presented in Figure 5 which shows the lender supply curve for the logistic distribution with $\bar{R}$=1.15, σ=.15, and probability limits equal to .05, .10, and .15, respectively. All points on the demand curves reflect equal probabilities for all (D,I) combinations. At points to the left of the demand-supply intersection borrowers would appear to be rationed by the lender. However, since the nature of the constraint is an $\leq$ inequality there is no reason to suspect that the iso-risk demand curves would be binding. Thus, loan contracts would follow along the lender's supply curve until the constraint becomes binding at the loan demand-supply intersection. Points to the right of this intersection indicate that borrowers ration lenders. A reverse proposition holds for profit maximizing borrowers and risk constrained lenders. If both borrowers and lenders impose internal risk constraints one of the two will be rationed unless risk constraints are equal.

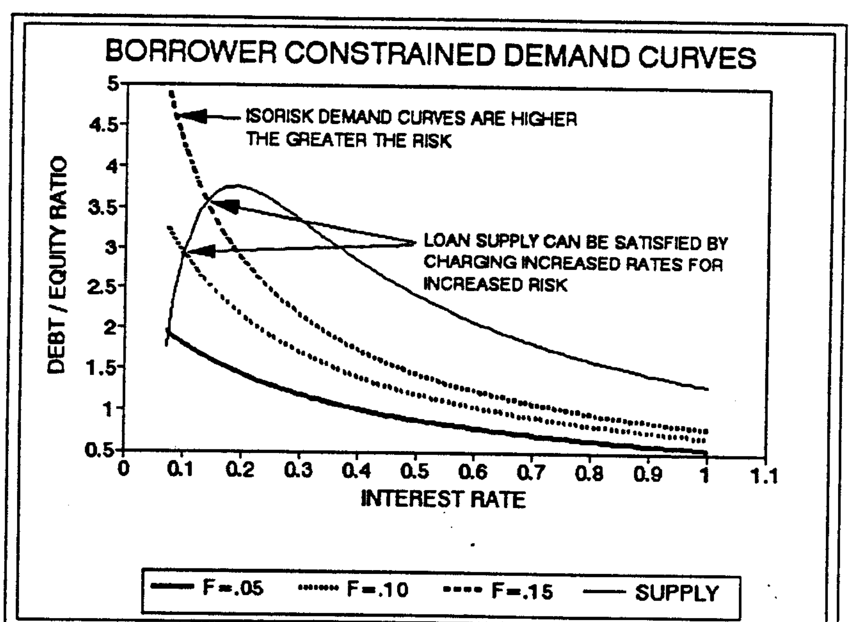

Figure 5

The notion of safety-first constraints implies the existence of a shadow-price for risk, which represents profits foregone by borrowing below the parieto optimal contract curve. This would appear irrational in a frictionless economy, but in reality there are many reasons why borrowers would impose such constraints including prohibition on future debt use which would come at a much higher opportunity cost, the loss of reputation due to public embarrassment of perceived failure, or risk. Aversion regulatory controls on maximum loan-loss provisions.

Loan Demand and the Empirical Significance of Credit Scoring

From the theoretical model of Section I, and the above results, it is clear that loan demand is indistinguishable from loan supply, even for disequilibrium loan contracts. From an empirical perspective, data on loans is generally censored because neither the actual loan request, the maximum loan possible, or loans applications denied are not generally observable. What is observed are the contract amounts at stated interest rates. Thus it is virtually impossible to directly test for the significance of internal or external credit rationing on the borrower/lender relationship.

However, it is possible to use loans contract data to estimate the two types of loan contract curves discussed if loan demand is estimated in conjunction with a credit scoring model, and loan amount and interest rates are included as arguments in both models. This is the intention of this section. In particular this section estimate for farm credit corporation loans data, a logistic credit scoring function and a quadratic loan contract equation, and thus derives estimates of the loan contract curve and iso-risk contract curves.

A total of 8,451 mortgage and refinancing loan records are used in the analyses covering about 25 percent of all FCC loans made between 1981 and 1988. Data used represent only those loans which were still outstanding as of January 1, 1992, were clean of possible errors, and had maintained the original loan.[3] The status of loans were categorized as being current, or noncurrent as at January 1, 1992, with noncurrent loans being defined as those being over $500 in arrears. No data on actual loan default was available[4], hence credit risk is proxied by the probability of being in arrears.

Agriculture, unlike most other industries, has the unique characteristic that seemingly homogenous products are produced with varying degrees of risk depending on the region of production, soil fertility and quality, heat, sunlight, moisture and other environmental/ecological factors. Hence it is questionable as to whether or not a single aggregate demand for loans is reasonable: As posited in the theoretical section investment classes defined by differential means and variance would be characterized by different levels of demand, all other things being equal.

Similarly, since the FCC is a federal lending agency its loan portfolio includes loans made to different types of farming. For example, cash crops - mostly grains and oilseeds, dairy and poultry, beef and hogs, and other horticultural, specialized farming operations must be served. Each one of these farming types faces unique risks which are not entirely systematically related to the others. Furthermore, similar commodities grown in different regions, have different associated risk, (Turvey; Turvey and Brown). This heterogeneity of risks across commodities and regions provides a unique opportunity to empirically investigate the borrower/lender relationship.

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3 Loans outstanding and free of error were 12,229. However, some loans had been adjusted, added to, or restructured over time. Because the nature of the loan adjustement was unknown, these loans were eliminated.

4 Approximately 20,000 of the loans made between 1981 and 1988 were retired, but there was no indication which of these were paid out or loan losses.

Accordingly the loans were categorized into four different regions, Pacific (British Columbia and Alberta), Prairies (Saskatchewan and Manitoba), Central (Ontario and Quebec) and Atlantic (Newfoundland, Price Edward Island, New Brunswick and Nova Scotia). Farm types were categorized into cashcrops (grains and oilseeds), supply managed[5] (dairy and poultry), livestock (hogs and beef) and other. Each region is defined by these commodities so there are a total of 16 commodity-region combinations. Some sample statistics are presented in Table 1.

The econometric formulation of the estimated loan contract curves is defined by two equations. The first equation is a logistic credit scoring model which includes variables reflecting liquidity, solvency, profitability, repayment capacity, security, and farm-type and regional risk differentials. It is of the for

$$F(Z) = 1/(1+e^{-z}) + \varepsilon \tag{14}$$

where

$$\begin{aligned} Z = {} & \propto_o + \propto_1 LR + \propto_2 DA + \propto_3 DCM + \propto_4 ROA \\ & + \propto_5 RR + \propto_6 LS + \propto_7 LOAN + \propto_8 RATE \\ & + \sum_{i=1}^{4} \sum_{j=1}^{4} \propto_{8+k} DR_iF_j + \sum_{y=1}^{7} \propto_{24+y} DY_y \end{aligned} \tag{15}$$

and LR is the liquidity ratio; DA is debt-asset ratio; DCM is the change in contribution margin; ROA is the return on assets; RR is the repayment ratio; LS is the loan to security ratio; LOAN is the loan contract amount; and RATE is the interest rate charged on the loan. The 16 dummy variables DR_iF_j represent covariance between regions (R_i) and farm type (F_j) while the dummy variables DY_y represent the year in which the loans were made. To avoid singularity a dummy variable for loans made in 1981 was not included.

The loan demand (contract) curve was estimated using a simple quadratic form:

$$\begin{aligned} LOAN = {} & B_o + B_1 RATE + B_2\, RATE^2 + B_3\, F(Z) \\ & + \sum_{i=1}^{4} \sum_{j=1}^{4} B_{5+r}\, DR_i\, F_j + \sum_{y=1}^{7} B_{17+y}\, DY_y + e\,. \end{aligned} \tag{16}$$

In (16), F(Z) is the estimated probability of default as proxied by the probability of a loan being noncurrent. The predicted value of (15), $\hat{F}(Z)$, is used as an instrumental variable in this equation. Both equations were estimated in SHAZAM (White, et.al.) with equation (16) being corrected for heteroscedasticity.

Results

Econometric results are presented in Table 2. As would be expected, loan default probabilities decrease with increased liquidity (LR), profitability (ROA), repayment ability (RR), and security (LS), and increase with financial leverage (DA), change in contribution margin (DCM), absolute loan amount (LOAN), nominal interest rate (RATE), and with refinancing (REFIN). Relative to loans made in 1981, loan default probabilities were higher for loans made in 1982 and 1985, but none of the year dummy variables were significantly different from zero at the five

[5] Supply managed commodities such as dairy, broilers, eggs and turkeys, face institutional quotas on individual farm and aggregate production. Farmers must purchase quotas at a fair market price, and are penalized for over producing. However, farmers holding quota receive monopoly-like prices, and hence tend to earn super-normal profits at low risk.

Table 1. **Sample Statistics for Empirical Model**

Risk Class	OBS	F(Z)	LR	DA	DCM	ROA	RR	LS	Loan Amount	Interest Rate
					Pacific					
Crops	777	.496	5.46	.512	.084	.053	1.27	.685	183400	10.82
	-	-	(3.94)	(.205)	(.184)	(.038)	(.399)	(.157)	(130150)	(2.94)
Dairy/Poultry	162	.198	5.15	.565	.015	.076	1.345	.724	237860	9.285
	-	-	(.399)	(.189)	(.101)	(.031)	(.369)	(.150)	(162240)	(3.002)
Livestock	366	.415	5.66	.483	.088	.053	1.343	.703	164970	10.784
	-	-	(3.784)	(.212)	(.194)	(.039)	(.511)	(.143)	(118080)	(2.868)
Other	53	.453	5.46	.477	.082	.055	1.261	.691	150450	11.398
	-	-	(3.89)	(.299)	(.184)	(.034)	(.356)	(.154)	(94338)	(2.951)
					Prairie					
Crops	4,139	.527	6.039	.414	.049	.056	1.289	.651	129300	11.458
	-	-	(3.73)	(.221)	(.144)	(.039)	(.336)	(.129	(91064)	(2.55)
Dairy/Poultry	124	.250	4.896	.518	.041	.08	1.325	.708	158130	11.006
	-	-	(3.82)	(.225)	(.120)	(.032)	(.326)	(.138)	(114360)	(2.883)
Livestock	486	.496	5.467	.444	.082	.061	1.367	.664	122750	11.210
	-	-	(3.701)	(.225)	(.172)	(.04)	(.35)	(.163)	(95236)	(2.53)
Other	68	.544	5.491	.462	.110	.065	1.335	.713	108120	12.076
	-	-	(4.078)	(.187)	(.148)	(.049)	(.336)	(.134)	(87762)	(2.035)
					Central					
Crops	606	.256	3.799	.544	.085	.085	1.310	.690	162840	11.897
	-	-	(3.815)	(.240)	(.165)	(.047)	(.441)	(.170)	(118910)	(2.059)
Dairy/Poultry	821	.107	4.284	.478	.019	.061	1.305	.677	149810	10.399
	-	-	(3.958)	(.197)	(.102)	(.026)	(.386)	(.179)	(106940)	(2.732)
Livestock	485	.177	4.094	.593	.054	.072	1.420	.737	123140	10.971
	-	-	(3.562)	(.231)	(.163)	(.045)	(.490)	(.156)	(85201)	(2.727)
Other	68	.324	4.068	.529	.115	.056	1.447	.755	106960	12.615
	-	-	(3.847)	(.268)	(.228)	(.058)	(.496)	(.139)	(72445)	(1.441)
					Atlantic					
Crops	119	.319	3.369	.513	.052	.0060	1.411	.700	115030	11.186
	-	-	(3.294)	(.189)	(.171)	(.046)	(.616)	(.160)	(97128)	(2.727)
Dairy/Poultry	99	.131	4.806	.422	.005	.054	1.297	.676	76037	11.419
	-	-	(4.116)	(.198)	(.092)	(.029)	(.316)	(.181)	(56970)	(2.342)
Livestock	46	.239	4.736	.474	.015	.040	1.381	.686	77204	11.57
	-	-	(3.753)	(.233)	(.110)	(.048)	(.392)	(.162)	(74101)	(2.234)
Other	4	.50	3.564	.459	-0.79	.056	1.356	.709	63000	12.350
	-	-	(4.303)	(.076)	(.137)	(.038)	(.229)	(.068)	(59099)	(.900)

percent level of confidence. The regional-farm type dummy variables indicate that the most risky loans are those made to Saskatchewan cash-crop farmers. In fact with the exception of the supply managed commodities in that province, Saskatchewan loans tend to be more risky than any other region in Canada. The supply managed commodities, dairy and poultry are in each region prone to the least amount of credit risk, while livestock and other crops pose modest credit risks.

Table 2. Results of Econometric Models

	Probability Model		Loan Demand Equation	
Variable	Coefficient	Standard Error	Coefficient	Standard Error
Constant	-2.542	.679	85735	22409
LR	-.033	.069	-	-
DA	.961	.152	-	-
DCM	.727	.164	-	-
ROA	-.304	.697*	-	-
RR	-.436	.084	-	-
LS	.457	.194	-	-
LOAN	.225E-5	.271E-6	-	-
RATE	.127	.018	-10570	4149
$RATE^2$	-	-	-411.13	232.77
REFIN	.507	.078	-	-
F(Z)	-	-	519480	10501
DPC	.876	.452	-53057	12473
DPS	-.415	.492*	118470	15605
DPL	.622	.460*	-29338	12719
DPO	.701	.531*	-50142	15837
DPRC	1.266	.447	-112400	12181
DPRS	-.299	.498*	54386	14019
DPRL	1.139	.456	-108110	12564
DPRO	1.056	.516	-116880	14795
DCC	-.605	.456*	75766	12601
DCS	-1.136	.460	96791	12433
DCL	-.786	.462	54550	12442
DCO	-.216	.525	10023	14820*
DAC	.108	.491*	-17738	14293*
DAS	-.813	.541*	35192	12977
DAL	-.197	.574*	-14907	15593
DAO	.778	1.136*	-125040	13301
D82	.051	.410*	-17466	41492
D83	-.244	.403*	57995	232.77*
D84	-.227	.411*	48483	8631.90
D85	.027	.416*	17019	8247.1
D86	-.635	.409*	86725	8981.6
D87	-.679	.409*	79096	9001.8*
D88	-.812	.436*	88812	8851.3
R^2	.20		.41	
Prediction Success				
Overall	61.3%			
Type = 0	56.9%			
Type = 1	53.3%			

The overall prediction accuracy of the credit scoring model is 61.3 percent, with 66.9 percent of current loans being correctly predicted, and 53.3 percent of noncurrent loans being correctly predicted. Typically, it is more difficult to predict bad loans because extraneous factors (such as death, divorce, drought, trade wars), etc., are much more difficult to assess *a priori* than objective measures of liquidity, profitability and solvency, etc.

The predicted values estimated by the credit scoring model were used, in the second stage, as an instrumental variable in the loan demand (contract) function. However, there is a major flaw in the procedure which cannot easily be overcome, and this may cause bias. The flaw is that the estimated logistic probabilities are defined in terms of posterior probabilities, whereas realistic credit scoring and credit rationing are theoretically dependent on prior probabilities. Thus it is assumed that the posterior, conditional probability estimates are perfectly correlated with subjectively or objectively determined prior probabilities.

The demand equation, corrected for heteroscedasticity, had an adjusted R^2 of 41.1 percent which is fairly high for cross-sectional data. Holding risk constant the demand elasticity (flexibility) is given by $E_I = -(10570\ \mathrm{RATE} + 411.13\ \mathrm{RATE}^2)/\mathrm{LOAN}$. When risk is variable (i.e., $\partial\hat{F}(Z)/\partial\mathrm{RATE} \neq 0$ and $\partial\hat{F}(Z)/\partial\mathrm{LOAN} \neq 0$) the elasticity is determined by the total derivative (dD/dI) rather than the partial derivative, $\partial D/\partial I$.

The total derivative is given by

$$\frac{dLOAN}{dRATE} = \frac{(B_1 + 2B_2\ RATE + B_3\ \partial\hat{F}(Z)/\partial RATE)}{(1 - B_3\ \partial\hat{F}(Z)/\partial LOAN)} \tag{17}$$

where $\partial\hat{F}(Z)/\partial\mathrm{RATE} = \alpha_8(1-\hat{F}(Z))\hat{F}(Z)$ and $\partial\hat{F}(Z)/\partial\mathrm{LOAN} = \alpha_7\ (1-\hat{F}(Z))\hat{F}(Z)$.
From Table 2 the estimated equation (16) B_1 = -10520, B_2 = -411.13, and B_3 = 519,480, and from equation (15), α_8 = .127, and α_7 = .226E-5. Hence

$$\frac{dLOAN}{dRATE} = -\frac{(10570+ 822.26\ RATE - (519{,}480)(.127)\ 1-\hat{F}(Z)))\ F(Z)))}{(1 - (519{,}480)\ (.00000226)\ (1-\hat{F}(Z))\ \hat{F}(Z))} \tag{18}$$

Since the denominator in (18) is, by these estimates, always positive, the shape of the demand curve is determined by the numerator. As proposed in the theoretical section, the demand curve is downward sloping when risk is held constant so it is the impact of credit risk on demand which causes a backward-bending curve. This possibility is allowed by equation (18) which will be positive (negative) when (10570 + 822.26 RATE) < (>) $(519{,}480)(-127)(1-\hat{F}(Z))F(Z)$.

In Figure 6 we plot the derived demand for credit for cash-crop farms in the Pacific, Prairie, and Central regions of Canada, as well as the average loan demand function. The first point of interest is that Prairie cash crops shows a backward-bending contract curve: up to approximately nine percent interest loan demand is increasing. Cashcrop loans in Ontario and Quebec seem to be more responsive to changes in interest rates than Prairie and Pacific which are equally responsive, except that Pacific region loans tend to be higher on average.

What is important about these contract curves is the different shapes they take. For relatively homogenous products (grains and oilseeds) regional characteristics, including risk, are significantly unique, to affect loan demand. Corresponding to this is the argument that regional supply of debt must also respond to regional and farm-type differences.

The elasticities, as derived in (17) and (18) are presented in Table 3 along with the average nominal interest rates, loan contract amounts and probabilities of default and success, used in the calculations. The fifth numerical column in Table 3 indicates the relative elasticities when default risk is excluded from the elasticity measure (i.e., equation (17)). These elasticities range from -.56

for Pacific region dairy producers to -3.067 for Atlantic producers of 'other' commodities. Evaluated at the means, all elasticities are negative. In general the 'other' category showed the more elastic response in all regions; that is a one percent increase in the nominal interest rate would decrease the loan amount by 1.16, 1.74, 1.85, and 3.067 percent for the Pacific, Prairie, Central, and Atlantic regions, respectively. With the exception of the Atlantic region, the smallest elasticity is attributed to dairy and poultry farms, which are the least risky of all commodity groups.

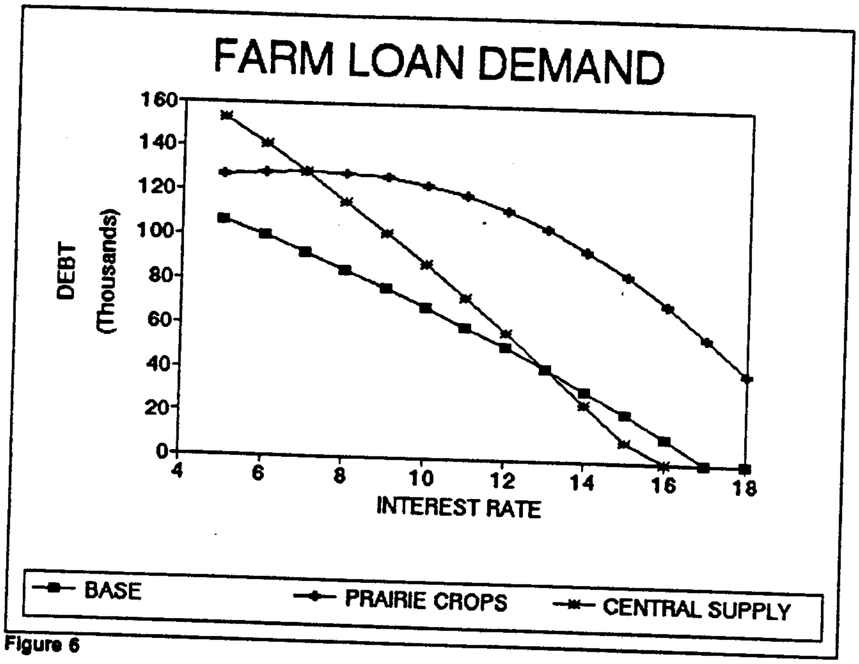

Figure 6

In the sixth numerical column in Table 3 we present the elasticities for loans which exclude the marginal change in default probabilities (i.e., equation (18)). The effect is to either increase or decrease the elasticity relative to those in column 5. Nine of the 16 classes showed a decrease in the elasticity amount. For example, the Central region crop elasticity falls from -1.35 to -1.18 when risk is included, whereas Central dairy farms showed an increase from -1.03 to -1.27.

That elasticities can either increase or decrease ought not to be surprising given the theoretical model. In essence, if default or bankruptcy risk is incorporated into the profit maximizing level of investment, then increasing the amount of debt -and the probability of default - may be optimal for some farmers. Recall that from the borrower's perspective, debt and equity are substitutes; for some farmers debt may be increasing as equity is decreasing, while for others, debt may be decreasing while equity is increasing.

Figure 7 shows the aggregate contract curve and two contract curves which have constant (hypothetical) risk of 20 and 30 percent, respectively. These iso-risk contract curves are more responsive to increased interest rates than the aggregate function. If we view the aggregate function as a demand curve and the 2 iso-risk curves as supply response functions consistent with credit scoring criteria, borrowers would be rationed for loans below and to the right of intersecting points. Alternatively, if the aggregate curve represents aggregate supply, and the two iso-risk curves borrower demand, then lenders would be rationed for all loans below and to the right of the intersecting points.

Table 3. Average Loan Demand (Contract) Elasticities

Variable	Probability of Default	Probability of Success	Loan Amount	Interest Rate	Default Risk Held Constant	Default Risk Allowed to Change
			Pacific			
Crops	.496	.503	183400	10.822	-.89	-.87
Dairy/Poultry	.197	.802	237860	9.285	-.56	-.55
Livestock	.415	.585	164970	10.784	-.98	-.59
Other	.453	.547	150450	11.398	-1.16	-.44
			Prairie			
Crops	.527	.472	129300	11.458	-1.35	-1.18
Dairy/Poultry	.250	.750	158130	11.006	-1.05	-.93
Livestock	.496	.504	122750	11.21	-1.39	-.26
Other	.544	.456	108120	12.076	-1.74	-1.51
			Central			
Crops	.256	.744	162840	11.897	-1.13	-1.40
Dairy/Poultry	.107	.893	149810	10.399	-1.03	-1.27
Livestock	.177	.823	123140	10.971	-1.34	-1.51
Other	.324	.676	106960	12.615	-1.85	-1.89
			Atlantic			
Crops	.314	.681	115030	11.186	-1.475	-1.74
Dairy/Poultry	.131	.869	76037	11.419	-2.29	-2.79
Livestock	.239	.761	77204	11.574	-2.29	-2.13
Other	.500	.500	63000	12.35	-3.067	-4.06

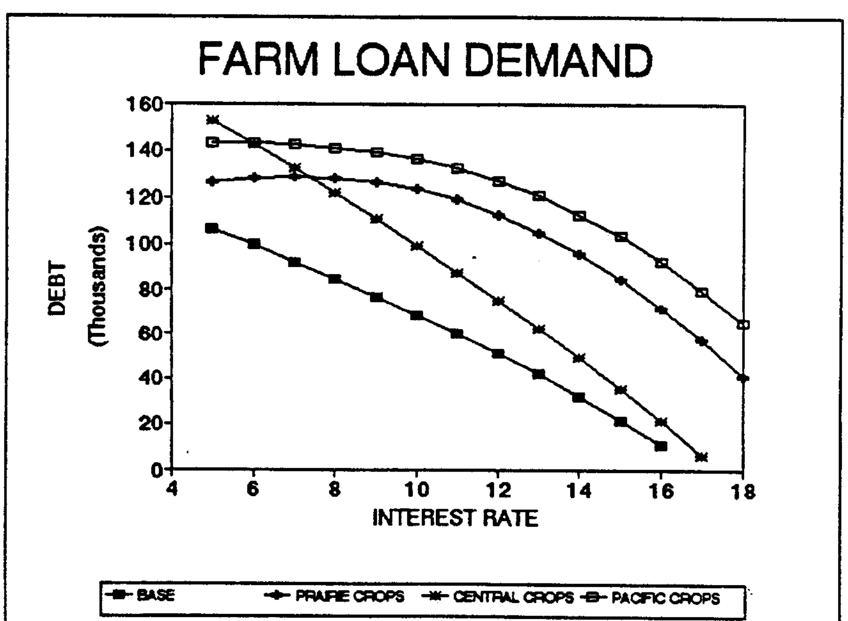

Figure 7

The iso-risk curves in Figure 7 are derived from the theoretically consistent loan contract curves and as such do not truly reflect demand or supply, nor is it possible to determine whether, for any given rate of interest, the borrower is being rationed, or the lender is being rationed or if any rationing is taking place at all. However, the derived iso-risk curves do span all points along the loan contract curve and thus they span the distribution of loan-rate observations about the average loan. It is thus feasible that the aggregate loan contract curve reflects loan-interest rate combinations for loans which are rationed by either the borrower or the lender. We are not, given unavailable data on lender-borrower intent, able to prove this and it is thus treated here as conjecture only.

Conclusions

This research developed first a series of propositions about the lender-borrower relationship and used these to develop and interpret empirical loan demand (contract) equations for Canadian Farm Credit Corporation Loans. From the theoretical model it was shown that loan demand and/or contract curves can have backward bending properties which are the result of bankruptcy probabilities. For the borrower, debt and equity are substitutes so that over some range of contracts debt relative to equity actually increases for risky investment, while personal holdings of risk-free investments, protected from bankruptcy by limited liability increases. Asymmetric information was posited as one reason why distinguishable loan demand and loan supply curves could emerge. It was also noted that loan demands differ substantially across different loan-type classifications, and asymmetric information can be resolved by identifying separate groups of borrowers.

From an empirical perspective, the irrefutable conclusion that loan demands cannot be estimated in isolation of default probabilities is an important one. In fact, screening devices such as credit scoring are employed by lenders to lessen informational asymmetries. For a contract to occur there must then be subjective agreement between the borrower and lender on the probability of success. This property is then used to estimate a loan contract function which includes a measure of loan default probability as an endogenous variable, where the loan default probability is estimated from an empirical credit scoring (logistic) regression.

The results confirm, for 16 pooled risk classes representing different regions and commodity groupings across Canada, that indeed different demand functions emerge according to risk. From a lender's perspective it is important to identify how loan demand might change given an increase in the nominal rate. For the data used in this study, elasticities ranged from a low of -.55 to a high of -4.06. It was also shown that for some classes, recognition of the marginal changes in default probabilities caused an increase in elasticity measures, while for others a decrease. This result is consistent with the theoretical model.

There are several points on which to conclude. First, the notion of farm level loan demand, as defined by the lender-borrower relationship should account for loan default probabilities, which differ across farm regions and types. Second, it is unlikely that any single loan demand curve exists which can describe the entire industry without taking into account farm type and regional differences. Finally, lenders should recognize that heterogenous risks imply heterogenous demands, and thus should be prepared to offer multiple loan contracts to each distinctive risk classes.

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