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# ASSET PRICING AND PORTFOLIO FUNDING STRATEGIES AT FARM CREDIT BANKS

Ananth Rao, Glenn Pederson, and Michael Boehlje

Bank earnings and capital positions are continuously changing due to variability of interest rates and interest rate risk management requires timely adjustments in the mix, level, and flow characteristics of assets and liabilities. Thus, interest rate risk becomes a strategic decision problem for bank management. Previous applied research has focussed on static management of bank assets or liabilities, but has not considered the integration of asset and liability management decisions in a dynamic and stochastic context. The bank decision problem lends itself to application of dynamic analysis, since banks make these adjustments in response to uncertainty in the economy.

The primary objectives of our paper are:

- 1) to model bank investment and funding decisions as a financial system, and
- 2) to evaluate the system model using an optimal stochastic control framework.

We address these objectives in four sections. Initially, we model asset rate decisions (using a multiple-factor, asset pricing framework), prepayment behavior, and debt callability. Second, the methodology for stochastic optimal control analysis is briefly presented. Third, we present empirical results based on an econometric estimation of the financial system in a Farm Credit Bank. Finally, the econometric model is simulated using the stochastic optimal control framework and historical data.

## CONCEPTUAL FRAMEWORK

We assume that all farm nonreal estate investments are short-term (with maturities less than one year) and all farm real estate investments carry maturities greater than one year.

### Asset Rate Decisions

The bank's decision to invest is made under conditions of uncertainty about future returns. Therefore, the borrower must compensate the bank with an economic return consistent with the bank's past performance and future outlook. The bank approximates its return expectations from risky investments by adding a risk premium to the expected rate of return on a riskless alternative investment (e.g., Treasury securities).

Multiple risk factors in agriculture motivated us to adopt a multi-beta framework in modelling loan rate decisions. Under a multi-factor asset pricing framework, the return on any risky asset is a linear combination of various common factors that affect asset returns.

In the case of a bank, these common factors could be; 1) a riskless rate of return, 2) a market rate of return, 3) default risk associated with loans, and 4) loan prepayment risk. Default risk in agricultural loans results from fluctuating farm incomes, land values and commodity prices, which are influenced in part by macroeconomic variables such as inflation. Similarly, prepayment risk in agricultural loans results from fluctuations in loan interest rates which are a function of market rates. We consider these two risks as components of systematic risk for which banks require a risk premium as compensation to invest.

### Expected Loan Prepayment

Long-term loan rates are affected by expected Treasury rates (as an indicator of riskless rates), expected market rates, expected default rates on loans, and expected loan prepayment rates (unscheduled principal payments). The expected prepayment rate assumes greater importance in the pricing of long-term fixed-rate investments, because variability of these prepayment amounts makes the cash flow actually received from a long-term investment highly uncertain.

Prepayment behavior has been modelled extensively in secondary mortgage markets. The actual cash flow a bank receives from a mortgage security depends on the particular termination pattern of individual mortgages making up the pool and real estate mortgagors employ a penalty schedule for early prepayments to discourage prepayments. Recently, the use of conditional prepayment rates (also known as constant prepayment rates or CPRs) has gained popularity as a technique for analyzing prepayments in secondary mortgage markets. The CPR is the percentage of principal (outstanding at the beginning of a period) that prepays during that period. At a constant interest rate level, prepayment on a new mortgage can be expected to increase during the first several years, largely as a function of mortgage age, before generally levelling off.

Hendershott and Buser found that the size of the prepayment penalty on loans depends on whether the interest rates are considered more likely to rise or fall and how uncertain rates are. When compared with the return on a nonprepayable loan portfolio, returns on prepayable loans are more sensitive to interest rate decreases than increases (a nonlinear and asymmetric response), because prepayment drastically shortens the duration of prepayable loans. For example, few borrowers are expected to prepay their mortgages if market rates drop 1 percentage point below comparable coupon rates, but a large number will prepay if market rates fall 4 percentage points below coupon rates. Thus, a significant increase in interest rate volatility is expected to alter the prepayment penalty. The relationship between the contract rate on the underlying loan mortgages and the mortgage rate currently available in the market has a dominant influence on prepayments and determines the incentive to refinance a high-rate mortgage.

## Callability of Debt

Callability is a risk associated with long-term fixed debt instruments in a falling interest rate environment. Provision could be made by the bank (at the time of issuing long-term debt) to call back high cost debt and refinance it at a lower rate when market interest rates are expected to fall. The finance literature suggests that the general level of interest rates exerts a direct effect on the yield required to include a call provision (Cook and Hendershott, Yawitz and Marshall). The basic argument is that when rates are at a historically high level, there is a greater than average probability that rates will fall some time after call protection expires, and that the firm will exercise its option to refinance the debt at more favorable terms. In order to compensate bondholders fully for this possibility a substantial yield premium may be required.

We use a simplified call (European) for a default-free bond as suggested by Yawitz and Marshall (Y-M). The terms on callable debt must be such that the investor has the same expected wealth (at the time the call is exercised) from either of two alternatives; 1) purchase only noncallable bonds, or 2) purchase identical callable bonds at a lower price and invest the difference. The initial price difference between the two bonds can be expressed as  $\Delta$ , the present value of the incremental coupon on the callable security discounted at the noncallable yield ( $r$ ). Computationally,

$$\Delta = \delta/r [ \{(1+r)^N - 1\}/(1+r)^N ] \quad (1)$$

where  $\delta$  is the coupon premium required for a callable bond to sell at par and  $N$  is the initial maturity of both bonds.

The purchaser of a callable bond has available  $\Delta$  at the time of purchase that can be invested to provide an accumulated value of  $\Delta(1+r)^M$ , after  $M$ -years of call protection. The accumulated value of coupons received is the same under both alternatives. The difference between the expected payoffs is

$$\gamma[E(P|\text{call}) - P_c] - \Delta(1+r)^M \quad (2)$$

where  $\gamma$  is the probability of call,  $P_c$  is the call price of the bond, and  $P$  is the uncertain price of the noncallable bond. We set (2) equal to zero to satisfy the requirement of investor indifference.

After combining and rearranging (1) and (2), the resulting expression for the incremental coupon is

$$\delta = [\gamma\{E(P|\text{call}) - P_c\} / (1+r)^M] * [\{r(1+r)^N/(1+r)^N - 1\}] \quad (3)$$

where the term in the first brackets is the present value of the expected loss from the call. The second brackets contain the interest factor that converts the expected loss to a coupon-

equivalent for an N-period bond. To derive an observable proxy for  $\delta$  in (3), values for M and N must be chosen, a decision must be made as to the rate series to use for r, and some means of estimating  $E(P|call)$  must be developed.

The theoretical framework developed by Y-M suggests that the rates used to determine  $\delta$  should correspond to noncallable bonds. Since  $E[(P|call) - P_c]$  is unobservable, Y-M assume that the relationship

$$E[(P|call) - P_c] = C + \alpha[p - 100] \quad (4)$$

is true. Here, C and  $\alpha$  are parameters to be estimated and p is a measure of  $E(P)$ , the unconditional expectation of P. The contingent liability is assumed to be linearly related to the difference between  $E(P)$  and the par value of the bond. Substitution of (4) into (3) and simplification yields the empirical model for estimating  $\delta$

$$\delta = \beta_0 + \beta_1 Y + \beta_2 K, \quad (5)$$

where  $Y = (p-100)[R_N(1+R_N)^N] / [(1+R_M)^M\{(1+R_N)^N - 1\}]$ ,  $K = Y/(p-100)$ , and  $\beta_1 = \alpha\gamma$ ,  $\beta_2 = C\gamma$ , and  $\beta_0$  is the intercept term. Finally, the associated forward rate is

$$p = 100R_N/E_{N-M} [1 - 1/(E_{N-M})^{N-M}] + 100/\{(1+E_{N-M})^{N-M}\} \quad (6)$$

where  $E_{N-M}$  is the (N-M) year forward rate inferred from the yield curve at time 0. Coefficient  $\beta_1$  provides an estimate of the size of the increase in coupon required on a callable debt instrument given a \$1 increase in the expected price of the underlying noncallable bond M-years hence.

## METHODOLOGY

We separate the preceding (and additional) variables into two groups; a vector of state variables (X), and a vector of control variables (U). State variables describe the financial system in the bank at any point in time. Control variables represent policy instruments which can be used by bank management. Table 1 provides an overview of the empirical model under development in this section.

Table 1. Asset/Liability Management System Specification

STATE VARIABLES	CONTROL VARIABLES	EXOGENOUS VARIABLES
<u>Short-term Assets:</u>	<u>Short-term Loan Rate:</u>	<u>Default Rate:</u>
$STA_t = STA_{t-1} + \text{New } STA_t - \text{STREP}_t$ where New $STA_t = f(\text{Const}, \text{EQUITY}_t, \text{STLR}, \text{INF}) + \epsilon$	$\text{STLR} = f(\text{Const}, \text{6-mo. Treas.}, \text{S\&P}, \text{DEFI})$	$\text{DEF} = f(\text{Const}, \text{LAND})$
Combining the two models yields,	<u>Long-term Loan Rate:</u>	<u>Long-term Repayment:</u>
$STA_t = f(\text{Const}, \text{EQUITY}_t, \text{STA}_{t-1}, \text{STLR}, \text{INF}, \text{STREP}_t) + \epsilon$		
<u>Long-term Assets:</u>		
$LTA_t = LTA_{t-1} + \text{New } LTA_t - \text{LTREP}_t - \text{VPP}_t$ where VPP is volume of prepayment, and New $LTA_t = f(\text{Const}, \text{PROFIT}_{t-1}, LTA_{t-1}, \text{LTLR}, \text{PPR}, \text{INF}, \text{LTREP}) + \epsilon$	$\text{LTLR} = f(\text{Const}, \text{10-yr. Treas.}, \text{S\&P}, \text{DEF}, \text{PPR})$ $\text{PPR} = f(\text{Const}, \text{SPREAD}, \text{SPREAD}^2)$	$\text{LTREP} = f(\text{Const}, LTA_{t-1}, \text{NFI}, \text{LAND})$
Combining the two models yields,	<u>Short-term Borrowing Rate:</u>	
$LTA_t = f(\text{Const}, LTA_{t-1}, \text{PROFIT}_{t-1}, \text{LTLR}, \text{PPR}, \text{INF}, \text{LTREP}) + \epsilon$		
<u>Short-term Liabilities:</u>		
$\text{STL}_t = \text{STL}_{t-1} + \text{New } \text{STL}_t - \text{STLRET}_t$ where New $\text{STL}_t = f(\text{Const}, \text{PROFIT}_{t-1}, \text{STBR}, \text{TREND}) + \epsilon$	$\text{STBR} = f(\text{Const}, \text{6-mo. Treas.})$	
Combining the two models yields,	<u>Long-term Borrowing Rate:</u>	<u>Retirement of Debt:</u>
$\text{STL}_t = f(\text{Const}, \text{PROFIT}_{t-1}, \text{STBR}, \text{STL}_{t-1}, \text{STLRET}_t, \text{TREND}) + \epsilon$		
<u>Long-term Liabilities:</u>		
$\text{LTL}_t = \text{LTL}_{t-1} + \text{New } \text{LTL}_t - \text{LTRET}_t$ where New $\text{LTL}_t = f(\text{Const}, \text{PROFIT}_{t-1}, \text{LTBR}, \text{CP}, LTA_{t-1}, \text{LTRET}, \text{TREND}) + \epsilon$	$\text{LTBR} = f(\text{Const}, \text{10-yr. Treas.})$ $\text{CP} = f(\text{Const}, Y, K)$	$\text{LTRET} = f(\text{Const}, \text{RET}_{t-1})$
Combining the two models yields,		
$\text{LTL}_t = f(\text{Const}, \text{LTL}_{t-1}, \text{PROFIT}_{t-1}, LTA_{t-1}, \text{LTBR}, \text{CP}, \text{LTRET}, \text{TREND}) + \epsilon$		
<u>Profit:</u>		<u>Non-interest Expenses:</u>
$\text{PROFIT}_t = f(\text{Const}, \text{STA}_t, LTA_t, \text{NOA}_t, \text{STL}_t, \text{LTL}_t, \text{NOE}_t) + \epsilon$		$\text{NOE} = f(\text{Const}, \text{STA}_t, LTA_t, \text{STL}_t, \text{LTL}_t)$
<u>Equity:</u>		
$\text{EQUITY}_t = \text{EQUITY}_{t-1} + \text{PROFIT}_t$		

Superscript † indicates that these variables are forecast using Bayesian vector autoregression.  
 Superscript \* indicates that these variables are derived from the option pricing framework.

### State Variables

State variables in period  $k$  are specified as; STA - short-term assets with maturity of 1 year or less ( $x_{1,k}$ ), LTA - long-term assets with maturity exceeding 1 year ( $x_{2,k}$ ), STL - short-term liability with maturity of 1 year or less ( $x_{3,k}$ ), LTL - long-term liability with maturity exceeding 1 year ( $x_{4,k}$ ), PROFIT - net profit ( $x_{5,k}$ ), and EQUITY - net equity ( $x_{6,k}$ ). These state variables represent bank balance sheet and income accounts. The objective of modelling agricultural banks is to observe and predict the behavior of the state vector over time. This requires the use of econometric modelling.

### Decision Variables

Decision variables are instruments of the decision process which are directly controllable by bank management. Given the set of time-dated requirements (such as a minimum level of capital) placed on the bank and the prevailing financial state, a decision strategy is needed which will cause the agricultural bank's state trajectory to be optimal in some sense. The decision vector ( $U_k$ ) represents the actions directly available in period  $k$  for control. The decision variables are broadly grouped into two categories. Loan pricing decision variables include: STLR - interest rate on short-term assets ( $u_{1,k}$ ), LTLR - interest rate on long-term assets ( $u_{2,k}$ ), PPR - expected prepayment rate ( $u_{3,k}$ ) in period  $k$ . Funding decision variables include: STBR - short-term liability rate ( $u_{4,k}$ ), LTBR - long-term liability rate ( $u_{5,k}$ ), and CP- call premium ( $u_{6,k}$ ) in period  $k$ .

Loan pricing and funding decisions are interrelated. It is assumed that STLR and LTLR are controllable by the bank. Assuming that the demand for loans is inelastic, banks set the appropriate interest rate on short- and long-term loans that clears the demand for these loans. Since variations in prepayments can have significant effect on market value of loan assets, the bank controls the PPR by estimating the expected prepayment rate and setting the LTLR, including a premium for prepayment risk. This is the practice prevailing in pricing Ginnie Mae mortgage pools (Arak and Goodman). Since prepayment has an influence on loan volume and loan rates, the expected prepayment rate is a control variable for the purpose of loan pricing and investment decisions.

It is assumed that the costs of funds (e.g., STBR and LTBR) are exogenous to the bank because of the requirement to compete with other credit institutions for deposits and other loanable funds at market rates. However, a bank can exert indirect control over the average rate paid for external funds by varying the maturities of bonds, notes, CDs, etc. and the volumes of those liabilities over time.

Based on the cost of funds and the demand for new loan volume, banks could elect to fund new loans either internally with equity or externally with debt. CP is the expected call premium for issuing callable debt. Similar to prepayment, callability is an interest rate risk management strategy. Should interest rates decline significantly, long-term bonds may be



immediately callable, otherwise the call is deferred for a period.<sup>1</sup> For example, suppose current market rates exhibit a downward-sloping yield curve. The bank could lend at the current short-term rate and borrow long-term by issuing callable bonds (with the expectation that short-term interest rates will fall in the future). Simultaneously, the mix and maturity of outstanding assets and liabilities undergo changes. If rates fall the debt is called and refinanced at the lower rate. If rates unexpectedly rise the call is allowed to expire and the short-term loan is rolled over at the higher rate.

### Exogenous Variables

Exogenous variables are predetermined and are not controllable by the bank. The vector of exogenous variables ( $Z_k$ ) is the set of regressors for the state variables. The set of exogenous variables is: INF - annual inflation rate ( $z_{1,k}$ ), LTREP - repayment of long-term assets ( $z_{2,k}$ ), LTRET - retirement of long-term liabilities ( $z_{3,k}$ ), NOA - net other assets ( $z_{4,k}$ ), NOE - non-interest operating expenses ( $z_{5,k}$ ), TREND - a trend variable ( $z_{6,k}$ ), and CONST - constant term ( $z_{7,k}$ ).

### Optimal Control Model

Uncertainty is commonly modelled along two lines; an additive error term ( $\epsilon_k$ ) and/or parameter uncertainty ( $\Theta_k$ ). Parameter uncertainty is typically treated with either of two assumptions (Kendrick). The simplest assumption is that the true parameters are constant over time, but that the estimates of the parameters are unknown and evolve through a stochastic process. The alternative is that the parameters are themselves stochastic (which is a more difficult problem to handle). Due to computational complexities involved in modelling stochastic parameters, we assume only parameter estimates are stochastic, but the true parameters are constant over time.

Formally, we state the bank asset/liability management decision problem as; choose the control path  $\{U_k\}$  to minimize the quadratic loss function (J)

$$\begin{aligned} \min J = E \{ & \frac{1}{2} [X_N - X_N^*]' W_N [X_N - X_N^*] + \sum_{k=0}^{N-1} \frac{1}{2} [X_k - X_k^*]' W_k [X_k - X_k^*] \\ & + \frac{1}{2} [U_k - U_k^*]' V_k [U_k - U_k^*], \end{aligned} \quad (7)$$

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<sup>1</sup>Typically, debt instruments issued for ten years are callable at par after seven years, seven year issues are callable at par in five years, and five year issues are callable at par in three years (Langerman and Gartland).

subject to the system of equations

$$\mathbf{X}_{k+1} = \mathbf{A}_k(\theta_k) \mathbf{X}_k + \mathbf{B}_k(\theta_k) \mathbf{U}_k + \mathbf{C}_k(\theta_k) \mathbf{Z}_k + \epsilon_k, \quad (8)$$

and  $\mathbf{X}_0$ , the initial conditions on state variables. The measurement relation of state variables is

$$\mathbf{M}_k = \mathbf{H}(\theta_k) \mathbf{X}_k + \mu_k, \quad (9)$$

and the random parameters and measurement errors follow a first-order Markov process

$$\theta_{k+1} = \mathbf{D}\theta_k + \eta_k. \quad (10)$$

We define notation in (7) through (10) as;

$E\{ \}$  is the expectation operator

$\theta_k$  is a vector of random parameters (6 x 1) in matrices  $\mathbf{A}_k$ ,  $\mathbf{B}_k$  and  $\mathbf{C}_k$ ,

$\mathbf{X}_k$  is a state variable vector (6 x 1) in period k,

$\mathbf{X}_k^*$  is a desired state variable vector (6 x 1) in period k,

$\mathbf{U}_k$  is a control variable vector (6 x 1) for period k,

$\mathbf{U}_k^*$  is a desired control variable vector (6 x 1) for period k,

$\mathbf{Z}_k$  is a vector of exogenous variables (7 x 1) in period k,

$\mathbf{W}_N$  is a state variable penalty matrix (6 x 6) in period N,

$\mathbf{W}_k$  is a state variable penalty matrix (6 x 6) for periods 0 through N-1,

$\mathbf{V}_k$  is a control variable penalty matrix (6 x 6) for periods 0 through N-1,

$\mathbf{A}_k$  is a state variable vector coefficient matrix (6 x 6) in period k,

$\mathbf{B}_k$  is a control variable vector coefficient (driving) matrix (6 x 6) in period k,

$\mathbf{C}_k$  is a exogenous variable vector coefficient matrix (6 x 7),

$\mathbf{I}_k$  is a matrix of measurement relations (6 x 6),

$\mathbf{H}$  is a matrix of measurement coefficients (6 x 6),

$\mathbf{D}$  is a known Markov process matrix (6 x 6),

$\epsilon_k$  is a vector of additive system error terms (6 x 1),

$\eta_k$  is a matrix of time-varying parameter errors (6 x 6), and

$\mu_k$  is a matrix of measurement error terms (6 x 6).

We assume the error vectors ( $\epsilon_k$ ,  $\eta_k$ ,  $\mu_k$ ,  $\mathbf{X}_0$  and  $\theta_0$ ) are mutually-independent, and normally-distributed with known means and covariances and that the identified matrices are positive, semi-definite. The distributional assumptions are

$$\begin{aligned} \mathbf{X}_0 &\sim N(\mathbf{X}_0, \Sigma_{0|0}^{xx}) \quad (\text{initial period states}) \\ \theta_0 &\sim N(\theta_0, \Sigma_{0|0}^{\theta\theta}) \quad (\text{initial parameters}) \\ \epsilon_k &\sim N(0, \mathbf{Q}) \quad (\text{system noise}) \end{aligned}$$

$$\begin{aligned} \eta_k &\sim N(0, G) && \text{(Markov process noise)} \\ \gamma_k &\sim N(0, R) && \text{(measurement noise)} \end{aligned}$$

where  $\Sigma_{0|0}^{xx}$  is a known var-covar matrix (6 x 6) for initial-period state variables,  
 $\Sigma_{0|0}^{\theta\theta}$  is a known var-covar matrix (6 x 6) for initial-period parameter estimates,  
 $Q$  is a known var-covar matrix (6 x 6) for system errors  $\epsilon_k$ .  
 $G$  is a known var-covar matrix (6 x 6) for Markov errors  $\eta$ , and  
 $R$  is a known var-covar matrix (6 x 6) for measurement error  $\mu$ .

The initial values of the state variables ( $X_0$ ) are an important part of the dynamic model since they affect the decisions in all but steady state equations.

### ***Solution Method***

Dynamic programming for a system comprising a quadratic objective with linear constraints requires the feedback rule to be of the form,  $U_k = G_k X_{k-1} + g_k$ . Parameters  $G_k$  and  $g_k$  are derived from parameter estimates in  $A$ ,  $B$ ,  $C$  and the Ricatti matrix. Details of the solution procedure are available in Kendrick. This feedback rule can be interpreted as; given that the bank is in state  $X_k$  (at time  $k$ ) the best set of policies is that in vector  $U_k$ . To derive the optimal feedback rule one begins in the terminal period and works backward in time. At each step of the solution only one unknown optimal control ( $U_k$ ) is determined and the problem of  $N$  unknowns ( $U_1, \dots, U_N$ ) is transformed to  $N$  problems each involving one unknown.

## **EMPIRICAL MODEL**

Bank-level data for the application was developed using the 1970-90 annual reports of a Farm Credit Bank. Macroeconomic and sectoral data were developed from secondary, published sources (Federal Reserve Board of Governors; U.S.D.A.). Information on the data set used for the analysis is available in Rao and from the authors.

### **Models of Decision Variables**

The following is a review of the empirical model results for the key decision (control) variables in the model.

#### ***Short-term Loan Rate (STLR)***

Regression results for the short-term loan rate are presented in Table 2.

Table 2. Short-term Loan Rate Model Results  
(average rate on outstanding short-term loans)

Variable	Coefficient	t-ratio
Constant	2.2575	2.50
Expected Treasury rate	.8639	9.39
Expected S&P (total) return	.0240	2.69
Expected default rate	.3080	2.64

Adj.  $R^2=0.89$  DW=2.01 F = 52.52

Short-term loan rates are directly related to expected Treasury rates, expected total market returns as measured by the S&P Index, and expected default rates. The results imply that when Treasury rates were expected to increase by 1 percentage point, the bank raised the rate on short-term loans by 0.864 percent. When the index of total market returns rose by 1 percentage point, the bank raised its short-term loan rate by 0.024 percent. Analogously, when the expected default rate on loans increased by 1 percentage point, the risk premium rose by 0.308 percent. Interestingly, the premium for default risk is of greater magnitude than the coefficient on market returns. Each of the coefficients carries a sign which is consistent with basic economic theory.

#### *Long-term Loan Rate (LTLR)*

The long-term loan rate and the expected prepayment rate (PPR) are modeled as a joint decision process, since the PPR depends on the difference (spread) between new loan rates and average loan rates on long-term loans. Time series information on the bank's prepayment volume was available for 1974-1984, but the series for 1970-73 and 1985-90 were constructed by extrapolation. The prepayment rate in period k was constructed as the ratio of prepayment volume in period k to the outstanding volume of long-term loans in period k-1. The expected prepayment rate model was estimated in quadratic form, which provided a superior fit. The 3SLS instrumental variable procedure is used to estimate the simultaneous model results in Tables 3 and 4. Increases in long-term Treasury bond rates, the S&P index of total market return, the default rate, and the expected prepayment rate all result in increases in the interest rate on long-term agricultural loans.

Table 3. Long-term Loan Rate Model (average annual rate on long-term loans)

Variable	Coefficient	t-ratio
Constant	-.7309	-1.22
Expected 10-year Treasury bond rate	.8131	16.35
Expected S&P (total) return	.0154	2.13
Expected default rate (DEF)	.5044	8.51
Expected prepayment rate (PPR)	.4072	4.78

Results in Table 4 confirm that spread and the prepayment rate were nonlinearly related. Since the spread is defined as the new loan rate minus the average (historical) loan rate, a negative spread produced an incentive for existing borrowers to prepay. For example, if new loan rates fell to 1 percentage point below the average loan rate the spread would be minus 1 percent, and the expected prepayment rate would increase by 3.72 percent. This prepayment response was enhanced by the quadratic term. Alternatively, when new loan rates were above average rates (and the spread was positive) there was no incentive to prepay and the expected PPR fell.

Table 4. Expected Prepayment Rate Model (annual prepayment rate)

Variable	Coefficient	T-ratio
Constant	7.2571	6.84
Spread	-3.7185	-2.72
Spread-squared	.6769	1.74

Other variables (change in net farm income, change in land values and the competitor's lending rate) were insignificant when included in the prepayment model.

### ***Short-Term Cost of Funds (STBR)***

The short-term cost of funds is primarily influenced by short-term Treasury rates; banks in the Farm Credit System were able to acquire funds at a fairly stable premium over the Treasury rate. Hence, a single-factor framework is used to model short-term fund costs. Results in Table 5 indicate a significant positive relationship exists between short-term cost and the short-term Treasury rate.

Table 5. Short-Term Fund Cost Model Results  
(average short-term borrowing rate)

Variable	Coefficient	t-ratio
Constant	2.4969	2.64
Expected Treasury (6 mo. auction) rate	.7531	7.57

Adj.  $R^2 = 0.86$     F-stat 124.14    DW = 1.62

***Call Premium (CP)***

Due to the lack of bank-level data on callable debts, rates on 30-year, noncallable Treasury bonds (the default-free security) and 30-year, A-rated utility bonds (with 5 years of call protection) were used as the proxies for noncallable and callable securities, respectively. The empirical model is based on equation (5). Regression coefficients in Table 6 carry expected signs and are significant, although the overall fit of the equation is low. These results imply that a \$1 increase in the expected price of an underlying 7 percent noncallable bond (after five years) would generate a premium of \$0.39. Thus, an additional coupon of \$0.39 is required by bond holders as compensation to invest in the callable bond.

Table 6. Call Premium Model Results

Variable	Coefficient	t-ratio
Constant	-1.2376	-1.07
Y	6.7979	2.53
K	43.7444	2.37

Adj.  $R^2 = 0.37$     F = 6.35    DW = 1.28

***Long-Term Cost of Funds (LTBR)***

Similar to the short-term model, the long-term cost of funds to the FCB is a function of the corresponding Treasury rate. Results indicate that the higher the long-term Treasury bond rate, the higher the bank's cost of long-term financing (Table 7).

Table 7. Long-term Fund Cost Model Results (average long-term borrowing rate)

Variable	Coefficient	t-ratio
Constant	5.9202	3.81
Expected Treasury (10-yr. bond) rate	.3083	2.98

Adj. R<sup>2</sup> = 0.67    F = 41.05    DW = 1.76

### System Estimation

Fitted values of expected short- and long-term loan and borrowing rates, expected prepayment rate, and expected call premium were used as regressors in the system model.<sup>2</sup> The reader is referred to Table 1 for the specifications. System estimation (3SLS) results are presented in Tables 8-12. Coefficient signs are all as expected.

### *Short-term Asset (STA) Model*

The estimated coefficient on STL<sub>R</sub> implies an own-price elasticity of 0.229 (Table 8). This is reasonably close to Lins' estimate of 0.16 for commercial banks in 1972. Although Lins did not separate short- and long-term loan estimates, commercial banks were predominantly short-term lenders in agriculture during his data period. The equity coefficient indicates that changes in the outstanding volume of short-term loans were directly associated with changes in the equity position of the bank. The positive coefficient is consistent with viewing bank equity as one source of funding for short-term assets. The expected annual rate of inflation acts as a shifter of supply and demand in the STA model. The positive coefficient suggests that expectations about inflation during 1970-90 resulted in a significant rightward adjustment in the quantity of short-term loans.<sup>3</sup>

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<sup>2</sup>The specification of system equations was verified using the Hausman specification test. The null hypothesis that actual values were not correlated with error terms was rejected at  $\alpha = 0.0001$ . Hence, using fitted values in the system model was appropriate.

<sup>3</sup>As Friedman pointed out in his analysis of the corporate bond market, changes in quantity are a function of the relative elasticities of supply and demand when the expected inflation rate changes.

Table 8. Short-term Asset Investment Model Results (STA outstanding)

Variable	Coefficient	t-ratio
Constant	-400.025	-2.01
Expected short-term loan rate (STLR)	45.584	1.44
Lagged STA (1-year lag)	.539	4.55
Inflation rate (INF)	72.329	3.89
Equity	.818	2.91

Adj.  $R^2 = .96$  DW = 1.5

*Long-term Asset (LTA) Model*

Lagged long-term assets (LTA) and profit were positively related to bank investment in long-term loans, as expected (see Table 9). The larger coefficients on lagged LTA (compared with that on lagged STA in the short-term investment model) are consistent with the longer maturity of the underlying assets. Since the coefficient on lagged LTA is less than one, the implication is that long-term assets were declining over time. Similarly, there was a positive short run response to increases in the level of lagged bank profits.

The long-term loan rate (LTLR) coefficient indicates a supply elasticity of 1.02 during 1970-90. This suggests that the FCBs supply of long-term loans was more rate sensitive than that of short-term loans. One explanation is that different entities in the St. Paul District were making the short- and long-term loan (rate and volume) decisions. Another explanation is that significant increases in long-term loan rates during 1970-81 resulted in increased expected returns to the bank, while the prospect of somewhat lower rates during the mid- and latter-1980s implied significant losses on the long-term asset portfolio and a curb on long-term lending.

Table 9. Long-term Asset Investment Model Results (LTA outstanding)

Variable	Coefficient	t-ratio
Constant	-3046.750	-4.76
Lagged LTA (1-year lag)	.920	9.06
Expected long-term loan rate (LTLR)	384.052	3.89
Expected prepayment rate (PPR)	199.399	2.16
Long-term loan repayment rate (LTREP)	-14.291	-4.88
Inflation rate (INF)	54.880	1.00
Lagged Profit (1-year lag)	.735	1.58

Adj.  $R^2 = .98$  DW = 1.58



Clearly, demand for new long-term loans was shifting during the period (rising during 1970\*-81, and falling during the 1980s). Our results (and that of Lins) indicate that demand for long-term loans was highly inelastic (-.14 to -.16) during this period. Therefore, shifts in loan demand (as captured by the rates of inflation, loan repayment, and loan prepayment) trace out a slightly elastic supply for long-term assets in the FCB.<sup>4</sup> The coefficient on the expected prepayment rate illustrates that as borrowers prepaid high rate loans, the new lower rate stimulated additional new long-term loan volume to replace the prepaid loans. Scheduled repayments of long-term loans (LTREP) carries the expected negative sign. Although not statistically significant, the positive coefficient on the annual inflation rate is theoretically consistent with a supply response to an increased interest rate margin on long-term loan investment.<sup>5</sup>

### *Short-term Liability (STL) Model*

Since an increase in net profit represents an alternative source of financing for short-term loans, the estimated coefficient in Table 10 carries the expected negative sign. This substitution relation implies that the bank relied both on internally generated funds and borrowing in the short-term market. The estimation also indicates that increases in the short-term borrowing rate were associated with increases in the availability of short-term loan funds. This occurred even though the bank was required to pay higher rates to acquire short-term funds. TREND is a significant variable and implies an increasing reliance of the bank on short-term purchased funds for investment. This result is consistent with the FCBs strategy of reducing average maturity of its debt portfolio through the 1980s. The TREND coefficient is interpreted as the nonlinear (drift) component, while the constant term is the linear component of the increasing shift toward short-term funds (Sims).

Table 10. Short-term Liability Model Results (STL outstanding)

Variable	Coefficient	t-ratio
Constant	-1177.390	-4.39
TREND	218.415	21.60
Expected short-term borrowing rate (STBR)	197.020	5.87
Expected profit	-1.153	-3.32

Adj. R<sup>2</sup> = .97      DW = 1.52

<sup>4</sup>The trend variable was used in the model to arrest the drifting behavior of short-term liabilities in the dynamic model and achieve stability in the dynamic path.

<sup>5</sup>The expected rate of inflation served as a demand shifter since borrowers were expected to invest in long-term, value-appreciating assets when inflation rates were rising. This shifting of the demand for long-term loans traced out the bank's supply schedule for loans.

### *Long-term Liability (LTL) Model*

Analogous to the previous (short-term liability) model, the expected long-term liability rate carries a positive and significant coefficient in the long-term liability model (Table 11). Both the coefficient and the implied elasticity are substantially higher than those in the short-term model. The coefficient on LTBR indicates that the bank was relatively rate responsive in its acquisition of long-term funds. The call premium coefficient suggests that the premium would have increased as the volume of noncallable debt declined. This implies an expectation of falling interest rates, since the bank would have issued callable debt (instead of noncallable debt) in anticipation of refunding at a lower rate. The call premium is not highly significant, which may be associated with the changing shape of the yield curve during 1970-90.

Table 11. Long-term Liability Model Results (LTL outstanding)

Variable	Coefficient	t-ratio
Constant	-9207.430	-5.01
TREND	-97.908	-2.73
Expected long-term borrowing rate (LTBR)	1271.530	4.78
Call premium (CP)	-485.341	-1.59
Long-term liabilities retired (RET)	-.287	-1.64
Lagged profit (1-year lag)	-.687	-1.68
Lagged long-term assets (1-year lag)	.864	13.52

Adj.  $R^2 = .98$     DW = 1.85

The coefficient on lagged long-term assets indicates that for a 1 percent change in long-term volume there was a 0.86 percent change in long-term liabilities in the subsequent period. Since the coefficient is less than one, the bank was not funding all of the increases in long-term loans with long-term bonds. Secondly, long-term loan volume actually declined during the 1980s and the bank was not able to costlessly reduce its outstanding long-term bonds. Thus, a 1 percent decline in long-term loan volume could not be met by a commensurate 1 percent reduction in the long-term debt portfolio. Lagged profit carries the appropriate negative sign but is not highly significant. While the bank used earnings as an alternative means of funding long-term liabilities the implied substitutability between profits and external funds is weaker than in the short-term liability model. Lagged profit could also be interpreted as an indicator of expected profitability. When bank profits were expected to rise, the bank relied less on acquired long-term debt and more on profits and short-term liabilities to fund long-term assets. Contrary to the short-term debt financing model, the TREND coefficient shows the decline of long-term funds borrowed by the bank during 1970-1990.

## Net Profit Model

Although the net profit equation is just reflective of an accounting relation, the results in Table 12 provide some interesting summary results on bank performance. For instance, the profit contributions of short-term and long-term loans were nearly equal, but both were higher than other bank assets. Since other assets represent nonearning assets, receivables, and acquired properties (among other things), it is reasonable to expect that other assets generated lower unit profits. Analogously, long-term liabilities carried a small premium over the cost of short-term liabilities, as evidenced by the difference between the estimated coefficients. The size of this cost differential could be interpreted as an incentive for the bank to rely on short-term funds for investment purposes. Non-interest operating expenses (NOE) carried the largest estimated coefficient and played a significant role in reducing FCB profits.

Table 12. Net Profit Model Results

Variable	Coefficient	t-ratio
Constant	-42.327	-3.97
Short-term assets (STA)	.238	9.88
Long-term assets (LTA)	.234	7.76
Net other assets (NOA)	.189	7.42
Short-term liabilities (STL)	-.218	-8.08
Long-term liabilities (LTL)	-.252	-8.20
Non-interest operating expenses (NOE)	-.822	-24.12

Adj.  $R^2 = .99$     DW = 2.50

System models exhibited interaction between asset and liability variables, profit and equity variables. Appropriateness of the system estimation of the bank decisions was confirmed by the cross-equation variance and covariance of the error structure. Since each of the system equations was over-identified, generalized least squares (3SLS) was used to obtain more efficient estimates (Kmenta).

### Penalty (Weighting) Matrices

The  $W_N$  and  $W_k$  matrices in equation (7) are the penalty matrices on the state variables. Similarly,  $V_k$  is the penalty matrix on the control variables. The higher the priority to a particular component in the state or control variable matrix, the greater is the penalty (weight) imposed on that component. Larger penalties prevent the optimal levels from being far away from their desired levels.<sup>6</sup>

<sup>6</sup>The economic justification is that the performance weights serve as Kuhn-Tucker multipliers, the higher these weights the more conservative the decision maker is, and the

One could develop priority weights by discussion with bank management (Krouse; Langen). Deriving the weighting matrix through an elicitation procedure involves trial and error and frequent interaction with management. The preference weights developed from such an elicitation process are not unique, since preferences can change, and preferences concerning a particular alternative may differ significantly across members in the group. Alternatively, the relative importance of state and control variables in the decision problem could be developed using a revealed preference (as is typically used in demand analysis). For instance, consider maximizing the present value of equity as the objective of the bank and the various components of short- and long-term assets and liabilities as the arguments of the objective function.<sup>7</sup> The relative importance of the components could be ordered in such a way that equity is maximized. The relative importance could be construed as preference weights the bank assigned to the composition of assets and liabilities.

If one analyzes the levels of equity, assets and liabilities over the last two decades, one could derive the relative weights on assets and liabilities. Assets and liabilities were regressed on bank capital. Since there is significant interaction between different components and deriving precise independent weights may be a problem, the components of assets and liabilities were first orthogonalized. The resulting regression coefficients indicate the relative importance of each component in the total composition. By normalizing the slope coefficients to a unit scale, the scaled coefficients represent revealed preference weights.

Table 13. Estimates of the State Variables Weighting Matrix

Variable	Coefficient	t-ratio
Constant	573.621	10.97
Short-term loan volume	184.539	5.01
Long-term loan volume	171.433	5.49
Short-term debt volume	113.689	2.60
Long-term debt volume	169.642	4.99
Profit	69.816	3.26

Adj.  $R^2 = .90$      $F = 28.5$      $DW = 1.21$

solution of the cost minimization problem will tend toward the desired solution (Kelley).

<sup>7</sup>Discussion with the bank personnel indicated that generating higher profit (so as to develop a strong equity base) was the most important consideration of management.

Slope coefficients reported in Table 13 indicate the importance of each component in the portfolio. The priority weights imply the following ranking (in decreasing order of importance): 1) short-term loans outstanding, 2) long-term loans outstanding, 3) long-term bonds outstanding, 4) short-term bonds outstanding, and 5) profit. This ordering could be used as the initial penalty weights and adjusted to minimize the deviation of the state variables from the historical (or target) levels.

This exercise is extended to derive the weights on control variables. Loan rates, debt rates, the call premium and the expected prepayment rate were orthogonalized and regressed on the log of equity capital outstanding. The results are presented in Table 14.

Table 14. Estimates of the Control Variables Weighting Matrix

Variable	Coefficient	t-ratio
Constant	6.0713	122.16
Expected short-term loan rate	.2370	4.94
Expected long-term loan rate	.3779	7.57
Expected prepayment rate	.0089	.18
Expected short-term debt rate	.1719	3.67
Expected long-term debt rate	.2952	6.07
Expected call premium	-.0012	-.03

Adj.  $R^2 = .88$      $F = 25$      $DW = 1.99$

Assuming the preference ordering is invariant to the scale of measurement, the negative value on the call premium was converted to a positive value by uniformly adding a bigger number to all the coefficients. The transformed coefficients imply the following ranking (in decreasing order of importance): 1) expected long-term loan rate, 2) expected long-term debt rate, 3) expected short-term loan rate, 4) expected short-term debt rate, 5) expected prepayment rate, and 6) expected call premium.

The lowest preference weights to the call premium and the expected prepayment rate was not unexpected since the bank did not have extensive experience with either feature in its long-term loan and debt portfolios.

### SIMULATION OF HISTORICAL BANK PERFORMANCE

Historical bank data and the estimated system of equations were used to simulate bank performance using the optimal control framework. The input matrices (A and B) in equation (8) represent coefficients of the reduced state transition equations, and are derived from the estimated structural models reported in Tables 2-12 following the procedure suggested by

Pindyck. The resulting graphs of optimal simulated values of state and control variables together with their actual, historical values provide a comparison which indicates how the model "tracks" historical asset and liability adjustments and overall bank performance. Since actual bank performance is not expected to be optimal, in the sense of equations (7) and (8), deviations between actual and optimal paths of the variables are to be expected. We suggest that these differences are attributable to the following primary factors (without reference to their rank-order of importance): 1) the state and control variable measures contain measurement error (proxy variables were used for some system variables); 2) information from the entire planning period was not available to the bank (in contrast with the optimal control model which used backward, recursive dynamic optimization); 3) the penalty matrices utilized in the model may vary from the corresponding preference weights used by the bank in selected subperiods (even though the weighting scheme effectively captures bank management's preference ordering for states and controls over the full historical period). Comparisons of optimal and actual time paths of financial variables are not a rigorous validation of the model. However, they do provide a means of investigating the dynamic properties of a stochastic control approach to bank management.

The figures which are presented in this section compare 3 time sequences; the historical (actual) path, the fitted model result (based on the system estimation), and the optimal stochastic path (the CE, or certainty equivalent solution). Our discussion of state variables focusses on the comparison of the "actual" and "CE" paths, since the historical values of the state variables were used as the target values for the optimal control simulations. The control variable comparison is between the fitted values of the controls and the corresponding CE paths. This was done since fitted values from the econometric models were used as target values for the control variables.<sup>8</sup>

### Short-term Assets

In Figure 1 we observe that the optimal (CE) path of short-term assets was significantly below the actual level during the years of rapidly rising short-term loan volume (1976-82), and remained lower through the 1980s. This suggests that optimal growth of short-term assets was slower than the actual rate of growth that occurred during the 1976-82 period.

Recall, the volume of short-term loans was sensitive to the rate of inflation and the short-term lending rate in the estimated STA model. The corresponding paths of the fitted and

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<sup>8</sup>The use of fitted variables in place of actual values comes from the fact that  $U_i^* = U_i + v_i$ , where the  $U_i^*$  are the measurements on control variables, the  $U_i$  are the actual values, and the  $v_i$  are the measurement errors. The true regression model is  $X_i = \beta U_i + \epsilon_i$ , while the regression which is actually run is of the form  $X_i = \beta U_i^* + (\epsilon_i + \beta v_i) = \beta U_i^* + \epsilon_i^*$ . To overcome the problems of bias and inconsistency associated with measurement errors, the fitted values were used as instruments in the system estimation.

optimal short-term lending rates (Figure 2) began to diverge in 1976. The optimal short-term rate remained above the fitted (and actual) rates throughout 1976-1990. As expected, higher optimal lending rates relative to fitted (or actual) rates resulted in lower optimal short-term loan volume. But large differences between the optimal and fitted short-term rate series during post-1976 resulted in relatively modest differences in loan volume due to the underlying price inelasticity. The optimal and fitted short-term loan rates rose sharply during 1977-81 (along with historical rates), primarily in response to increases in the short-term Treasury rate and the underlying expected rate of inflation. Short-term rates fell in the post-1981 period (as inflation subsided), but the fitted and optimal rate series continued to differ. We interpret this as an indication of the risk premium for expected loan default, which is captured in the optimal short-term loan rate series.

### Long-term Assets

The paths of long-term assets (Figure 3) indicate faster optimal growth of long-term loans than actually occurred during 1981-90. Since LTA is sensitive to the long-term lending rate (LTLR) and the expected rate of prepayment (PPR), we first evaluate the degree to which the disparity between the actual and optimal LTA paths is attributable to differences between the fitted and optimal series for LTLR and then for PPR. During 1972-77 the fitted long-term loan rate tended to exceed the optimal rate (see Figure 4). This pattern occurred again during 1982-1990. The result was that optimal long-term loan volume exceeded the actual level of long-term assets during the period of declining loan volume.

The estimated long-term loan rate model indicated that the long-term Treasury rate, the expected rate of default, and the expected rate of prepayment were (in decreasing order of importance) significant determinants of the LTLR. A primary reason for the differences in fitted and optimal long-term loan rates during 1982-90 (Figure 4) was that the optimal prepayment rate was sharply lower than the fitted (or actual) rate of prepayment in the post-1981 period (see Figure 5). In fact the optimal prepayment rate became negative during 1982-86.<sup>9</sup> Fitted (and actual) prepayment rates increased from 1982 through 1989. The optimal prepayment rate was more erratic and escalated only during 1985-89, as new loan rates rose above average historical long-term interest rates and the resultant rate spread became positive. A second reason for the increase in the optimal new loan rate was the higher rate of expected default on long-term loans. Due to higher optimal long-term loan rates and the underlying rate elasticity of loan volume, the optimal path of long term loans exceeded the historical level during the 1980s.

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<sup>9</sup>The negative prepayment rate can be interpreted as no incentive to prepay the loan.

### Short-term Liabilities

Optimal short-term liabilities exceeded the actual level throughout the simulation period (Figure 6). The major determinants of STL were TREND, the short-term borrowing rate (STBR), and the level of bank profits. TREND captured the (nonlinear) shift to short-term funds over time and improved the ability of the optimal STL series to track the actual series.

STBR and bank profits help explain why the two series consistently differed. The STBR simulation (Figure 7) indicates that the optimal short-term borrowing rate was greater than the fitted rate series during 1970-83 and 1988-90. Since the borrowing rate carried a positive coefficient in the STL model, this suggests that the optimal STL series should tend to exceed the actual STL level. However, the constant term and bank profits both carried negative signs and their effects tended to dominate the optimal STL series. FCB profit performance during 1970-90 was highly erratic. Profits were positive and steadily increasing during 1970-80, but declined and then became negative during 1981-87. The effects of sharply negative bank earnings in 1984-86 is reflected in the increase in actual and optimal STL during the same years. This is evidence that short-term borrowings of the bank and bank profits served as substitute sources of funding.

We conclude that the most significant factor explaining the consistent difference between the actual and optimal paths of STL was the constant term, which represented the linear component of the shift toward short-term funding of loans. While it would have been optimal for the FCB to use less short-term borrowed funds and rely more on bank profits, other factors such as the bank's desire to retire high cost, long-term debt and replace it with lower cost, short-term liabilities were captured by the constant term.

### Long-term Liabilities

One striking difference between the short-term liability and long-term liability (LTL) paths is the decline of LTL (in Figure 8) versus the continued expansion of STL (in Figure 7) during 1984-89. The FCBs shift toward short-term financing of loans is quite apparent and is captured by the optimal control model. Figure 8 also illustrates that the actual level of noncallable long-term liabilities was greater than the optimal level throughout the simulation period.

Why would optimal LTL consistently lie below actual LTL? First, the fitted (and actual) long-term borrowing rates equalled or exceeded the optimal rate in most years (see Figure 9), and particularly during 1979-85 (the rapid growth years for LTL). Since the expected long-term liability rate carried a positive coefficient in the LTL model, the actual level of noncallable liabilities is expected to be higher than the optimal level during those years. Secondly, the optimal funding strategy for the bank was to substitute retained earnings for external borrowed funds and to issue more callable long-term debt when borrowing rates were at historical highs and expected to decline. This interpretation is consistent with the



estimated negative sign on the call premium variable in the estimated LTL model. Greater use of callable debt in the optimal funding strategy implies a higher call premium and is consistent with the above scenario. Greater use of callable debt in the optimal funding strategy implies a higher call premium and a higher cost of acquired funds in the high rate years (see Figure 10) and optimally lower use of long-term, noncallable liabilities than the bank actually used.

### Net Profit and Equity

Interesting results occur from the foregoing simulation of optimal paths of assets and liabilities. While optimal short-term assets are lower and optimal long-term assets are higher than actual levels and optimal liabilities are lower than actual levels, the optimal path of FCB profits is nearly identical to the actual profit performance of the bank (Figure 11). In fact, the stochastic control model closely tracks the historical path of profits through the most erratic years of bank performance (1982-1988), when significant operating losses were reported. During the 5-year interval 1982-86, the noninterest operating expenses of the bank accelerated from \$11.1 million to \$586.4 million. These noninterest expenses were associated with the deterioration of performance in the loan portfolio and successive years of large provisions for loan losses and nonperforming assets. These noninterest expenses were a significant determinant in the profit model (as reflected by the large estimated coefficient reported in Table 12).

Given the bank would have shifted optimally from borrowed (external) funds to retained earnings (as an internal source), would not the optimal path of bank profits be expected to remain significantly above the actual path of profits? This result is not observed in Figure 11. The optimal paths of STA and LTA indicated a less aggressive rate of growth of assets during 1976-84 than actually occurred. The corresponding optimal growth of liabilities was also slower. Thus, the implied optimal paths of revenues and expenses were both lower and approximately the same level of profits was achieved in each period (when compared to actual profits). The advantage of an optimal management of assets and liabilities is that the variability of optimal profits was somewhat lower than that implied by the actual profit series. Thus, the results in Figure 11 imply that an optimal policy can achieve comparable levels of bank profit with potentially less risk. Finally, it is important to remember that non-interest operating expenses were a dominant factor in the determination of FCB profits during the mid-1980s. Optimal funding and asset pricing policies could have had some impact on maintaining loan quality through optimal loan rates, but widespread farm financial stress and loan default in the FCB portfolio overwhelmed this effect causing optimal profits to decline along with actual profits.

As expected, the actual and simulated optimal paths of bank equity capital (Figure 12) are quite close. The optimal path of equity capital was derived by updating prior period equity with current period profits. Interestingly, the optimal level of bank equity was consistently below the actual level of equity during 1974-84 due to slower optimal growth, but exceeded the actual level during 1985-1990.

## Model Tracking

The optimal control simulation tracked the historical values well during 1972-90. Tracking performance of the model can be evaluated using the RMSE (root mean square error) measure. RMSE for variable  $Y_k$  is defined as

$$\text{RMSE} = \left[ \frac{1}{K} \sum_{k=1}^K (Y_k^s - Y_k^a)^2 \right]^{0.5}$$

where  $Y_k^s$  is the simulated value of  $Y_k$  and  $Y_k^a$  is the actual value.

The RMSE is a measure of the deviation of the simulated variable from its actual time path.

Since the magnitude of RMSE can be evaluated only by comparing it with the average size of the variable in question, Theil's inequality coefficient (U) can be used as an alternative measure for models that are primarily used for forecasting purposes (Pindyck and Rubinfeld).

The numerator of U is the RMSE, but scaling is such that U will always fall between 0 and 1. If  $U = 0$ , there is a perfect fit and as U approaches 1.0 the predictive (tracking) performance of the model is poor. The calculated RMSE and U-stat values for the state and control variables are;

<u>Variables</u>	<u>RMSE</u>	<u>U-stat</u>
STA	260.03	0.06
LTA	266.72	0.03
STL	635.20	0.11
LTL	263.21	0.04
PROFIT	35.79	0.11
EQUITY	92.86	0.07
STLR	4.20	0.17
LTLR	1.07	0.05
PPR	2.56	0.39
STBR	1.93	0.10
LTBR	1.74	0.10
CP	0.69	0.21

Thus, the model tracked all the variables relatively well, except for the prepayment rate and the call premium where estimates and a proxy variable were used to develop time-series data for estimation of the fitted values.

## **CONCLUDING REMARKS**

Our objectives were to develop an econometric model of bank investment and funding decisions and to demonstrate that the estimated system model could be used to simulate

optimal asset-liability management strategies in a stochastic control framework. The paper emphasized: 1) the conceptual basis for asset (loan) rate decisions, use of callable debt by the bank, and loan prepayment by borrowers. The empirical methodology involved developing the structural model of a Farm Credit Bank (FCB) during 1970-1990. This paper serves as the background for subsequent analyses of how stochastic optimal control can be used by a financial institution to carry out strategic planning in the area of asset-liability management.

Estimated equations for selected control variables provide new estimates of the relationships that existed between short-term and long-term loan rates, underlying market interest rates, and rates of loan default and prepayment during a period of significant variability in financial markets and in agriculture. In particular, the joint estimation of the prepayment rate and long-term loan rate models provides a useful interpretation of the loan rate pricing decision. System estimation of the asset, liability, and net bank profit (state variable) models generated a set of parameter estimates which are internally consistent and statistically significant. These results have not been derived previously for a Farm Credit Bank or other agricultural credit institution. This estimation demonstrated that FCB lending and borrowing decisions were functions of changes in expected market interest rates, expected inflation, loan prepayment rates, premia on callable debt, and lagged bank profit performance. In addition it also pointed out that the roles of these explanatory variables tended to vary across the bank's loan and debt portfolios.

A stochastic control program and the estimated system equations were used to generate optimal paths for control and state variables. When compared with historical bank data and the predicted series, the simulated optimal values revealed that the FCBs actual asset and liability decisions led to faster growth of assets and liabilities than was optimal. Slower growth of the bank would have achieved comparable, certainty equivalent levels of bank profitability and equity capital during 1970-1990. In addition the bank would have optimally raised its level of callable long-term debt and use of internal financing, and reduced the use of noncallable long-term debt. This strategy would have positioned the FCB for stronger financial performance in the 1980s.

## REFERENCES

- Bellman, R. Adaptive Contest Processes: A Guided Tour. Princeton, NJ:Princeton University Press, 1961.
- Board of Governors of the Federal Reserve System, Federal Reserve Bulletin (various issues): Washington D.C.
- Caputo, M.R. "How to Do Comparative Dynamics on the Back of an Envelope in Optimal Control Theory." Journal of Economic Dynamics and Control. 14(1990):655-683.
- Cohen, K.J. and F.S. Hammer. "Linear Programming Models for Optimal Bank Dynamic Balance Sheet Management." Optimization Models for Strategic Planning, T.H. Naylor and C. Thomas (eds.), Elsevier Science Publishers. (North-Holland), 1984.
- Cook, T.Q. and P.H. Hendershott. "The Impact of Taxes, Risk and Relative Security Supplies on Interest Rate Differentials." Journal of Finance. 33(1978):1173-1186.
- Copeland, E.T and J.F. Weston. Financial Theory and Corporate Policy. Third Edition. New York: Addison-Wesley Publishing Company. 1988.
- Dorfman, R. "An Economic Interpretation of Optimal Control Theory." American Economic Review. 59(1969):817-831.
- Hendershott, H.P and S.A. Buser. "Spotting Prepayment Premiums." Secondary Mortgage Markets. August (1984):21-25.
- Kelley, J.H. Optimization Techniques With Applications to Aerospace Systems. New York: Academic Press, 1962.
- Kendrick, D. Stochastic Control for Economic Models. New York: McGraw-Hill Inc., 1981.
- Krouse, C.G. "A Model for Aggregate Financial Planning." Management Science. 18.10(1972):B555-B566.
- Langen, D. "An Interactive Decision Support System for Bank Asset Liability Management." Decision Support Systems. 5(1989):389-401.
- Langerman, D.A. and W.J. Gartland. "Callable Corporate Bonds Pricing and Portfolio Considerations." Fixed Economic Research/Portfolio Strategy. Drexel Burnham Lambert, New York, October 1986.

Lins, D.A. "Determination of Net Changes in Farm Real Estate Debt." Agricultural Economics Research. 24.1(1972):1-8.

Merton, C.R. "An Intertemporal Capital Asset Pricing Model." Econometrica. 41(1973):867-887.

Rao, A. "Dynamic Asset and Liability Management at Agricultural Banks." Unpublished Ph.D. Thesis. University of Minnesota, September 1991.

Sims, C. "Policy Analysis with Econometric Models." Brookings Papers on Economic Activity. 1:1982.

\_\_\_\_\_. "Macroeconomics and Reality." Econometrica. 48(1980):1-48.

Theil, H. Optimal Decision Rules for Government and Industry. Chicago: Rand McNally and Co. 1964.

U.S.D.A. Agricultural Income and Finance, Situation and Outlook Report. Economic Research Service, AF-40, June 1991.

Yawitz, J.B., and W.J. Marshall. "Measuring the Effects of Callability on Bond Yields." Journal of Money, Credit and Banking. 13(1981):60-71.

Figure 1. Short-Term Assets Outstanding (STA)

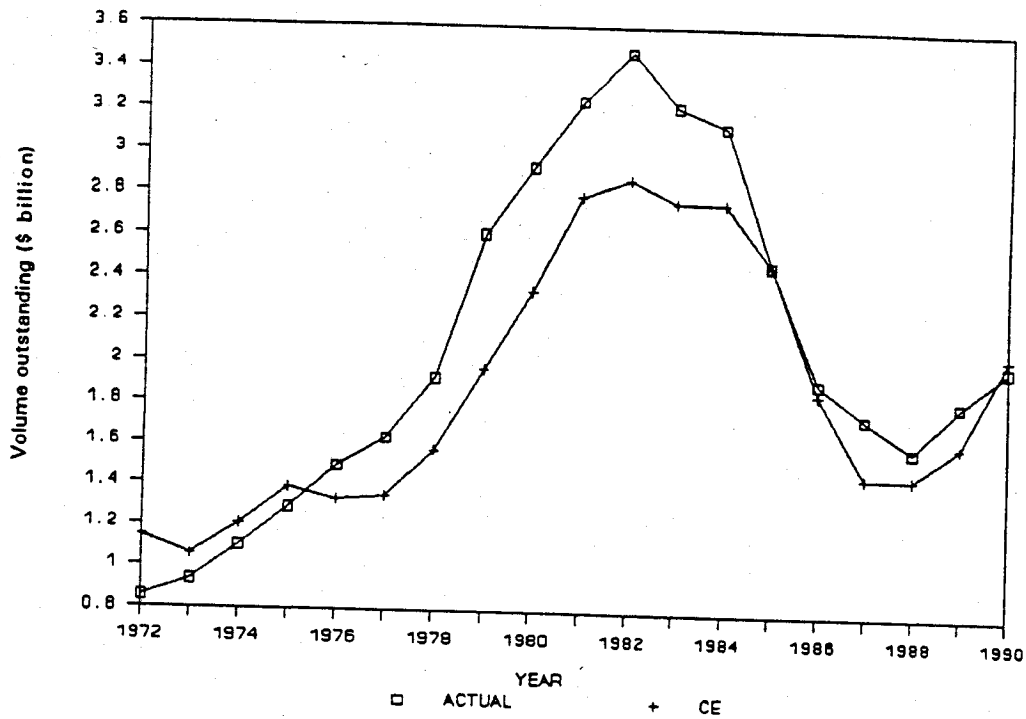


Figure 2. Short-Term Loan Rate (STLR)

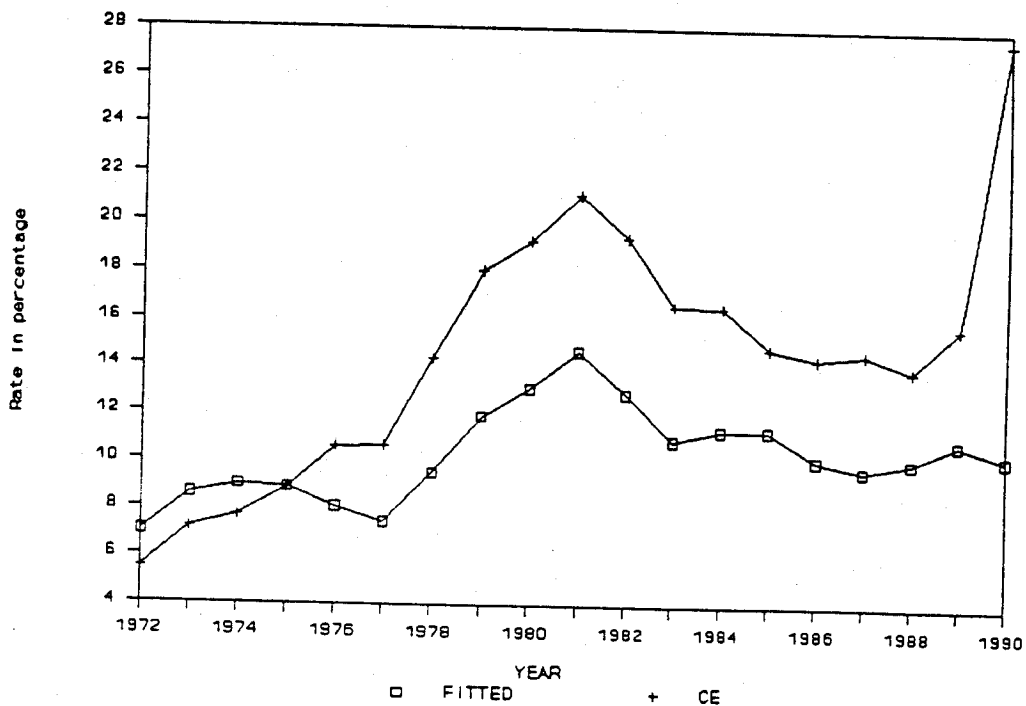


Figure 3. Long-Term Assets Outstanding (LTA)

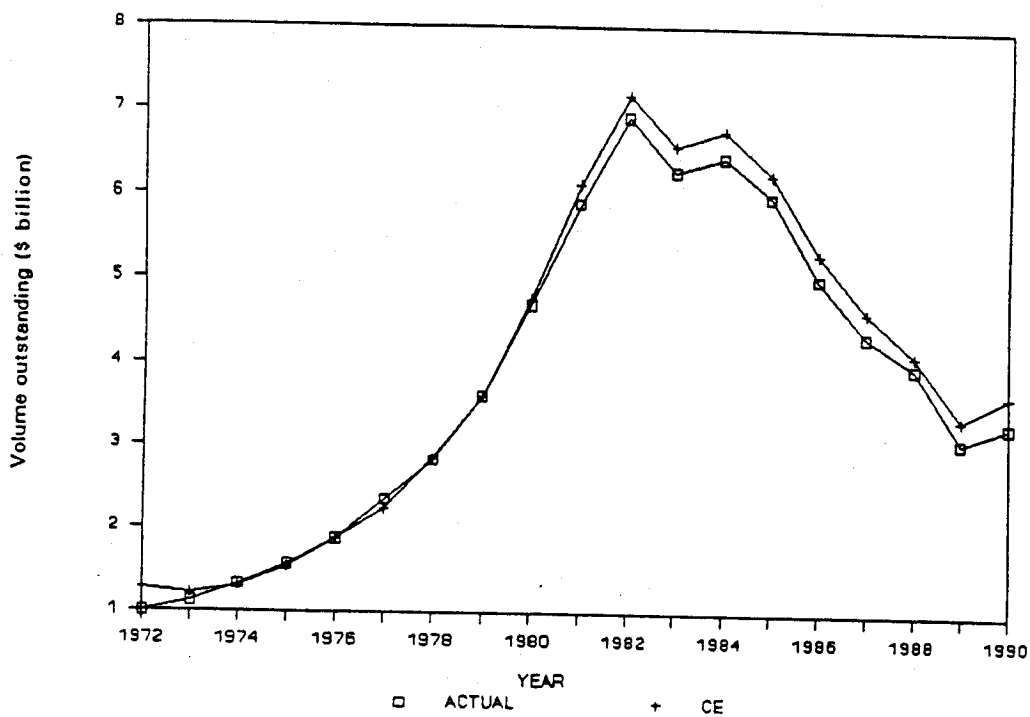


Figure 4. Long-Term Loan Rate (LTLR)

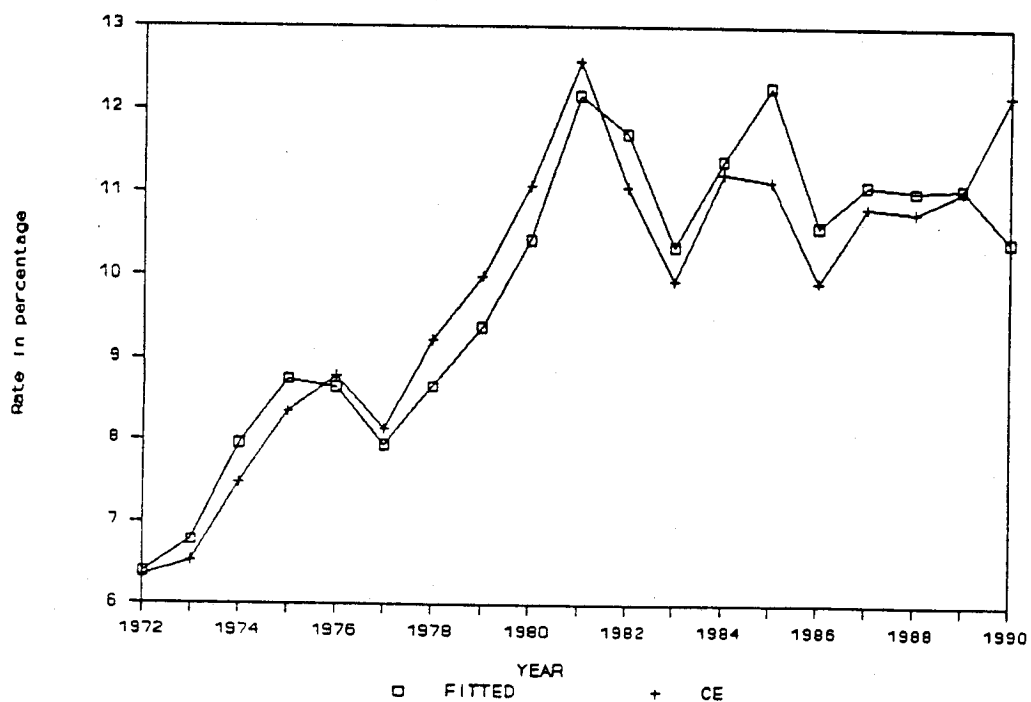


Figure 5. Prepayment Rate on Long-Term Assets (PPR)

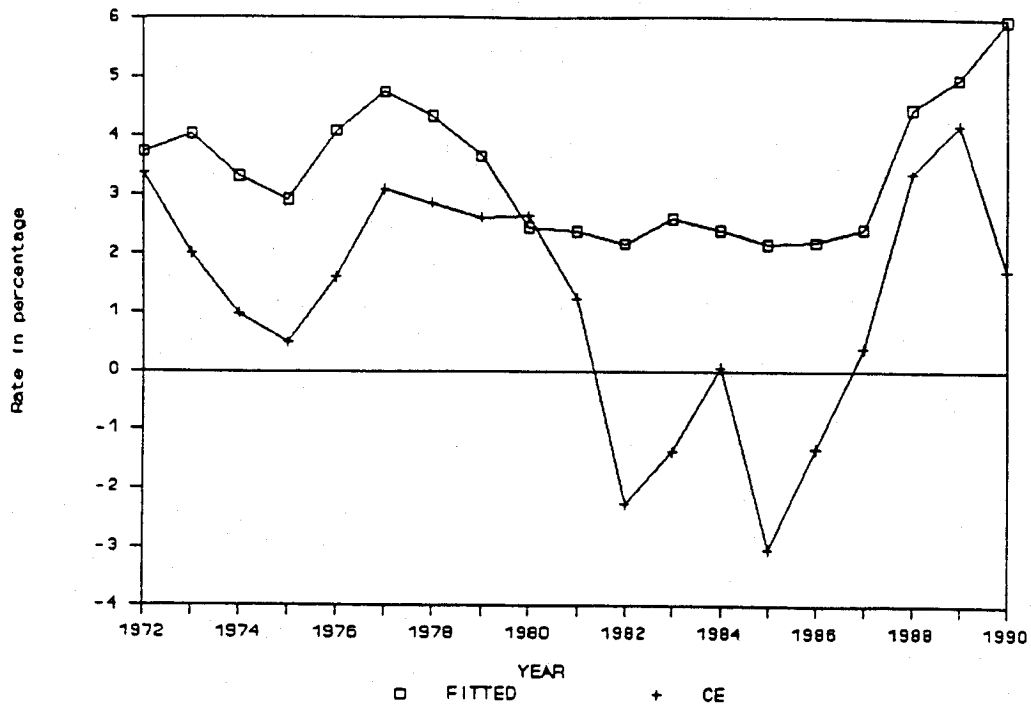


Figure 6. Short-Term Liabilities Outstanding (STL)

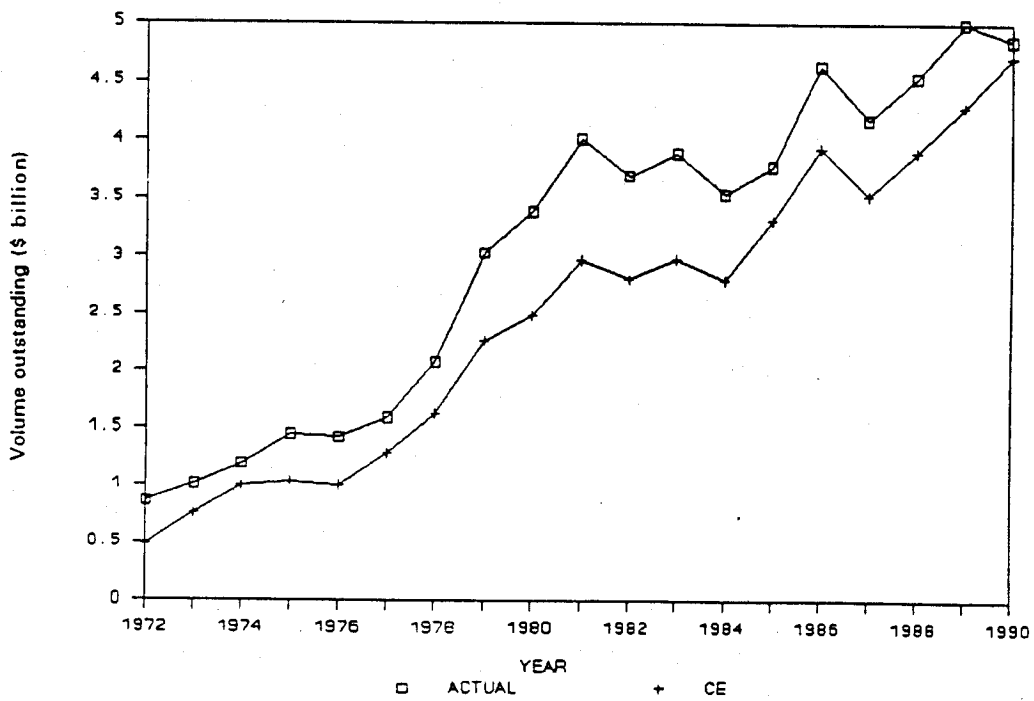




Figure 7. Short-Term Borrowing Rate (STBR)

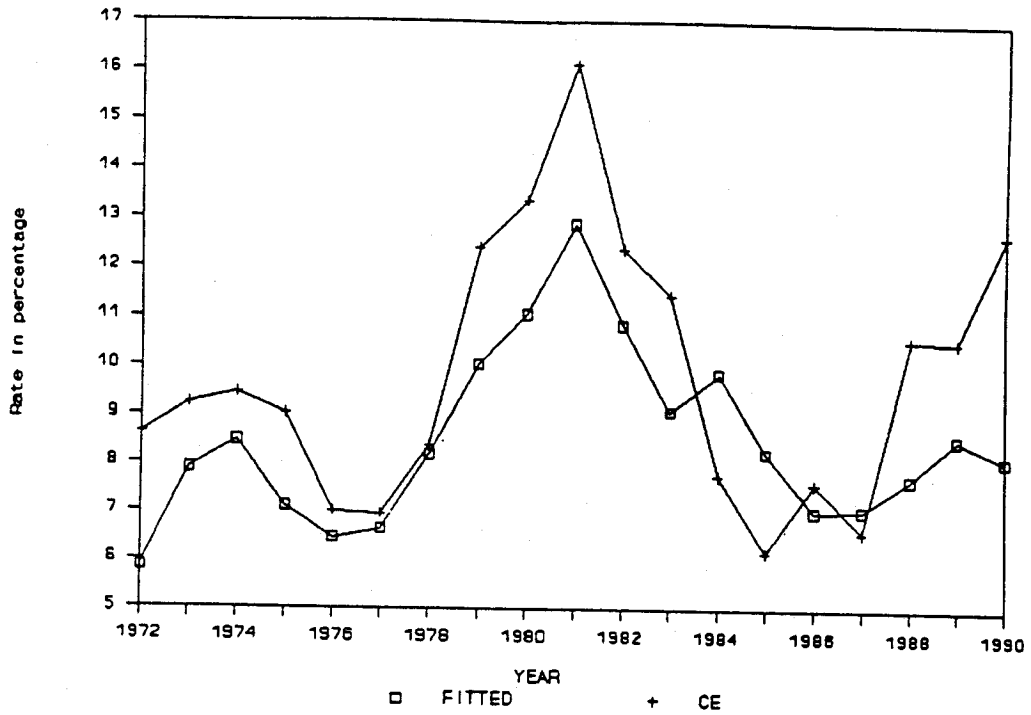


Figure 8. Long-Term Liabilities Outstanding (LTL)

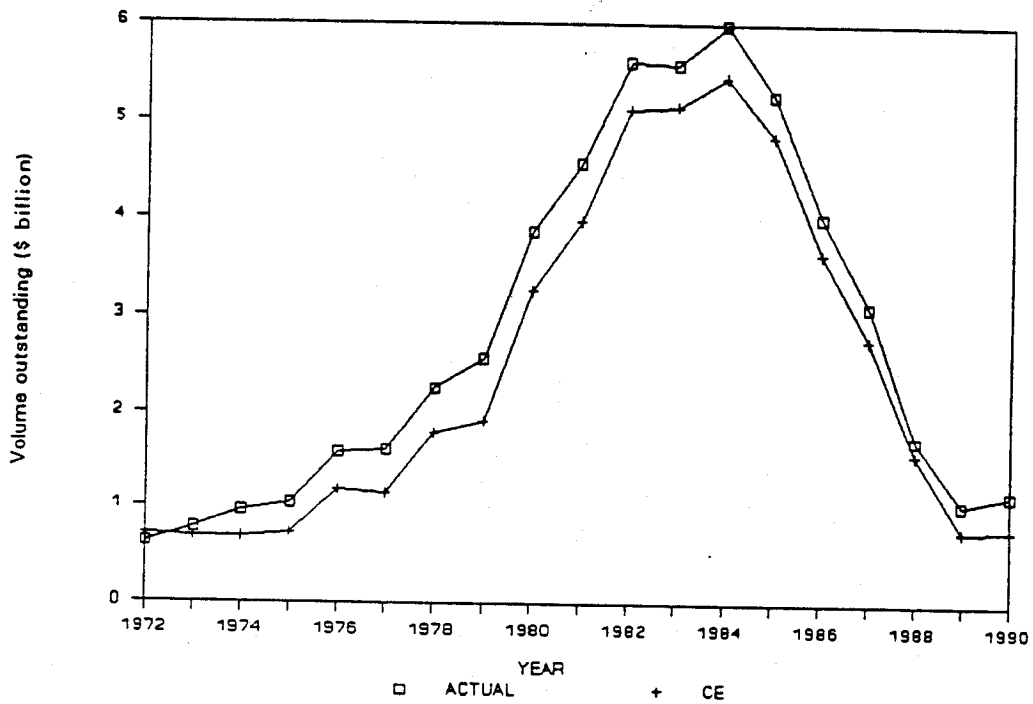


Figure 9. Long-Term Borrowing Rate (LTBR)

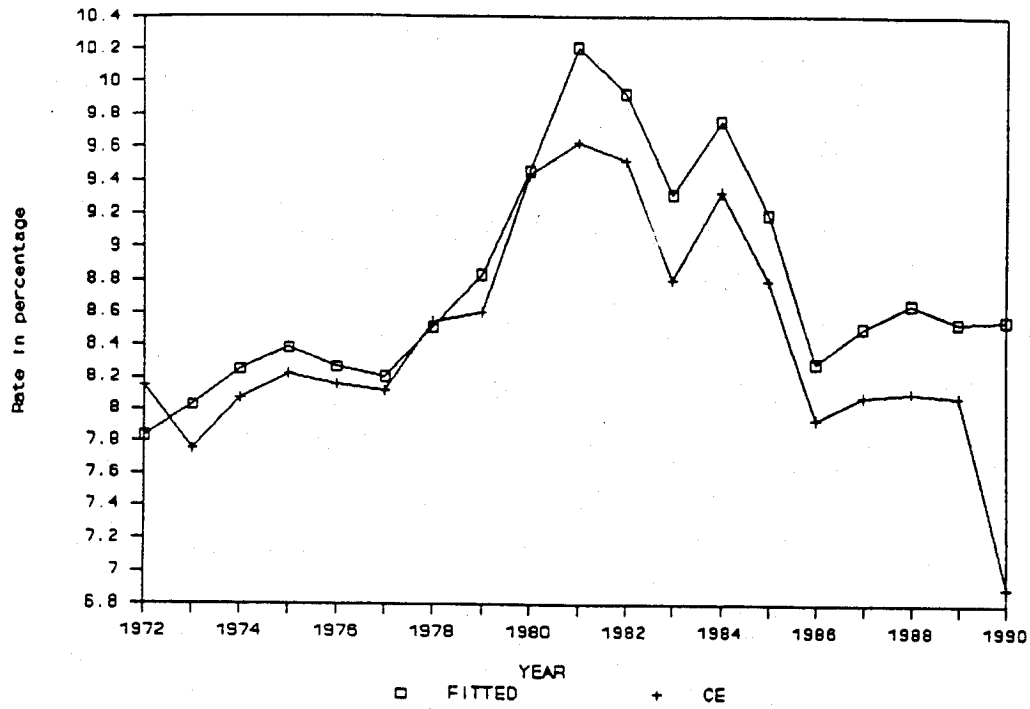


Figure 10. Call Premium on Long-Term Liabilities (CP)

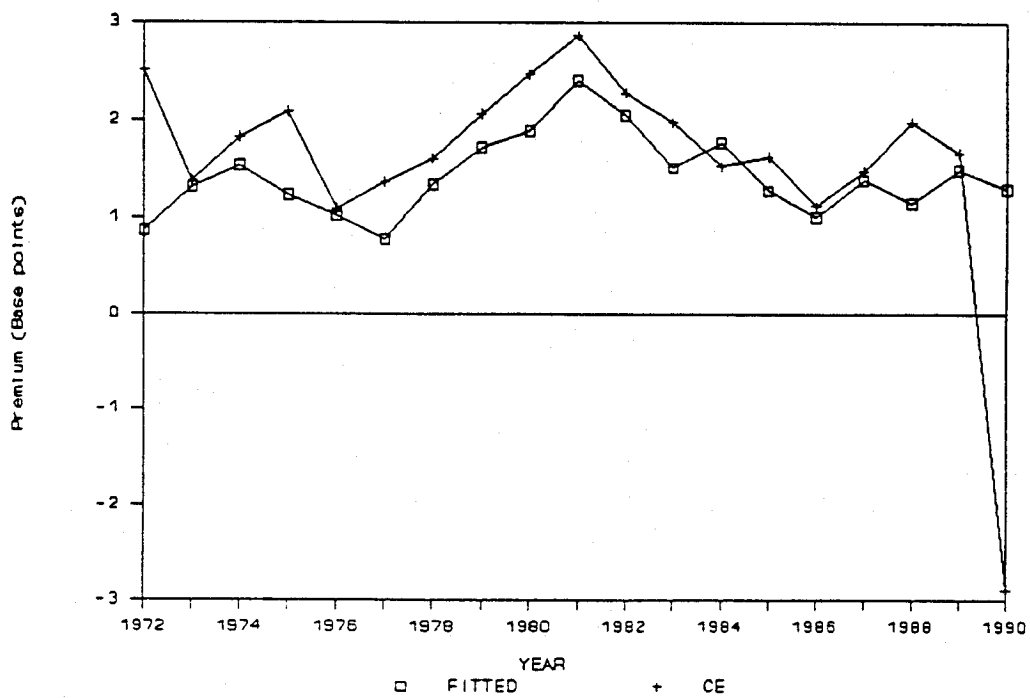


Figure 11. Net Profits

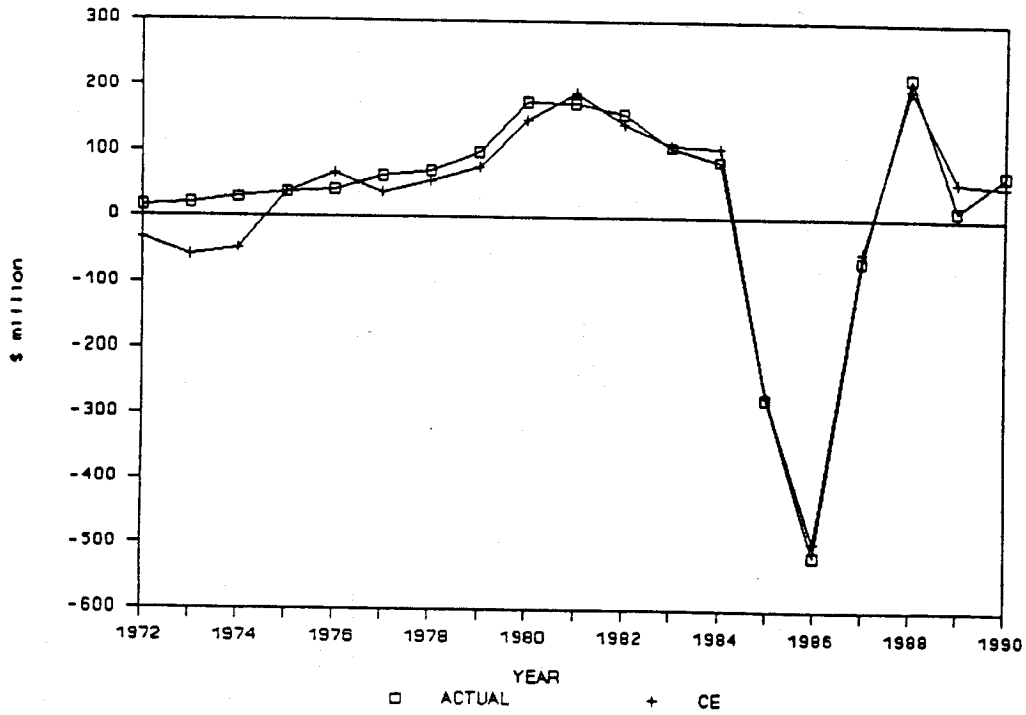


Figure 12. Equity Capital

