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The effect of bifurcation error in small area intercommunity input-output models: an example from north central Idaho

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Abstract. The economics of regional input-output model construction hinge on the cost of collecting better data versus the effect of data error on the accuracy of impact analyses. This paper considers the impact of data errors on the multipliers of a rural community and intercommunity input-output model. Estimates of sales to visitors in a tourism economy are gathered in an inexpensive and informal fashion. These sales estimates are built into input-output accounts of the community and intercommunity model. To test the impact of error, we assume the estimates are off 100 percent. We then reconstruct the models and examine the impact of error on the model's multipliers. We find that a 100 percent error in this part of the model has a generally negligible impact on community and intercommunity multipliers. The finding has important implications for the allocation of resources in the construction of community and intercommunity models.

1. Introduction

The effect of model estimation error on the accuracy of regional input-output (I-O) multipliers is the subject of considerable research (Evans 1954; Jensen and West 1980; Jilek 1971; Sherman and Morrison 1950; Stevens and Lahr 1992; Stevens and Trainer 1976; West 1981; West 1982; Xu and Madden 1991). Our examination of error effects differs from other studies in two respects. First, our concern is with error effects in small area intercommunity I-O models, including spillover intercommunity effects. Second, our analysis focuses on the problem of bifurcating sectoral activity between export and local and the errors that result from incorrect bifurcation.

The empirical data for our study come from a set of intracommunity models for a rural region in north central Idaho. Our study involves a practical problem in the economics of data collection: where accuracy comes at a cost, what is the effect of survey error on the results of impact analysis?

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A strategic step in building a community I-O model is the bifurcation of industry sales between export and local. For sectors such as farming, mining, and wood products, it is usually reasonable to assume all sales are export. On the other hand, in the many cases of small scale service sectors (such as beauty and barber shops, accounting services, and utilities) it is reasonable to assume all sales are local, particularly where the community is remote and defined broadly to include its entire trade area.

More problematic are hybrid sectors that are likely to serve both an export and local market. In communities frequented by tourists, hybrid sectors include retail trade, eating and drinking, automobile repair and services, amusement and recreation services, and others. The portion of sales to tourists is export, while the portion to residents is local. But how does the model builder determine these portions, and what effect does bifurcation error have on results in impact analyses?

One way to obtain information on tourist versus local sales is to survey businesses directly. This can be expensive, however. While still qualifying as a business survey, the least expensive way to obtain information is to conduct a partial and unscientific survey of businesses. For example, contacted businesses are asked to make a spontaneous estimate of the percentage split in sales between tourists and locals. These percentages are averaged across sampled businesses in some informally weighted or unweighted fashion and applied to the total sales indicated in the I-O model database. While inexpensive to obtain, the accuracy of such estimates is questionable.

We have designed a simple test to gauge the effect of bifurcation error on community and intercommunity I-O multipliers. Our analysis is not focused on the accuracy of alternative estimating procedures or data sources as such, although these are certainly worthy issues. Rather our analysis assumes error in the bifurcation estimate and then considers the effect of that error on I-O impact exercise results.

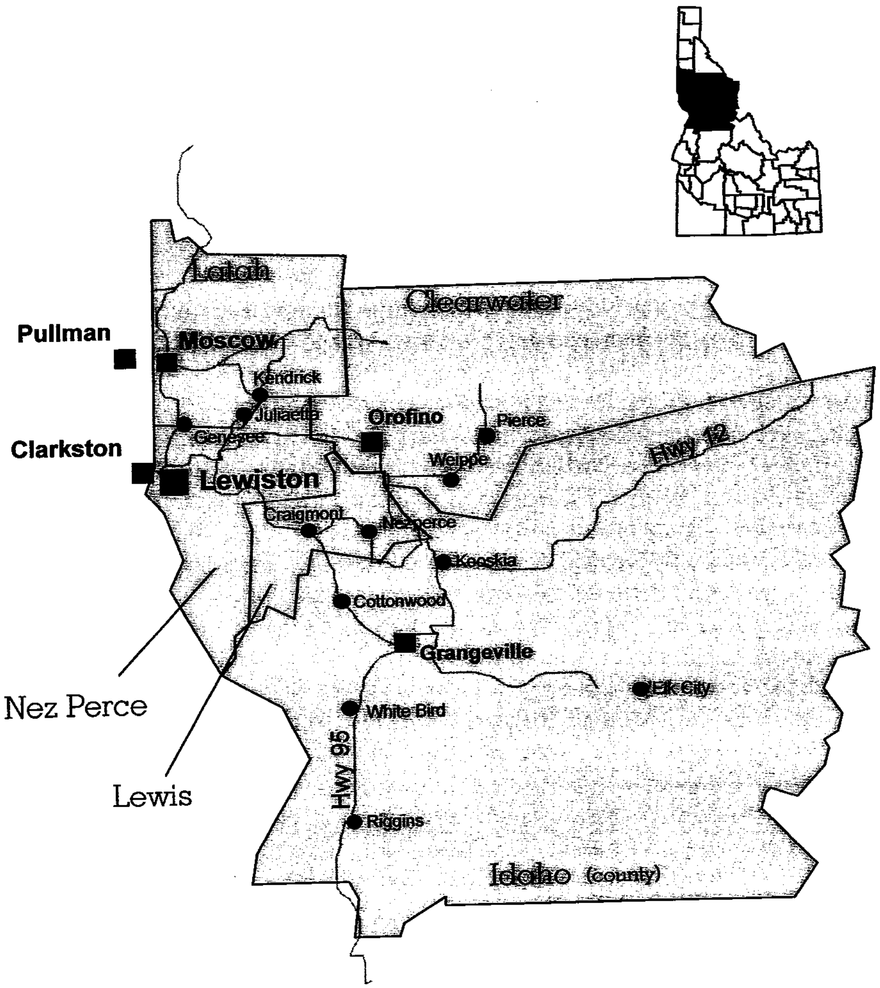
Using models from a north central Idaho study, we focus on bifurcation of retail trade in one of our third order communities. For the purposes of our test, we assume estimates provided by local businesses are off 100 percent and recalibrate the community and intercommunity I-O models accordingly. To judge the effect of error in impact analyses, we recompute and compare multipliers before and after the 100 percent perturbation.

While our test is not comprehensive (in the sense of providing a large number of alternative test settings), it does provide an intensive look at one case of bifurcation error. The test also provides insight into the workings of the intercommunity I-O modeling approach. It is instructive of impacts that may be expected from bifurcation error in other sectors.

2. Empirical setting and model building procedures

Our test of bifurcation error relies on a set of community and intercommunity I-O models constructed for the five county region in north central Idaho shown in Figure 1. The models are constructed as part of a larger study, funded by the Idaho

Figure 1. Lewiston functional economic area



Legislature, to examine the impact of changing timber harvests on rural communities in the area (Robison *et al.* 1996).

Models for 15 communities are constructed. The communities range in size from Lewiston, Idaho (with 1990 census population of 28,082) to White Bird, Idaho (with 1990 census population of 108). Based on field visits, the perceptions of local residents, and the distance and travel time between places, we develop Figure 2 as capturing the underlying structure of the region's intercommunity trade hierarchy. Figure 2 can be thought of as the intercommunity I-O gross-flows matrix, where X's represent square (on the main diagonal) and rectangular (on the off diagonal) matrices of interindustry transactions.

Figure 2 exhibits a three order trade hierarchy. Lewiston dominates all communities and thereby occupies the top of the trade hierarchy. Orofino and Grangeville are second order centers, each dominating their own collection of first order places.

The north central Idaho intercommunity I-O model is constructed with a full six digit I-O code level of sectoral detail. Individual community models vary in size from the Lewiston model, with 107 sectors, to the White Bird model, with 17 sectors. The overall model, e.g., the intercommunity I-O transactions matrix in Figure 2, has 597 rows and columns.

For our test of bifurcation error, we focus on the small first order community of Elk City. We compute employment multipliers for Elk City, including intercommunity spillover multipliers. To simulate bifurcation error, we perturb the model by increasing Elk City's retail trade sales to tourists from 25 percent of all sales, as indicated by the original business survey, to 50 percent. We then recompute the several employment multipliers and examine the impact of our simulated error.

The north central Idaho community and intercommunity I-O models are built according to a modeling approach discussed elsewhere (e.g., Robison 1992; Robison 1997; Robison and Miller 1991; and Robison *et al.* 1993). Our test examines errors in models built according to that approach and exhibits the workings of the approach itself. Two features of the intercommunity I-O modeling approach have particular bearing on our analysis: 1) closure of community models in an economic base/I-O fashion, and 2) the modified supply-demand-pool procedure used for estimating intercommunity trade. Below we focus on these two aspects of community and intercommunity modeling. Concepts are developed with reference to our test area (namely, to the Elk City model) and the hierarchical path from Elk City through Grangeville to the region's highest order place, Lewiston (Figure 2).

2.1 Bifurcating the economic base/I-O model

The kinship between regional I-O and economic-base models is well-known (Billings 1969; Hirsch 1973, p. 206). The models traditionally have differed, however, in the way they are closed. Regional I-O models traditionally are closed to make households endogenous, providing a model of so-called type II regional I-O multipliers (Miller and Blair 1985). Economic base models, on the other hand, are closed with regard to households, local government, and investment (Andrews 1953; Tiebout 1962; Isard 1960).

Figure 2. Intercommunity trade pattern in the north central Idaho intercommunity input-output model

	Lewiston	Clarkston	Grangeville	Cottonwood	Kooskia	White Bird	Elk City	Orofino	Kamiah	Pierce	Weippe	Juliaetta/Kendrick	Craigmont	Nezperce	Riggins
Lewiston	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Clarkston	.	X	.	.	.	.	.	.	.	.	.	.	.	.	.
Grangeville	.	.	X	X	X	X	X	.	.	.	.	.	.	.	.
Cottonwood	.	.	.	X	.	.	.	.	.	.	.	.	.	.	.
Kooskia	.	.	.	.	X	.	.	.	.	.	.	.	.	.	.
White Bird	.	.	.	.	.	X	.	.	.	.	.	.	.	.	.
Elk City	.	.	.	.	.	.	X	.	.	.	.	.	.	.	.
Orofino	.	.	.	.	.	.	.	X	X	X	X	.	.	.	.
Kamiah	.	.	.	.	.	.	.	.	X	.	.	.	.	.	.
Pierce	.	.	.	.	.	.	.	.	.	X	.	.	.	.	.
Weippe	.	.	.	.	.	.	.	.	.	.	X	.	.	.	.
Juliaetta/Kendrick	.	.	.	.	.	.	.	.	.	.	.	X	.	.	.
Craigmont	.	.	.	.	.	.	.	.	.	.	.	.	X	.	.
Nezperce	.	.	.	.	.	.	.	.	.	.	.	.	.	X	.
Riggins	.	.	.	.	.	.	.	.	.	.	.	.	.	.	X

The economic base/I-O model extends endogeneity to include a portion of local government and investment. With the model closed in this fashion, sales of a given sector are either internal to model accounts (i.e., interindustry sales and sales to the endogenous household, government, and investment sectors) or they are export. For specifics on the economic base/I-O approach to model closure, see Robison (1992), Robison (1997), and Robison and Duffy-Deno (1996).

A variety of information is used to bifurcate sectors. Some sectors are assumed to be entirely export, while others are assumed to be entirely local. In the case of hybrid sectors, we rely on mechanical export estimates indicated by the supply-demand-pool or simple-location-quotient approaches (see Miller and Blair 1985). In the north central Idaho study, tourist-serving hybrid sectors are bifurcated based on informal survey.

Where it is determined that exports are greater than the excess of supply over regional requirements (Isard 1953), borrowed national coefficients are scaled downward. Where it is determined that exports are less than the excess of supply over requirements, we employ a biproportional procedure similar to the familiar RAS approach (Miller and Blair 1985) to scale borrowed national coefficients upward. See Robison (1996) for a discussion of the biproportional procedure.

2.2 Modeling intercommunity trade

Christaller (1966) describes centrality as the matching of "excess importance" in trade-dominating higher order places with "deficit importance" in trade-dominated lower order places (p. 18). Motivated by this view, Robison and Miller (1991) and Robison *et al.* (1997) suggest that a spatial extension of the supply-demand-pool nonsurvey technique (Schaffer and Chu 1969) is appropriate to estimate trade among places in hierarchically structured regions. We illustrate the intercommunity trade estimating technique for the Elk City-Grangeville-Lewiston path in the north central Idaho economic model (Figure 2). We start with estimation of two order trade from Grangeville to dominated places, including Elk City, and then consider three order trade from Lewiston to dominated places, including Grangeville and Elk City.

2.2.1 Estimating two order trade

As Figure 2 shows, Grangeville dominates four smaller communities: Cottonwood, Kooskia, White Bird, and Elk City. Let G denote Grangeville and E Elk City. Intercommunity I-O coefficients A_{GE} capture the Grangeville to Elk City trade that needs to be estimated.

Let G_{GE} be a matrix of coefficients indicating demand unmet at Elk City for the commodities available at Grangeville. Let H_{GE} be an array of borrowed (usually national model) I-O coefficients and N_{GE} be an array that maps a matrix with E rows into one with G rows. For industries present at both communities, columns of H_{GE} contain a one in the row for that industry and zeros elsewhere. H_{GE} allows us to subtract elements of the Elk City I-O coefficient matrix A_{EE} from their counterparts among the matrix of select national model I-O coefficients N_{GE}. An estimate of G_{GE} is obtained as follows:

(1) $G_{GE} = \{N_{GE} - H_{GE} A_{EE}\}$.

The total value of Elk City's demand for Grangeville commodities is given by:

(2) $R_{GE} = G_{GE}X_E$

where:
X_E = The vector of Elk City's total gross outputs.

Let T = E, C, K, W, be an index covering communities dominated by Grangeville, where C denotes Cottonwood, K Kooskia, W White Bird, and E Elk City. Also let Y_{G_i} denote exports of the ith commodity from Grangeville. The collection of vectors like (2) for the places T dominated by Grangeville serves to form a vector of scalars ρ_G. The ith element of that vector represents the ith sector at Grangeville and is estimated according to the following:

$$(3)\ \rho_{G_i} = \begin{cases} Y_{G_i}/\Sigma_T R_{GT_i} \text{ if } Y_{G_i} < \Sigma_T R_{GT_i} \\ 1.0 \text{ otherwise.} \end{cases}$$

Scalars (3) multiply unmet demand coefficient matrices (1) providing matrices of intercommunity I-O coefficients. For example, coefficients for Grangeville-Elk City trade are given as:

$$(4)\ A_{GE} = \{\hat{\rho}_G\}G_{GE}$$

In the case of Grangeville, our baseline data indicate sufficient retail trade to satisfy all Grangeville demands as well as the retail trade demands of its tourists with some excess left to serve the unmet demands of dominated communities. In bifurcating Grangeville retail trade, it is deemed unrealistic to have all of Grangeville's demand for retail trade served by the Grangeville retail trade sector. Accordingly, we increase retail trade exports, Y_{G_i} (assuming retail trade as the ith commodity), increasing the portion to supply-dominated communities, while decreasing the portion serving Grangeville residents. This move necessitates a downward adjustment in Grangeville's retail trade coefficients, with the difference being met by the retail trade sector at Lewiston.

2.2.2 Estimating three order trade

Consider estimation of three order trade from Lewiston. Lewiston dominates second order centers Grangeville and Orofino and all the third order places including Elk City. Coefficients indicating Grangeville's unmet demand for commodities available from Lewiston appear in a matrix G_{LG}, estimated following a procedure similar to the one illustrated in equation (1). A similar procedure provides a matrix indicating Orofino's unmet demand for commodities available from Lewiston.

The next step is estimating unmet demand coefficients for the several third order places. The demand by these places satisfied by the second order centers, Grangeville and Orofino, is already estimated in equation (4). Remaining potential demand from Lewiston is estimated according to the following using Elk City as an example:

$$(5)\ G_{LE} = \{N_{LE} - H_{LG}A_{GE} - H_{LE}A_{EE}\}.$$

where elements are similar to those in equation (1), but defined with L representing Lewiston, E representing Elk City, and G representing Grangeville.

Let M be an index that includes all of the communities dominated by Lewiston, including Grangeville and Orofino, and the several third order places. Applying procedures similar to equation (2) through equation (4) provides a vector of scalars ρ_L:

$$(6)\ \rho_{L_i} = \begin{cases} Y_{L_i}/\Sigma_M R_{LM_i} & \text{if } Y_{L_i} < \Sigma_M R_{LM_i} \\ 1.0 & \text{otherwise} \end{cases}$$

Intercommunity I-O coefficients for Lewiston to third order places now can be estimated as in the following for Lewiston-Elk City trade:

$$(7)\ A_{LE} = \{\rho_L\}G_{LE}.$$

In our applications of the procedures described above for north central Idaho, Elk City's demand for retail trade is satisfied partly from Elk City, partly from Grangeville, and partly from Lewiston. Grangeville's demand for retail trade is satisfied partly from Grangeville and partly from Lewiston. Lewiston's demand for retail trade is mainly satisfied from Lewiston, with the remainder supplied by places outside our north central Idaho region.

3. Testing the effect of bifurcation error

To test the effect of bifurcation error on impact analyses using rural intercommunity models, we change the bifurcation of retail trade in our Elk City model and examine the impact on that community's employment multipliers. A perturbation in the Elk City model will affect not only multipliers for Elk City, but multipliers in the Grangeville and Lewiston models. We examine this changes as well.

According to our baseline data, the Elk City retail trade sector has model-year sales (i.e., trade margin sales) of $297,000. On the basis of an informal survey, we estimate retail trade sales to tourists to be roughly 25 percent, or $74,000, of the total. Suppose 25 percent is the true portion of sales to visitors, but due to unrepresentative sampling or incorrect reporting by local businesses we overstate the value by 100 percent and estimate visitor sales at $148,000 (50 percent) rather than $74,000 (25 percent). This would result in an erroneous bifurcation of Elk City's retail trade sector. What effect would this bifurcation error have on model multipliers and economic impact estimates?

We assess the effect of bifurcation error on Elk City employment multipliers, formally given by the following:

$$(8)\ m_{EE} = \lambda_E B_{EE} \hat{\lambda}_E^{-1}$$

where:

m_{EE} = Row vector of economic base/I-O employment multipliers for Elk City;
B_{EE} = Matrix of economic base/I-O output multipliers for Elk City; and
λ_E = Vector of employment-sales ratios for Elk City.

E denotes Elk City, as defined before.

We also examine bifurcation error effects on Elk City's spillover multipliers (Hamilton and Jensen 1983; Miller and Blair 1988). There are two sets of these: one for the spillover to the second order center Grangeville and one for the spillover to the first order center Lewiston. It is easily shown that where trade is strictly hierarchical, as in the case of Figure 2, intercommunity multipliers are computed as a linear combination of intracommunity multipliers and intercommunity trade matrices. Accordingly, Grangeville from Elk City spillover multipliers are given as:

$$(9)\ m_{GE} = \lambda_G B_{GG} A_{GE} B_{EE} \hat{\lambda}_E^{-1}$$

where:

m_{GE} = Row vector of economic base/I-O employment spillover multipliers, Grangeville from Elk City;
B_{GG} = Matrix of economic base/I-O output multipliers for Grangeville;
A_{GE} = Grangeville/Elk City intercommunity I-O coefficient matrix; and
λ_G = Vector of employment-sales ratios for Grangeville.

G denotes Grangeville, and other terms are as defined previously. Lewiston from Elk City spillover multipliers are given by:

$$(10)\ m_{LE} = \lambda_L B_{LL} \{A_{LE} + A_{LG} B_{GG} A_{GE}\} B_{EE} \hat{\lambda}_E^{-1}$$

where:

m_{LE} = Row vector of economic base/I-O employment spillover multipliers, Lewiston from Elk City;
B_{LL} = Matrix of economic base/I-O output multipliers for Lewiston;
A_{LE} = Lewiston/Elk City intercommunity I-O coefficient matrix;
A_{LG} = Lewiston/Grangeville intercommunity I-O coefficient matrix; and
λ_L = Vector of employment-sales ratios for Lewiston.

L denotes Lewiston, and other terms are as defined previously.

3.1 The effect of perturbation on Elk City multipliers

Table 1 shows multipliers computed prior to perturbation, i.e., with 25 percent of Elk City retail trade sales to tourists as per the original (and assumed accurate) survey of businesses. Column 1 multipliers are computed as per equation (8) and reflect employment effects at Elk City. Without entering a discussion of specific multiplier values, we note that these vary for the usual reasons, with wage rates and the extent to which an industry and its immediate suppliers are linked to other parts of the community economy. For readers interested in comparing Table 1 with multipliers with those of other I-O models, it must be recalled that Table 1 multipliers are economic base/I-O, and therefore larger than would be if computed with the model closed in type II I-O multiplier fashion.

Table 1's second column shows spillover multipliers to Grangeville computed as per equation (9). The third column shows spillover multipliers to Lewiston computed as per equation (10). The final column shows region-wide multipliers that are computed as the Elk City intracommunity multiplier plus its two spillover multipliers.

Table 2 shows multipliers computed with the imbedded bifurcation error, i.e., computed postperturbation. We increase the portion of Elk City retail trade sales to tourists from 25 percent to 50 percent, recalibrate the model, and recompute the multipliers shown in Table 1. Table 3 shows the percentage error in the multipliers of Table 2, i.e., the error modeled by our perturbation. Percentages errors are computed as the erroneous value (Table 2) minus the correct value (Table 1) divided by the correct value.

Column 1 of Table 3 shows the percentage error in intracommunity multipliers for Elk City. Multipliers decline because the local retail trade sector devotes more of its output to tourists and therefore less to local households and businesses. The principal finding is that a large (100 percent) error in the portion of retail trade sales assigned to tourists has a negligible effect on small community multipliers. Declines in column 1 multipliers vary from a low of -0.03 percent in the case of commercial fishing (a fish-for-fee trout farm) to a high of -0.75 percent in automobile repair and services. The mean average percentage error is -0.29 percent. Assuming the multipliers in Table 1 are correct, use of erroneous Table 2 multipliers to model an event that

Table 1. Baseline employment multipliers for Elk City, Idaho

I-O code	Name	Elk City multiplier	Grangeville spillover multiplier	Lewiston spillover multiplier	Region-wide multiplier
	Farm	1.07268	0.32253	0.07377	1.46898
3.0001	Forestry products	1.20825	0.92560	0.13953	2.27338
3.0002	Commercial fishing	1.01034	0.01669	0.00641	1.03344
4.0001	Agricultural, forestry, fishery	1.04890	0.10958	0.05115	1.20963
4.0002	Landscape & horticultural service	1.05309	0.11738	0.04200	1.21247
6.0200	Nonferrous metal ores	1.19537	0.29135	0.09313	1.57985
20.0100	Logging camps & logging contractors	1.26754	0.58919	0.19562	2.05235
20.0200	Sawmills & planing mills	1.21441	0.50144	0.14707	1.86292
68.0301	Water supply & sewer systems	1.08033	0.22692	0.09560	1.40285
69.0100	Wholesale trade	1.14394	0.25343	0.08857	1.48594
69.0200	Retail trade	1.08463	0.15988	0.06072	1.30523
70.0300	Security & commodity brokers	1.08195	0.13259	0.04873	1.26327
72.0100	Hotels & lodging places	1.07000	0.15521	0.06739	1.29260
74.0000	Eating & drinking	1.07271	0.10182	0.04879	1.22332
75.0002	Automobile repair & services	1.11123	0.35325	0.10922	1.57370
76.0206	Amusement & recreation services	1.08155	0.14415	0.05260	1.27830
77.0502	Labor & civic organizations	1.04144	0.10273	0.03105	1.17522
	Local government	1.08560	0.18139	0.06420	1.33119
	State government	1.08006	0.18304	0.07155	1.33465
	Federal government	1.12809	0.28143	0.18516	1.59468

Table 2. Employment multipliers for Elk City, Idaho with perturbation in retail trade sector

I-O code	Name	Elk City multiplier	Grangeville spillover multiplier	Lewiston spillover multiplier	Region-wide multiplier
	Farm	1.07062	0.32449	0.07397	1.46908
3.0001	Forestry products	1.20319	0.93040	0.14001	2.27360
3.0002	Commercial fishing	1.01007	0.01695	0.00643	1.03345
4.0001	Agricultural, forestry, fishery	1.04727	0.11121	0.05126	1.20974
4.0002	Landscape & horticultural service	1.05145	0.11903	0.04211	1.21259
6.0200	Nonferrous metal ores	1.19167	0.29501	0.09341	1.58009
20.0100	Logging camps & logging contractors	1.25872	0.59805	0.19618	2.05295
20.0200	Sawmills & planing mills	1.20725	0.50859	0.14753	1.86337
68.0301	Water supply & sewer systems	1.07818	0.22904	0.09575	1.40297
69.0100	Wholesale trade	1.14008	0.25731	0.08880	1.48619
69.0200	Retail trade	1.08241	0.16210	0.06086	1.30537
70.0300	Security & commodity brokers	1.08010	0.13445	0.04884	1.26339
72.0100	Hotels & lodging places	1.06786	0.15734	0.06753	1.29273
74.0000	Eating & drinking	1.07135	0.10318	0.04888	1.22341
75.0002	Automobile repair & services	1.10291	0.36201	0.10963	1.57455
76.0206	Amusement & recreation services	1.07930	0.14642	0.05273	1.27845
77.0502	Labor & civic organizations	1.04037	0.10378	0.03113	1.17528
	Local government	1.08264	0.18437	0.06437	1.33138
	State government	1.07716	0.18595	0.07172	1.33483
	Federal government	1.12392	0.28562	0.1854	1.59494

Table 3. Percent change in Elk City, Idaho employment multipliers from baseline to perturbated values

I-O code	Name	Elk City multiplier	Grangeville spillover multiplier	Lewiston spillover multiplier	Region-wide multiplier
	Farm	-0.19%	0.61%	0.27%	0.01%
3.0001	Forestry products	-0.42%	0.52%	0.34%	0.01%
3.0002	Commercial fishing	-0.03%	1.56%	0.31%	0.00%
4.0001	Agricultural, forestry, fishery	-0.16%	1.49%	0.22%	0.01%
4.0002	Landscape & horticultural service	-0.16%	1.41%	0.26%	0.01%
6.0200	Nonferrous metal ores	-0.31%	1.26%	0.30%	0.02%
20.0100	Logging camps & logging contractors	-0.70%	1.50%	0.29%	0.03%
20.0200	Sawmills & planing mills	-0.59%	1.43%	0.31%	0.02%
68.0301	Water supply & sewer systems	-0.20%	0.93%	0.16%	0.01%
69.0100	Wholesale trade	-0.34%	1.53%	0.26%	0.02%
69.0200	Retail trade	-0.20%	1.39%	0.23%	0.01%
70.0300	Security & commodity brokers	-0.17%	1.40%	0.23%	0.01%
72.0100	Hotels & lodging places	-0.20%	1.37%	0.21%	0.01%
74.0000	Eating & drinking	-0.13%	1.34%	0.18%	0.01%
75.0002	Automobile repair & services	-0.75%	2.48%	0.38%	0.05%
76.0206	Amusement & recreation services	-0.21%	1.57%	0.25%	0.01%
77.0502	Labor & civic organizations	-0.10%	1.02%	0.26%	0.01%
	Local government	-0.27%	1.64%	0.26%	0.01%
	State government	-0.27%	1.59%	0.24%	0.01%
	Federal government	-0.37%	1.49%	0.13%	0.02%

would change employment in Elk City by 100 jobs would result in understatement of about one third of a job.

Column 2 of Table 3 shows the percentage error in Elk City to Grangeville spillover multipliers. The effect of perturbation is positive. Demand by Elk City consumers and businesses formerly met by the city's retail trade sector is met instead by Grangeville retail trade. In terms of spillover multiplier equation (9), the increase in Grangeville to Elk City retail trade appears as an increase in the mass of intercommunity I-O coefficient matrix A_{GE}, and this is reflected in the positive changes in Table 3's column 2. On average, the effect of perturbation on Elk City to Grangeville spillover multipliers, in absolute terms, is roughly four times greater than its effect on the Elk City-Elk City multipliers shown in column 1. The errors are small, with a mean average percentage error of 1.38 percent.

Column 3 shows the percentage error in the Elk City-Lewiston spillover multipliers. As the share of Grangeville retail trade devoted to Elk City increases, Grangeville satisfies a greater portion of its own needs from Lewiston. Increased trade from Lewiston is reflected in an increase in the mass of equation (10)'s intercommunity I-O coefficients matrix A_{LG}. Similarly, Elk City's dependence on Lewiston for retail trade increases, and this is reflected in an increase in the mass of equation (10)'s intercommunity I-O coefficients matrix A_{LE}. The effect on Elk City to Lewiston multipliers is small: the mean average percentage error is approximately 0.25 percent.

The final column of Table 3 shows the effect of bifurcation error on region-wide multipliers. Region-wide changes are no greater than 0.05 percent. From the standpoint of the overall economy, the cumulative effect of our simulated bifurcation error is largely self-canceling. The reduction in retail trade demand satisfied at Elk City is offset by an increase in its demand satisfied at higher order places. While trade effects shift among communities, the overall magnitude of trade stays roughly the same.

3.2 The effect of perturbation on Grangeville multipliers

The effect of bifurcation error in the Elk City model is not limited to Elk City multipliers. Multipliers for other communities along Elk City's hierarchical path to the highest order place are affected as well. These include multipliers for Grangeville and Lewiston. Grangeville employment multipliers are given by the following:

$$(11)\ m_{GG} = \lambda_G B_{GG} \hat{\lambda}_G^{-1}$$

where:

m_{GG} = Row vector of economic base/I-O employment multipliers for Grangeville;

B_{GG} = Matrix of economic base/I-O output multipliers for Grangeville; and

λ_G = Vector of employment-sales ratios for Grangeville.

There is also a Grangeville to Lewiston spillover multiplier:

$$(12)\ m_{LG} = \lambda_L B_{LL} A_{LG} B_{GG} \hat{\lambda}_G^{-1}$$

where:

m_{LG} = Row vector of economic base/I-O employment spillover multipliers, Lewiston from Grangeville;
B_{LL} = Matrix of economic base/I-O output multipliers for Lewiston;
A_{LG} = Lewiston to Grangeville intercommunity I-O coefficient matrix; and
λ_L = Vector of employment-sales ratios for Lewiston.

Other terms are as defined previously.

We construct pre- and postperturbation multipliers for Grangeville according to equations (11) and (12). The Grangeville model has 79 sectors; we do not report full multiplier tables here. The mean average percentage error in multipliers appear in Table 4.

In building the Grangeville model, retail trade sales are allocated to tourists as per informal survey of local businesses. Remaining retail trade is allocated to the demands of dominated places, including Elk City, and to the demands of Grangeville residents. Unmet demand remains, both by Grangeville residents and by Grangeville's dominated places. This demand is satisfied by retail sales from Lewiston.

With our change in the bifurcation at Elk City, Grangeville to Elk City retail trade is increased at the expense of Grangeville to Grangeville retail trade. Referring to the multipliers of equation (11), this decreases Grangeville intracommunity trade, A_{GG}, which is reflected in a slightly reduced Grangeville to Grangeville output multiplier matrix B_{GG}. The percentage error in Grangeville to Grangeville employment multipliers runs from -0.15 percent (logging camps and logging contractors) to less than -0.01 percent (several sectors). Table 4 shows that the mean average percentage error in these multipliers is -0.06 percent.

Newly unmet retail trade demand at Grangeville is shifted to Lewiston. Referring to multiplier equation (12), this increases slightly the mass of intercommunity I-O coefficients matrix A_{LG}, and this, overcoming the decrease in B_{GG}, results in an increase in Grangeville to Lewiston spillover multipliers. The percentage error in these multipliers runs from 0.96 percent (automobile repair and services) to 0.10 percent (ready-mixed concrete). Table 4 indicates that the mean average percentage error in the Grangeville to Lewiston spillover multipliers is 0.57 percent.

The effect on regionwide multipliers for Grangeville is small. As shown in Table 4, the mean average percentage error in these multipliers is -0.02 percent. The negative sign reflects the fact that trade along this particular link, including Grangeville to Grangeville trade, is lower postperturbation, reflecting the redirection of Grangeville retail trade to Elk City.

Table 4. Mean average percentage errors in Grangeville employment multipliers

	Grangeville multiplier	Lewiston spillover multiplier	Region-wide multiplier
Mean average percentage error	-0.06%	0.57%	-0.2%

3.3 The effect of perturbation on Lewiston multipliers

Bifurcation error at Elk City has a final set of effects, this time on multipliers for Lewiston. Reflecting its place as the region's highest order place, the northern Idaho model includes multipliers for Lewiston intracommunity trade only. Employment multipliers for Lewiston are given by:

$$(13)\ m_{LL} = \lambda_L B_{LL} \hat{\lambda}_L^{-1}$$

where:

m_{LL} = Row vector of economic base/I-O employment multipliers for Lewiston;
B_{LL} = Matrix of economic base/I-O output multipliers for Lewiston; and
λ_L = Vector of employment-sales ratios for Lewiston.

The Lewiston model has 107 sectors; we do not present these multipliers here. The effect of our simulated bifurcation error at Elk City on employment multipliers for Lewiston is insignificant. Percentage errors range from -0.02 percent (several sectors) to less than -0.005 percent (several sectors). The mean average percentage error is -0.01 percent. The effect of our Elk City bifurcation error on Lewiston multipliers is insignificant because Elk City is small relative to Lewiston. Lewiston multipliers show slight decline, because a greater share of Lewiston retail trade is devoted to Grangeville and Elk City.

4. Conclusions

Given our modeling procedure, the effect of the bifurcation error is to erroneously shift attainment of local retail trade services from lower order to higher order places. The effect on the multipliers of any particular place is small. From the standpoint of the overall region, effects at individual communities tend to be self-canceling—the impact on region-wide multipliers is all but zero.

It is reasonable to ask to what extent our findings can be generalized to other sectors, to communities of larger size, and to communities differently placed in the trade hierarchy. While there is room for additional empirical research, we suggest that our findings likely will hold wherever the sector of interest is small relative to the overall community economy and/or where the community is small relative to dominating higher order places. The effect of error in large industries located in larger higher order places is an important although separate matter, and our findings should not be extrapolated to these noncomparable cases.

With the above caveat, the applied rural community modeler can be encouraged by the results of our simulation. It appears that large bifurcation errors in the construction of models generally will have negligible effects on multiplier magnitudes and results in applied impact analyses. Moreover, slight negative errors in the multipliers for one place tend to be offset by nearly equal slight positive errors in the multipliers of other places. For the economy overall, the impact of bifurcation at smaller lower order rural communities appears to be small.

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