AGGLOMERATION AND MARKET AREA DIVISION IN A SIMULATED TWO LEVEL CENTRAL PLACE SYSTEM

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Introduction

A central place hierarchy is the product of an evolutionary process in which the suppliers of central goods make locational decisions on the basis of current conditions and limited foresight. Each decision is contingent upon the ones that already have been made by competitors, suppliers of other goods and services, and consumers. The resulting spatial structures are the outcomes of successive actions made by agents whose abilities to maximize their objectives are at best local, both spatially and temporally. This paper uses simulation to examine the development of central places supplying two goods to a region, with particular emphasis on the conditions under which agglomeration will occur and which will generate a hierarchy.

The Role of Agglomeration in Systems of Cities

Central place theory holds the promise of explaining the complexity of systems of settlements. It appeals to the common observation that cities vary enormously in their populations, complexity, and service bundles. It provides fertile grounds for theoretical developments in which geometric, algebraic, and analytic techniques can be employed. The theory is also appealing as a basis for empirical research, which often involves primary data collection, mapping, and statistics. There is, however, a gap between the geometrically pure lattices with clean hierarchies of theory and the odd-shaped market areas with overlapping and incomplete service bundles found by empiricists. In part this is because, as noted by Beguin (1992, 227), Christaller's "theory is rather weak, verbal, sometimes contradictory, and most often not clearly sep-

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arated from a number of empirically based comments." This research attempts to bridge that gap through an examination of how a theoretically simple system may evolve over time.

A crucial element of central place theory is agglomeration, the tendency for firms to locate in the same place. Once agglomeration has begun, it is self-reinforcing because employees and firms create markets for suppliers of other goods and services as the economic base literature describes.[1] But why does agglomeration start? One commonsense explanation that has received considerable attention may be called the *attraction of one stop shopping*. A consumer who needs to purchase several different kinds of goods can save time and travel expense by traveling to a single point that offers several different goods. This is the basis of Christaller's (1966) seminal marketing principle. Others, such as Eaton and Lipsey (1982) and Thill (1992), have examined the importance of combining trips in more systematic theoretical studies. Most of these studies, however, have used the handy abstraction of one dimensional geography and have sought optimal, static distributions of firms. There are, of course, exceptions to this line of research, such as Braid's (1992) consideration of mixed population distributions and Greenhut and Smith's (1993) attempt to formulate empirically testable predictions about spatial competition.

These studies are of limited usefulness as guides to empirical analysis. Real regions have two spatial dimensions, a complication that often adds enormous complexity to calculations based on one dimensional models. Empirical research also must account for the influence of a third temporal dimension. Settlements have life spans of decades or centuries, and systems of cities change slowly. Any individual agent must view his or her own options against the pattern developed by the previous decisions of a myriad of other agents. Under these circumstances, only the most limited and local optimization is to be expected.

What is needed is a method of analysis that can trace the development of a two dimensional region over time that allows agents with bounded rationality and theoretical objective functions to enter and exit. From this, one can derive how a central place system should look under a variety of circumstances. This picture can be compared with empirical observation to test the theory and suggest areas in which it needs to be refined or modified. This study is an early stage in the development of such a method of analysis. It is an attempt to simulate the process of agglomeration in a region that corresponds to the rigid assumptions of classical central place theory.

[1] See, for example, Parr, Denike, and Mulligan (1975).

Description of the Model

The model used in the simulation experiments is an extension of a single industry spatial competition model described in Vandenbroucke (1992). The region is a square grid, 50 units per side. One household lives at each point in the grid. Movement is equally easy in all directions. Firms must locate at these points. Each firm produces one of two goods for sale to the households. Demand is perfectly inelastic, with each household purchasing one unit of each good per time period.

Households

Households generally will purchase the goods from the closest suppliers. A key assumption of central place theory is that households prefer to patronize centers that offer a variety of services. This is one of the forces that drive the development of a hierarchy of places and functions. One reason for this preference is that multipurpose trips allow a household to economize on transportation costs. This preference is simulated by an exogenous variable representing the consumer's propensity for one stop shopping, α ($0 \leq \alpha \leq 1$). The perceived distance from household i located at (x_i, y_i) to firm j at (x_i, y_i) is defined as:

$$(1)\ d_{ij} = \alpha^M \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

where M is a dummy variable that is equal to 1 if there is one firm of each type at j and zero if not. In effect, the travel distance perceived by the household is discounted by $1 - \alpha$ to account for the convenience of multipurpose trips.

Firms

Each unit of each good is sold at a constant price, which for convenience is normalized to unity. The price is paid at the firm; transportation costs are the responsibility of households. Although there is no explicit treatment of transportation costs, these costs are implied by each household's preference for the nearest firm. Firms have fixed costs but no variable costs.[2] In the computer simulations the fixed cost per iteration for low level firms is set at 78 and at 452 for high level firms (this is what defines the different kinds of firms).

A firm exits if it fails to cover its costs. Given the values of the parameters, this means that low level and high level firms must have at least 78 and 452 customers, respectively, in order to survive. Because the households are distributed with a density of one household per square unit, a low level firm's minimum market radius is

[2] Alternatively, this may be interpreted as meaning that the unit price is the price net of a constant average variable cost.

$$(2)\ r_L = \sqrt{\frac{78}{\Pi}} = 4.98,$$

which is rounded up to 5, and a high level firm's is

$$(3)\ r_H = \sqrt{\frac{452}{\Pi}} = 11.99$$

which is rounded to 12. Note that the ratio r_L/r_H is 0.42, which is approximately equal to the 0.43 of Christaller's marketing principle.[3]

Equilibrium

Equilibrium in the context of this model occurs when the region reaches a state of stability for a sufficiently long period of time. *Stability* is defined as an iteration in which the entering firms fail to survive, and no existing firm exits. Preliminary experimentation reveals that stability is seldom lost once achieved. Consequently, requiring the region to be stable for ten iterations is an adequate criterion. The simulation also may end if the simulation enters an endless loop,[4] which is considered to have occurred if 100 iterations have passed since the maximum number of firms in the region was reached. In principle, the program also allows for the simulation to end if all of the firms in the region have the minimum number of customers, indicating an economic equilibrium with zero profit. In practice, however, this condition was never satisfied.

Sequence of Events

Each simulation begins with one firm of each level being randomly located in the region, more than its minimum market radius from any

[3] Under Christaller's marketing principle, a lower level place will be located at the center of the equilateral triangle formed by the surrounding higher level places. Thus, the minimum market radius is one-half the distance from the center of the triangle to any vertex. Because the higher level centers are spaced two of their own minimum market radii from one another, it follows from a simple exercise in plane geometry that the radius of the lower level place is approximately 0.43 times that of the higher.

[4] An endless loop will occur if, for example, there are two possible locations for firms. Given a firm at the first location, the next firm will enter at the second. This can so reduce the market areas, however, that both firms are unable to survive. The exit of both firms may make the location of the first attractive again, starting the cycle over.

edge.[5] There follows a series of iterations until equilibrium is achieved, and the simulation ends. During each iteration, these events occur:

- One firm of each level enters the region. The location of each entrant is the grid point at the maximum distance from competing firms. This maximum distance computation takes into account the possible multifirm weights of potential sites and those of existing competitors;
- Each household in the neighborhood[6] of an entrant is examined to determine if it will change its allegiance to the new firm. It does so if the distance d_{ij} to the new firm is less than the distance to the current supplier. At this point all potential trips to places with more than one firm are assumed to be combined trips, and thus the perceived distances are modified by α, as discussed above. This may cause households in the neighborhood of a single entrant to change its supplier of both goods;
- All households that changed any supplier in the previous step are rechecked to ensure that the perceived distances to only those firms for which the households are actually combining trips are adjusted by α. This may cause some households to be reallocated to their previous suppliers;
- Firms are checked for exit, by determining which have fewer than the minimum number of customers;
- After all exits are determined, the customers of the defunct firms are allocated to the closest surviving firms. All multifirm places are treated as if every household were combining its trips;
- Another reconciliation is performed to confirm that changed households would combine trips; and
- The region is checked for equilibrium, as defined above. If no equilibrium exists, the sequence is repeated.

Simulation Results

Iterations to Equilibrium

The simulation is run 20 times for each value of α. The median number of iterations before reaching equilibrium, less the ten or 100 iterations necessary to confirm it, is 21. Although the minimum number of such iterations is 13 and the maximum is 88, 95 percent of the runs reach their final configuration in 16 to 31 iterations. There is some evidence that decreasing the attraction of multifirm places (by increasing α) lengthens the process of equilibration. Regressing iterations against α yields a 95 percent confidence interval that increasing α between 0.045 and 0.167 will increase the number of iterations by one. This regression has a low R^2 of only 0.06, however. An average of 21.1 percent of all the runs enters endless loops, the rest ending in stability.

[5] This buffer zone essentially captures the effect of competition from other firms outside the region being simulated. It prevents firms from choosing nonviable locations on the edge of the world.

[6] The neighborhood to be examined is determined by starting from the location of the entrant and working outward in larger and larger rectangles until a complete circuit is made with no households changing allegiance.

Table 1—Mean Number of Firms at Equilibrium

Weight (α)	Low Level	High Level	Percent Multifirm
0.60	18.50	4.00	97.4
0.65	20.50	4.15	92.3
0.70	21.40	4.00	80.0
0.75	21.55	4.00	55.0
0.80	21.20	4.00	38.8
0.90	22.20	3.95	11.3
0.97	19.95	4.00	2.5
0.99	17.15	3.90	0.0
All	19.71	3.98	42.1

Although the proportion of loops vary between 5 percent and 40 percent for different values of α, there is no systematic relationship.

Firms at Equilibrium

Given that demand and costs are fixed, the number of firms at equilibrium is the primary measure of efficiency and accessibility. Table 1 shows the number of high and low level firms and the percentage of high level places that are also low level places as they vary with α. The table reveals a tendency to a decreasing number of low level firms as α increases. Figure 1 shows a scatter diagram of the number of low level firms resulting from each simulation run, together with a quadratic regression curve that is estimated from the data. The equation for this curve is:

$$(4)\quad f_l = -27.407 + 104.9331\alpha - 55.6756\alpha^2$$
$$\qquad (-4.34) \qquad (6.45) \qquad (-5.520)$$

where f_l is the number of low level firms at equilibrium, and the numbers in parentheses are t-statistics, all of which are significant at the 95 percent level. The adjusted R^2 of this equation is 0.6019. Thus, decreasing the attraction of multifirm places increases the number of low level firms. This is to be expected because this improves the competitive positions of isolated firms relative to those in multifirm places.

Almost all of the simulation runs result in four high level places. This appears to be an artifact of the simulation parameters, specifically the 50×50 size of the region and the constraint that firms are to locate at least their minimum radius away from any edge. This essentially reduces the region for high level firms to 26×26, and there is little room for variation. Almost all of the high level firms locate at or near the corners of the permitted region, points (12, 12), (12, 38), (38, 12), or (38, 38).

Figure 1—Low Level Firms At Equilibrium by Multifirm Weight

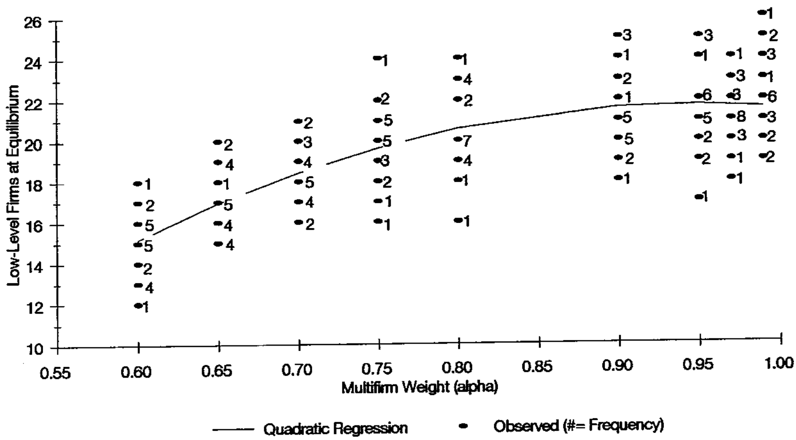

Table 1 also illustrates that the percentage of multifirm places steadily declines as α increases. This is consistent with expectations. As the advantage of colocation decreases, so does the number of multifirm places. A regression on the proportion of multifirm places as a function of α results in this equation:

$$(5)\ p_m = 2.6470 - 2.7419\alpha$$
$$(17.02)\quad (-32.62)$$

which has an R^2 of 0.8567. Thus, one would expect multifirm places to disappear when α is near unity and to dominate when the parameter falls below about 0.50.

Spatial Patterns

Vandenbroucke (1992, pp. 27-28) notes that the spatial patterns produced by competition among the firms of a single type exhibit considerable variation, from almost textbook hexagonal lattices to tiled squares. The most common pattern is a set of irregular polygons, each having from three to nine sides. The addition of a second set of firms produces a similar pattern, provided that there are no multifirm places. If this provision does not hold, the spatial patterns become more complex.

Figure 2—A Typical Two Level System (α = 0.70)

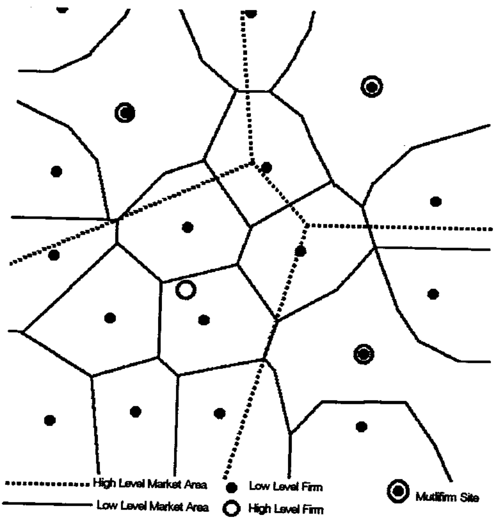

Figure 2 illustrates this with a map of the equilibrium market areas of one simulation run. In the lower left of the region the high level firm has not colocated with a low level firm, and we see that the low level market areas are all made up of convex polygons. In the market areas that are dominated by the other high level firms, however, the low level market areas form enlarged concave polygons around the multifirm places. This particular shape is caused by the fact that the model treats the attraction of multifirm places as a percentage decrease in perceived distance. Households that are further away from any center perceive a larger absolute decrease in travel, making the multifirm places relatively more attractive for these households than those that are closer to both types of places. Thus, the low level market areas of the multifirm places bulge outward at the centers of the triangles formed by any three adjacent market centers.

Concluding Remarks

The current study establishes a framework for examining how a central place system becomes established and reaches stability. It confirms that competition that includes a spatial element does not necessarily drive economic profits to zero, a point made by Scotchmer and Thisse (1992) in their survey of location theory. On the contrary, the locations chosen by early entrants with an eye toward profit maximization can leave gaps between firms that are too small to support new firms but large enough for positive profits. The general locational pattern that results is similar to that found in empirical studies, with polygonal but not regular market areas, most often with five or six immediate competitors.

The simulations illustrate that it is not difficult to specify conditions that consistently result in multifirm central places. Thus, the development of dual nested hierarchies of places and central place functions is confirmed. The firms in these places have larger market areas, which in this model implies larger profits. Not only are the market areas expanded, in this model they change shape from convex polygons to more complicated star-shaped structures as the propensity for one stop shopping increases.

The model can be extended so that more substantive and complex questions can be examined. There are some simple extensions that can be made easily that will enable the current model to address some interesting questions of central place development. These include enlarging the size of the region to allow for more flexible placement of high level firms and increasing the number of different kinds of firms to allow for more than two levels of central place and more than one kind of firm at each level.

The economic agents represented by the firms and households in the current model are limited. Before any reliable conclusions can be obtained, there must be a more sophisticated microeconomic treatment of production, demand, and transportation. This will free the model from its reliance on fixed cost, prices, and quantities. A more robust equilibrium could be obtained, with consumption, prices, and travel derived endogenously. Similarly, a more explicit treatment of the benefits of one stop shopping to consumers would better explain the development of complex high level centers.

Simulation can be thought of as an intermediate method of analysis, more flexible than theory but less noisy than empirical studies. Some possible extensions of the model include tracing demographic change as centers develop; examining the effects of improved transportation routes, either exogenously specified or constructed within the model; examining the effects of production for export; and simulating the effects of major topographical features, such as mountains and rivers. Many of these extensions are common distortions for which empirical studies must account. It is hoped that a simulation demonstrating what their effects should be will prove to be an aid to further research.

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