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AN EX-POST ANALYSIS OF THE FLOOD CONTROL PROJECT IN RUSHFORD, MINNESOTA

by

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INTRODUCTION

The United States suffers an average of about one billion dollars in flood damages every year. More than 10 billion dollars have been spent on structural measures alone to mitigate flood damages since passage of the Flood Control Act of 1936 (Cline, 1968). Flood control projects of the U.S. Army Corps of Engineers prevented flood damages estimated at 38 billion dollars during the period 1936-1974 (Corps of Engineers,1974). In fiscal year 1985, the flood damages prevented by Corps Projects in Minnesota alone, were 2.86 billion dollars with floods concentrated in March, April and August. Despite this effort, annual flood damages are increasing every year. Thus, flood control measures, both structural and non-structural , need to be considered in years to come. The importance of "accurate" estimates of flood damages (benefits attributed to a project) can hardly be exaggerated.

The major objective of this report is to do an ex-post evaluation of the urban flood damage reduction project in Rushford, Minnesota. The city of Rushford is located in southeastern Minnesota at the confluence of Rush Creek and the Root River, the latter being a tributary of the Mississippi River. Prior to the project, on the average of once each year the city experienced damages caused by separate or concurrent overbank flows on Root River and Rush Creek.

The U.S. Army Corps of Engineers found that the only feasible solution was a project including levees along the two streams, channel excavation, temporary pondage areas for interior drainage waters, and other modifications.

Rushford consists of a main business and residential district along the right bank of Rush Creek and two residential areas along the left bank of Rush Creek. The principal occupation of residents in the rural area surrounding Rushford is agriculture and essentially all of the businesses and the small industries in Rushford serve the farm needs.

The population of Rushford has remained almost constant during the past 35 years with an increase of only 4.7 % between 1950 and 1985 or an annual population growth of less than 1.5%. Population projections done by the Corps of Engineers in 1977 indicate the same growth rate for the next 50 years.

The basic rationale for the ex-post evaluation, as Palanisami and Easter (1983) state, is to help improve ex-ante planning rather than merely criticize project implementation. Therefore, an important aspect of this ex-post evaluation is to provide feedback to help improve future ex-ante planning procedures.

Project Description

Flood protection from the Root River and the Rush Creek was authorized by the 1958 Flood Control Act. Construction was started in June 1967 and completed in 1969. The Root River was realigned and Rush Creek was deepened. The project included construction of almost two miles of levee on the left bank of the Root River and right bank of Rush Creek to protect the principal commercial and residential areas (see Map of Rushford, Figure 1). In addition, structures for drainage, traffic crossing over the levees, bridge alteration or removal, and utility and sewer system changes were built, as well as five pumping stations.

The improvements were designed to provide protection against river flows nearly 80 percent greater than the peak flood on Rush Creek, recorded in 1950. If a flood similar to 1950 should recur,the water would crest with 3 feet of additional design protection on the levees and the improvements would prevent damages of $ 2,367,000.

Total Federal Cost of the project was $ 2,610,979. Cost to local interests was $ 326,000 for lands and alteration of bridges and utilities. The work was substantially completed in the fall of 1968. A bridge relocation and the raising of a railroad track completed the project in 1969.

In response to a request from local authorities, the Rushford project was inspected on May 18th, 1972. Bank erosion was found to be prevalent, and remedial work was conducted to halt erosion. Repair work consisted of shaping and riprapping banks and was accomplished by late 1974 at a cost of $ 160,354.

A construction contract for additional remedial work was awarded in September 1977 to correct severe erosion problems and to prevent further damage to the project upstream of Minnesota Highway bridge No 43. Also included in the contract was

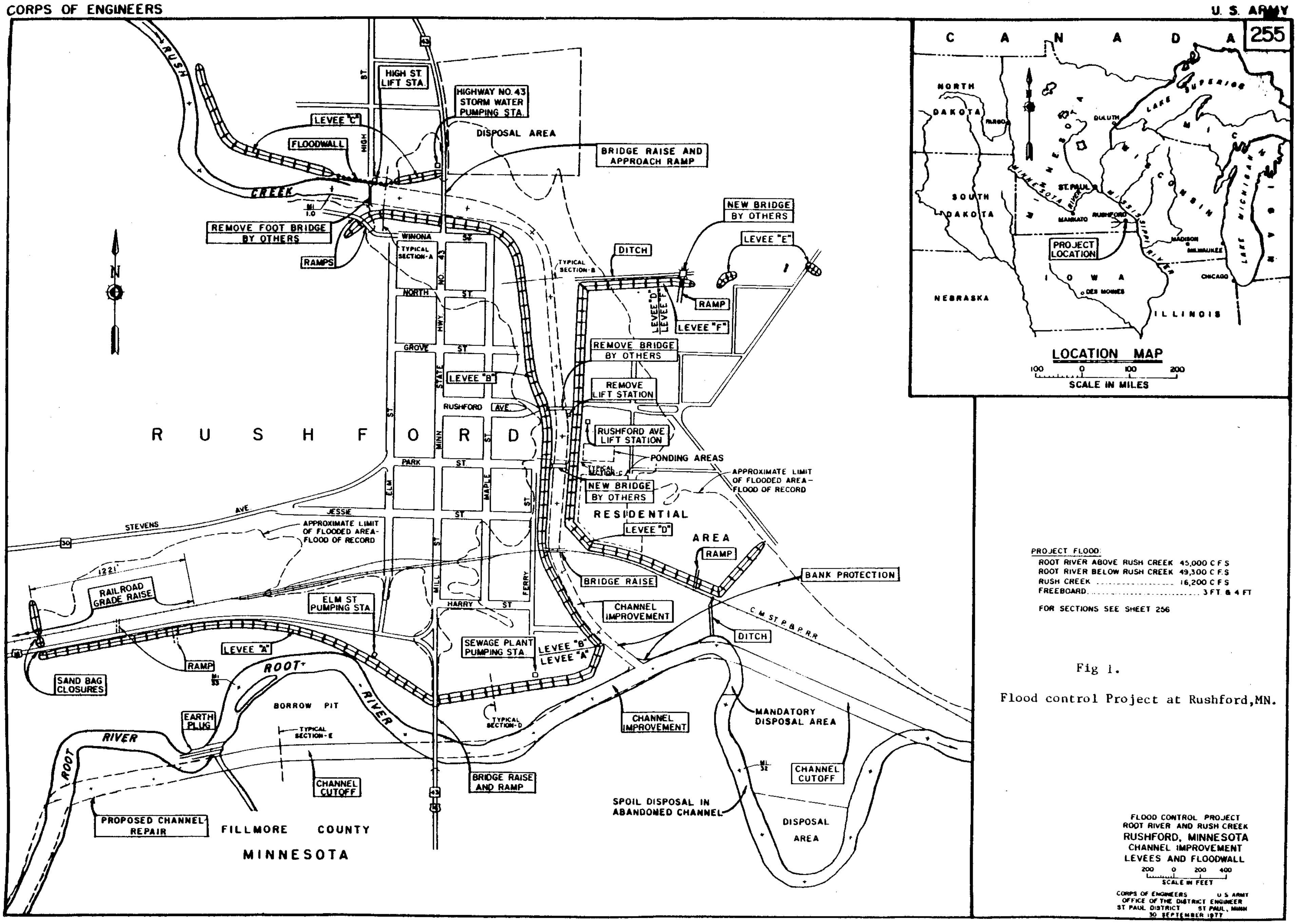

Fig 1.

Flood control Project at Rushford,MN.

construction of a ditch outlet near Rush Creek and a roadway safety improvement. Construction was completed in 1979 at a cost of $421,000.

Project Design Flood

Because of the flashy nature and frequency of flooding at Rushford the Corps determined that the minimum protection to be considered would be for floods having a frequency of once in 100 years. The plan, which would provide the 100-year protection, very nearly approximated the upper limit of protection which could be justified by anticipated benefits. Later on, in the general design memorandum of 1965, modifications were made in the project design which changed the degree of protection. The final project design floods were 45,000 c.f.s.[1] on Root River above the confluence of Rush Creek and 16,200 c.f.s. on Rush Creek. These two values correspond to the 200-year flood for the Root River and the 100-year flood[2] for the Rush Creek. The design flood on the Root River below the confluence of Rush Creek was 49,300 c.f.s. The key years for the ex-post evaluation of this project are shown in table 1.

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[1] cubic feet per second

[2] The 100-year flood is the flood flow which can be expected to be exceeded once in 100 years, twice in 200 years, etc.

Table 1. Project History.

1956	Project Document Plan
1965	General Design Memorandum
1967	Construction started
1969	Construction ended
1974	Repair work
1979	Repair work
1986	Ex-post analysis
2068	End of project

EX-ANTE BENEFITS AND COSTS OF THE PROJECT

The first benefit-cost analysis done by the Corps of Engineers was presented in the 1956 Project Document Plan. The results are summarized in table 2. Estimated annual capital charges were based upon an assumed project life of 50 years and interest rates of 2.5%.

Table 2. Ex-ante Benefit-Cost Analysis (1956 and 1965).

	1956	1965
Total Federal and Non-Federal annual capital charges	$ 32,810	$ 76,590
Average annual Benefits	$ 37,960	$ 163,000
Benefit-Cost ratio	1.16	2.1

The second benefit-cost analysis was presented in the 1965 General Design Memorandum. The ex-ante cost estimates include the investment costs and the annual capital charges (tables 3 and 4). Estimated annual capital charges are based on a discount rate of 3 1/8 percent for both Federal and non-Federal works over a 100 year amortization period. The average annual benefits were about $ 163,000 (Table 2).

The Corps of Engineers up-dated the benefit-cost analysis in 1967 (tables 5 and 6). The new attempt produced a benefit-cost ratio close to the one estimated for the 1956 project document plan (1.16) but lower than the one estimated in the 1965 general design memorandum (2.1).

Ex-ante Benefit Analysis in Project Document Plan

Benefits from the proposed flood-control improvements represented the difference between damages which would be experienced under existing conditions and those which would occur with the proposed flood-control plan. Estimated damages included a minor allowance for anticipated normal growth and development in the flood plain without flood protection.

Average annual benefits were estimated by the Corps of Engineers using frequency-damage relationships (Figure 2). The area under this damage curve is the expected annual damages for Rush Creek. For the Rush Creek flood plain, annual urban flood damages of $30,750 were estimated for existing conditions while damages dropped to $4,400 under proposed conditions, resulting in annual urban flood control benefits of $26,350. For the Root River flood area, annual flood damages were estimated at $16,060

Table 3. Ex-Ante Cost Estimation, 1965.
(General Design Memorandum)

Federal and Non-Federal Costs	Estimate	
Federal First Costs		
Relocations	$ 30,000	
Channels	350,000	
Levees and flood walls		
Levee A-Root River	48,000	
Levee-Rush Creek		
B Right bank, west area	115,000	
C Left bank, north area	30,000	
D Left bank, east area	52,000	
Total levee work	245,000	
Flood wall	45,000	
Closure structure	39,000	
Drainage facilities		
Root River	131,000	
Rush Creek		
West area	237,000	
North area	53,000	
East area	113,000	
Total D.F.	534,000	
Total levees and flood walls	863,000	
Pumping plants	197,000	
Engineering and design	215,000	
Supervision and administration	115,000	
Total Federal first costs		1,770,000
Non-Federal first costs		
Land and Damages	91,000	
Relocations	162,000	
Total non-Federal first costs		253,000
Total Federal and non-Federal costs		2,023,000

(1) All prices are based on 1964 price levels.

SOURCE: Flood Control,Root River at Rushford, Minnesota; General Design Memorandum, U.S. Army Engineer District, Corps of Engineers, St Paul, Minnesota, March 1965.

Table 4. Estimated Investment Costs and Annual Capital Charges, 1965. (General Design Memorandum)

Investment Costs		
Federal Investment		
First Cost	$1,770,000	
Interest during construction (31/8% x $1,770,000x2/2)	55,200	
Total Federal investment		$1,825,200
Non-Federal investment		
First cost	253,000	
Interest during construction (31/8%x$253,000x2/2)	7,900	
Total Non-Federal investment		260,900
Total Federal and non-Federal investment		2,086,100
Annual charges		
Federal annual charges		
Interest, $1,825,200 at 3.12%	56,950	
Amortization(100 years)	2,740	
Inspection	260	
Total Federal annual charges		59,950
Non-Federal annual charges		
Interest	8,140	
Amortization(100 years) $260,900 at 0.15%	390	
Major replacements, mechanical installations(25 year life) Interest and amortization at 3.98%	1,440	
Major replacements, sluice and flap gates(50 year life) Interest and amortization at 3.98%	670	
Operation, maintenance and insp.	6,000	
Total non-Federal annual charges		16,640
Total Federal and non-Federal annual charges		$76,590

Source: Flood Control. General Design Memorandum, U.S. Corps of Engineers, 1965.

Table 5. Average Benefits Estimation for 1967 Benefit-Cost Analysis.

Source	Average Annual Benefits	
Rush Creek	$ 111,000	
Root River	25,400	
Duplication of damages	- (4,600)	
Rural benefits	400	
TOTAL	$ 132,200	
Correction for Future Growth		
(25% of total benefits, discounted over life of the project)	+ 16,500	
Total after correction	$ 148,500	
Additional benefits	19,900	
Total annual benefits	$ 168,400	(July 1966 price level)
	172,900	(July 1967 price level)

Source: Update files of the Flood Control Project at Rushford. U.S. Army Corps of Engineers, St. Paul, Minnesota, July 1967.

Table 6. Ex-Ante Benefit-cost Estimation, 1967[a].

Non-Federal Cost	Corps Estimates
Interest	$ 10,200
Amortization	2,800
Operation & maintenance	6,800
Major replacement	2,300
Total non-Federal annual cost	$ 22,100
Federal Cost	
Interest	$ 84,400
Amortization	23,200
Annual inspection	300
Total Federal annual cost	$ 107,900
Total annual costs	$ 130,000
Average annual benefits	$ 172,900
Benefit-cost ratio	1.3

[a] Discount rate: 3-1/8
Price level: July 1967

Source: Update files of the flood control project at Rushford. U.S. Army Corps of Engineers, St. Paul, Minnesota, July 1967.

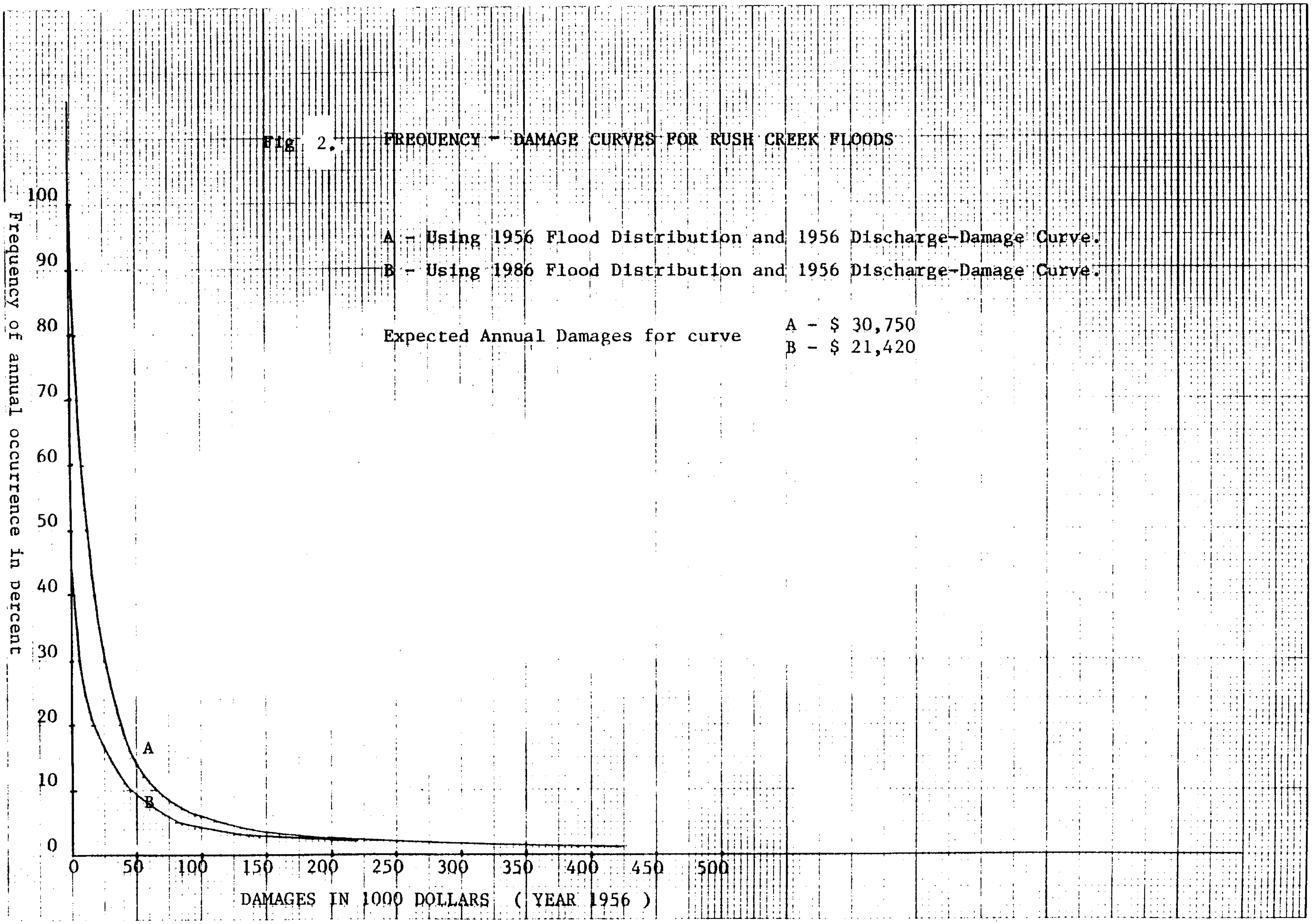

Fig 2. FREQUENCY - DAMAGE CURVES FOR RUSH CREEK FLOODS

A - Using 1956 Flood Distribution and 1956 Discharge-Damage Curve.

B - Using 1986 Flood Distribution and 1956 Discharge-Damage Curve.

Expected Annual Damages for curve A - $ 30,750 B - $ 21,420 for existing conditions while under proposed conditions, the annual urban flood control benefits would be $14,140. The total estimated annual urban flood control benefits were $40,490. However, this was reduced by $2,830 to eliminate a duplication of estimated damages in that portion of the flood plain which is affected by both the Root River and Rush Creek overflows.

HYDROLOGY FOR ESTIMATING BENEFITS

Hydrology is one of the most important inputs into ex-post analysis because it transforms the climatologic characteristics of a particular watershed into models and information concerning flood levels. There are two confluent waterways (Root River and Rush Creek) that determine the two most important random variables of the project. These are the peak discharges of both streams. Any flood over this town would be a function of these two particular variables.

One objective of this section will be to actualize the estimation of the probability distributions of these random variables and make comparisons with those made in the feasibility stage.

Time Series

The available time series data for the three hydrologic gauging stations[3] are shown in table 7. The objective of the hydrologic analysis is to obtain probability distributions at two specific points: a) Rush Creek at Rushford just before the confluence with Root River and b) Root River at Rushford just before the confluence. The past studies used probabilities distributions at these two points to estimate benefits.

There are two main obstacles to estimating these distributions. First, there is no hydrologic station in the Root River at Rushford, and second, the length of record in each station is different. To deal with the second obstacle, information is transferred from one station to the other. The solution for the first problem is to estimate the probability distributions at the points where there is information and then, through probability interpolation, estimate the probability distribution for the Root River at Rushford.

Transferring Information Between Stations

Since the length of the records are different in each station, information must be used from the station with longest record. A procedure described by Matalas (1964) was used to adjust the logarithmic mean and standard deviation of the short record (Lanesboro) on the basis of a regression analysis with the long-term record at Houston. The adjusted estimates are presented in table 8.

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[3] For this project, data from three hydrologic gauge stations were analyzed. These stations are the following:

1) Station 05384500. Rush Creek near Rushford, MN.
2) Station 05384000. Root River at Lanesboro, MN.
3) Station 05385000. Root River at Houston, MN.

Table 7. Peak Discharge for the Three Hydrologic Stations in cfs.

	----ROOT RIVER------		RUSH CREEK
Year	Houston	Lanesboro	Rushford
1910	2500	2040	
1911	15200	13300	
1912	10600	7930	
1913	10000	11800	
1914	11700	9670	
1915	7330		
1916	7970	5020	
1917	17000	12000	
1930	5100		
1931	4580		
1932	6900		
1933	26600		
1934	19000		
1935	11700		
1936	14000		
1937	14500		
1938	15600		
1939	6620		
1940	7860	5070	
1941	6280	5460	
1942	23700	15000	11000
1943	10600	8490	3600
1944	6120	5570	1660
1945	23900	13900	4000
1946	13700	10400	7130
1947	9300	7620	2590
1948	11700	7220	2000
1949	8450	6470	3640
1950	31000	20500	11600
1951	14800	16400	6580
1952	37000	20400	6740
1953	10400	8370	3750
1954	5370	4090	920
1955	3760	4090	1180
1956	9660	5430	1380
1957	2230	4530	1980
1958	9600	17800	420
1959	10100	9170	2000
1960	8800	8100	3460
1961	31400	19500	4920
1962	29500	22100	4550
1963	10700	7250	1530
1964	1110	409	53
1965	31000	19000	5490
1966	18500	16200	7490
1967	14200	12200	5170
1968	3210	1790	370
1969	8280	7340	620
1970	2250	2430	1640
1971	8970	7650	1290
1972	10200	8260	2300
1973	11700	11400	2030
1974	19800	17500	4400
1975	9430	7340	1220
1976	19800	14100	6040
1977	2290	1910	1300
1978	12200	14400	7930
1979	10400	8310	1500
1980	16400	6930	3930
1981	12600	7950	800
1982	4460	3920	600
1983	9500	8860	700
1984		3680	900
1985		4620	

Table 8. Adjusted Estimates of Means and Standard Deviations for Peak River Discharges.

	---------Stations--------		
	ROOT RIVER		RUSH CREEK
AT	Houston	Lanesboro	Rushford
Mean of logs	4.01	3.91	3.38
Standard Dev. of logs	0.29	0.27	0.39

Log-Pearson Type III Distribution[4]

Flood events are a succession of natural events which, as far as can be determined, do not fit any one specific known statistical distribution. To make the problem of defining flood probabilities tractable, it is necessary, however, to assign a distribution.

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[4] This distribution has a cumulative distribution function as follows:

$$F(x) = \int_{x0}^{x} \frac{(E - x0)^{y-1} \quad e^{-(E - x0)/\beta}}{\beta^{y} \ \Gamma(y)}$$

where β, y and $x0$ are parameters, and Γ is the gamma function, namely:

$$\Gamma(t) = \int_{0}^{\infty} x^{t-1} \ e^{-x} \ dx, \quad t > 0$$

In an important study done by the Hydrology Subcommittee for the U.S.G.S.(1982), it was concluded that the Log-Pearson type III distribution was the most accurate for annual peak discharges after studying 14,200 station-years of record in the United States. They concluded that:

> "In essence, then, regardless of the methodology employed, substantial uncertainty in frequency estimates from station data will exist, but the log-Pearson type III method with regional skew coefficients will produce unbiased estimates when the adjustment to expected probability is employed, and will reduce uncertainty as much as or more than other methods tested"

Fitting the Distribution

The recommended technique for fitting a log-Pearson type III distribution to observed annual peaks is to compute the base 10 logarithms of the discharge, Q, at selected exceedence probability [5], P, by the equation:

$$\text{Log } Q = X + KS \quad [1]$$

where X = mean logarithm, S = Standard deviation of logarithms, and K is a factor that is a function of the skewness coefficient and the selected exceedance probability. Instead of using the skewness coefficient from the station directly, it is weighted with a generalized skewness coefficient developed for subregions of the United States by U.S.G.S. This weighted skewness coefficient is estimated in the following way:

$$Gw = \frac{MSEo\ (Go) + MSEu\ (Gu)}{MSEo + MSEu} \quad [2]$$

---

[5] Probability that the river discharge exceeds Q in the period of one year.

where Gw = weighted skewness coefficient

Go = station skewness coefficient

Gu = generalized skewness coefficient

MSEu= Mean-square error of generalized skewness

MSEo= Mean-square error of station skewness

Table 9 shows the weighted skewness for the three stations considered in this analysis.

Table 9. Skewness and Weighted Skewness Coefficients for the Hydrologic Stations.

Station	Length of Record N	Skewness G	Weighted Skewness Gw
Houston	61	-0.40	-0.40
Lanesboro	52	-0.64	-0.50
Rushford	43	-0.19	-0.20

Table 10 illustrates the computation of frequency curve coordinates for the three stations. The first column is the exceedance probability, the second column is the K factor, which is a function of the weighted skewness coefficient (Gw) and the probability. The third column shows the estimated value of Log Q and the last column is the discharge in cubic feet per second corresponding to each probability.

The drainage area ratio transfer method was used to estimate the frequency curve coordinates for Root River at Rushford. The curve coordinates are estimated using the following expression:

Table 10. Discharge Frequency for the Root River at Houston and Lanesboro and for Rush Creek at Rushford.

ROOT RIVER AT HOUSTON

Prob	Kg,p	Log Q(*)	Q (cfs)
0.9999	-4.597	2.667	464.42
0.999	-3.666	2.937	864.64
0.995	-2.949	3.145	1395.68
0.9	-1.317	3.618	4151.01
0.8	-0.816	3.763	5797.62
0.5	0.066	4.019	10454.13
0.3	0.569	4.165	14618.88
0.1	1.231	4.357	22752.57
0.05	1.524	4.441	27658.92
0.025	1.764	4.511	32481.66
0.02	1.833	4.532	34020.98
0.01	2.029	4.588	38770.88
0.005	2.200	4.638	43477.72

ROOT RIVER AT LANESBORO

0.9999	-4.821	2.656	453.35
0.999	-3.811	2.919	830.16
0.995	-3.041	3.119	1316.24
0.9	-1.323	3.566	3681.26
0.8	-0.808	3.699	5010.08
0.5	0.083	3.931	8542.50
0.3	0.578	4.060	11491.76
0.1	1.216	4.226	16834.75
0.05	1.491	4.297	19845.52
0.025	1.713	4.356	22675.22
0.02	1.777	4.372	23553.83
0.01	1.954	4.418	26195.53
0.005	2.109	4.458	28717.39

Table 10 (con't)
RUSH CREEK AT RUSHFORD

0.9999	-4.153	1.760	57.58
0.999	-3.377	2.063	115.60
0.995	-2.763	2.302	200.60
0.9	-1.301	2.873	745.74
0.8	-0.830	3.056	1137.96
0.5	0.033	3.393	2471.53
0.3	0.547	3.593	3922.40
0.1	1.258	3.871	7425.29
0.05	1.586	3.999	9967.06
0.025	1.864	4.107	12788.15
0.02	1.945	4.138	13757.70
0.01	2.178	4.229	16966.01
0.005	2.388	4.311	20478.61

(*) Calculated using formula [1]

[3] $$Q = Y1 + ((Y2-Y1)x(S-X1)/(X2-X1))$$

where Q= Log of discharge at Rushford

S= Log of drainage Area at Rushford

$X1$= Log of drainage area at Lanesboro

$X2$= Log of drainage area at Houston

$Y1$= Log of discharge at Lanesboro

$Y2$= Log of discharge at Houston.

Expected Probability Estimation

The probabilities shown in table 10 have to be corrected to obtain an estimate of the expected probabilities. These probabilities are defined as the average of all the true probability estimates for any specified flood frequency that might be made from successive samples of a specified size. For any specified flow, it is considered to be the most appropriate estimate of probability or frequency of future flows for water resources planning and management (Bulletin 17B, U.S. Dept. of Interior, 1982).

It has been shown that for the normal distribution the expected probability Pn can be obtained from the formula

[4] $$Pn = Prob [tN-1 > Kn (N/(N+1))^{1/2}],$$

where Kn is the standard normal variate of the desired probability of exceedance, N is the sample size, and tN-1 is the Student's t-statistic with N-1 degrees of freedom. The final probability distributions are shown in Figure 3, simultaneously with the original distributions from past studies.

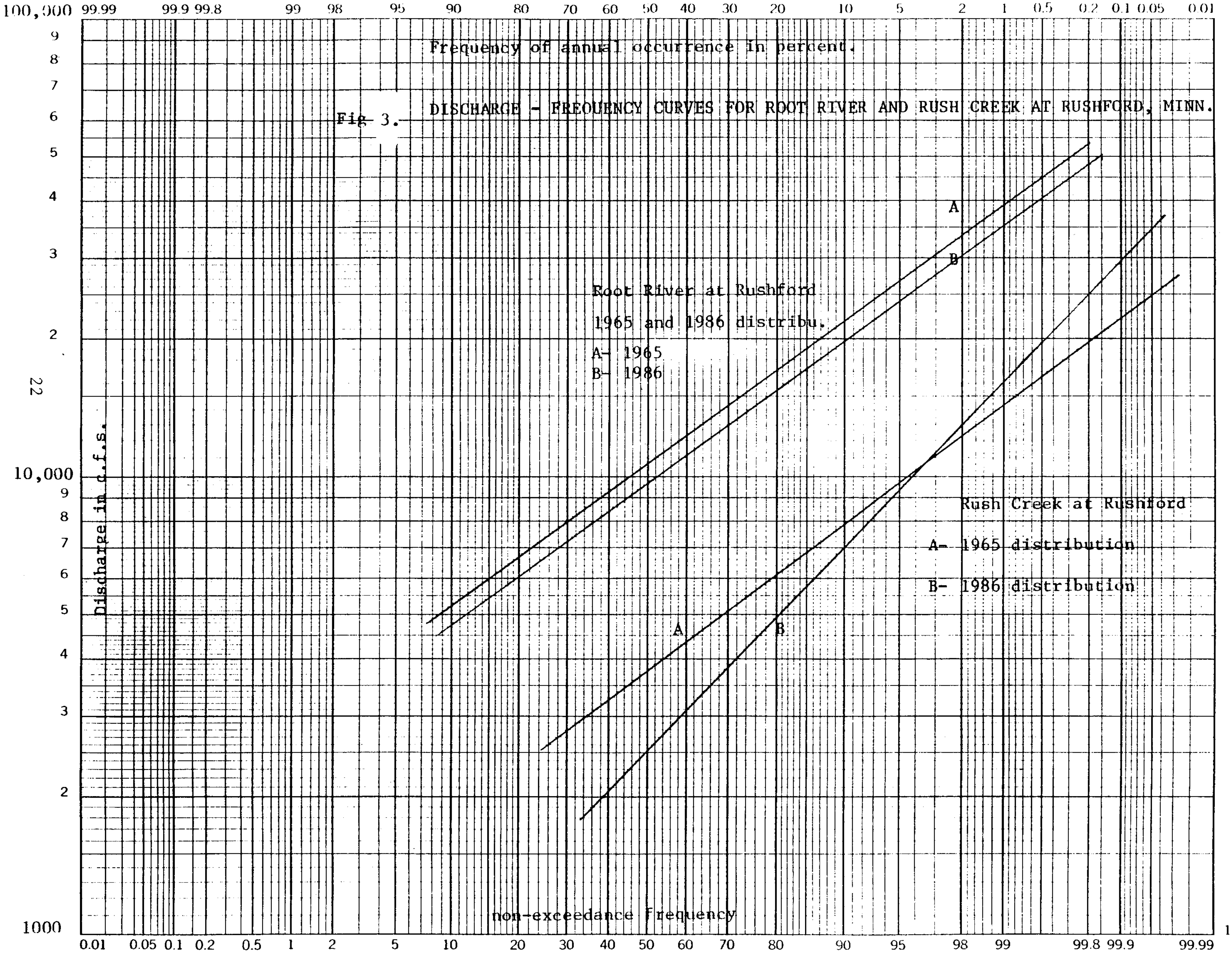

Fig 3. DISCHARGE - FREQUENCY CURVES FOR ROOT RIVER AND RUSH CREEK AT RUSHFORD, MINN.

Even though there is more information and the methodology is improved, the probability distribution for the Root River at Rushford has remained almost the same. But this is not the case with Rush Creek at Rushford, where the new estimate is quite different from the "original" estimate done in the feasibility study (Figure 3).

The 100 and 200-year floods for both locations are shown in table 11. The new estimates are compared with the same estimates done 30 years ago. For the Root River the new flood estimates are lower while they are higher for Rush Creek.

Table 11. Estimated Flood Discharge Levels.

	Root River at Rushford	Rush Creek at Rushford
	(new estimates, 1986)	
100-year flood	36,000 c.f.s	16,000 c.f.s
200-year flood	42,000 c.f.s	20,000 c.f.s
	(1956 estimates)	
100-year flood	39,000 c.f.s	14,000 c.f.s
200-year flood	45,000 c.f.s	16,500 c.f.s

EX-POST BENEFIT-COST ANALYSIS

Benefits

The Economic and Environmental Principles and Guidelines for water and related land resources, which has been a guide to formulate and evaluate federal water resource development projects, recognizes the following three flood protection benefit categories:

(1) Inundation reduction benefit: If floodplain use is the same with and without the project, the benefit is the increased net income generated by the use.

(2) Intensification benefit: If the type of floodplain use is unchanged but the method of operation is modified because of the plan, the benefit is the increased net income generated by the floodplain activity.

(3) Location benefit: If an activity is added to the floodplain because of the project, the benefit is the difference between aggregate net incomes (including economic rent) in the economically affected area with and without the project.

The report also classifies flood damages in the following way:

1) Physical damages include damages to or total loss of buildings or parts of buildings; loss of contents, etc.

2) Income loss either in terms of loss in wages or net profits to business over and above physical flood damages. This loss of income by commercial, industrial, and other business firms was not considered in the analysis because of the complexity involved in determining whether the loss is recovered by the firm at another location or at a later time.

3) Emergency Costs include those expenses resulting from a flood that would not otherwise be incurred.

i. Flood Plain and Damages

Hydraulic calculations are used in a three step standard procedure for obtaining two essential variables, flood profile and flood-plain delineation. The three steps used to obtain the flood profile and flood-plain delineation are:[6]

1. Obtain survey cross sectional information directly from topographic maps.

2. Calculate the flood profile. The flood profile is a graph showing the water-surface elevation or height of a particular flood event for any point along a stream. The flood profile is determined by using the standard open-channel hydraulic calculations. These calculations, in effect, determine the height of a flood through the confinement of a given flood discharge within the cross-sections that were obtained in step 1. For the flood profile calculations, sections every 700 feet on average were used along both Root River and Rush Creek.

3. The flood-plain is delineated on topographic maps. The flood plain is delineated by finding ground points on both sides of the stream that correspond to the flood-profile elevations at a sufficient number of points for accurate determination of boundaries.

In this particular project there are two main parts in the estimation of benefits. The first part consists of estimating the expected future benefits which is called the stochastic (future)

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[6] The analysis of response to a flood hazard is based on a probability weighing of floods of various magnitudes. This implies that floodplain occupants are risk-neutral, but many occupants, individually or as a group, either avert or accept risk. An important research topic would be to see how economic parameters would change if the risk neutral assumption was not used.

part. The second is to estimate the past benefits which is the deterministic part of the analysis. In the case of Rushford, the flood frequency analysis is developed using coincident frequency analysis. What is needed is the frequency distribution of a random variable C, where C is dependent on two random variables, A and B, whose individual occurrence patterns are known. Ideally, we would need to determine the stage on Rush Creek as a function of the stage on the Root River and the flow in Rush Creek. However, this cannot be done due to a lack of data. Therefore a different approach is used.

ii. Coincident Frequency Analysis

The new methodology consists of using discrete values for the discharges of the Root River and Rush Creek, which determine discrete areas in the two dimensional space. A bivariate probability distribution is then fitted to this space. For instance, each area would correspond to the following set :

$$\{ (Q1,Q2) / Q1a < Q1 < Q1b \; ; \; Q2a < Q2 < Q2b \}$$

where Q1,Q2 represent the peak discharges in the Root River and Rush Creek, and Q1a,Q1b,Q2a,Q2b are the bounds of the intervals of the discrete variables. Next, the mean values of each interval (W1,W2) are taken for each discrete area,(ie. $W1=(Q1a+Q1b)/2$ and $W2=(Q2a+Q2b)/2$), and the estimated flood plain for these values would be the representative flood plain value for that specific discrete area.

Figure 4 illustrates how peak discharge variables Q1 and Q2 define the flood plain. This figure shows water surface profiles along a tributary near the junction with a main river. The stage[7] on the tributary is a function of main river stage and tributary discharge.

In region I, the main river stage will tend to have the dominant influence on the tributary stage, whereas in region II, the tributary discharge will tend to dominate. The boundary between regions I and II cannot be precisely defined and will vary with exceedance frequencies. Stage-frequency determinations will be least accurate in the vicinity of the boundary where both variables have a substantial impact on the combined result.

The next step for this coincident frequency analysis is to estimate a flood plain for the center values (W1,W2) of each area-interval. Once the flood plain is determined for each flood event, then damages can be related to the flood plain. Fitting a probability distribution to the space defined by an interval is the way of associating each flood plain with its occurrence probability and then estimating the expected annual benefits for the remaining life of the project.

Fitting the lognormal function to empirical distributions of annual flood peak discharges may be more attractive than fitting other distribution functions. The bivariate lognormal probability function allows the analytical derivation of distributions of conditional variables which is an advantage when fitting the

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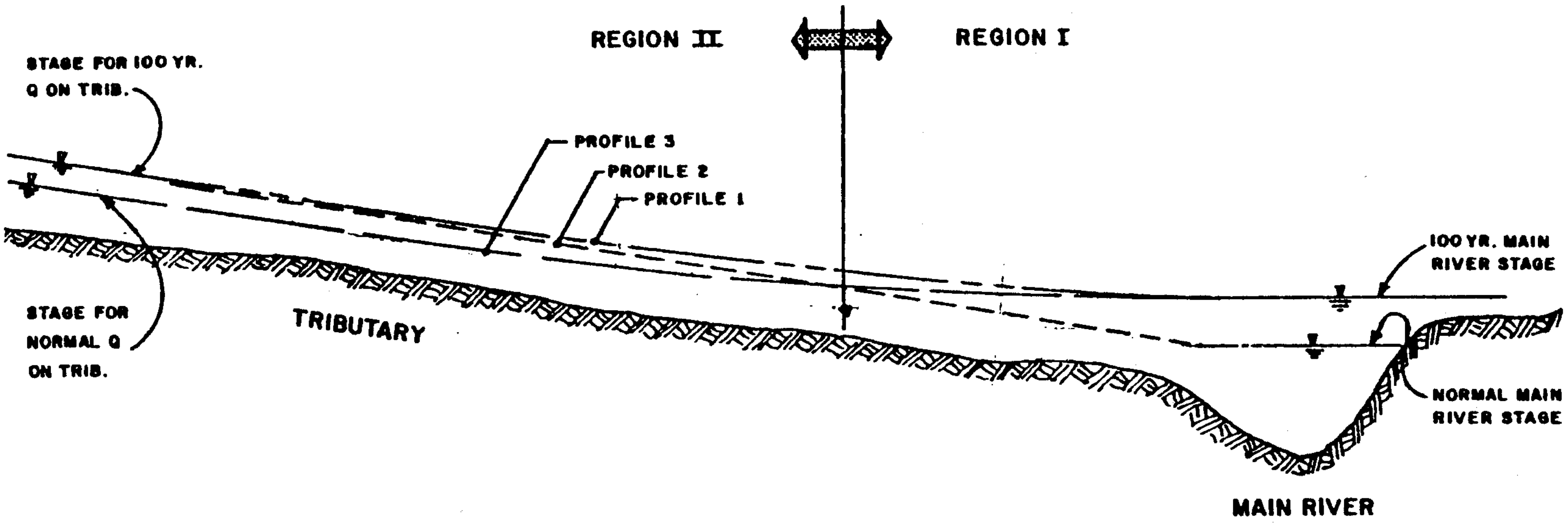
[7] Water level in the main channel.



PROFILE 1	100 YR MAIN RIVER STAGE AND 100 YR Q ON TRIB.
PROFILE 2	NORMAL MAIN RIVER STAGE AND 100YR Q ON TRIB.
PROFILE 3	100 YR MAIN RIVER STAGE AND NORMAL Q ON TRIB.

Fig. 4 WATER SURFACE PROFILES ON A TRIBUTARY AT THE JUNCTION WITH A MAIN RIVER

distribution of flood peaks. The bivariate lognormal function was fitted to random variables Q1 and Q2; peak discharges in the Root River and Rush Creek respectively.

iii. Bivariate Lognormal Probability Distribution

For a bivariate (q1,q2) with simple correlation coefficient, r(q1,q2), of components q1 and q2, the bivariate lognormal probability density function is,

$$[5] \quad f(x,y) = \frac{1}{2\,\pi x y\,\sigma(x)\,\sigma(y)\sqrt{1-r^2}}\, e^{-QN/2(1-r^2)}$$

$$QN = \{(x-\mu x)^2/\sigma(x)^2 + (y-\mu y)^2/\sigma(y)^2 - 2r\,\frac{(x-\mu x)(y-\mu y)}{\sigma(x)\,\sigma(y)}$$

in which q1= log Q1, and q2= log Q2, and μx, $\sigma(x)$ and μy, $\sigma(y)$ are the means and standard deviations of the logarithms. As discussed in the benefit section, the probability for each area-interval is needed. This means that function [5] must be integrated over every area-interval. A polynomial approximation for bivariate normal probabilities methodology introduced by Moskowitz, et al (1986) is used and presented in appendix A. The results of this integration are shown in Table 12. An example from this table would be, there is a probability of 0.48 that the realization of the random variable Q1 be in the interval [0,9000] and Q2 be in the interval [0,5000].

Table 12. Lognormal Bivariate Probability Distribution.

RUSH CREEK DISCHARGE (Q2)	ROOT RIVER DISCHARGE (Q1)				
	0-9000	9000-18000	18000-27000	27000-36000	36000-45000
0-5000	(0.48)*	(0.244)	(0.058)	(0.018)	(0.005)
5000-10,000	(0.01)	(0.060)	(0.028)	(0.009)	(0.005)
10,000-15,000	(0.005)	(0.018)	(0.008)	(0.006)	(0.001)
15,000-20,000	(0.002)	(0.005)	(0.005)	(0.000)	(0.000)

Q1 = Peak discharges in the Root River in c.f.s
Q2 = Peak discharges in the Rush Creek in c.f.s

$$P[\ 0 <= Q1 <= Q1(200) \text{ and } 0 <= Q2 <= Q2(200)\ ] = 0.967$$

$$P[\ Q1 > Q1(200) \text{ and } Q2 > Q2(200)\ ] = 0.033$$

Q1(200)=45,000 c.f.s. (200-year discharge for the Root River)
Q2(200)=20,000 c.f.s. (200-year discharge for the Rush Creek)**

(*) Values within parentheses are areas under the bivariate lognormal probability distribution for each specific two dimensional interval. The procedure used to obtain these values is shown in Appendix A.

(**) The frequency of these discharges was estimated using the Log Pearson type III distributions.

iv. Residential, Industrial, Commercial and Public Units

To estimate flood damages in Rushford one must identify residential, industrial, commercial and public units that might be flooded. In order to accomplish this, an inventory of structures is required. A base map is used which details the affected area and contains information such as blocks and lots. A topographic map is also used to estimate the elevation of each unit.

The inventory for residential structures is sometimes called a "windshield survey". Such a survey is done by an analyst who drives up and down the streets, using maps and a computation sheet. The houses are numbered as a means of identification. Information needed includes the number of floors in the house, whether or not it has a basement, if it is split level and the elevation of the ground and first floor.

Ground elevation is obtained from maps, and first floor elevation is ground elevation plus (or minus if the house is a split level) the number of steps leading to the front door.In this study a height of 4 inches per step was used. Ground elevation is the point at which water comes into contact with the structure and first floor elevation is the elevation at which water causes damage.

For commercial, public and industrial structures, interviews with business owners or managers were necessary. The principle variables obtained were the area of the unit, value of merchandise susceptible to damage and the kind of equipment that might be in the building. For some businesses in which it was not possible to conduct interviews, the required data was obtained by sampling similar businesses elsewhere.

v. Market and Assessed Values

To estimate the economic value of these units, assessed values were obtained from the county assessor, and some market values were obtained from a local real estate agent. These latter values were used to construct a correction factor for the assessed values which are almost always below the real economic value of the units. One conclusion drawn from this information is that the higher the market value of the house, the larger the difference between market and assessed value of the unit. A correction factor based on the market values was used to adjust the original estimates (1.38 for units with assessed value less than $25,000 and 1.54 for units assessed at more than this amount).

vi. Damage Susceptibility

An important step in measuring flood damages is to determine the damage susceptibility of units. Once the number of physical units and the value associated with each unit are known, a damage susceptibility relationship for these units is needed. This unit damage function shows the fraction of its market value that would be lost if these units were inundated to a certain level.

Figure 5 shows a simplified version of the relationships that the Corps of Engineers (St. Paul) uses for residential units. For some residential, commercial and public units, individual damage

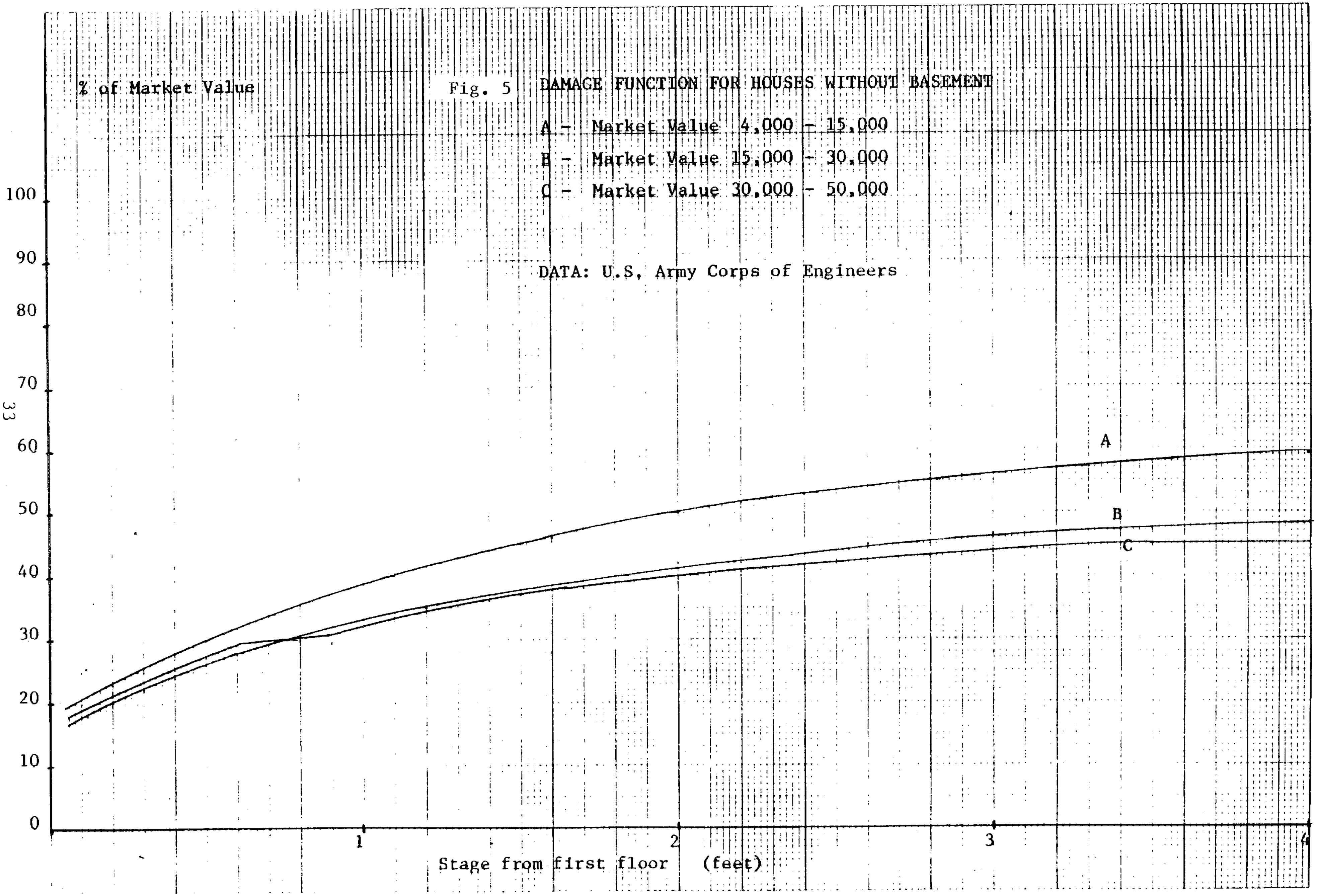


Fig. 5 DAMAGE FUNCTION FOR HOUSES WITHOUT BASEMENT

DATA: U.S. Army Corps of Engineers

susceptibility relationships were developed using the survey data. The remaining units had to be grouped on the basis of similar characteristics, and a standard damage susceptibility relationship was used.

Knowing these relationships and the flood plain for a particular flood event, a computer program was developed to compute damages for each individual unit and to aggregate these damages for the flood event.

vii. Stochastic Benefits

The last step in estimating damages from the flood events (W1,W2) is to relate damages to discharges (table 13). The values in parentheses are the estimated representative damages for each two dimensional discharge interval. This table also shows (values in brackets) the expected damages (probability from table 12 times damages) for each interval. The expected annual avoided damages is the sum of all values in brackets.[8]

viii. Deterministic Benefits

These benefits are the damages prevented by the project since construction was completed (1968) and 1985 (last year with

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[8] Assuming benefits as the expected value of damages implies risk neutrality. Samuelson (1964) has suggested that the government should be risk neutral because of the diversity of investments, also Arrow and Lind (1970) demonstrate that the spreading of risk associated with a single public project over a large number of tax-payer beneficiaries, each having a small share in the net returns, can also result in the disappearance of a (social) premium.

TABLE 13. AVERAGE DAMAGES FOR SPECIFIC DISCHARGE INTERVALS

RUSH CREEK DISCHARGE (Q2)	ROOT RIVER DISCHARGE (Q1)				
	0-9000	9000-18000	18000-27000	27000-36000	36000-45000
0-5000	(2,045,100)* [981,648]	(2,219,500) [541,558]	(2,705,000) [156,890]	(2,853,100) [51,356]	(3,915,700) [19,579]
5000-10000	(7,487,100) [74,871]	(7,659,000) [459,540]	(7,706,000) [215,768]	(7,840,000) [70,560]	(7,840,000) [39,200]
10000-15000	(10,013,900) [50,069]	(10,025,000) [180,450]	(10,350,000) [82,800]	(10,053,900) [60,323]	(10,053,900) [10,054]
15000-20000	(11,031,000) [22,062]	(11,090,000) [55,450]	(11,120,000) [55,450]	(11,120,000) [0]	(11,120,000) [0]

Expected annual damages prevented = $ 2,764,628

Expected annual damages not prevented = $ 363,000

(*) Values in parentheses are the estimated damages for every two dimensional discharge interval in 1986 U.S. dollars. Values in brackets represent the expected damages (Probability times damages) for every two dimensional discharge interval. They are also given in 1986 U.S. dollars.

Q1 = Peak discharge in the Root River in c.f.s
Q2 = Peak discharge in the Rush Creek in c.f.s

available hydrologic data).

With the help of the computer program (shown in appendix C of Ramirez (1987)), Figures 6 and 7 were constructed. Figure 6 is the damage vs. discharge relationship for the Root River given a normal discharge (discharge not causing damages) from Rush Creek. Figure 7 is the same individual relationship for Rush Creek provided that there are no discharges causing damages from the Root River.

Once these relationships and Q1,Q2 (peak discharges in the Root River and Rush Creek) are known, the damages prevented can be estimated either by a) using Figure 6 and 7 when either Q1 or Q2 is low enough so that it can be considered a normal discharge, or b) running the computer program which follows the methodology explained earlier (when both discharges have damage potential).

The aggregate flood damages prevented for each year during the period 1968-85 are shown in Table 14. The present value of these benefits as of 1986 is $ 18.0 million. In contrast, the expected present value of benefits for this period using the bivariate log-normal distribution approach is $24.2 million (table 14). Benefits are discounted at 8-7/8% for the 18 year period.

The project has accomplished its main objective so far, as no flood damage has occurred since the project was completed and as one old citizen of Rushford said about the project: "This is the best thing that has happened in the history of Rushford".

Since the primary objective of this study is to compare the ex-post estimates with the original ex-ante estimates, both costs

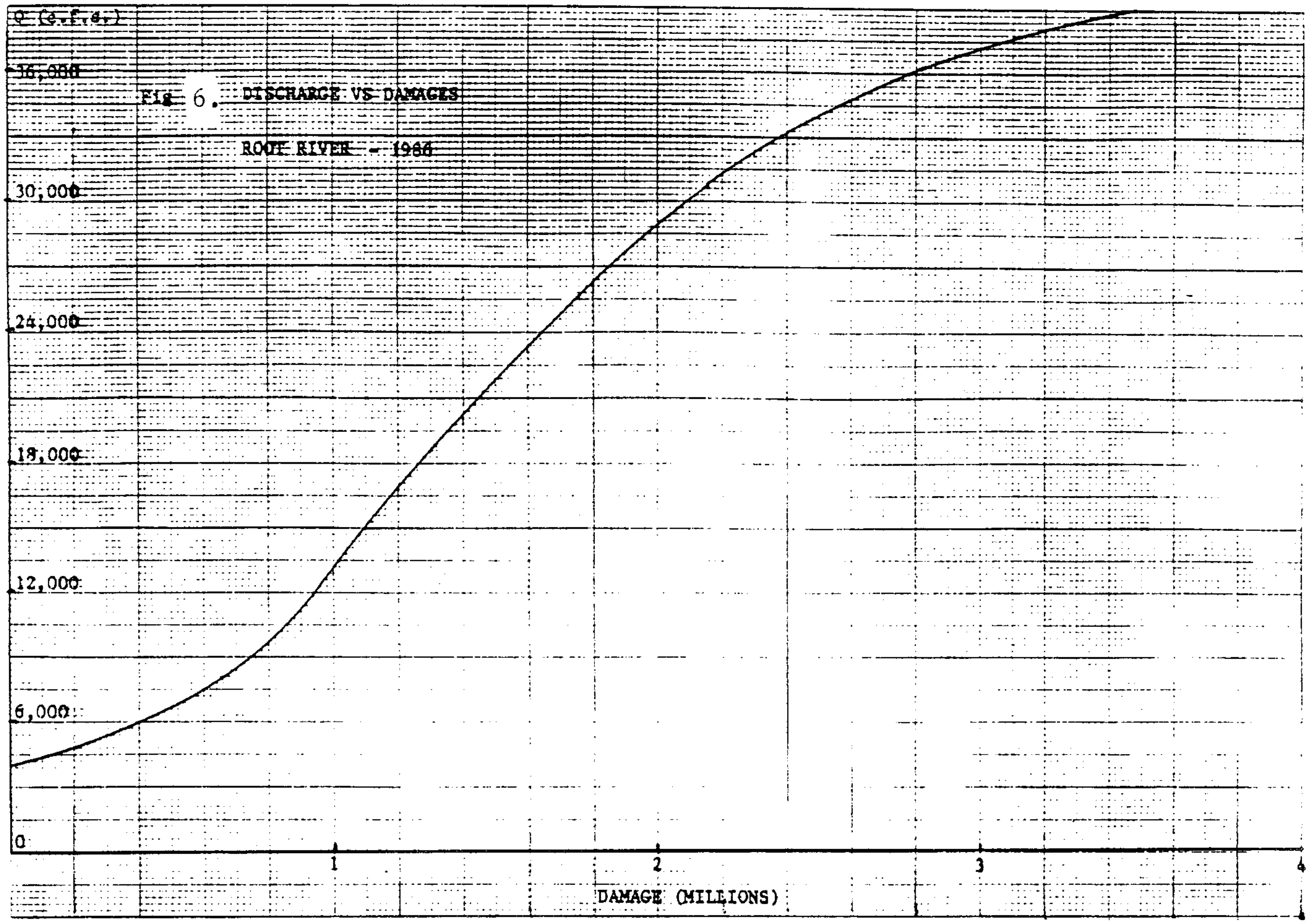


Fig 6. DISCHARGE VS DAMAGES

ROOT RIVER - 1986

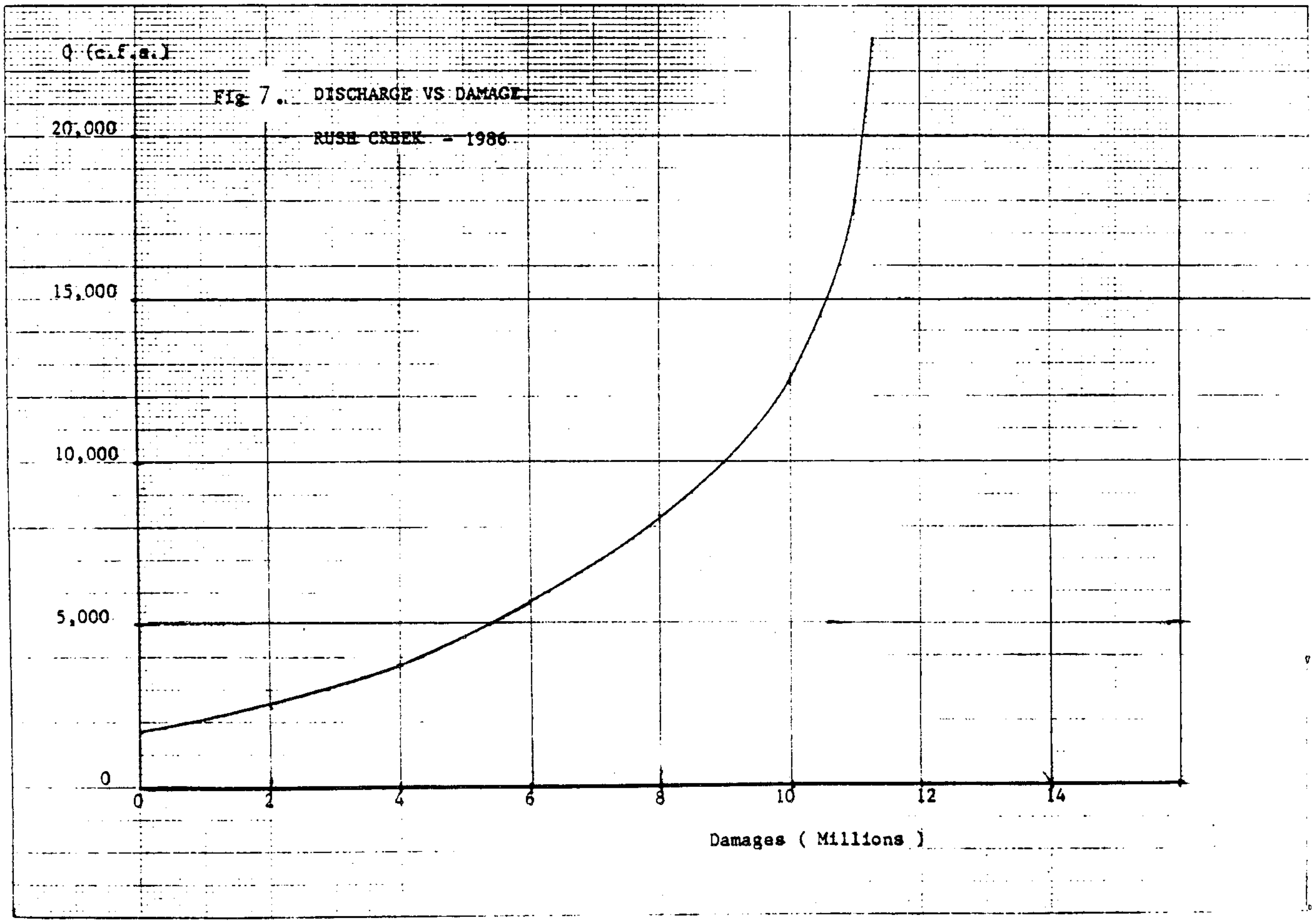


Fig 7. DISCHARGE VS DAMAGE

RUSH CREEK - 1986

Table 14. Estimated Project Benefits for the Period 1968-1985.

YEAR	Benefits (1986 price level)
1968	$ 0
1969	559,400
1970	0
1971	639,100
1972	1,688,320
1973	2,070,613
1974	1,762,886
1975	667,660
1976	6,550,000
1977	0
1978	8,463,900
1979	2,126,813
1980	7,735,380
1981	1,507,140
1982	41,440
1983	2,079,926
1984	60,900
1985	208,320
TOTAL	$ 36,161,798

Using 8-7/8% discount rate, this corresponds to 1986 present value of $18,012,754.

Using the same discount rate and time period, and the expected annual damages prevented from Table 13, one obtains an expected present value of $24,206,563.

and benefits reported in actual prices had to be converted to 1967 prices through the use of price indexes. The Engineering News Record's (ENR) construction index was used to deflate costs and the ENR building index was used to deflate project benefits.

The average annualized benefits calculated for the Rushford project, are $533,862, $570,262, $610,038 and $648,736 respectively for 8-7/8, 7, 5 and 3-1/8 percent discount rates[9] (See table 15).

Costs

The total Federal cost was $2.78 million, and the non-Federal cost was $326,000 (1967 price level). The cost of repair work finished in 1974 was $160,354 and the cost of construction completed in 1979 was $421,000. The total annualized costs of the project were $377,054, $300,274, $218,835 and $141,504 respectively for 8-7/8, 7, 5 and 3-1/8 percent discount rates (see table 15).

Benefit-cost Analysis

The ex-post estimates show that flood control benefits are considerably more than costs, with an ex-post benefit-cost ratio of more than unity even with a 8 7/8% real discount rate (see table 15). Table 16 shows the distribution of structures directly damaged by floods in specific interval areas.

[9] In all the cases, the rates are real rates since all benefits and costs have been deflated.

Table 15. Ex-Post Benefit-Cost Analysis of the Flood Control Project without Contents Growth Factor (1967 price level).

Benefits and Costs	Discount Rates (%) 8-7/8	7	5	3-1/8
Deterministic Benefits (past)	4,180,238	4,950,169	5,969,462	7,162,779
Net Stochastic Benefits (future)	1,834,857	3,187,136	6,139,281	12,706,616
TOTAL Benefits	6,015,095	8,137,305	12,108,743	19,869,395
Annualized Benefits	533,862	570,262	610,038	648,736
Annualized Costs	352,829	278,769	200,405	127,292
Additional repairs	24,225	21,505	18,430	14,212
TOTAL Annualized Costs	377,054	300,274	218,835	141,504
Benefit-Costs ratios	1.4	1.9	2.8	4.6

Sensitivity Analysis

Multiple sources of uncertainty can be recognized in this analysis given the complexity of the hydraulics and hydrology of Rushford's flood plain. With the coincident frequency methodology used in this analysis, one of the major sources of uncertainty is the damage susceptibility relationship for industrial, commercial and public units due to the lack of homogeneity and the inability of people interviewed to estimate these relationships. On average, 45 percent of expected flood damages in Rushford are attributed to residential damages and 55 percent to industrial, commercial and public non-residential damages. A sensitivity analysis was developed for the damages to these non-residential units. The non-residential damages were increased and decreased by 20%. Table 17 illustrates that the B/C ratio has a low sensitivity to this 20% change in non-residential damages.

Another source of uncertainty is the selection of a time horizon or planning period for the physical and economic life of the improvements to prevent or control flooding. Initially, in 1956, this period was considered to be 50 years, but later the Corps of Engineers changed it to 100 years.

The last row of table 17 presents the B/C ratio when the life of the project is only 50 years. The lower the discount rate the more sensitive the B/C ratio is to the time horizon. However, changing the life of the project does not affect the economic results of the analysis.

A third source of uncertainty is the contents growth for

Table 16. Distribution of Structures Directly Damaged by Floods in Some Specific Interval Flood Areas.

RUSH CREEK DISCHARGE (Q2)	ROOT RIVER DISCHARGE (Q1)				
	0-9000(61%)	9,000-18,000(18%)	18,000-27,000(5%)	27,000-36,000(1.6%)	36,000-45,000(0.5%)
0-5000(20%)**	(92)* [8]	(103) [9]	(106) [11]	(108) [15]	(145) [20]
5000-10000(4%)	(236) [30]				
10000-15000(1.4%)	(290) [32]				
15000-20000(0.5%)	(315) [34]				

(*) Values in parentheses are the identified residential units that are inundated with floods of that magnitude. Values in brackets are the commercial, business and public units identified that are inundated with floods of that magnitude.

(**) Frequency

Q1= Discharges in the Root River in c.f.s.
Q2= Discharges in the Rush Creek in c.f.s.

Table 17. Sensitivity Analysis on the Evaluation of the Flood Control Project.

	Discount Rates (%)			
Change	8-7/8	7	5	3-1/8
	benefit-cost ratios			
Without contents growth rate	1.4	1.9	2.8	4.6
With contents growth rate	1.5	2.1	3.1	5.6
20% increase in commercial, business and public damages	1.6	2.1	3.1	5.1
20% decrease in commercial, business and public damages	1.2	1.7	2.5	4.1
Project's life is change to 50 years	1.4	1.9	2.7	4.0

Note: All of these sensitivity cases are with respect to the benefit-cost analysis presented in table 15.

residential units. The Corps of Engineers suggests a value of typical contents of approximately 40 percent of structure value. They apply a growth rate for contents in residential units to estimate future damages. This growth factor is a function of Rushford's income per capita projections. The ex-post economic analysis presented in table 15 assumes a zero contents growth factor. For a sensitivity analysis, a 2% contents growth rate for the next 50 years was applied (table 17). The B/C ratio is only marginally increased by this change, except at the low discount rate.

A final source of uncertainty considered is the discount rate. Selection of an appropriate discount rate is important because this rate has a substantial impact on B/C ratios. Selection should reflect at least the cost of borrowed capital for the entities involved, in this case the government. In this project the recommended value from the Corps of Engineers of 8 7/8% per year was used. However, for purposes of the sensitivity analysis, other rates were tried. Lowering the discount rate from 8 7/8% to 3 1/8% increased the B/C ratios three fold.

COMPARISON OF COSTS AND BENEFITS RESULTS

In this section comparisons are made: a) between estimates of ex-ante and ex-post costs and benefits, and b) between probability distributions. It will indicate how accurately the ex-ante estimates were in predicting project performance.

Cost Comparisons

The first comparison is between the 1967 ex-ante estimates and the ex-post estimates, while the second is between the 1956 ex-ante estimates and the ex-post estimates (table 18). In both comparisons, a discount rate of 8-7/8% and a project life of 100 years are used. For 1967, total annualized ex-ante costs of $319,011 are significantly below the ex-post costs of $ 377,054. The difference is due to the additional repairs in 1974 and 1979. For the second comparison, the total annualized ex-ante costs of $82,581 are well below the ex-post costs of $ 225,253. The ex-ante cost estimate is based on a project that provides protection against a 100-year flood while the ex-post cost is based on a project that protects the city of Rushford from a 200-year flood. This, along with repairs, explain the difference between these two estimates.

Benefit Comparisons

A comparison of ex-ante and ex-post benefits has to be done for just flood protection benefits, because they were the only benefits considered in the ex-ante benefit-cost analyses of 1956, 1965 and 1967. Thus the actual benefit-cost ratio for the project could be higher because other benefits are excluded.

The 1967 comparison show ex-ante benefits are $110,343 lower than ex-post benefits (see table 18). Some reasons for this difference are: 1) the coincident frequency methodology applied in

Table 18. Comparison of Ex-ante and Ex-post Estimates, Rushford Project.

A) Comparison between ex-post estimates and 1967 ex-ante estimates

(1967 price level and 8 7/8 % discount rate)

	EX-ANTE	EX-POST
Total annualized Cost	$ 319,011	$ 377,054
Total annualized flood benefits	$ 423,519	$ 533,862
B/C ratio	1.3	1.4
Protection	(200-year floods)	(200-year floods)

B)Comparison between ex-post estimates and 1956 ex-ante estimates

(1955 price level and 8 7/8 % discount rate)

	EX-ANTE	EX-POST
Total annualized cost	$ 82,581	$ 225,253
Total annualized benefits	$ 95,543	$ 363,017
B/C ratio	1.16	1.6
Protection	(100-year floods)	(200-year floods)

the ex-post analysis, 2) the change in discharge-damage relationships for both the Root River and Rush Creek. This can be observed by comparing figure 8 with figures 6 and 7. Had the town remained the same, these relationships would have had similar shapes. This change is explained by the residential, commercial and industrial growth in Rushford since the project was constructed. 3) The 1967 ex-ante benefit estimate includes a correction for future growth of 1.25.

A comparison for 1956 indicates that ex-ante benefits were $267,474 less than the ex-post estimates. Reasons for this difference are: 1) and 2) above, plus the fact that the ex-ante estimate is associated with protection against a 100-year flood while the ex-post estimate is for a 200-year flood.

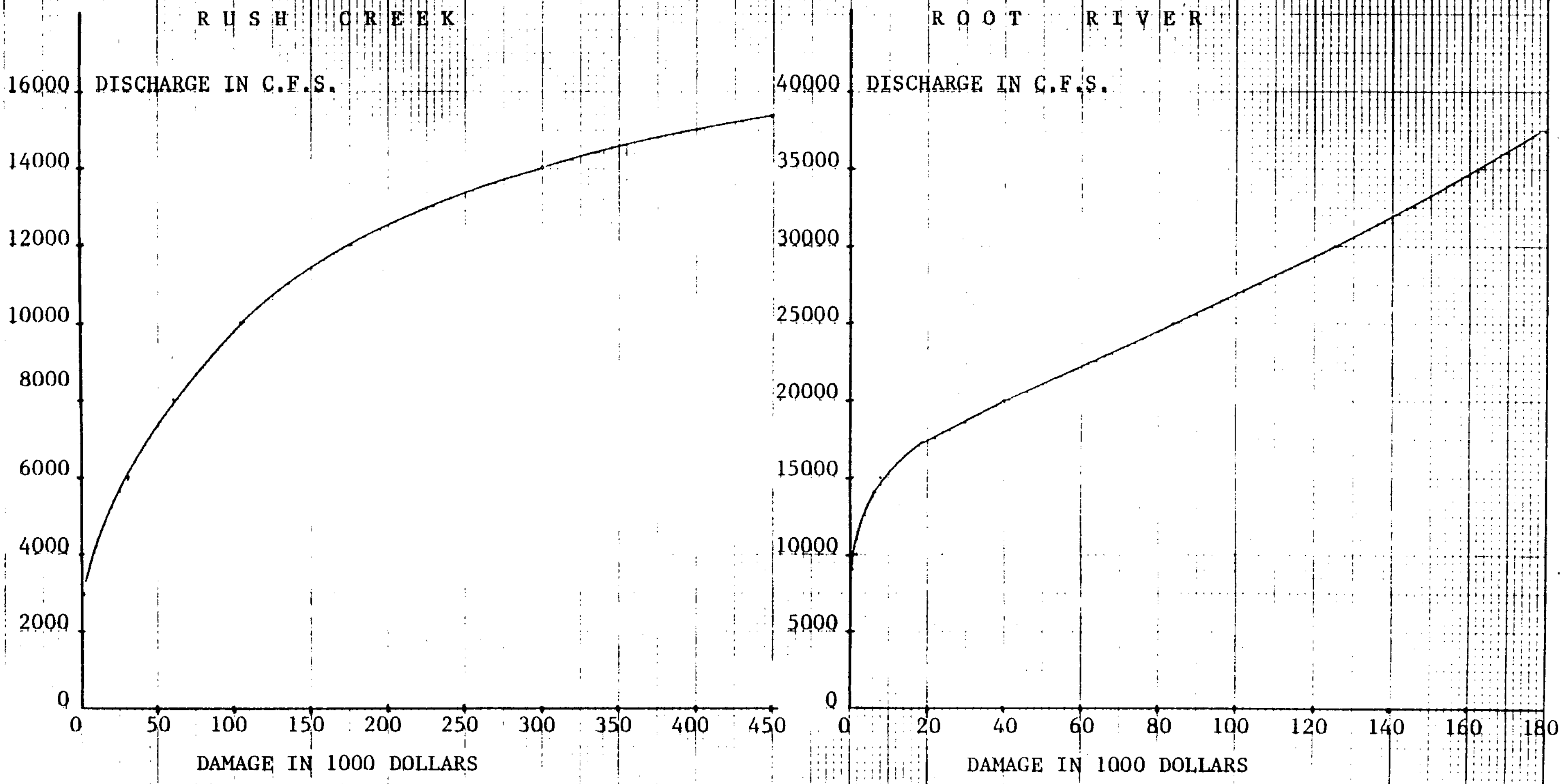
Comparison of Probability Distributions

The ex-post estimated peak discharge distribution for the Root River is almost the same as the ex-ante estimate in 1956 and 1965, but this was not the case for the Rush Creek. Figure 3 illustrates the departure of peak discharges in Rush Creek for a given exceedance probability. The new distribution is steeper than the old distribution presented in the general design memorandum of 1965. The deviation between these two distributionsmakes the estimated 100 and 200-year floods higher than the initial estimates. This is equivalent to saying that the design flood for Rush Creek has a higher flood frequency than what was thought to be the case when the project was proposed.

The new methodology and the availability of additional data

Fig 8. DISCHARGE-DAMAGE CURVES FOR RUSH CREEK AND ROOT RIVER IN 1956

SOURCE: General Design Memorandum. U.S. Army Corps of Engineers, 1956.



explain this difference. The methodology used in 1956 consisted of fitting the best line to the discharge-probability scatter diagram. The methodology used in this study fits recorded data to the log-Pearson type III distribution.

An interesting implication of this deviation is shown in Figure 2. This figure shows the frequency-damage curves for Rush Creek floods using both distributions and the 1956 Rushford conditions. The area under these curves is the expected annual damages for Rush Creek. With the old distribution, the expected annual damages is $ 30,750, while for the new distribution, the estimated value is only $21,420. This shows that the new methodology estimated a 30% decrease in expected benefits using the 1956 discharge-damage relationships. Thus, if one had used this new estimating procedure for the first benefit-cost analysis the benefit-cost ratio would have been less than one, 0.87. This new B/C ratio would have had negative implications for construction of the project.

SUMMARY AND CONCLUSIONS

The flood control project in Rushford, Minnesota was authorized by the 1958 Flood Control act to provide flood control on the two main streams in the Rushford flood plain. The project planning document showed a favorable benefit-cost ratio of 1.2 and an expected life of 100 years. The project was finally built by the Corps of Engineers between 1967 and 1969. The ex-ante benefit-cost ratio close to one, and 18 years of operating experience, made this project attractive for an ex-post evaluation.

The two stream flood plain at Rushford suggested a coincident frequency analysis of the peak discharges to estimate the coincident occurrences of floods. Therefore, the ex-post hydrology analysis is done using an estimated bivariate lognormal probability distribution of the peak discharges. A numerical integration of this function was performed using an approach developed by Moskowitz and Tang Tsai (1986).

To further improve the benefit estimates, damage susceptibility relationships were developed for some residential, commercial and public facilities based on surveys done in the flood plain. The other units were grouped according to similar characteristics and evaluated based on standard susceptibility relationships. The survey results improved the estimates of potential damages to property.

The ex-post analysis indicates that the ex-post flood benefits were higher than ex-ante flood benefit estimates. This was due to the application of the new methodology used to estimate expected annual flood benefits and the 18 years of additional information available to estimate peak discharge probability distributions. The estimated increases in costs were due to repair work done in 1974 and 1979. The ex-post benefit-cost analysis indicates that the project's performance was better than expected. The benefit-cost ratios, for different real discount rates ranging from 3-1/8 to 8-7/8, are all 1.4 or greater.

Ex-ante project planning could be improved by: (1) having a longer period on which to base predictions of the probability distribution for peak discharges, (2) improving the methodology of estimating expected flood damages, (3) making projections of residential, commercial and industrial growth, and (4) adopting appropriate discount rates for the analysis.

Additional research is needed to improve flood damage estimation procedures, particularly the relations between the value of contents and the value of the structures and the relationship of time variation of structure values and the value of the contents. Also, more information is needed concerning commercial and industrial damages. In this project, 55% of the total flood damages were attributed to non-residential damages, but this percentage can go as high as 70% (Cornell, 1972). Currently, little information is available concerning flood levels and actual damages to different types of commercial and industrial property.

APPENDIX A

A Polynomial Approximation for Bivariate Normal Probabilities.

The Bivariate Normal Distribution is frequently employed, but the computational formulae for bivariate normal probabilities are quite complicated.

An approximation for the cummulative standard bivariate normal probability was developed by Moskowitz and Tang Tsai (1986). They developed a polynomial approximation for computing cumulative bivariate normal probabilities. Let $BvN(c,d,\rho) = P(y \leqslant c, x \leqslant d)$ where x and y have a standard bivariate normal distribution with correlation ρ.

The computational procedure is stated below :

STEP

0. Let $Z = \begin{matrix} 1 \text{ if } \rho \geqslant 0 \\ -1 \text{ otherwise} \end{matrix}$

1. Compute $A = c/\sqrt{1-\rho^2}$, $S = \rho/\sqrt{1-\rho^2}$ and $Max = A - Sd$

2. If $Max < e3$, then $F(\rho) = 0$

3. Compute $K2 = (A-e2)/S$, $K3 = (A-e3)/S$

   $B = \Phi(-Zd)$, $B2 = \Phi(-ZK2)$, $B3 = \Phi(-ZK3)$

4. If $e3 < Max < e2$ then $F(\rho) = p(B-B3)$

5. If $e2 < Max < e$ then :

   Compute $T = (c0 + c1A + c2A^2 + c3A^3) + (c2 + 3c3A)S^2$

   $P = (c1 + 2c2A + 3c3A^2)S + 2c3S^3$

   $Q = (c2 + 3c3A)S^2$

   $R = c3S^3$

   $F(\rho) = TB - Z(P - Qd + Rd^2)\Phi'(d)$

   $\quad - TB2 - Z(P - QK2 + RK2^2)\Phi'(K2) + p(B2 - B3)$

6. Finally, let

   $F(c,d,\rho) = F(\rho)$ for $\rho < 0$

   $F(c,d,\rho) = \Phi(c) - F(\rho)$ for $\rho > 0$

The algorithm for approximating BvN (c,d,ρ) by F (c,d,ρ) is the following :

i) Choose c and d such that $|c| \leq |d|$

ii) If $\rho \neq 0$ and

(1) If $c \leq 0$, then approximate BvN (c,d,ρ) = F (c,d,ρ)

(2) If $c > 0$, then approximate BvN $(c,d,\rho) = \Phi(d) - F(-c,d,-\rho)$

iii) If $\rho = 0$, then BvN $(c,d,\rho) = \Phi(c)\ \Phi(d)$

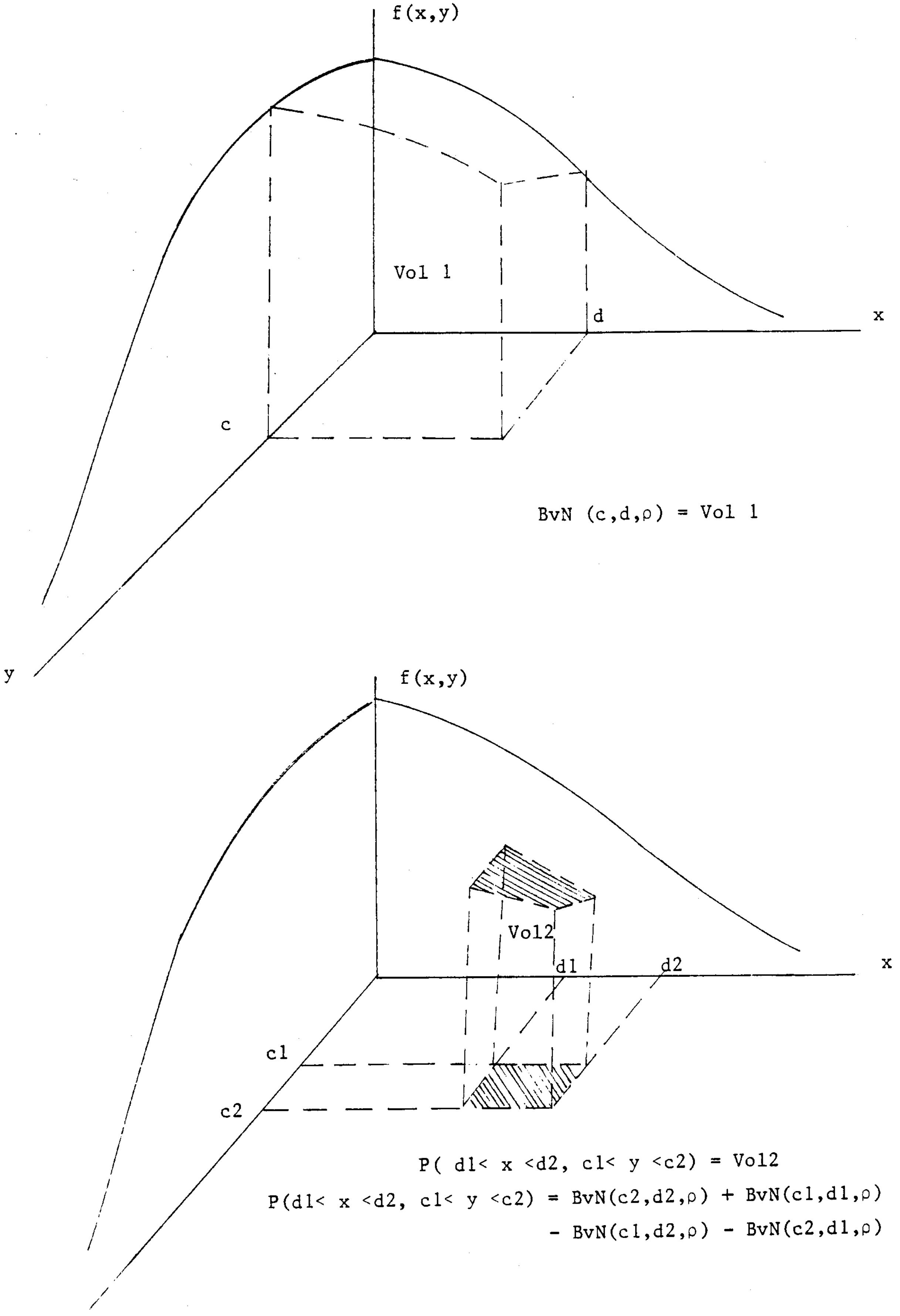
where : $\Phi(.)$ is the standard normal density function ,

$\int x d\Phi(x) = \Phi'(x)$

$c0 = 0.48458$, $c1 = 0.47776$, $c2 = 0.15920$, $c3 = 0.01787$

$e2 = -3.2, e3 = -3.5, p = 0.0005$

Graphical Representation.



BvN (c,d,ρ) = Vol 1

P(d1< x <d2, c1< y <c2) = Vol12

P(d1< x <d2, c1< y <c2) = BvN(c2,d2,ρ) + BvN(c1,d1,ρ)
 - BvN(c1,d2,ρ) - BvN(c2,d1,ρ)

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