AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

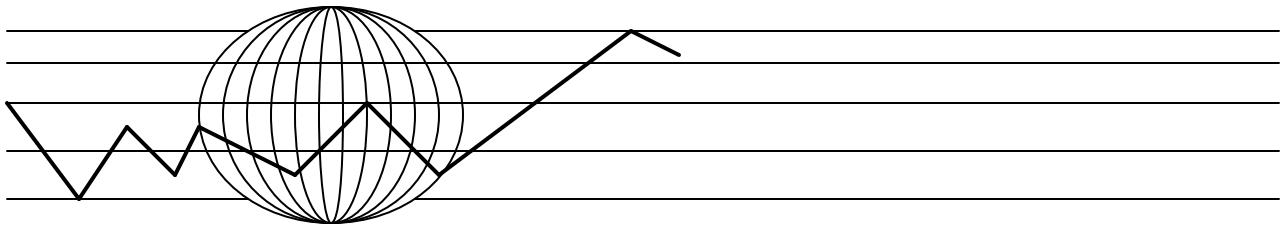
Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Bulletin Number 05-1 **June 2005**

ECONOMIC DEVELOPMENT CENTER



NATURAL RESOURCE ABUNDANCE AND ECONOMIC GROWTH IN A TWO COUNTRY WORLD

BEATRIZ GAITAN
TERRY ROE

ECONOMIC DEVELOPMENT CENTER
Department of Economics, Minneapolis
Department of Applied Economics, St. Paul

UNIVERSITY OF MINNESOTA

Natural Resource Abundance and Economic Growth in a Two Country World

Beatriz Gaitan[*] and Terry L. Roe[**,+]

June 21, 2005

Abstract

We investigate the dynamics of nonrenewable resource abundance on economic growth and welfare in a two-country world. One country is endowed with a nonrenewable-resource, otherwise, countries are identical, except possibly for their initial endowments of capital. Unlike previous studies analyzing small open economies, we show that once interactions between resource-rich and resource-less economies are considered the effect of the nonrenewable resource on the resource rich economy's performance can be positive. We derive the necessary condition for the nonrenewable resource to have a positive (negative) effect on the growth rate of the resource-rich economy. The endowment of the nonrenewable resource has a positive effect on the growth rate of the resource-rich country provided the elasticity of the initial price of the resource with regard to the initial stock of the resource is greater than minus one. An analytical solution to the model confirms that this elasticity is greater than minus one, and numerical simulations with a very large range of parameter values confirm the same.

Key Words: Growth, Non-renewable Resources.

JEL Classification: O13, O41

[*] Economics Department, University of Hamburg. Von-Melle-Park 5, IWK, 20146 Hamburg, Germany. e-mail address: Gaitan_Soto@econ.uni-hamburg.de

[**] Department of Applied Economics, University of Minnesota. 231 ClaOff Building, 1994 Buford Av., St. Paul, MN 55108-6040. e-mail address: Troe@apec.umn.edu

[+] The authors would like to thank Timo Trimborn for his very helpful hints and suggestions.

1. Introduction

The economics of nonrenewable resources has received increased attention following the empirical observations of Jeffrey D. Sachs and Andrew M. Warner (1995). They perform cross-country growth regressions and find that economies with large natural resource based exports to GDP ratios in 1971 tended to experience relatively low growth rates in the subsequent period (1971-89). More recent research has led to contradicting results. For example Daniel Lederman and William F. Maloney (2003) account for measures of exports' concentration and intra-industry trade and find that the coefficient of resource abundance has a positive sign on the growth regression. Jean-Phillip Stijns (2001) employs different indicators of natural resource abundance than those used by Sachs and Warner and finds no evidence that resource abundance is a detriment for economic growth. Ning Ding and Barry C. Field (2004) distinguish between natural resource endowments and natural resource dependence and find that, across countries, natural resources do not affect growth. A common feature of these empirical studies is that they also control for corruption (except for Lederman and Maloney, 2003) and openness, despite this, contradicting results about the effect of resource abundance on growth are found. Given this puzzling empirical evidence, the challenge is whether a theoretical model can help advance our understanding of how resource abundance affects economic growth.

Several hypothesis explaining why resource-rich economies have underperformed resource-poor economies have been proposed. Aaron Tornell and Philip. R. Lane (1995) argue that this underperformance may be explained by the struggle of groups attempting to extract natural resource rents. Sachs and Warner (1995 and 2001) argue in favor of Dutch Disease causes. Francisco Rodríguez and Jeffrey D. Sachs (1999) argue that "resource-rich countries may grow more slowly because they are likely to be living beyond their means..." (p. 278). In particular, they indicated that a sufficiently large initial natural resource stock will generate an overshoot (above long-run) of capital and GDP. In such a case, an economy will converge from above to its long-run value of GDP and thus, will perform "poorly in terms of GDP per capita growth." (p. 289). Moreover, they show that, even in the absence of any type of distortions (trade barriers, rent-seeking activities, Dutch-Disease possibilities etc.), larger initial stocks of a nonrenewable resource are sufficient to generate negative growth. They consider a single economy which exports all of the oil extracted, eventually totally depleting the nonrenewable resource stock. In their setting the economy does not use oil for their own production or consumption, implying that the rest of the world must be different than the economy they study, and that the rest of the world's growth cannot impact the resource-rich country.

We study a two country world economy and investigate if in the absence of distortions resource abundance can be a detriment to economic growth, as some studies indicate. In particular, this paper analyzes how initial factor endowments of nonrenewable resources and accumulable resources influence income, growth and welfare in a two country world. We provide the necessary condition for a nonrenewable resource to have a positive (negative) effect on the relative economic performance of a resource-rich economy.

Previous work in this area includes Geir B. Asheim (1986) and John M. Hartwick (1995) who study a two country world and investigate whether constant consumption paths are achievable when the economies invest resource rents into new capital. Carl Chiarella (1980) focuses on international trade aspects of a world economy with two countries and shows that if countries are equally patient then consumption growth across countries is identical. This implies that the ratio

between consumption levels (across countries) must be constant over time. Chiarella determines the long-run ratio (steady state) of consumption levels between the two countries when the utility function is logarithmic and discount factors across countries are different. Jan H. van Geldrop and Cees A. Withagen (1993) generalize Chiarella's (1980) approach, by introducing n number of trading partners. Among others things, they concentrate on equilibrium existence, but they do not study cross-country differences on consumption, income or growth.

Our analysis provides insights into how differences in nonrenewable resources and capital endowments affect economic performance. Consequently, we ignore other differences across countries. The economies considered are thus identical, except for the initial endowments of assets (capital) and the nonrenewable resource. As in other models of trade, factor price equalization across countries occurs. The rental rate of capital across countries is equal even in the absence of international borrowing and lending. This result implies that the growth rate of consumption across countries is equal. Thus, as in Chiarella (1980), the ratio of consumption across countries is constant over time. Regardless of *income* growth, relative welfare across countries is shown to remain unchanged. The ratio of the consumption levels between the two countries is determined by the ratio of their respective value of assets at any point in time. The wealth of the nonrenewable resource-rich country, and hence its relative welfare, is shown to increase with the initial stock of the resource. This effect is counterbalanced, to some degree, by the negative effect of the initial stock of the depletable resource on the price of the nonrenewable.

We also prove that the initial endowment of the nonrenewable resource has a *positive* effect on the GDP growth rate of the *resource-rich* country as long as the elasticity of the initial price of the resource with regard to the initial stock of the resource is greater than minus one. That is, depending upon structure, a non-renewable resource rich economy can grow faster or slower than an economy without this resource. An analytical solution of the model under a parameter restriction demonstrates that indeed this elasticity is greater than minus one[3]. Thus, the result of Rodríguez and Sachs (1999) that the initial stock of the resource negatively influences the GDP growth of an economy exporting a nonrenewable resource is shown not to hold in general. This result is mostly derived from the fact that Rodríguez and Sachs (1999) consider a small open economy and do not account for inter-country linkages, via the terms of trade, that influence the resource-rich economy's performance. While it is possible that nonrenewable resources rich economies are growing slower because of rent-seeking activities and failures to invest in market supporting institutions. Most importantly, we confirm that previous conclusions, suggesting that owning a large amount of the resource alone is sufficient to generate negative growth, do not hold.

An overview of the model is as follows. Similar to Chiarella (1980), we model a world economy with two countries that are engaged in international trade. Different to Chiarella (1980), we allow the two countries to be engaged in the production of a final good as in Geldrop and Withagen (1993). Capital and a nonrenewable resource are used as factors of production and we assume that both countries have access to the same technology to produce the final good. A single country owns the entire stock of the nonrenewable. This country has an extracting sector that depletes the nonrenewable by maximizing discounted profits. The economies trade the final good and the nonrenewable internationally, but international borrowing and lending is not allowed. Each country has a representative consumer that derives satisfaction from consuming the final good and maximizes discounted instant utility, subject to a budget constraint. The rental rate of capital

[3] Numerical simulations indicate the same.

of each country is determined endogenously and equals the marginal physical product of capital in that country. Finally, a market clearing condition for the nonrenewable resource endogenously determines its price.

The paper is organized as follows. In section two we introduce the model where a nonrenewable resource and capital are used as inputs in production. In section three we characterize the equilibrium, prove the stability of the system and look at the impact of the nonrenewable resource on income growth and relative welfare. In section four we provide an analytical solution under a restriction on parameter values. In section five we provide numerical simulations and conclude in section six.

2. The model

2.1. Set-up

Consider the environment of a two-country world in which one of the countries is endowed with a nonrenewable-resource (referred to as country one) and the other is a nonrenewable-resource-less country (referred to as country two). The representative consumer of each country seeks to maximize discounted utility of consumption subject to a budget constraint. Across countries consumers have identical preferences and identical discount factors. Whereas country one owns and can deplete the nonrenewable, both economies have an amount of capital that each combines with the nonrenewable resource to produce an identical final good. The final good technology is given by

$$Y_i = F\left(K_i, R_i\right) = K_i^{\alpha}\left(e^{\eta t} R_i\right)^{1-\alpha} \tag{1}$$

where K_i and R_i for $i = 1, 2$ denote the amount of capital and the amount of the nonrenewable resource employed in the production of output Y_i of country i and η is the growth rate of a resource saving technological progress. The price of the final good is numéraire. Let r_i denote the rental rate of capital in country i. To maximize profits the final good sector of country i sets the marginal physical product of each input factor equal to its rental rate/price as follows:

$$r_i = \alpha \frac{Y_i}{K_i} \qquad\qquad q = (1-\alpha)\frac{Y_i}{R_i}, \tag{2}$$

where q denotes the price of the nonrenewable resource. Since country one exports the nonrenewable to country two, in the absence of trade distortions, the nonrenewable is traded at the same price in both countries. Rearranging these expressions we obtain

$$Y_i = q\frac{R_i}{1-\alpha} = r_i \frac{K_i}{\alpha} \qquad \Rightarrow \qquad K_i = \frac{\alpha}{1-\alpha}\frac{q}{r_i} R_i. \tag{3}$$

Substituting for K_i from (3) in (1) yields

$$Y_i = \left(\frac{\alpha}{1-\alpha}\frac{q}{r_i}\right)^{\alpha} e^{(1-\alpha)\eta t} R_i. \tag{4}$$

Equating this expression to Y_i from (3) provides a relationship between the rental rate of capital r and the price of the nonrenewable resource:

$$r_i = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)^{\frac{1}{\alpha}} \left(\frac{e^{\eta t}}{q}\right)^{\frac{1-\alpha}{\alpha}}. \tag{5}$$

Equation (5) suggests that, the Heckscher-Ohlin tendency for factor price equalization is satisfied. Using $r_1 = r_2 = r$ and (3) implies

$$R_i = \frac{1-\alpha}{\alpha}\frac{r}{q}K_i \tag{6}$$

indicating that the ratio of R_i/K_i across countries is the same.[4]

2.2. Extracting sector

Since the final good serves as numéraire, r is the gross interest rate of country one. Define

$$\omega(t) = e^{-\int_0^t (r(\tau)-\delta)d\tau}, \tag{7}$$

where δ is the constant depreciation rate of capital. Note that holding one unit of capital yields a gross return $r(\tau)$ at instant of time τ, but, since capital depreciates, the net return from holding one unit of capital at time τ equals $r(\tau)-\delta$. Presuming perfect capital markets in country one, as in Geldrop and Withagen (1994 p. 1014), $\omega(t)$ is a present-value factor that converts a unit of revenue at time t into an equivalent unit of revenue at time *zero*. We presume that extractions are costless. Using $\omega(t)$ to discount profits, the extracting sector finds the optimal path of extractions $R(t)$ that maximizes the present value of profits subject to the constraint that cumulative extractions do not exceed the initial stock of the nonrenewable, i.e.,[5]:

$$\max\{\int_0^{\infty} q(t)\,R(t)\,\omega(t)\,dt \,\Big|\, \int_0^{\infty} R(t)\,dt \leq S_0\}. \tag{8}$$

The Lagrangian of this isoperimetric problem is given by

$$\mathcal{L} = qRe^{-\int_0^t (r(\tau)-\delta)d\tau} - \lambda R \tag{9}$$

and the necessary conditions for a maximum and transversality condition are, respectively, given by

[4] If the profit maximization problem of the final good sector were set as a dynamic problem, as in Geldrop and Withagen (1994), the same first order conditions would be obtained.

[5] If we had integrated the extracting activity into the optimization problem of the consumer of country one the same results would had been obtained.

$$q = \lambda e^{\int_0^t (r(\tau)-\delta)d\tau}, \qquad \dot{\lambda} = 0, \qquad \lim_{t\to\infty} qRe^{-\int_0^t (r(\tau)-\delta)d\tau} = 0. \tag{10}$$

A differential equation for q is obtained by applying Leibniz's rule to (10) which yields

$$\frac{\dot{q}}{q} = r - \delta. \tag{11}$$

This condition indicates that the real price q, of the nonrenewable resource must grow at the real interest rate, or equivalently, the Solow-Stiglitz (Robert M. Solow, 1974 and Joseph E. Stiglitz, 1974) efficiency condition holds. That is, (11) indicates that the nonrenewable is an asset and for the economy to have incentives to hold the nonrenewable its price must grow at the real rate of interest. Substituting r from (5) into (11), we obtain an equation describing the motion of the nonrenewable resource price q:

$$\frac{\dot{q}}{q} = r - \delta = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)^{\frac{1}{\alpha}} \left(\frac{e^{\eta t}}{q}\right)^{\frac{1-\alpha}{\alpha}} - \delta. \tag{12}$$

Indeed, this differential equation has an analytical solution given by

$$q(t) = \left(\frac{\Lambda}{\eta+\delta} e^{\frac{1-\alpha}{\alpha}\eta t} + \frac{1}{e^{\frac{1-\alpha}{\alpha}\delta t}}\left(q(0)^{\frac{1-\alpha}{\alpha}} - \frac{\Lambda}{\eta+\delta}\right)\right)^{\frac{\alpha}{1-\alpha}}, \tag{13}$$

where $\Lambda = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)^{1/\alpha}$ and $q(0)$ remains to be determined. Note that the price of the nonrenewable resource can transitionally decline depending on the size of $q(0)$. In particular, $\dot{q}/q < (\geqq)\, 0$ if

$$\left(\frac{\Lambda}{\eta+\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\eta e^{\frac{1-\alpha}{\alpha}(\eta+\delta)t} + \delta}{\delta}\right)^{\frac{\alpha}{1-\alpha}} < (\geqq)\, q(0) \tag{14}$$

Observe from (5), (12) and (13) that the long-run growth rate of q equals

$$\lim_{t\to\infty} \frac{\dot{q}}{q} = \eta \tag{15}$$

Thus, the rate of growth of the resource saving technological change, η, positively influences the long-run growth rate price of the nonrenewable resource. Similar to the standard Ramsey model in which labor augmenting technological change positively influences the long-run labor wage rate, here a resource saving technological change positively affects the long-run price of the nonrenewable resource. Moreover, if (14) holds for the case $<$ then the price of the nonrenewable resource is U-shaped over time.

2.3. Consumers' optimization problems

2.3.1. Resource-rich country

The representative consumer of country *one* solves the problem of maximizing discounted utility of consumption subject to an intertemporal budget constraint as follows:

$$\max \int_0^\infty \frac{c_1(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \tag{16}$$

subject to

$$\dot{K}_1(t) = (r_1(t) - \delta) K_1(t) + \pi(t) - c_1(t), \tag{17}$$

given $K_1(0) = K_{0,1} > 0$,

where $1/\theta > 0$ is the elasticity of intertemporal substitution, $\rho > 0$ is a discount factor, and δ is the constant rate of capital depreciation. π denotes the profits of the extracting sector. Initial and instant t stock of capital for country one are $K_{0,1}$ and $K_1(t)$, respectively. Since extraction is costless, $\pi(t) = q(t) R(t)$ holds. Thus, the budget constraint of country one (equation 17) can be rewritten as

$$\dot{K}_1 = (r_1 - \delta) K_1 + qR - c_1. \tag{18}$$

Since only country one owns the resource, the rate of extraction consists of what is used for domestic consumption plus an amount that is exported.

2.3.2. Resource less country

The representative consumer of country two solves the problem of maximizing discounted utility of consumption subject to an intertemporal budget constraint, i.e.,

$$\max \int_0^\infty \frac{c_2(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \tag{19}$$

subject to

$$\dot{K}_2(t) = (r_2(t) - \delta) K_2(t) - c_2(t), \tag{20}$$

given $K_2(0) = K_{0,2} > 0$

where $K_{0,2}$ and $K_2(t)$ are, respectively, the initial and instant t capital stocks of country two.

The Euler condition of the consumer's problem of country i is given by

$$\frac{\dot{c}_i}{c_i} = \frac{r_i - \delta - \rho}{\theta} \quad \text{for } i = 1, 2 \tag{21}$$

with transversality condition

$$\lim_{t\to\infty} \frac{K_i}{e^{\rho t} c_i^{\theta}} = 0 \quad \text{for } i = 1, 2. \tag{22}$$

The result r_1 equaling r_2, implies that c_1 and c_2 grow at identical rates. Using $r_1 = r_2 = r$, and $r - \delta = \dot{q}/q$, (21) yields

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\theta}\left(\frac{\dot{q}}{q} - \rho\right) \quad \text{for } i = 1, 2. \tag{23}$$

The solution to this differential equation is

$$c_i(t) = \left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\theta}} \frac{c_i(0)}{e^{\frac{\rho}{\theta} t}} \quad \text{for } i = 1, 2. \tag{24}$$

Since $q(t)$ can transitionally decline or increase, $c_i(t)$ can follow a similar pattern, albeit influenced by the discount factor and the elasticity of intertemporal substitution. Using (24), the transversality condition of country i can be rewritten as

$$\lim_{t\to\infty} \frac{K_i(t)}{q(t)} = 0 \quad \text{for } i = 1, 2. \tag{25}$$

3. Equilibrium characterization

3.1. Equilibrium

An equilibrium for this economy are paths of quantities $c_i(t)$, $Y_i(t)$, K_i,(t) $R_i(t)$, $R(t)$ and prices $q(t)$, and $r_i(t)$ for $i = 1, 2$ such that $c_i(t)$, and $K_i(t)$ solve the consumer's optimization problem of country i, $Y_i(t)$, $K_i(t)$, and $R_i(t)$ solve the optimization problem of the final good sector of country i, $R(t)$ solves the maximization problem of the extracting sector of country one, and the *nonrenewable resource* market clears, i.e.,

$$R_1(t) + R_2(t) = R(t). \tag{26}$$

Proposition 1. The ratio $c_1(t)/c_2(t)$ is constant for all t and equals the present value of the ratio of the assets of country one and country two $(q(t) S(t) + K_1(t))/K_2(t)$ at time t.

Proof. See proof in Appendix A.

Let $\mu \equiv c_1(t)/c_2(t)$ where μ is a constant. Since $\mu = (q(t)S(t) + K_1(t))/K_2(t)$ for all t then,

$$\mu = \frac{c_1(t)}{c_2(t)} = \frac{q(0)S_0 + K_{0,1}}{K_{0,2}} \tag{27}$$

Remark: This result implies that relative consumption is determined from time zero. This result suggests that consumption is influenced forever by each country's initial wealth, i.e., by the value of each country's assets in time $t = 0$. Since S_0, $K_{0,1}$ and $K_{0,2}$ are given, then μ can be determined if $q(0)$ were known.

We have shown thus so far that:

Corollary 1.
i) Factor price equalization across countries results. The rental rate of capital across countries is equal even in the absence of international borrowing and lending.
ii) The Solow-Stiglitz efficiency criterion is satisfied, that is, the price of the nonrenewable resource grows at the real interest rate.
iii) The price of the nonrenewable resource can transitionally decline.
iv) The Harrod rate of growth positively influences the long-run growth rate of price of the nonrenewable resource.
v) The ratio of consumption levels of the resource-rich to the resource less country is constant over time.
vi) The ratio $c_1(t)/c_2(t)$ equals the ratio of the value of the assets of each country $q(t)S(t)/K_2(t) + +K_1(t)/K_2(t)$.

3.2. The reduced system

To study the long-run stability properties of the model, it is useful to normalize the variables of the model, and decrease the dimensionality of the system.

Proposition 2. An equilibrium, if it exists, converges to a balanced growth path with growth rates

$$g_q = \eta, \qquad g_{K_i} = g_{c_i} = \frac{\eta - \rho}{\theta} \equiv g_K, \qquad g_{R_i} = g_R = g_S = \frac{(1-\theta)\eta - \rho}{\theta} \tag{28}$$

for $i = 1, 2$ and r is constant. Where g_v denotes the long-run growth rate of variable v.
Proof. See appendix A.

Remark: Thus, as in Chiarella (1980) a sufficiently large exogenous technological change can lead to sustainability. Here the economies are sustainable in the long run ($g_{c_i} \geqq 0$) if the growth rate of the resource saving technological change is equal to or greater than the consumers' discount factor.

Let $K = K_1 + K_2$. Variables are now normalized by their corresponding growth rate as follows

$$\hat{K} = \frac{K}{e^{g_K\, t}}, \qquad \hat{K}_i = \frac{K_i}{e^{g_K\, t}}, \qquad \hat{c}_i = \frac{c_1}{e^{g_K\, t}}, \tag{29}$$

$$\hat{q} = \frac{q}{e^{\eta\, t}}, \qquad \hat{S} = \frac{S}{e^{g_S\, t}}, \qquad \hat{R} = \frac{R}{e^{g_S\, t}}, \qquad \hat{R}_i = \frac{R_i}{e^{g_S\, t}}$$

In this way the normalized variables ($\hat{}$) will remain constant along the growth path. To study the stability of the model we find it useful to reduce the system of equations to the smallest possible number of differential equations. Let

$$T \equiv \frac{\hat{S}}{\hat{K}}, \qquad g \equiv \frac{\hat{c}_2}{\hat{S}}, \qquad \hat{c}_1 = \mu \hat{c}_2 \tag{30}$$

where T is a state variable and g is a control like variable, and $\mu > 0$ is a constant. Since all the variables have been normalized, T and g will remain constant in the long-run. Recall that since c_1 and c_2 grow (forever) at the same rate, then $\hat{c}_1(t) = \mu \hat{c}_2(t)$ for all t. Taking the time derivative of $\hat{q}$, T and g we obtain:

$$\dot{\hat{q}} = (r - \delta - \eta)\, \hat{q} \tag{31}$$

$$\dot{T} = -\frac{r}{\hat{q}} \frac{1-\alpha}{\alpha} - T\left(\frac{r}{\alpha} - \delta - \eta - (1+\mu)\, gT\right). \tag{32}$$

$$\dot{g} = \left(\frac{r - \delta - \rho}{\theta} - \eta + \frac{r}{\hat{q}} \frac{1-\alpha}{\alpha} \frac{1}{T}\right) g \tag{33}$$

with

$$r = \left(\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{\hat{q}^{1-\alpha}}\right)^{\frac{1}{\alpha}}. \tag{34}$$

In deriving $\dot{T}$ we have employed the following $\dot{\hat{S}} = -\hat{R} - g_S \hat{S}$, (6) and (26) so that $\hat{R} = (1-\alpha)(r/\hat{q})\left(\hat{K}/\alpha\right)$, $\hat{c}_2 = g\hat{S}$ and $\hat{c}_2/\hat{K} = gT$. Thus, the first order conditions of the model can be reduced to a system of three differential equations, (32), (33) and (31) in three variables T, g and $\hat{q}$.

3.2.1. Stability

Here we investigate the stability properties of the long-run equilibrium of the reduced system. The equilibrium is locally unique if the Jacobian of the reduced system has two eigenvalues with positive real parts and one with a negative real part. The reason is because the initial condition $T(0)$ is given, but $q(0)$, and $g(0)$ are free. Let

$$\begin{pmatrix} \dot{\hat{q}} \\ \dot{g} \\ \dot{T} \end{pmatrix} \equiv \begin{pmatrix} \mathrm{f}^{\hat{q}}(\hat{q}, g, T) \\ \mathrm{f}^{g}(\hat{q}, g, T) \\ \mathrm{f}^{T}(\hat{q}, g, T) \end{pmatrix}. \tag{35}$$

By setting $\dot{\hat{q}}$, $\dot{T}$ and $\dot{g}$ equal to zero and employing (34) we obtain the long-run or steady-state values of $\hat{q}$, T, g, and r, respectively, given by

$$\hat{q}^* = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(\delta+\eta)^{\alpha}} \right)^{\frac{1}{1-\alpha}}, \qquad T^* = \frac{r^*}{\hat{q}^*} \frac{1-\alpha}{\alpha} \frac{1}{\eta - g_K} \tag{36}$$

$$g^* = \left(\frac{\eta + \delta(1-\alpha)}{\alpha} - g_K \right) \frac{1}{(1+\mu)T^*}, \qquad \text{and} \qquad r^* = \eta + \delta, \tag{37}$$

where variables with the superscript $*$ denote steady-state values. The eigenvalues (ξ_i) of the Jacobian matrix of (35) are given by

$$\xi_1 = -\frac{1-\alpha}{\alpha} r^* = -\frac{1-\alpha}{\alpha}(\eta + \delta) \qquad \xi_2 = -g_S > 0 \qquad \xi_3 = (1+\mu) g^* T^* > 0. \tag{38}$$

This leads to the next proposition.

Proposition 3. The equilibrium is locally unique.

Proof. Notice that $\xi_1 = -(1-\alpha)(\eta+\delta)/\alpha$ is negative. In Appendix A we show that ξ_1, ξ_2 and ξ_3 are the eigenvalues of the Jacobian matrix of (35). Since there is a single negative eigenvalue the equilibrium is locally unique and stable.

Remark: This result facilitates an empirical analysis as it eliminates the need to search for other equilibria.

3.3. Growth

To provide insights on how the nonrenewable resource affects GDP growth, we proceed by examining the share of country i GDP in aggregate (global) GDP. If the share of country $i's$ GDP in the world's GDP changes along the growth path, then one of the countries transitionally grows slower than the other. We, therefore, want to obtain information about country $i's$ GDP share in the world's GDP at the begging of time and at the steady state.

Let $s_i(t)$ denote the share of GDP of country i on global GDP as follows

$$s_i(t) = \frac{GDP_i(t)}{GDP^w(t)}, \tag{39}$$

where $GDP^w(t)$ equals $GDP_1(t)+GDP_2(t)$. In particular,[6] $s_2(t) = \alpha K_2(t)/K(t)$ where as before K equals $K_1 + K_2$.

Proposition 4. At time *zero,* the share of GDP of country i for $i = 1, 2$ on global GDP^w does not depend on the level of the nonrenewable resource and

$$s_1(0) = \frac{K_{0,1} + (1-\alpha) K_{0,2}}{K_0}, \qquad s_2(0) = \alpha \frac{K_{0,2}}{K_0} \tag{40}$$

where K_0 denotes the sum of the initial stocks of capital of country one and two $(K_{0,1} + K_{0,2})$.

Proof . Country-i's GDP is, respectively, given by

$$GDP_1 = rK_1 + qR \qquad GDP_2 = rK_2. \tag{41}$$

Equation (6) and the market clearing condition (26) yield $R = (1-\alpha)(r/q)(K_1 + K_2)/\alpha$. Substituting this result into the definition of GDP of country *one* and computing GDP^w yields the proposition.

Remark: Proposition 4 indicates that the shares $s_i(0)$ do not depend on the stock of the resource S_0, but rather on capital, and any price effect can be eliminated.

Let κ_i be the share of capital of country i in total (global) capital as follows

$$\kappa_i = \frac{K_i}{K} = \frac{\hat{K}_i}{\hat{K}} \tag{42}$$

Since K_i and K growth at the same rate in the long run, the share of capital of country i in the long run is given by

$$\kappa_i = \kappa_i^* = \frac{\hat{K}_i^*}{\hat{K}^*} \tag{43}$$

where at time zero $\kappa_{0,i} = K_{0,i}/K_0$ are given to the world economy.

In Proposition 1 we showed that the ratio of consumption across countries equals

$$\frac{c_1(t)}{c_2(t)} = \mu = \frac{q(t) S(t) + K_1(t)}{K_2(t)}, \tag{44}$$

where μ is a constant for all t. This also implies that

$$\frac{q(0) S_0 + K_{0,1}}{K_{0,2}} = \frac{\hat{q}^* \hat{S}^* + \hat{K}_1^*}{\hat{K}_2^*} \tag{45}$$

[6] Note that $GDP^w = rK + qR = rK + (1-\alpha) rK/\alpha = rK/\alpha$.

where, as before, variables with the superscript $*$ denote steady-state values and $\hat{}$ denote normalized variables. Equation (45) can be rewritten as

$$\frac{q(0) S_0 + K_{0,1}}{K_{0,2}} = \frac{\hat{q}^* \hat{S}^*}{\kappa_2^* \hat{K}^*} + \frac{1 - \kappa_2^*}{\kappa_{2^*}}. \tag{46}$$

Using $T^* = \hat{S}^* / \hat{K}^*$ we obtain

$$\frac{q(0) S_0 + (1 - \kappa_2(0)) K_0}{\kappa_2(0) K_0} = \frac{\hat{q}^*}{\kappa_2^*} T^* + \frac{1 - \kappa_2^*}{\kappa_2^*}. \tag{47}$$

Solving for κ_2^* we obtain

$$\kappa_2^* = \frac{\kappa_2(0) K_0 (1 + q^* T^*)}{K_0 + q(0) S_0}. \tag{48}$$

Since the share of GDP of country two in total GDP equals $s_2(t) = \alpha K_2(t) / K(t)$, in the long run s_2 equals

$$s_2^* = \alpha \kappa_2^* = \alpha \frac{\kappa_2(0) K_0 (1 + q^* T^*)}{K_0 + q(0) S_0}. \tag{49}$$

Thus, if

$$s_2(0) = \alpha \frac{K_{0,2}}{K_0} < s_2^* = \alpha \frac{\kappa_2(0) K_0 (1 + q^* T^*)}{K_0 + q(0) S_0}, \tag{50}$$

we have the result that the resource-less country is transitionally growing faster than the resource-rich economy. This is a result that would seem consistent with the results of Sachs and Warner (1995).

But, to what degree is s_2^* affected by the natural resource and how? Our problem relies on the fact that in general it is not possible to solve for the price of the resource at time zero ($q(0)$). If $q(0)$ is a relation of S_0, (perhaps) also of K_0, lets presume that such solution for $q(0)$ exists and denote this solution as

$$Q_0(S_0, K_0) \tag{51}$$

The effect of S_0 on s_2^* is given by

$$\frac{\partial s_2^*}{\partial S_0} = -\alpha \frac{\kappa_2(0) K(0) (1 + q^* T^*)}{(K(0) + q(0) S_0)^2} q(0) \left(1 + \frac{\partial Q_0}{\partial S_0} \frac{S_0}{q(0)}\right). \tag{52}$$

Proposition 5. If $s_2(t)$ is a monotonous function of time and if $1+(\partial Q_0/\partial S_0)(S_0/q(0))>0$, then the *nonrenewable resource enhances the GDP growth of the resource-rich* economy.

Proof. Inspection of (52) shows that if $1+(\partial Q_0/\partial S_0)(S_0/q(0))>0$ holds the effect of S_0 on s_2^* is negative. Since

$$s_1^* = 1 - s_2^* \tag{53}$$

the effect of S_0 on s_1^* is positive.

Remark: We have thus established conditions whereby either empirical results of Sachs and Warner (1995) or the contrary results of Stijns (2001) are obtainable.

However, we cannot claim that $1+(\partial Q_0/\partial S_0)(S_0/q(0))>0$ since we expect $\partial Q_0/\partial S_0<0$. That is, as the stock of the resource increases we expect the price $q(0)$ to decline. While in general we are not able to prove that $(\partial Q_0/\partial S_0)(S_0/q(0))>-1$, our numerical solutions, for a very large range of parameter values, confirm this inequality. Moreover, an analytical solution indicates that $(\partial Q_0/\partial S_0)(S_0/q(0))=-\alpha$.

We next proceed to an analytical solution for the special case where $\alpha=\theta$.

4. Analytical solution

So far integration techniques have provided a general analytical solution. We now proceed with an analytical solution for the case where the parameter values are restricted to $\alpha=\theta$. The first step is to determine what the solution for this case reveals about evolution of the two economies.

Proposition 6. If $\alpha=\theta$ the price of the nonrenewable resource and aggregate consumption $(C\equiv c_1+c_2)$ at time zero equal

$$q(0)=\frac{(1-\alpha)\,\alpha^{\alpha}}{(\rho-(1-\alpha)\,\eta)^{\alpha}}\left(\frac{K_0}{S_0}\right)^{\alpha} \qquad C(0)=K_0\left(\delta\frac{1-\alpha}{\alpha}+\frac{\rho}{\alpha}\right), \tag{54}$$

and the rate of extraction and aggregate capital $(K\equiv K_1+K_2)$ for all t equal

$$R(t)=\frac{\rho-(1-\alpha)\,\eta}{\alpha}S_0e^{\frac{(1-\alpha)\eta-\rho}{\alpha}t} \qquad K(t)=K_0\left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\alpha}}e^{-\frac{\rho}{\theta}t}. \tag{55}$$

Proof. See appendix B.

Remark: Notice that

$$\frac{dQ_0}{dS_0}\frac{S_0}{q(0)}=-\alpha. \tag{56}$$

Proposition 5 implies that with a sufficiently large initial endowment of the nonrenewable resource, the resource-rich economy can experience larger GDP growth rates than the resource-less economy. This result contradicts previous results (e.g. Rodríguez and Sachs (1999)). This contradiction comes from the fact that previous analysis have considered isolated countries (small open economies), and thus they do not take into account the inter-country linkages and the effects of these linkages on the world's transition dynamics. In particular, the effects of the terms of trade (the evolution of the price of the nonrenewable resource) on the resource-rich economy, has previously been neglected.

From (54) we observe that aggregate consumption at time zero ($C(0)$) depends on the aggregate level of capital K_0. Moreover, consumption at time zero is not influenced by the initial level of the nonrenewable resource. In Appendix B, we show that the transversality condition holds if $\rho - (1-\alpha)\eta > 0$, which also guarantees $q(0) > 0$. Finally, the restriction $\alpha = \theta$ also implies that the rate of depletion of R declines at the negative constant rate of $((1-\alpha)\eta - \rho)/\alpha$.

Proposition 7. If $\alpha = \theta$, the constant μ ($\equiv c_1(t)/c_2(t)$) equals

$$\mu = \frac{\frac{(1-\alpha)\alpha^{\alpha}}{(\rho-(1-\alpha)\eta)^{\alpha}}\left(K_{0,1}+K_{0,2}\right)^{\alpha} S_0^{1-\alpha} + K_{0,1}}{K_{0,2}}. \tag{57}$$

Proof. Substituting $q(0)$ into $(q(0)S_0 + K_{0,1})/K_{0,2}$ we yield the result.

Remark. From simple calculations one can obtain the effect of $K_{0,2}$ on μ. The effect of country two's initial capital endowment on μ is negative. Thus, the capital stock of country two negatively influences the consumption of the resource rich country *relative* to the resource less economy. Furthermore, the ratio c_1/c_2 is positively influenced by the stock of the resource at time zero. The effect of S_0 in μ equals

$$\frac{\partial \mu}{\partial S_0} = \frac{(1-\alpha)(1-\alpha)\alpha^{\alpha}}{(\rho-(1-\alpha)\eta)^{\alpha}} \frac{1}{K_{0,2}} \left(\frac{K_{0,1}+K_{0,2}}{S_0}\right)^{\alpha}. \tag{58}$$

However, this effect is important to the extent by which the share of the resource is large. In particular, if $(1-\alpha)$ is small, country one consumer's benefit from owning the resource is relatively small. Our conclusion comes from observing that $(1-\alpha)$ is the exponent of the stock of resource $\left(S_0^{1-\alpha}\right)$. Thus, a relatively small $(1-\alpha)$ severely reduces the effect of the resource.

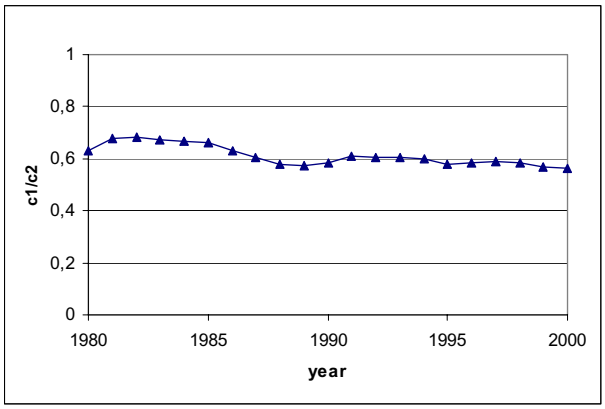
Interestingly, the consumption of country one (the resource-rich economy) relative to the consumption of country two (the resource-less economy) increases with the size of the growth rate of the resource saving technological change η.

5. Simulations

At this point our main concern is that the analytical solution's result indicating that larger amounts of the resource positively affect the growth rate of the resource rich economy (presented in section 4) may only be coincidental. To verify if this is the case we simulate the model under parameter values that are more consistent with other features of the data. First, we want to verify if indeed the ratio c_1/c_2 is constant over time.

Later we find initial endowments of the nonrenewable resource stock and capital that are consistent with $c_1/c_2 = (q(0) S_0 + K_{0,1}) / K_{0,2}$ as the model indicates. To this end we use the consumption levels in purchasing power parity obtained from the World Bank's (WB) World Development Indicators (WB, 2004) of all countries for which data is available for the period 1980-2000. To obtain consumption levels in purchasing power parity (PPP) we have computed the share of consumption in total GDP for each country in the sample, and multiplied this share by GDP in purchasing power parity. When it was impossible to obtain the share of consumption on GDP from the WB (2004) we have obtained consumption share data from the International Monetary Fund International Financial Statistics Yearbook (2002). Combining both sources of data allows us to include most of the non-oil-exporting and oil-exporting countries, including Saudi Arabia, Kuwait and Iran. To follow the model's setting as much as we can, we have divided the world into two regions, a nonrenewable-resource-rich region (countries whose oil exports are more than 25 percent of total merchandise exports) and a resource-less region otherwise. Our sample includes 71 resource-less and 19[7] resource-rich countries. We have added the consumption level in purchasing power parity of the countries of each region and have divided this number by its respective population, and obtained what we call per-capita consumption of a resource-rich (c_1) economy and resource-less (c_2) economy. Figure 1 shows the ratio $\mu = c_1/c_2$ from 1980-2000:

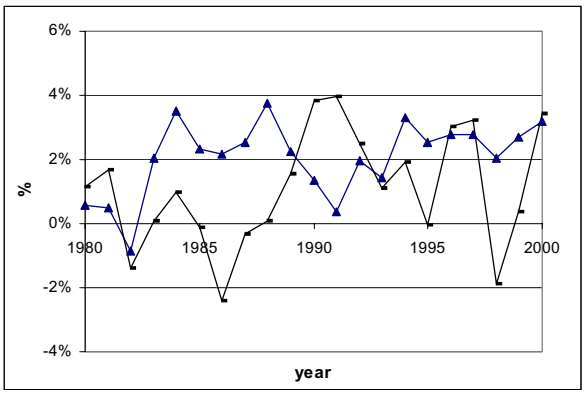
Figure 1. c_1/c_2 ratio



The larger ratio c_1/c_2 observed at the beginning of the 80's likely relates to the oil crises experienced during that period. Interestingly, the ratio of the per-capita consumption levels remains within the 56-68 percent interval. In particular, during the 1990's this ratio only fluctuated between the 56-61 percent interval. For illustrative purposes, Figure 2 shows the growth experience of the two regions. The oil-rich countries on average grew slower during the 1980-2000 period (the yearly average of the oil-rich economies was 1.1 percent versus 2 percent for the oil-less economies). Since our objective is to verify if, in the absence of any distortion, owning larger amounts of the resource is sufficient to generate smaller or negative growth rates, as Rodriguez and Sachs indicate, Table 2 does not speak against or in favor of any argument.

[7]Since counties with large mining activities represents another set of nonrenewable-resource-rich countries, we have excluded Bolivia, Chile, Jordan and South Africa due to their large mining activity. Their inclusion, however, does not affect the results.

Figure 2. Per-capita GDP in PPP growth rates



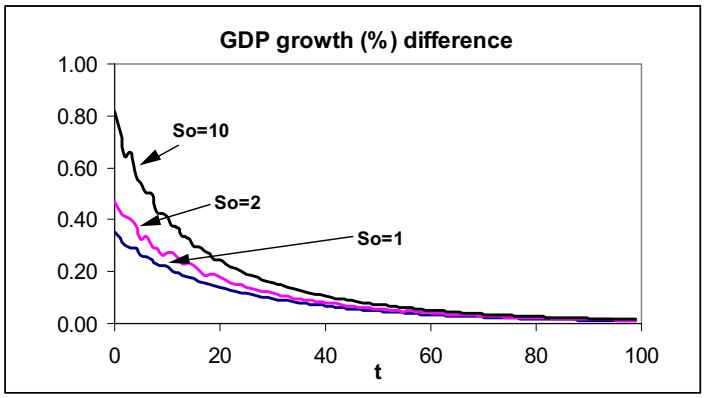
▲ nonrenewable-resource-less region, - nonrenewable-resource-rich region

The following parameter values are assumed for the simulations: $\alpha = 0.82$, $\theta = 2.5$ (a relatively large number, but we want to depart from the value of α) $\delta = 0.04$, $\rho = 0.03$ and $\eta = 0.1$ (the value $\eta = 0.1$ generates a long run growth of 2.8 percent ($g_K = (\eta - \rho)/\theta$)). We then choose values for $S_0 (= 1)$ [8], $K_{0,1} (= 10)$, and $K_{0,2} (= 90)$ that generate a ratio of $c_1/c_2 = 0.62$[9]. The simulations consist of increasing the endowment of the nonrenewable resource to assess whether larger endowments generate lower growth rates for the resource-rich region than the growth rates of the resource-less region. In Figure 3 we have plotted the GDP growth (percent) difference between the resource rich and the resource less country $\left(\dot{GDP}_1/GDP_1 - \dot{GDP}_2/GDP_2\right) * 100$ for three *different* levels of the stock of the nonrenewable resource (1, 2 and 10). From Figure 3 we observe that increasing the endowment of the nonrenewable resource increases the difference between the growth rates, indicating that as the initial stock of the resource increases the more the resource-rich economy grows. We have performed many other simulations, using a large range of parameter values, that are not reported here, but in all the cases larger endowments of the resource do not negatively affect the GDP growth rate of the resource-rich country as our analytical solution indicates.

[8] An initial resource stock equal to one ($S_0 = 1$) implies a value of the resource stock ($q(0) S_0$) equal to 45.88.

[9] The value of 0.62 corresponds to the average between the smallest and the largest value of the ratio c_1/c_2 observed during 1980-200.

Figure 3 $\left(\dot{GDP}_1/GDP_1 - \dot{GDP}_2/GDP_2\right) * 100$



In Table 1 we also report how the price of the nonrenewable resource at time zero and the value of μ change as a result of a change in the resource endowment S_0.

Table 1 Effect of S_0 on $q(0)$ and $\mu = c_1/c_2$

	S=1	S=2	S=10
q(0)	45.8833	23.3479	4.8472
μ	0.6200	0.6300	0.6497

Table 1 shows that, for an initial stock of the resource equal to one, the price of the resource equals 45.88. If we double the stock of the resource to $S_0 = 2$ the price of the resource declines and equals 23.35. Note, however, that this decline is less than proportional to the increase of the stock of the resource, indicating that the elasticity of the price of the resource is negative and greater than minus one. This result is thus consistent with Proposition 5, which states that if the elasticity of the price of the resource at time zero with regard to the stock of the resource is greater than minus one, the stock of the resource will positively influence the relative income growth rate of the resource-rich economy. In Table 1 we have also included the value of $\mu = c_1/c_2$ for the different levels of the initial stock of the resource considered. Our simulations show that the effect of the increase of the resource on relative welfare (c_1/c_2) is small but positive. Interestingly, increases on the stock of the resource (even by ten times) as indicated in Table 1 affect the ratio μ only by small amount (from 0.62 to 0.65). This reflects the fact that larger endowments of the nonrenewable negatively affect the price of the resource and thus its overall effect on $\mu = (q(0) S_0 + K_{0,1}) / K_{0,2}$ is highly counterbalanced.

6. Conclusions

This paper examines the impact of nonrenewable resources on economic growth and relative welfare in a two country world economy. We introduce a nonrenewable-resource-rich economy and a nonrenewable-resource-less country in an otherwise standard neoclassical growth model. As in

other models of trade, factor price equalization across countries is shown to result. In particular, the rental rate of capital across countries is equal even in the absence of international borrowing and lending. This result implies that the growth rates of consumption across countries are equal and therefore, the ratio of consumption levels is constant. We show that the ratio of the consumption of the resource-rich country to the consumption of the resource-less economy equals the ratio of the value of assets at any point in time. While the resource-rich country wealth increases with the initial stock of the resource, this effect is counterbalanced, to some degree, by the negative effect of the initial stock of the resource on the price of the nonrenewable.

We show that the initial endowment of the nonrenewable resource has a positive effect on the GDP growth rate of the *resource-rich* country as long as the elasticity of the initial price of the resource with regard to the initial stock of the resource is greater than minus one. An analytical solution of the model under a parameter restriction indicates that indeed this elasticity is greater than minus one. Numerical simulations under a large rage of parameter values provide the same conclusion about the elasticity of the initial price of the resource with regard to the initial stock of the resource. Thus, we find the result of Rodríguez and Sachs (1999), indicating that the initial stock of the resource influences negatively the GDP growth of an economy that exports a nonrenewable resource, does not to hold in general. This contradiction comes from the fact that Rodríguez and Sachs (1999) consider an isolated country, and thus fail to account for any iteration between the economy they analyze and the rest of the world transitional dynamics. In particular, they failed to account for the dynamic price effect of the nonrenewable resource on the oil-exporting economy.

While data indicates that GDP growth of resource-rich economies has underperformed that of resource-poor economies, the model presented here indicates that owning a nonrenewable is not sufficient to generate relatively lower GDP growth. We believe that the point of view of Warner and Sachs (1995 and 2001) in the lines of Dutch-Disease arguments or "crowding-out stories" such as rent seeking or over compensation of workers in the oil industry can offer an explanation. On the other hand the explanation presented by Rodríguez and Sachs (1999) that the resource alone can generate lower GDP growth rates does not seem to provide the answer once interactions across countries are introduced.

References

Asheim, Geir B. (1986). Hartwick's Rule in open economies. The Canadian Journal of Economics, 19, 395-402.

Chiarella, Carl (1980). Trade between resource-poor and resource-rich economies as a differential game. Exhaustible Resources, Optimality, and Trade. Ed. M.C. Kemp and N.V. Long. North-Holland Publishing Company, 219-246.

Dasgupta, Partha, and Geoffrey Heal (1974). The Optimal Depletion of Exhaustible Resources. Review of Economic Studies (Symposium on the economics of exhaustible resources), 41, 3-28.

Ding, Ning and Barry C. Field (2004). Natural resource abundance and economic growth. Department of Resource Economics, University of Massachusetts Amherst, Working paper no. 2004-7, (www.umass.edu/resec/workingpapers).

Geldrop, Jan H. van and Cees A. Withagen (1993). General equilibrium and international trade with exhaustible resources. Journal of International Economics, 34, 341-357.

Geldrop, Jan H. van and Cees A. Withagen (1994). General equilibrium in an economy with exhaustible resource and an unbounded horizon. Journal of Economic Dynamics and Control, 18, 1011-1035.

Hartwick, John M. (1995). Constant consumption paths in open economies with exhaustible resources. Review of International Economics, 3, 275-283.

International Monetary Fund (2002). International Financial Statistics Yearbook, Washington DC, USA.

Lederman, Daniel and William F. Maloney (2003). Trade structure and growth. The World Bank, Policy Research Working Paper 3025.

Rodríguez, Francisco and Jeffrey D. Sachs (1999). Why do resource-abundant economies grow more slowly? Journal of Economic Growth, 4, 277-303.

Sachs, Jeffrey D. and Andrew M. Warner (1995). Natural resource abundance and economic growth. NBER Working Paper 5398.

Sachs, Jeffrey D. and Andrew M. Warner (2001). Natural resource abundance and economic development the curse of natural resources. European Economic Review, 45, 827-838.

Solow, Robert M. (1974). Intergenerational Equity and Exhaustible Resources. Review of Economic Studies (Symposium on the economics of exhaustible resources), 41, 29-45.

Stiglitz, Joseph E. (1974). Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths. Review of Economic Studies (Symposium on the economics of exhaustible resources), 41, 123-37.

Stijns, Jean-Phillip (2001). Natural resource abundance and economic growth revisited. Unpublished manuscript. Department of Economics, University of California Berkeley.

Tornell, Aaron and Philip. R. Lane (1995). The voracity effect. American Economic Review, 89, No. 1, 22-46.

World Bank (2004). World Development Indicators CD-ROM. International Bank for Reconstruction and Development / The World Bank, Washington, DC, USA.

Appendix A

Proof of Proposition 1.
Using (24) , and $r-\delta=\dot{q}/q$ the budget constraint of country two can be rewritten as

$$\frac{\dot{K}_2(t)}{q(t)}-\frac{\dot{q}(t)}{q(t)}\frac{K_2(t)}{q(t)}=-\left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\theta}}\frac{c_2(0)}{e^{\frac{\rho}{\theta}t}}\frac{1}{q(t)}. \tag{59}$$

Note that the left hand side of expression (59) equals the derivative of K_2/q with respect to time $\left(d\left(K_2/q\right)/dt\right)$, integrating (59) we obtain

$$\lim_{t\to\infty}\frac{K_2\left(t\right)}{q\left(t\right)}-\frac{K_2\left(\tau\right)}{q\left(\tau\right)}=-\frac{c_2\left(0\right)}{q\left(0\right)^{\frac{1}{\theta}}}\int_\tau^\infty\frac{q\left(t\right)^{\frac{1-\theta}{\theta}}}{e^{\frac{\rho}{\theta}t}}dt. \tag{60}$$

Employing the transversality condition (25) implies

$$K_2\left(\tau\right)=q\left(\tau\right)\frac{c_2\left(0\right)}{q\left(0\right)^{\frac{1}{\theta}}}\int_\tau^\infty\frac{q\left(t\right)^{\frac{1-\theta}{\theta}}}{e^{\frac{\rho}{\theta}t}}dt. \tag{61}$$

Since the ratio c_1/c_2 is constant for all t, let $c_1\left(t\right)=\mu c_2\left(t\right)$ where μ is a constant. (6) and (26) imply

$$R=\frac{r}{q}\frac{1-\alpha}{\alpha}\left(K_1+K_2\right). \tag{62}$$

The budget constraint of the consumer of country one can be rewritten as

$$\dot{K}_1-\left(r-\delta\right)K_1+\mu c_2=q\frac{r}{q}\frac{1-\alpha}{\alpha}\left(K_1+K_2\right). \tag{63}$$

Using $\dot{q}/q=r-\delta$, $\dot{S}=-R$, setting $c_2\left(t\right)=\left(q\left(t\right)/q\left(0\right)\right)^{\frac{1}{\theta}}c_2\left(0\right)e^{-(\rho/\theta)t}$, dividing by q and integrating we get:

$$\begin{aligned}\int_\tau^\infty\left(\frac{\dot{K}_1\left(t\right)}{q\left(t\right)}-\frac{\dot{q}\left(t\right)}{q\left(t\right)}\frac{K_1\left(t\right)}{q\left(t\right)}+\frac{\mu}{q\left(t\right)}\left(\frac{q\left(t\right)}{q\left(0\right)}\right)^{\frac{1}{\theta}}\frac{c_2\left(0\right)}{e^{\frac{\rho}{\theta}t}}\right)dt &= \int_\tau^\infty R\left(t\right)dt \\ &= S\left(\tau\right)-\lim_{t\to\infty}S\left(t\right).\end{aligned} \tag{64}$$

Since depletion is costless $\lim_{t\to\infty}S\left(t\right)=0$. Therefore, (64) can be written as

$$\int_\tau^\infty\frac{d\frac{K_1}{q}}{dt}dt+\mu\frac{c_2\left(0\right)}{q\left(0\right)^{\frac{1}{\theta}}}\int_\tau^\infty\frac{q\left(t\right)^{\frac{1-\theta}{\theta}}}{e^{\frac{\rho}{\theta}t}}dt=S\left(\tau\right) \tag{65}$$

or

$$\lim_{t\to\infty}\frac{K_1\left(t\right)}{q\left(t\right)}-\frac{K_1\left(\tau\right)}{q\left(\tau\right)}+\mu\frac{c_2\left(0\right)}{q\left(0\right)^{\frac{1}{\theta}}}\int_\tau^\infty\frac{q\left(t\right)^{\frac{1-\theta}{\theta}}}{e^{\frac{\rho}{\theta}t}}dt=S\left(\tau\right). \tag{66}$$

Using the transversality condition (equation 25) and rearranging we obtain

$$\mu q(\tau)\frac{c_2(0)}{q(0)^{\frac{1}{\theta}}}\int_\tau^\infty \frac{q(t)^{\frac{1-\theta}{\theta}}}{e^{\frac{\rho}{\theta}t}}dt = q(\tau)S(\tau)+K_1(\tau). \tag{67}$$

From (61) we yield

$$q(\tau)\frac{c_2(0)}{q(0)^{\frac{1}{\theta}}}\int_\tau^\infty \frac{q(t)^{\frac{1-\theta}{\theta}}}{e^{\frac{\rho}{\theta}t}}dt = K_2(\tau), \tag{68}$$

and therefore,

$$\mu = \frac{q(\tau)S(\tau)+K_1(\tau)}{K_2(\tau)}. \tag{69}$$

Proof of Proposition 2

First note from (5), (12) and (13) that the long-run growth rate of q and the long-run value of r equal

$$\lim_{t\to\infty}\frac{\dot{q}}{q} = \eta = g_q, \qquad \lim_{t\to\infty} r = \eta+\delta \equiv r^* \tag{70}$$

hence, r is constant in the long run. Thus, the per country growth rate of consumption is constant in the long run and given by

$$\lim_{t\to\infty}\frac{\dot{c}_i}{c_i} = \lim_{t\to\infty}\frac{r-\delta-\rho}{\theta} = \frac{\eta-\rho}{\theta} = g_{c_i}. \tag{71}$$

The first order conditions of the final good sectors in each country (equation (6)) and the market clearing condition for R (equation (26)) imply

$$R = \frac{1-\alpha}{\alpha}\frac{r}{q}(K_1+K_2) \qquad \Rightarrow \qquad \frac{\alpha}{1-\alpha}q\frac{R}{K} = r \tag{72}$$

where $K = K_1 + K_2$. Since r is constant in the long run, (72) indicates that in the long run the growth rate of K and R must satisfy the following relation

$$\lim_{t\to\infty}\frac{\dot{q}}{q}+\lim_{t\to\infty}\frac{\dot{R}}{R}-\lim_{t\to\infty}\frac{\dot{K}}{K}=0 \qquad \Rightarrow \qquad \lim_{t\to\infty}\frac{\dot{K}}{K}-\lim_{t\to\infty}\frac{\dot{R}}{R}=\eta. \tag{73}$$

Next notice that $\dot{S} = -R$, thus the growth rate of S equals the ratio $\dot{S}/S = -R/S$. Since extraction is costless, as time goes to infinity both R[10] and S approach zero. Using L'Hôpital's rule we have that

$$\lim_{t\to\infty} \frac{\dot{S}}{S} = \lim_{t\to\infty} -\frac{R}{S} = \lim_{t\to\infty} -\frac{\dot{R}}{\dot{S}} = \lim_{t\to\infty} \frac{\dot{R}}{R}. \tag{74}$$

Implying that as $t \to \infty$ both R and S grow at the same rate, (this result was previously derived by Partha Dasgupta and Geoffrey Heal, 1974). What remains to be shown is that $g_{c_i} = g_{K_i}$. Observe that (72) implies that qR/K is constant in the long run, since in the long run S and R grow at the same rate, then, it is also the case that $\chi \equiv qS/K$ is constant in the long run. Taking the log time derivative of χ, using $\dot{S} = -R$, and $qR = (1-\alpha) Kr/\alpha$ we obtain

$$\frac{\dot{\chi}}{\chi} = \dot{q}\frac{S}{K}\frac{1}{\chi} + \frac{q\dot{S}}{K}\frac{1}{\chi} - \frac{qS}{K}\frac{\dot{K}}{K}\frac{1}{\chi}. \tag{75}$$

Taking the limit we obtain

$$\lim_{t\to\infty} \frac{\dot{\chi}}{\chi} = 0 = \eta - \lim_{t\to\infty} \frac{1-\alpha}{\alpha}\frac{r}{\chi} - \lim_{t\to\infty} \left(r - \delta + r\frac{1-\alpha}{\alpha} - \frac{C}{K} \right) \tag{76}$$

where $C = c_1 + c_2$. Since both c_1 and c_2 grow at the same rate, then C also grows at the same rate as c_i. Note that since χ is constant in the long run $(\dot{\chi}/\chi = 0)$, thus, for (76) to hold it must be that C/K is constant in the long run and therefore, C and K must asymptotically grow at the same rate. Therefore, $g_{c_i} = g_K$. Employing (73) we obtain

$$\lim_{t\to\infty} \frac{\dot{R}}{R} = \frac{\eta(1-\theta) - \rho}{\theta} = g_R. \tag{77}$$

For the transversality condition (10) to hold it must also be the case that $g_R = g_S < 0$. Finally, since $K = K_1 + K_2$ then it is straightforward to show that

$$g_{K_i} = g_K. \tag{78}$$

Proof of Proposition 3. The Jacobian matrix of (35) evaluated at the steady state equals

$$\mathbf{J}^* = \begin{pmatrix} -\frac{1-\alpha}{\alpha} r^* & 0 & 0 \\ \mathrm{f}_q^{\,g} & 0 & -\frac{g^*}{(T^*)^2}\frac{r^*}{q^*}\frac{1-\alpha}{\alpha} \\ \mathrm{f}_q^{\,T} & (1+\mu)(T^*)^2 & (1+\mu) g^* T^* - g_S \end{pmatrix}, \tag{79}$$

[10] For the tranversality condition (10) to hold it must be that $\lim_{t\to\infty} R = 0$.

where f_q^g denotes the partial derivative of f^g with respect to q evaluated at the steady state, and similarly for f_q^T. The eigenvalues of $\mathbf{J}^*$ are the values of ξ that solve the characteristic equation

$$\left(-\frac{1-\alpha}{\alpha}r^* - \xi\right)\left(-\xi\left(\left(1+\mu\right)g^*T^* - g_S - \xi\right) + \frac{r^*}{q^*}\frac{1-\alpha}{\alpha}\left(1+\mu\right)g^*\right) = 0. \tag{80}$$

One of the solution to (80) is given by

$$\xi_1 = -\frac{1-\alpha}{\alpha}r^* < 0. \tag{81}$$

The other two are values of ξ that solve the quadratic equation in ξ given by

$$\xi^2 - \xi\left(\left(1+\mu\right)g^*T^* - g_S\right) + \frac{r^*}{q^*}\frac{1-\alpha}{\alpha}\left(1+\mu\right)g^* = 0 \tag{82}$$

or

$$\xi^2 + b\xi + d = 0 \tag{83}$$

where

$$b = -\left(\left(1+\mu\right)g^*T^* - g_S\right), \qquad d = \frac{r^*}{q^*}\frac{1-\alpha}{\alpha}\left(1+\mu\right)g^*. \tag{84}$$

Using g^* from (36) we obtain

$$\frac{\eta + \delta\left(1-\alpha\right)}{\alpha} - g_K = \left(1+\mu\right)T^*g^*. \tag{85}$$

Substituting T^* from (37) into (85) we get

$$\frac{r^*}{\hat{q}^*}\frac{1-\alpha}{\alpha}\left(1+\mu\right)g^* = \left(\eta - g_K\right)\left(\frac{\eta + \delta\left(1-\alpha\right)}{\alpha} - g_K\right). \tag{86}$$

Substituting (85) into (86) yields

$$\frac{r^*}{\hat{q}^*}\frac{1-\alpha}{\alpha}\left(1+\mu\right)g^* = \left(\eta - g_K\right)\left(1+\mu\right)T^*g^* \qquad \Rightarrow \qquad \frac{r^*}{\hat{q}^*}\frac{1-\alpha}{\alpha} = \left(\eta - g_K\right)T^*. \tag{87}$$

From (28) it is the case that $\left(\eta - g_K\right) = -g_S$. Using (87) note that d can be rewritten as

$$d = -g_S\left(1+\mu\right)g^*T^*. \tag{88}$$

Thus the other two eigenvalues of $\mathbf{J}^*$ equal

$$\xi_{2,3} = \frac{-b \pm \sqrt{D}}{2} \tag{89}$$

where $D = ((1+\mu)\, g^* T^* - g_S)^2 + 4 g_S (1+\mu)\, g^* T^*$. Further manipulation of D yields

$$D = (1+\mu)^2 (g^* T^*)^2 + 2 g_S (1+\mu)\, g^* T^* + g_S^2 = ((1+\mu)\, g^* T^* + g_S)^2, \tag{90}$$

substituting into (89)

$$\xi_{2,3} = \frac{-b \pm ((1+\mu)\, g^* T^* + g_S)}{2} \tag{91}$$

the remaining eigenvalues of $\mathbf{J}^*$ equal

$$\xi_2 = -g_S \qquad\qquad \xi_3 = (1+\mu)\, g^* T^* \tag{92}$$

which are positive.

Appendix B

Proof of Proposition 6. Here we show that

$$q(0) = \frac{(1-\alpha)\,\alpha^\alpha}{(\rho - (1-\alpha)\,\eta)^\alpha} \left(\frac{K_0}{S_0}\right)^\alpha \tag{93}$$

when $\alpha = \theta$. Since $R = (1-\alpha)\, r\, (K_1 + K_2)\, q^{-1}/\alpha$. To determine R we need to find a solution for aggregate capital $K = K_1 + K_2$. Since $\dot{K} = \dot{K}_1 + \dot{K}_2$ using both budget constraints and denoting $C = c_1 + c_2$, $\dot{K}$ equals

$$\dot{K} = (r - \delta)\, K + \frac{1-\alpha}{\alpha} r K - C, \tag{94}$$

which is a first order differential equation with a variable coefficient. Using $\dot{q}/q = r - \delta$, its general solution is given by

$$K(t) = K_0 e^{\int_0^t \left(\frac{r(\varepsilon)}{\alpha} - \delta\right) d\varepsilon} - \int_0^t C(\tau)\, e^{\int_\tau^t \left(\frac{r(\varepsilon)}{\alpha} - \delta\right) d\varepsilon} d\tau \tag{95}$$

$$= K_0 \left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\alpha}} e^{\delta\left(\frac{1-\alpha}{\alpha}\right)t} - \int_0^t C(\tau) \left(\frac{q(t)}{q(\tau)}\right)^{\frac{1}{\alpha}} e^{\delta\left(\frac{1-\alpha}{\alpha}\right)(t-\tau)} d\tau. \tag{96}$$

Using $\alpha = \theta$ and $C(t) = (q(t)/q(0))^{1/\theta} \frac{C(0)}{e^{(\rho/\theta)\, t}}$ from (24) we get

$$K(t) = K_0\left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\alpha}} e^{\delta\left(\frac{1-\alpha}{\alpha}\right)t} - \frac{C(0)}{q(0)^{\frac{1}{\theta}}}\int_0^t q(t)^{\frac{1}{\alpha}} \frac{q(\tau)^{\frac{1}{\theta}}}{q(\tau)^{\frac{1}{\alpha}}} \frac{e^{\delta\left(\frac{1-\alpha}{\alpha}\right)(t-\tau)}}{e^{\frac{\rho}{\theta}\tau}} d\tau$$

$$= K_0\left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\alpha}} e^{\delta\left(\frac{1-\alpha}{\alpha}\right)t} + \frac{C(0)}{q(0)^{\frac{1}{\theta}}} \frac{q(t)^{\frac{1}{\alpha}}\left(e^{-\frac{\rho}{\theta}t} - e^{\delta\left(\frac{1-\alpha}{\alpha}\right)t}\right)}{\delta\left(\frac{1-\alpha}{\alpha}\right) + \frac{\rho}{\theta}}. \tag{97}$$

Using $r = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha} e^{(1-\alpha)\eta t} q^{-(1-\alpha)}\right)^{1/\alpha}$ from (5), we obtain that the ratio r/q equals

$$\frac{r}{q} = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} e^{\left(\frac{1-\alpha}{\alpha}\right)\eta t}}{q^{\frac{1}{\alpha}}}. \tag{98}$$

Using $R = (1-\alpha) rKq^{-1}/\alpha = (1-\alpha)^{1/\alpha} e^{((1-\alpha)/\alpha)\eta t} Kq^{-1/\alpha}$, $\int_0^{\infty} R dt = S_0$ and (97) we obtain

$$\int_0^{\infty} R(t)\, dt = (1-\alpha)^{\frac{1}{\alpha}} \int_0^{\infty} \left(\frac{K_0}{q(0)^{\frac{1}{\alpha}}} e^{\frac{1-\alpha}{\alpha}(\eta+\delta)t} + \frac{C(0)}{q(0)^{\frac{1}{\theta}}} \frac{\left(e^{\left(\frac{1-\alpha}{\alpha}\eta - \frac{\rho}{\theta}\right)t} - e^{\frac{1-\alpha}{\alpha}(\eta+\delta)t}\right)}{\frac{1-\alpha}{\alpha}\delta + \frac{\rho}{\theta}}\right) dt \tag{99}$$

$$= \frac{(1-\alpha)^{\frac{1}{\alpha}}}{q(0)^{\frac{1}{\alpha}}} \lim_{t\to\infty} \left(\frac{\alpha\left(e^{\frac{1-\alpha}{\alpha}(\eta+\delta)t} - 1\right)}{(\eta+\delta)(1-\alpha)}\left(K_0 - \frac{C(0)}{\frac{1-\alpha}{\alpha}\delta + \frac{\rho}{\theta}}\right) + \frac{C(0)\left(e^{\left(\frac{1-\alpha}{\alpha}\eta - \frac{\rho}{\theta}\right)t} - 1\right)}{\left(\frac{1-\alpha}{\alpha}\eta - \frac{\rho}{\theta}\right)\left(\frac{1-\alpha}{\alpha}\delta + \frac{\rho}{\theta}\right)}\right).$$

For (99) to converge to the constant S_0 it must be that

$$C(0) = K_0\left(\delta\frac{1-\alpha}{\alpha} + \frac{\rho}{\theta}\right). \tag{100}$$

For the transversality condition to hold $\left(((1-\alpha)/\alpha)\eta - \frac{\rho}{\theta}\right) < 0$ must be satisfied, this implies that (99) converges. Solving for $q(0)$ we get,

$$q(0) = \frac{(1-\alpha)\alpha^{\alpha}}{(\rho - (1-\alpha)\eta)^{\alpha}}\left(\frac{K_0}{S_0}\right)^{\alpha} \tag{101}$$

also

$$K(t) = K_0\left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\alpha}} e^{\delta\left(\frac{1-\alpha}{\alpha}\right)t} - \frac{C(0)}{q(0)^{\frac{1}{\theta}}}\int_0^t q(t)^{\frac{1}{\alpha}}\frac{q(\tau)^{\frac{1}{\theta}}}{q(\tau)^{\frac{1}{\alpha}}}\frac{e^{\delta\left(\frac{1-\alpha}{\alpha}\right)(t-\tau)}}{e^{\frac{\rho}{\theta}\tau}}d\tau$$

$$= K_0\left(\frac{q(t)}{q(0)}\right)^{\frac{1}{\alpha}} e^{-\frac{\rho}{\theta}t} = q(t)^{\frac{1}{\alpha}}\frac{(\rho-(1-\alpha)\eta)}{e^{\frac{\rho}{\theta}t}(1-\alpha)^{\frac{1}{\alpha}}\alpha}S_0. \tag{102}$$

Substituting (102) into $R = ((1-\alpha)/\alpha) rKq^{-1}$ and the solution for $q(0)$ we get

$$R = \frac{1-\alpha}{\alpha}\frac{r}{q}K = \frac{\rho-(1-\alpha)\eta}{\alpha}S_0 e^{\frac{(1-\alpha)\eta-\rho}{\alpha}t}. \tag{103}$$