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**Graduate School of Agricultural and Resource Economics
&
School of Economics**

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A Simplified Approach**

by

J. Brian Hardaker and Gudbrand Lien

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Stochastic Efficiency Analysis With Risk Aversion Bounds: A Simplified Approach

J. Brian Hardaker and Gudbrand Lien**

Abstract

A method of stochastic dominance analysis with respect to a function (SDRF) is described and illustrated. The method, called stochastic efficiency with respect to a function (SERF), partitions a set of risky alternatives in terms of certainty equivalents for a specified range of attitudes to risk. It can be applied for any utility function with risk attitudes defined by corresponding ranges of absolute, relative or partial risk aversion coefficients. SERF involves comparing each alternative with all the other alternatives simultaneously, not pairwise as with conventional SDRF. Hence it yields a subset of the efficient set found by SDRF. Moreover, the method is readily implemented in a simple spreadsheet with no special software needed.

Key Words: risk analysis, stochastic dominance with respect to a function, risk aversion.

JEL Classification: D81.

** Brian Hardaker is Emeritus Professor in the Graduate School of Agricultural and Resource Economics, University of New England. Gudbrand Lien is Senior Researcher at the Norwegian Agricultural Economics Research Institute, Oslo, Norway. E-mail: gudbrand.lien@nifl.no. The authors are grateful to Jock Anderson for helpful comments on an earlier draft.

Contact information: School of Economics, University of New England, Armidale, NSW 2351, Australia. Email: hardaker@northnet.com.au (corresponding author).

1. Introduction

Risk assessment requires coming to grips with both probabilities and preferences for outcomes held by the decision maker (DM). Chances of bad versus good outcomes can only be evaluated and compared knowing the DM's relative preferences for such outcomes. According to the subjective expected utility (SEU) hypothesis (Anderson, Dillon, Hardaker 1977: 66-69), the DM's utility function for outcomes is needed to assess risky alternatives. The SEU hypothesis states that the utility of a risky alternative is the DM's expected utility for that alternative, meaning the probability-weighted average of the utilities of outcomes.

The shape of the utility function reflects an individual's attitude to risk. Several attempts have been made to elicit such utility functions from relevant DMs in order to put the SEU hypothesis to work in the analysis of risky alternatives (Robison *et al.* 1984; Hardaker *et al.* 1997). Usually the results have been rather unconvincing (King and Robison 1984; Anderson and Hardaker 2003).

Partly to avoid the need to elicit a specific single-valued utility function, methods under the heading of stochastic dominance or efficiency criteria have been developed. Stochastic dominance criteria are useful in situations involving a single DM whose preferences are not known precisely, in situations where more than one DM may be involved, and in analysing policy alternatives or extension recommendations for a group of many individual DMs.

A stochastic dominance criterion is a decision rule that provides a partial ordering of risky alternatives for DMs whose preferences conform to specified conditions about their utility functions (preferences for consequences). There is an important trade-off to be made in conducting a stochastic dominance analysis. The fewer restrictions that are placed on the

utility function, the more general applicability the results will have, but the less powerful will be the criterion in selecting between alternatives. Usually, efficiency analysis will result in only a partial ordering of alternatives into efficient and dominated sets. The DM must then make the final choice from among the members of the efficient set. Criteria that identify small efficient sets usually require more specific information about preferences.

Hadar and Russell (1969) and Hanoch and Levy (1969) presented the concepts of first-degree stochastic dominance (FSD) and second-degree stochastic dominance (SSD). FSD is used to partition alternatives for DMs who prefer more wealth to less and have absolute risk aversion with respect to wealth, $r_a(w)$, between the bounds $-\infty < r_a(w) < +\infty$ (King and Robison 1984). SSD requires the additional assumption that DMs are not risk preferring, i.e., that absolute risk aversion bounds are $0 < r_a(w) < +\infty$. This means that SSD accounts for DMs who possess an absolute risk aversion parameter that is so large that the utility of a small difference at the lowest observation is extraordinary important. In empirical work it is often found that these two forms of analysis are not discriminating enough to yield useful results, meaning that the efficient set can still be too large to be easily manageable (King and Robison 1981, 1984).¹

More powerful than FSD and SSD is stochastic dominance with respect to a function (SDRF), which was introduced by Meyer (1977). For SDRF the absolute risk aversion bounds are reduced to $r_1(w) \leq r_a(w) \leq r_2(w)$, i.e., the criterion is defined for all DMs whose absolute risk aversion function lies anywhere between lower and upper bounds $r_1(w)$ and $r_2(w)$.

¹ There are third to t -th degree stochastic dominance criteria but they are seldom much more discriminating than SSD, and so are not reviewed in this paper. A good review of ordinary stochastic dominance and stochastic dominance with respect to a function is given by Zentner *et al.* (1981). Within the stochastic dominance paradigm, Levy (1992) reviewed the theoretical developments and empirical applications in economics, finance and statistics.

Eliciting from the DMs (or inferring) the bounds on their risk aversion coefficients may be simpler than eliciting a complete utility function. For SDRF there is no solution in closed form so a numerical evaluation of the optimal control problem is used.

FSD, SSD and SDRF are all pairwise comparison methods that identify a subset of dominated alternatives, leaving the remainder of undominated ones that are described as 'efficient'. However, convex stochastic dominance (CSD), developed by Fishburn (1974a, 1974b), can be used to exclude further alternatives from the efficient set by comparing each alternative in turn with all possible convex combinations of the others. Convex forms of FSD and SSD have been implemented using linear programming, involving formulating and solving a different model to test each alternative for possible dominance (e.g., Drynan 1977; Bawa *et al.* 1985). A multiobjective linear programming model has been used to implement convex SDRF (Cochran *et al.* 1985). The rather tedious nature of this analytical task may explain why CSD appears to have been rarely used by practitioners.

The logic of convex stochastic dominance depends on forming probability mixes of alternatives, not real mixes as in portfolio analysis. Forming a convex combination of two alternatives is equivalent to making a single random drawing from the appropriate probability distribution to decide which to use. This then does not require the stochastic dependencies between the alternatives to be taken into account because they are not implemented in combination. If a real mix or portfolio of risky prospects is possible, individual prospects cannot be ordered by stochastic dominance analysis (though properly defined portfolios can be). For portfolio selection, stochastic dependencies between portfolio members (such as correlations) must be accounted for. Methods typically used for portfolio selection include E,V formulations (requiring strong assumptions about the form of the distribution and/or the form of the utility function) solved by quadratic programming, or non-linear utility efficient programming, solved for discrete states of nature (Patten *et al.* 1988). Applications of

stochastic dominance methods to cases where the risky prospects being evaluated are not genuine alternatives are likely to be flawed except in the unlikely case of stochastic independence between all the alternatives.

Some software packages are available for SDRF (e.g., McCarl 1988, 1990; Goh *et al.* 1989). It seems that, for many users, SDRF software, if not the concept itself, is somewhat of a 'black box'. The available software (except Richardson's (2003) software) gives users no choice of functional form, and most accept input on risk aversion only in terms of absolute risk aversion, r_a . Therefore we suspect that most of the existing software uses constant absolute risk aversion (CARA) functions, although this is not particularly clear in the program descriptions.

In this paper we introduce a more straightforward and more discriminating SDRF method, which we call stochastic efficiency with respect to a function (SERF). The name is chosen to distinguish it from conventional SDRF and to indicate that the method works by selecting utility efficient alternatives, not by finding (a subset of) dominated alternatives. SERF partitions alternatives in terms of certainty equivalents as a selected measure of risk aversion is varied. SERF can be applied for any utility function based on ranges in the absolute, relative, or partial risk aversion coefficient, as appropriate. Since conventional SDRF picks only the pairwise dominated alternatives, we can expect that pairwise SDRF will not isolate the smallest possible efficient set. By contrast SERF will potentially identify a smaller efficient set than SDRF because it picks only the utility efficient alternatives, comparing each with all the other alternatives simultaneously. In addition to its important advantage of being more discriminating, SERF can easily be implemented in a simple spreadsheet with no special software needed.

The paper is structured as follows: Section 2 describes the SERF method; the relationship between conventional SDRF and SERF is discussed in Section 3; some

applications of the SERF method are presented in Section 4; Section 5 contains a short discussion and some concluding comments.

2. The SERF method

Let $U(w)$ be the utility function of a DM with performance criterion w (wealth).² We assume that the risky alternatives to be compared have uncertain outcomes so that values of w are stochastic. Let $f_1(w), f_2(w), \dots, f_n(w)$ be the probability density functions (PDFs) describing the outcomes for n risky alternatives. The corresponding cumulative distribution functions (CDFs) are denoted by $F_1(w), F_2(w), \dots, F_n(w)$. The SEU hypothesis is that $U(w) = EU(w) = \int U(w)f(w)dw = \int U(w)dF(w)$, i.e., the utility of any risky alternative is its expected value. Since we do not know the exact shape of the utility function or, in other words, the DM's risk aversion, we solve the problem where the absolute, relative or partial risk aversion function $r(w)$ of the DM lies everywhere between lower and upper bounds $r_1(w)$ and $r_2(w)$.

So for each risky alternative and for a chosen form of the utility function, we define the function for utility in terms of risk aversion and the stochastic outcome w as:

$$U(w, r(w)) = \int U(w, r(w))dF(w) = \sum_{i=1}^m U(w_i, r(w))P(w_i), \quad r_1(w) \leq r(w) \leq r_2(w) \quad (1)$$

where the second term in equation 1 represents the continuous case and the continuous case is converted to its discrete approximation in the third term for computational purposes. In the discrete case $P(w_i)$ is the probability for states i and there are m states for each risky

² Although we use wealth, w , as the performance criterion in this paper, w can be replaced by x (for loss/gain or transient income) provided x is small relative to w and also provided we measure risk aversion consistently with the outcome measure (Anderson and Hardaker 2003).

alternative. We are assuming here that we start with CDFs for a set of risky alternatives, convert points on the CDF for a set of finite values of w , each of which is converted to its utility for selected values of the risk aversion coefficient, then each finite utility is multiplied by its associated probability to calculate a weighted average of the utilities of outcomes. In this way we can evaluate this discrete function for a sufficient number of discrete points of $r(w)$ to describe the relationship between U and $r(w)$ for that alternative.

Partial ordering of alternatives by certainty equivalent (CE) will be the same as a partial order of them by utility values. However, we chose to convert the utilities to CEs by taking the inverse of the utility function:

$$CE(w, r(w)) = U^{-1}(w, r(w)) \quad (2)$$

We prefer the CE representation to leaving results in utilities not only because CEs are easier to interpret than utility values, but also because this method allows inclusion of expected monetary value in cases where $U(w, r(w))$ is undefined for $r(w) = 0$.

By this method we end up with a set of CEs for each of the n alternatives calculated for a set of $r(w)$ values within the bounds $r_1(w) \leq r(w) \leq r_2(w)$. For easy interpretation of results when the number of alternatives is sufficiently small, we suggest graphing the CEs of the alternatives on the vertical axis against risk aversion on the horizontal axis. Such a graph allows ready identification of the efficient set and also provides an immediate insight into how the method works, as illustrated by the examples to follow. The efficient set contains only the alternatives that yield the highest CE for some value of $r(w)$ within the range of interest. We can partition alternatives using the following rule:

- Only those alternatives which have the highest (or equal highest) CE for some value in the range of $r(w)$ are utility efficient. All other alternatives are dominated in the SERF sense.

[Figure 1 about here]

In the example in Figure 1 the SERF method is used to compare three alternatives simultaneously for all values in the range of $r_1(w)$ and $r_2(w)$, not pairwise as for SDRF, and identifies alternatives 1 and 2 as the utility-efficient set. Because of the simultaneous comparisons of all alternatives, the SERF efficient set may be a subset of the efficient set found by conventional SDRF.

The SERF rule can readily be implemented within a spreadsheet application if the alternatives are too numerous for graphical analysis.

McCarl (1988) suggested that instead of just partitioning the set of risky alternatives for a range of risk aversion, one should solve to find the risk aversion coefficient where the preference between a pair of efficient alternatives changes. He called the value of the risk aversion coefficient at which the preference changes the breakeven risk root, BRAC. For values of the risk aversion coefficient less than BRAC one alternative is preferred and for values greater than the BRAC the other is preferred. In SERF it is simple to identify where the CE curves cross or, for large data sets, to use, e.g., Solver in Excel to find this crossover for $r(w)$, by varying $r(w)$ to minimise the difference between two CEs.

The results of a SDRF analysis may depend on the choice of utility function. The SERF method can be applied for any utility function³, although we suggest it will usually be best to adopt the CARA function (negative exponential) as a reasonable approximation of the actual but presumably unknown utility function. Such an approximation will be appropriate provided that the risky alternatives being compared are small relative to the DM's wealth. The main advantage of the CARA function is that, as Anderson and Hardaker (2003) show, coefficients of absolute risk aversion can be validly applied to consequences measured in

terms of wealth, losses and gains, or (transient) income. These authors point out some traps in deriving relative or partial risk aversion measures needed for other functional forms if the consequences are not measured in terms of wealth.

3. Relations between SERF and SDRF

The conventional SDRF method is sequentially to select a risk-averse utility function, $U(w)$, which has

$$r_l(w) \leq r_a(w) \leq r_2(w), \quad (3)$$

and then discover for which of these values of $r_a(w)$ equation 4

$$\int [F_2(w) - F_l(w)] U'(w) dw \quad (4)$$

is minimised for all values of w , where the cumulative density functions $F_l(w)$ and $F_2(w)$ represent two risky alternatives. If, for a given class of decision makers (or attitudes to risk), the minimum of the above expression is positive, then alternative $F_l(w)$ is preferred to $F_2(w)$. That means that the utility (or CE) of $F_l(w)$ is greater than the utility (CE) of $F_2(w)$ for all values of $r_a(w)$ in the set for the particular form of $U(w)$ used. If the minimum is zero, some DM within the group may be indifferent between the two alternatives. Thus the two alternatives cannot be ranked. If the minimum is negative, $F_2(w)$ could be preferred to $F_l(w)$. To check, the difference $F_l(w) - F_2(w)$ is introduced in the square brackets term in equation 4 and the evaluation procedure is repeated.

³ Examples of different utility functions are given in, e.g., Hardaker *et al.* (1997) and Lin and Chiang (1978).

If we look closely at equation 4 we observe that it is equivalent to measuring the difference between utilities of distributions $F_1(w)$ and $F_2(w)$. To show this let the difference in utility between $F_1(w)$ and $F_2(w)$ be

$$\int U(w)f_1(w)dw - \int U(w)f_2(w)dw = \int U(w)[f_1(w) - f_2(w)]dw \quad (5)$$

Applying the change-in-variable technique to integrate, let $dv = f_1(w) - f_2(w)$, $v = F_1(w) - F_2(w)$, and $u = U(w)$. Then, recalling $udv = uv|_{-\infty}^{+\infty} - \int u dv$, we write (Robison and Barry 1987: 55-56)

$$\begin{aligned} \int U(w)[f_1(w) - f_2(w)]dw &= U(w)[F_1(w) - F_2(w)]_{-\infty}^{+\infty} + \int [F_2(w) - F_1(w)]U'(w)dw \\ &= \int [F_2(w) - F_1(w)]U'(w)dw \end{aligned} \quad (6)$$

In other words, this method orders the utility of alternatives $F_1 \left(\int U(w)f_1(w)dw \right)$ and $F_2 \left(\int U(w)f_2(w)dw \right)$ within defined bounds of $r_a(w)$. By comparing this method with SERF as described in Section 2 we can see we are making the same comparison, though more directly and informatively than with conventional SDRF.

4. Application

In this section, as an example of its application, the SERF method outlined above is used and compared with the SDRF method on two constructed examples.

Example 1

The first example is a hypothetical one using four constructed risky alternatives, A to D (Table 1).

[Table 1 about here]

The means of the alternatives vary from about 122 for alternative D to about 154 for alternative B. The overall range of outcomes is from 50 to 230. Both extremes are associated with alternative B. Alternative A has the largest minimum outcome of 100. Figure 2 shows the graphs of CDFs for each of the four alternatives.

[Figure 2 about here]

A relevant range of $r_a(w)$ is assumed be from 0.0006667 to 0.0266666 (which approximately corresponds to a $r_r(w)$ in the range 0.1 to 4, given an average wealth of about 150). The software computer programme developed by Goh *et al.* (1989) was used for the computational task of ranking the alternatives using the SDRF approach. Implementation of this SDRF approach involves using a negative exponential utility function. The result of the analysis is a risk-efficient set with three members, alternatives A, B and C (Table 2).

[Table 2 about here]

Our SERF approach, when using a negative exponential utility function and the same range for $r_a(w)$ as in the SDRF analysis, gave the CE-graph shown in Figure 3.

[Figure 3 about here]

With the SERF approach the efficient set is alternatives A and B only. The value of $r_a(w)$ where CE curves for alternative A and B cross is $r_a(w) = 0.0085$ (i.e., where $r_r(w) \approx 1.27$). As a check, McCarl's (1988) software named RISKROOT was used on the same dataset. This program estimated the crossover to be at $r_a(w) = 0.0085$ between alternatives A and B, exactly the same as we found with the SERF method.

This hypothetical example was constructed to illustrate our claim that the efficient set with the SERF approach can be a subset of the efficient set found by conventional SDRF.

We also did an experiment with the SERF approach with a constant relative risk aversion (CRRA) power function on the same hypothetical example data. The efficient set was identical to that described above and the implied value of $r_a(w)$ where CE curves for alternative A and B cross over was almost identical ($r_r(w)=1.13$) to that found using the negative exponential function (1.27).

Example 2

A second hypothetical example represents net returns from six risky arable rotation alternatives, F to K (Table 3).

[Table 3 about here]

The means of these alternatives vary from about 296 for alternative F to about 446 for alternative I. The overall range of outcomes is from 45 to 905. Alternative K and I have the most extreme values. Alternative H has the largest minimum outcome of 180. Figure 4 shows the graphs of CDFs for each of the six alternatives.

[Figure 4 about here]

Use of the Goh *et al.* (1989) software on these alternatives shows both the SSD set and the SDRF set with $r_a(w)$ within the bounds 0 and 0.01 is I, J and K. Figure 5 shows the results with the SERF approach, using a negative exponential utility function and the same range for $r_a(w)$.

[Figure 5 about here]

With the SERF approach the efficient set is rotation alternatives I and J. The BRAC where CE curves for rotation I and J cross over is $r_a(w) = 0.0033$, which is exactly the same as we found with the RISKROOT software. As in the previous example, in this example the efficient set is smaller with the SERF method than with the SDRF method.

5. Discussion and concluding comments

The main advantage of SERF over SDRF is that the utility efficient set is obtained directly, and is potentially smaller than the SDRF efficient set. SDRF would produce the same, potentially smaller, efficient set only if that method is extended to include convex dominance.

Otherwise, whether conventional SDRF and SERF applied using the same form of utility function will give comparable results will depend on differences in data handling. There are many different ways one might approach the discrete approximation of continuous functions as may be needed for a stochastic dominance analysis. Using the SDRF approach, it is generally only possible to process the risky alternatives specified for the same set of fractile values. That may require some pre-processing of data to get them into this format. There is an issue of how many fractiles to take and how to get them. In deriving fractile values from data (abundant or sparse) one faces a choice between using the raw data or smoothing a CDF and then deriving fractile values. While we would normally advise that smoothing is best, there is the related issue of how specialist SDRF software processes the fractile values entered, particularly whether any interpolation or further smoothing is done. We suspect that some of these issues could be as important as choice of functional form in influencing results at the margin, i.e., in comparing risky alternatives that have very close expected utilities.

With the SERF method there is no need to define the same probability intervals for all alternatives. The method works both with the same intervals on w for all alternatives with

different probabilities, or it could have both values of w and of $P(w)$ uniquely defined for each alternative. That is another advantage with the SERF method.

In cases where the risky prospects to be analysed not are genuine alternatives (as assumed in this paper) but are members of a portfolio, the stochastic dependency between the real mix of prospects needs to be accounted for. This problem can also be solved comparing CEs for a bounded range of risk aversion by using a utility-efficient programming approach (Patten *et al.* 1988).

There is nothing particularly novel in SERF. It depends on concepts such as certainty equivalents and measures of risk aversion that will be understood by most people who are familiar with the basics of decision analysis. The basic idea is so simple that it is surprising that it has not been widely adopted. There may be more, but the only application we have found in searching the agricultural economics literature is in the decision analysis software of Richardson (2003). He illustrates the method without noting its particular advantages.

Conventional SDRF has been widely used in applied work, yet the underlying concept of SDRF and its implementation are not easy to understand. The SERF method illustrated in this paper includes all the advantages of SDRF yet is much more transparent, is easier to implement and has a stronger discriminating power. These seem to be powerful advantages which suggest that it is time for the more widespread use of this simpler method.

References

- Anderson, J.R., Dillon, J.L. and Hardaker, J.B. 1977, *Agricultural Decision Analysis*. Iowa State University Press, Ames.
- Anderson, J.R. and Hardaker, J.B. 2003, 'Risk aversion in economic decision making: pragmatic guides for consistent choice by natural resource managers'. In: Wesseler, J.,

- Weikard, H.P. and Weaver, R. (Eds.), *Risk and Uncertainty in Environmental and Resource Economics*. Edward Elgar Publishing, UK. forthcoming.
- Bawa, V.S., Bodurtha Jr., J.N, Rao, M.R. and Suri, H.L. 1985, 'On determination of stochastic dominance optimal sets'. *Journal of Finance*, vol. 40, pp. 417-431.
- Cochran, M.J., Robison, L.J. and Lodwick, W. 1985, 'Improving the efficiency of stochastic dominance techniques using convex set stochastic dominance'. *American Journal of Agriculture Economics*, vol. 67, pp. 289-295.
- Drynan, R.G. 1977, *Experimentation - its value to the farm decision maker*. Unpublished PhD thesis, University of New England, Armidale, NSW.
- Fishburn, P.C. 1974a, 'Convex stochastic dominance with continuous distribution functions'. *Journal of Economic Theory*, vol. 7, pp. 143-158.
- Fishburn, P.C. 1974b, 'Convex stochastic dominance with finite consequence sets'. *Theory and Decision*, vol. 5, pp. 119-137.
- Goh, S., Shih, C-C., Cochran, M.J. and Raskin, R. 1989, 'A generalised stochastic dominance program for the IBM pc'. *Southern Journal of Agricultural Economics*, vol. 21, pp. 175-182.
- Hadar, J. and Russell, W.R. 1969, 'Rules for ordering uncertain prospects'. *American Economic Review*, vol. 49, pp. 25-34.
- Hanoch, G. and Levy, H. 1969, 'Efficiency analysis of choices involving risk'. *Review of Economic Studies*, vol. 36, pp. 335-345.
- Hardaker, J.B., Huirne, R.B.M. and Anderson, J.R. 1997, *Coping with Risk in Agriculture*. CAB International, Wallingford.
- King, P.K. and Robison, L.J. 1981, 'An interval approach to measuring decision maker preferences'. *American Journal of Agricultural Economics*, vol. 63, pp. 510-520.

- King, P.K. and Robison, L.J. 1984, 'Risk efficiency models'. In: Barry, P.J. (Ed.), *Risk Management in Agriculture*. Iowa State University Press, Ames, Iowa, pp 68-81.
- Levy, H. 1992, 'Stochastic dominance and expected utility: survey and analysis'. *Management Science*, vol. 38, pp. 555-583.
- Lin, W.R. and Chang, H.S. 1978, 'Specification of Bernoullian utility function in decision analysis'. *Agricultural Economics Research*, vol. 30, pp. 30-36.
- McCarl, B.A. 1988, 'Preference among risky prospects under constant risk aversion'. *Southern Journal of Agricultural Economics*, vol. 20, pp. 25-33.
- McCarl, B.A. 1990, 'Generalized stochastic dominance: an empirical examination'. *Southern Journal of Agricultural Economics*, vol. 22, pp. 49-55.
- Meyer, J. 1977, 'Choice among distributions'. *Journal of Economic Theory*, vol. 14, pp. 326-336.
- Patten, L.H., Hardaker, J.B. and Pannell, D.J. 1988, 'Utility-efficient programming for whole-farm planning'. *Australian Journal of Agricultural Economics*, vol. 32, pp. 88-97.
- Richardson, J. 2003, Simulation for Applied Risk Management. Program documentation. Department of Agricultural Economics, Texas A&M University.
- Robison, L.J. and Barry, P.J. 1987, *The Competitive Firm's Response to Risk*. Macmillan, New York.
- Robison, L.J., Barry, P.J., Kliebenstein, J.B. and Patrick, G.F. 1984, 'Risk attitudes: concepts and measurement approaches'. In: Barry, P.J. (Ed.), *Risk Management in Agriculture*. Iowa State University Press, Ames, Iowa, pp. 11-30.
- Zentner, R.P., Greene, D.D., Hickenbotham, T.L and Eidman, V.R. 1981, Ordinary and generalised stochastic dominance: a primer. Dept. Agr. And Appl. Econ. Staff Pap. P81-27, University of Minnesota.

Table 1 A hypothetical example with four alternatives specified for the same set of fractiles values.

F[w]	W			
	Alternative A	Alternative B	Alternative C	Alternative D
0.0	100	50	83	78
0.1	125	100	113	102
0.2	135	128	130	111
0.3	142	145	140	117
0.4	147	152	147	121
0.5	150	157	151	123
0.6	153	162	155	125
0.7	158	171	161	129
0.8	163	183	170	133
0.9	175	207	186	144
1.0	195	230	214	163

Table 2 Pairwise comparison matrix^a to investigate SDRF for a set of bounds for the hypothetical example (range: $0.0006667 \leq r_a(w) \leq 0.0266666$).

Alternative	A	B	C	D
A	-	?	?	1
B	?	-	?	1
C	?	?	-	1
D	0	0	0	-

^a Comparison by row, across columns

? = no dominance of the alternative in this row

1 = row item dominates the respective column item

0 = row item is dominated by the respective column item.

Efficient set consists of rows with no 0 indicators.

Table 3 A hypothetical example with net returns from six rotation alternatives specified for the same set of fractiles values.

F[w]	Rotation alternative					
	F	G	H	I	J	K
0.0	45	80	180	110	165	170
0.1	158	186	232	252	278	239
0.2	205	230	257	311	325	282
0.3	241	264	281	356	361	322
0.4	272	294	305	394	392	365
0.5	299	326	332	432	421	412
0.6	324	361	361	474	455	464
0.7	353	401	395	522	492	523
0.8	387	449	434	578	537	593
0.9	432	511	486	657	595	685
1.0	540	660	610	850	735	905

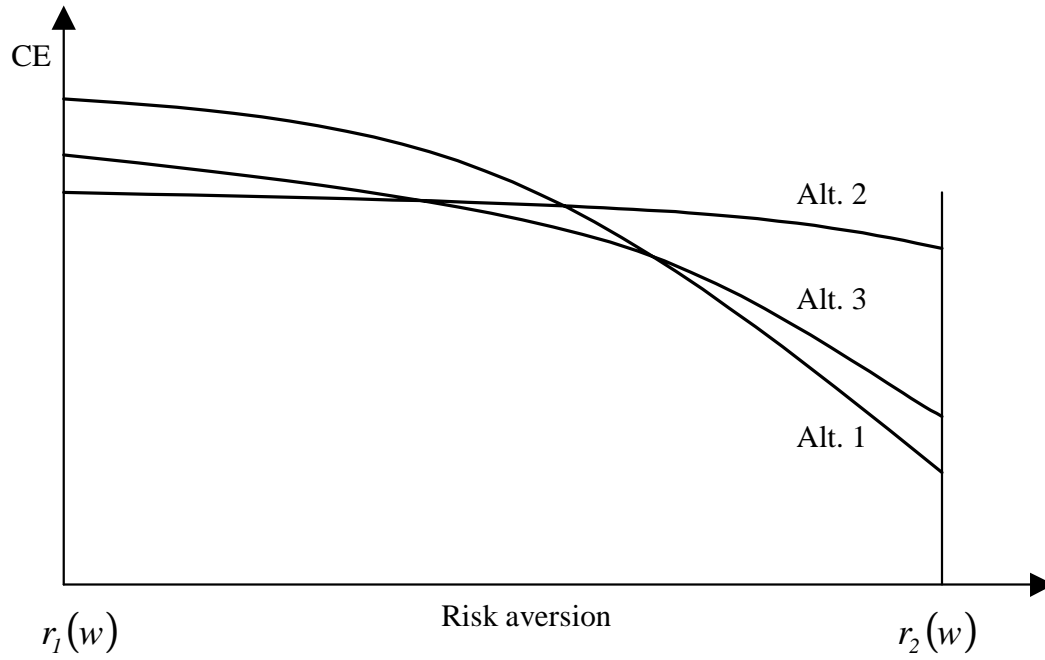


Figure 1 The principles of the SERF method illustrated. In this example three risky alternatives are considered simultaneously. Partial ordering of the risky alternatives is done in terms on certainty equivalents (CEs) for all values of risk attitudes in the range of $r_1(w)$ and $r_2(w)$.

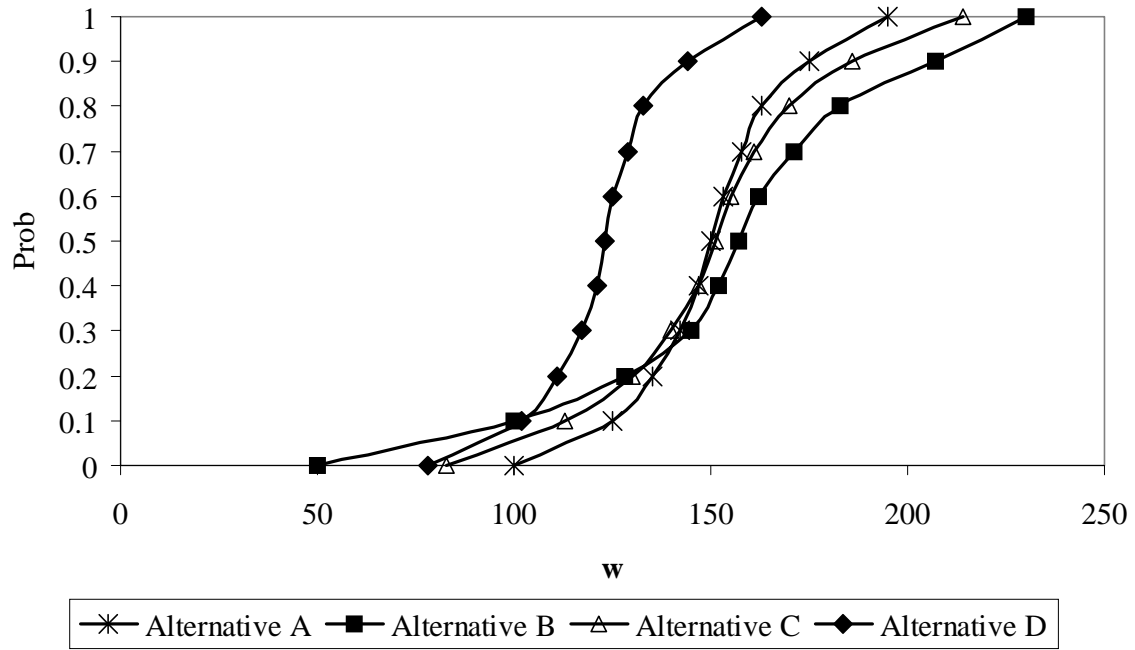


Figure 2 Cumulative probability distributions for alternatives A to D.

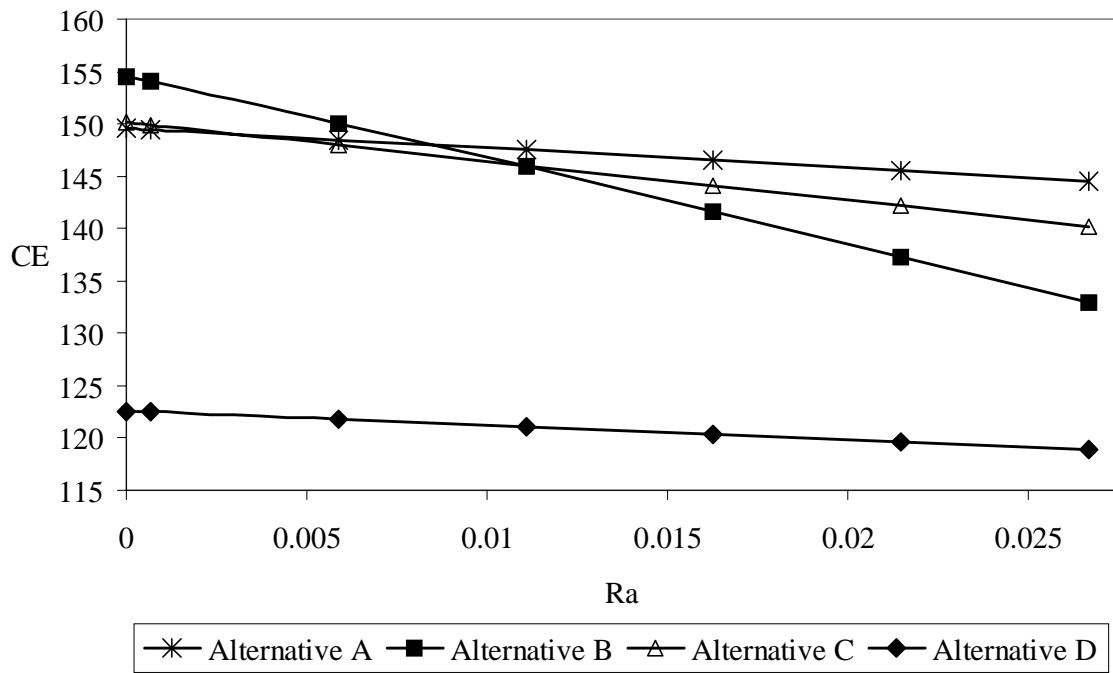


Figure 3 CE-graph for the constructed example.

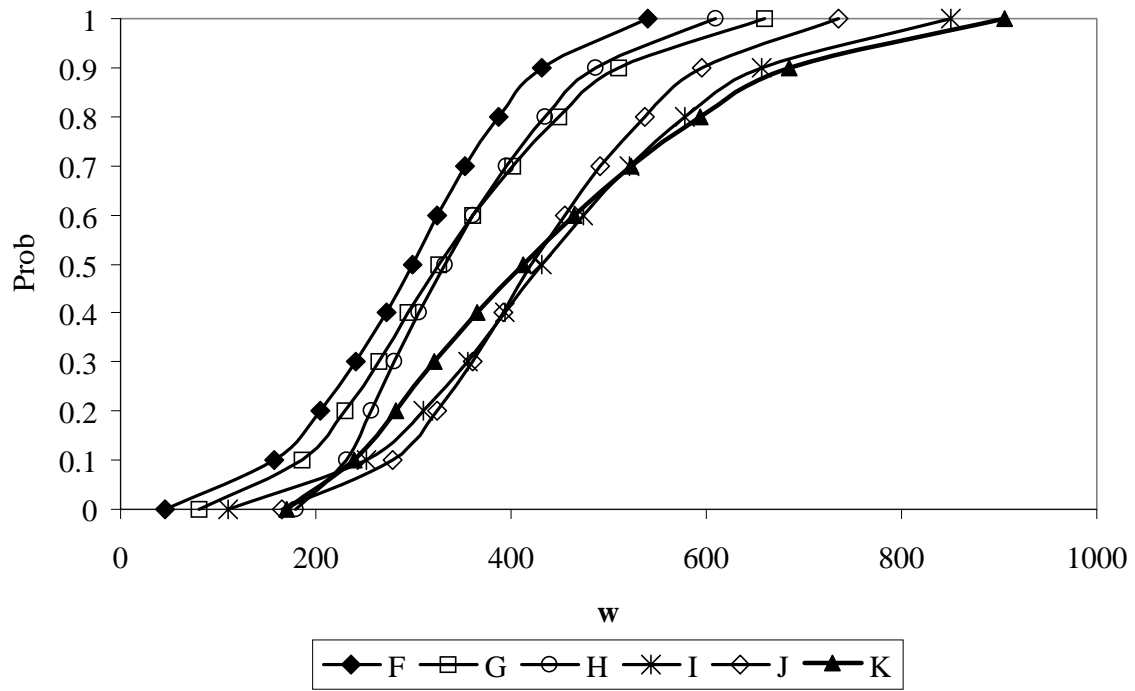


Figure 4 Cumulative probability distributions for rotation alternatives F to K.

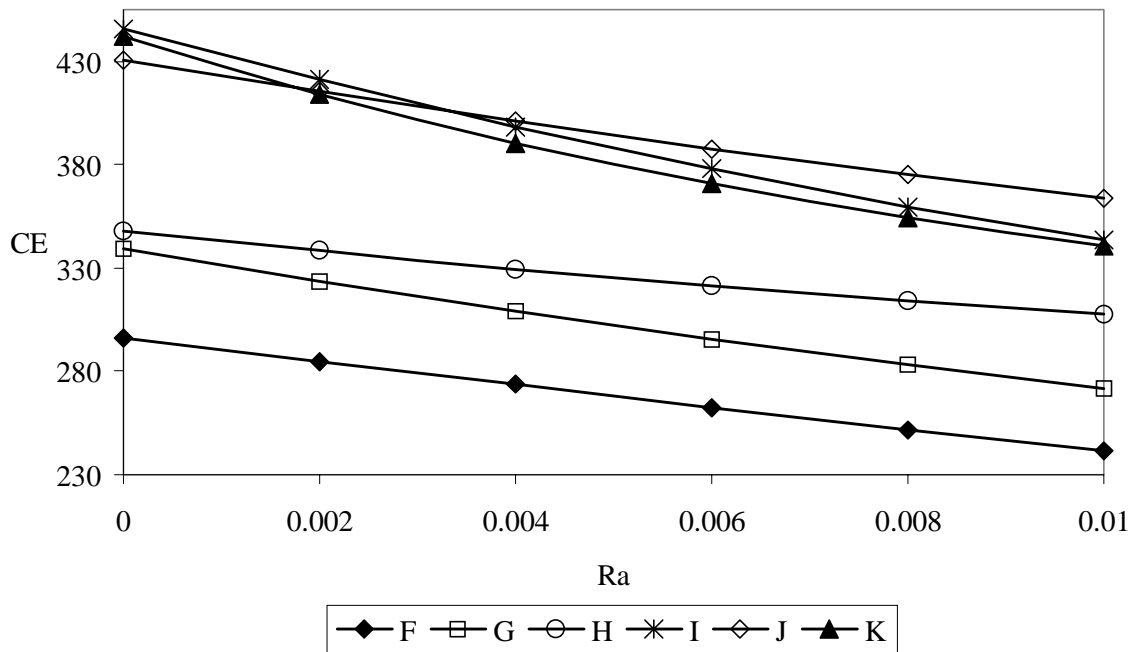


Figure 5 CE-graph for the constructed rotation example, when using a negative exponential utility function.