Dynamic PEATSim Model
Documenting Its Use in Analyzing Global Commodity Markets

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A Report from the Economic Research Service

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Abstract

This report documents the updated version of the Partial Equilibrium Agricultural Trade Simulation (PEATSim) model developed by USDA's Economic Research Service. PEATSim is a global model, covering 31 commodities and 27 countries/regions. The model, consistent with economic theory, provides a flexible country and commodity aggregation and accounts for cross-commodity linkages and interactions. The report includes a presentation and discussion of the structure and specific features of the revamped model, along with the theoretical underpinnings. It also documents an application of the model to illustrate its dynamic structure and to demonstrate the differential behavior.

Keywords: Partial equilibrium, dynamic, trade, multi-commodity markets, agriculture, global model

Acknowledgments

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Summary

Background

PEATSim (Partial Equilibrium Agricultural Trade Simulation) is a dynamic, partial equilibrium, mathematical-based model that enables users to reach analytical solutions to problems, given a set of parameters, data, and initial conditions. This theoretical tool developed by ERS incorporates a wide range of domestic and border policies that enables it to estimate the market and trade effects of policy changes on agricultural markets. PEATSim captures the economic behavior of agricultural producers, consumers, and markets in a global framework. It includes variables for production of crops and livestock activities, consumption, exports, imports, stocks, world prices, and domestic producer and consumer prices.

In 2010, ERS updated and modified the model extensively. The original model, static in its specification, was developed through a collaborative effort between ERS and Pennsylvania State University. This report supports requirements for documentation and information quality as specified by the U.S. Office of Management and Budget.

What Is the Contribution?

PEATSim's innovative and flexible design enables users to analyze a variety of domestic and trade policy issues. The model is written in GAMS (General Algebraic Modeling System) using PATH, a Mixed Complementarity Problem (MCP) solver. MCP enables PEATSim to account for a discontinuous policy regime, such as a tariff rate quota or a trade ban. This report makes the model transparent and available to a larger audience.

In 2010, PEATSim was modified to include new features and enhanced capabilities:

- PEATSim was augmented to incorporate different sets of production activities; links between upstream and downstream sectors; and interactions of producers, processors, and consumers at a global level.

- Unlike previous versions, the updated model's dynamic specification and enhanced flexibility enable researchers to analyze short-term and long-term effects of domestic and border policies.

- The model accounts for simultaneous interactions between livestock and crop activities and has the ability to capture and solve different sets of production activities worldwide.

- PEATSim now includes 27 countries/regions, up from 12 in the previous version. The updated version also allows flexible aggregation of the countries in the model to accommodate various modeling needs.

- The updated model includes 31 agricultural commodities in addition to 3 biofuel-related commodities (ethanol, biodiesel, and distillers' dried grains with solubles).

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- The data in PEATSim calibrate to *USDA Agricultural Projections to 2019* and *OECD-FAO Agricultural Outlook 2010-2019*, while an innovative econometric method (i.e., cross entropy) equilibrates supply and use.

- PEATSim uses transparent, clearly listed programming codes, data inputs, equations, data rules, policies, and parameters.

How Was the Study Conducted?

This report documents the latest version of PEATSim. To illustrate the model's capabilities, we evaluate the effects on the biofuels sector of alternative macroeconomic conditions and crude oil prices, taking into account biofuel production on a global scale from different feedstocks across countries.

Introduction

This report documents the latest version (2010) of the Dynamic Partial Equilibrium Agricultural Simulation (PEATSim) model and discusses specific modified features and structures, such as commodity/country coverage and updates of trade policies. It also presents an empirical application of the model on the global biofuels sector under alternative macroeconomic conditions and, especially, crude oil prices (testing for high/low levels).

PEATSim is a simulator in the sense that it is a mathematical-based system that attempts to find analytical solutions to problems, given a set of parameters, data, and initial conditions. Previous versions of the PEATSim model have been used in numerous studies (see Blayney et al., 2006; Dyck et al., 2008; Langley et al., 2003, 2005, 2006; Stillman et al., 2005; Zahniser et al., 2010; Valdes et al., 2010, Meade et al., 2010; and Shane et al., 2009). For example, PEATSim was used to analyze global dairy markets, examine the effects of agricultural trade liberalization, estimate the effects of biofuels expansion on global agricultural production and trade, and analyze the impacts of biofuel mandates on worldwide grain, livestock, and oilseed sectors (see Peters et al., 2008, 2009, 2010; and Stillman et al., 2007, 2008a, 2008b).

In previous versions of the model, biofuels were represented by "stylized" biofuels activities, which do not fully capture the complexity and interaction of biofuels production patterns. An analysis of global expansion of biofuels production calls for an innovative way to incorporate these activities into the model and capture their impacts and links between upstream and downstream activities of the biofuels "module." For this reason, PEATSim was augmented to incorporate a module that links farm activities/sectors with downstream industries on a global basis.

The documentation of the static version of the model, the ERS/Penn State Trade model, was developed by ERS's James Stout in collaboration with David Abler of the Pennsylvania State University (2004).[1] In fall 2005, the model was revamped and extended to include the complementarity features of PATH, a state-of-the-art program (Dirkse, 1994; Dirkse and Ferris, 1994b, 1994c). In 2010, the dynamic version of the model was developed and extended to include the global biofuels module and the use of maximum entropy techniques for data consistency.

[1]The theoretical underpinnings for the static version of the model were developed by a modeling team, including David Abler, David Blandford, Karl Meilke, Ian Sheldon, GianCarlo Moschini, Mary Bohman, and Praveen Dixit.

Methodology and Data

PEATSim's multiple-commodity, multiple-region structure enables it to account for simultaneous interactions between livestock and crop activities while maintaining identities such as supply and use. PEATSim covers major crops, oilseed and oilseed products, livestock, and dairy activities. It also incorporates explicit representation of each country's domestic and trade policies pertaining to agricultural commodities. PEATSim has the ability to model different sets of production activities; links between various crops and livestock sectors both upstream (at the farm gate) and downstream (such as biofuels and dairy processing); and interactions of producers, processors, and consumers on a global level. The model's innovative and flexible specification enables researchers to analyze a variety of domestic and trade policy issues.

PEATSim is a reduced-form model that captures the economic behavior of producers, consumers, and markets in a global framework. It includes variables for production of crops and livestock activities, consumption, exports, imports, stocks, world prices, and domestic producer and consumer prices. Commodity-based markets are modeled such that quantities and prices clear the market (Hamilton, 1994, pp. 324-327). This implies that market-clearing quantities are the sum of beginning stocks, production, and imports and equal to the sum of exports, consumption, and ending stocks. These equilibrating conditions hold at commodity levels and at world markets. The model is used to simulate "what-if" scenarios for comparison with base year(s) results. Constant elasticity functions are selected because of their ease of interpretation and well-behaved properties. They can be viewed as first-order approximations to underlying supply and demand functions (see Stout and Abler, 2004).

The PEATSim model is written in GAMS (General Algebraic Modeling System, Brooke et al., 1988) programming language using PATH, a Mixed Complementarity Problem (MCP) solver developed by Dirkse (1994a), Dirkse and Ferris (1994b), and Dirkse et al. (1994c) (see appendix). MCP enables PEATSim to generate a model with different production-consumption regimes and functional form discontinuities. Thus, PEATSim can incorporate discontinuous functional forms such as tariff-rate quotas (TRQs) and discontinuous demand issues created by mandates, targets, and other complicated policy instruments. MCP also allows for endogenous determination of active regimes and the consequences of regime shifts, such as the shift from an "in-quota" tariff to "over-quota" tariff. For example, PEATSim endogenously determines the TRQ price and quantity and makes the need for the arbitrary quota rent allocation obsolete.

Model Structure, New Features, and Modifications

Given the complexity of global agricultural markets, it is not easy to evaluate the implications of a growing global market for biofuels because a model would need to account for biofuel production and its links with other agricultural and nonagricultural sectors worldwide. PEATSim was extensively modified to include additional sectors involved in biofuels and their byproducts, including conventional ethanol and biodiesel, on a global basis, such as the AGLINK-COSIMO (OECD-FAO, 2010) and FAPRI (FAPRI, 2010) models.

The interactions of biofuel and agricultural markets are inherently multisectoral because of the interactions between energy, farm inputs, crops, feed, food consumption, and trade. Continued long-term growth in the use of food and feed products and in the production of fuel have made it difficult to assess the impacts of such growth on the global market. For these reasons, ERS augmented and revised the model with a global biofuels module.

Theoretically, the model was modified incorporating detailed global ethanol and biodiesel markets. The new global biofuel component of PEATSim includes ethanol and biodiesel from a variety of feedstocks, such as corn, wheat, rapeseed, soybeans, sugarcane, and sugar beets, as well as downstream production activities related to biofuels for all countries in the model.

Also, the expanded database includes biofuel production from sources other than feedstocks. For the nonfeedstock-related biofuel production processes, such as that for cellulosic ethanol, detailed technical data are unavailable. However, nonfeedstock sources, or "other" biofuels, are accounted for in PEATSim given data availability in the AGLINK-COSIMO (OECD-FAO, 2010) model and its database.

Geographic Scope and Flexible Country Aggregation

PEATSim was updated to include 27 countries/regions. The country coverage is Argentina, Australia, Bangladesh, Brazil, Canada, China, European Union (EU-27), Egypt, Indonesia, India, Iran, Japan, South Korea, Mexico, Malaysia, New Zealand, Pakistan, Philippines, Russia, Saudi Arabia, South Africa, Thailand, Turkey, Ukraine, United States, Vietnam, and the rest of the world (see fig. 1). The latest version allows flexible aggregation of the countries in the model. For example, a user can aggregate the model into fewer countries if needed.

Commodity Coverage

The model covers 31 agricultural commodities: 11 crops (rice, wheat, corn, other coarse grains, soybeans, sunflower, rapeseed, cotton, sugarcane, sugar beets, and sugar (semi-processed)); 10 oilseed, oil, and meal products (soybeans, soybean oil, soybean meal, sunflower seed, sunflower oil, sunflower meal, rapeseed, rapeseed oil, rapeseed meal, and other oilseed oil); 4 livestock and livestock-related products (beef and veal, pork, poultry, and raw milk); and 6 dairy products (fluid milk, butter, cheese, nonfat dry milk,

Figure 1
PEATSim countries/regions, 2009

World biodiesel production

- U.S.–17%
- Argentina–9%
- Australia–5%
- Brazil–6%
- Canada–1%
- EU27–50%
- India–2%
- Rest of world–10%

World ethanol production

- Argentina–0.41%
- Australia–0.74%
- Brazil–29.82%
- Canada–1.93%
- China–5.14%
- EU27–7.08%
- India–2.66%
- Mexico–0.08%
- Rest of world–2.67%
- U.S. 49.46%
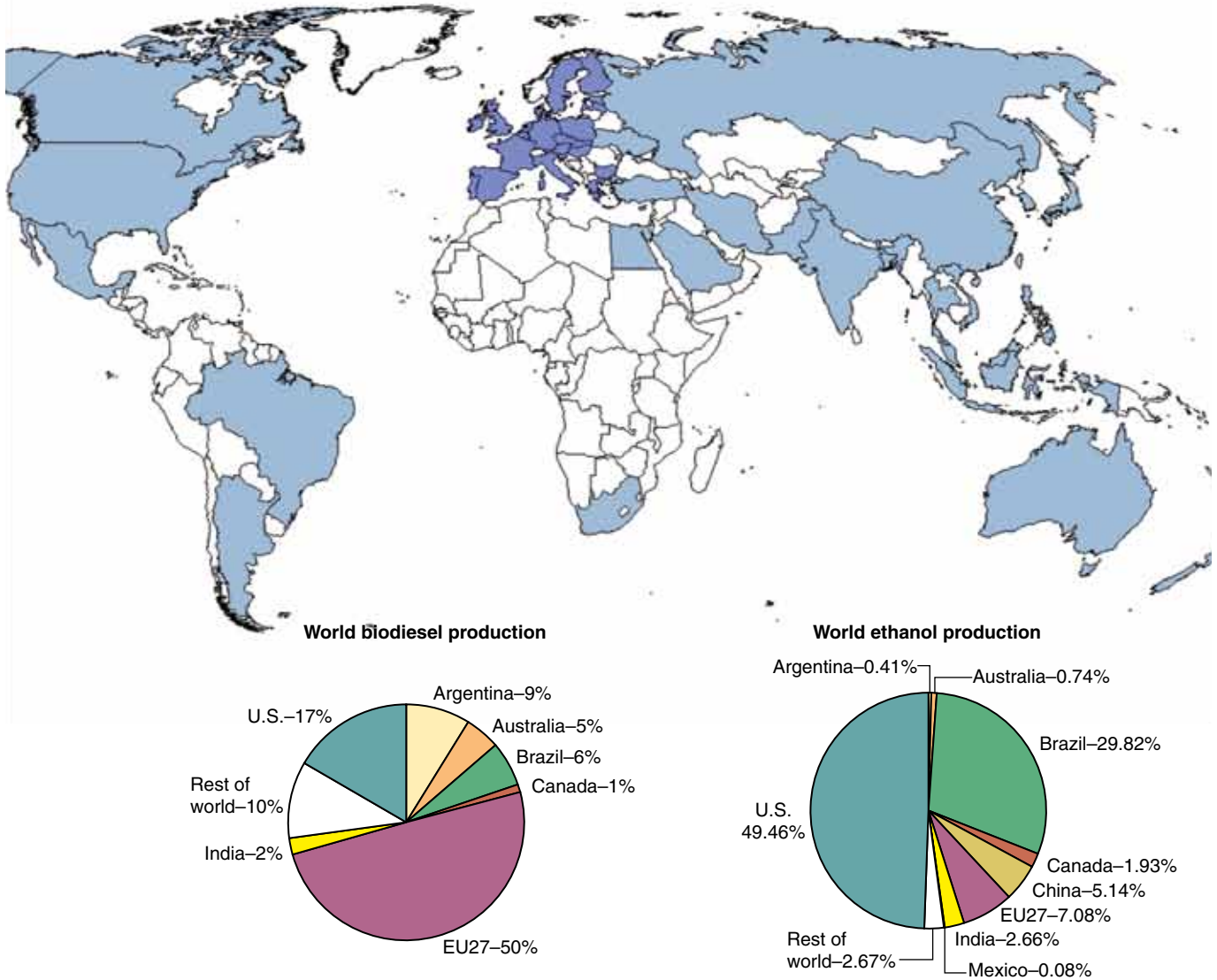
whole dry milk, and other dairy products). In addition, coverage includes three biofuel commodities and byproducts (ethanol, biodiesel, and dried distillers' grains with solubles (DDGs)).

Sectors

The model includes the crop sector, livestock and products sector, and the food processing sector. These sectors are related to each other through production processes, commodity prices, and their use of shared resources, such as land. The model includes four types of production activities: crop production (includes sugarcane and sugar beets), livestock production (includes raw milk), oilseed product production, and dairy product production.

The crop sector comprises grains (rice, wheat, and corn), oilseeds, upland cotton, and sugarcane and sugar beets. "Other coarse grains" are primarily barley, sorghum, millet, and oats. Livestock sector includes beef and veal, pork, poultry, and raw milk. Sugarcane and sugar beet crops are newly introduced to the model, as are the complementarity conditions between sugar feedstocks (sugarcane and sugar beets) and processed/refined sugar. The revised model also includes sugar that is processed/refined, with cane and beet sugar combined into a single commodity.

Supply/Production

Crop production in the model is determined by acreage harvested and yield per harvested acre, while both acreage harvested and yield are determined endogenously. The latest version of the model is dynamic, and in an attempt to capture the relative profitability of farmers, acreage response equations are specified to reflect farmers' perceptions of expected prices and yields. In documenting the Food and Agricultural Policy Simulator (FAPSIM), Gadson et al. (1982) wrote: "Farmers can base future yield perceptions on past or experienced yield levels or alternatively discount abnormal weather conditions in past years and base their expected yield perceptions on yields realized under normal weather conditions" (see also Westhoff et al., 1990). At planting time, acreage harvested and yields are not known. Assuming an adaptive expectations hypothesis of farmers' perceptions, we use lagged or past acreage harvested, yields, and price to explain crop production. Following FAPSIM, the reduced form equations of acreage response and yields are specified to represent farmers' price expectations based on past or experienced yield levels and market prices prior to planting.

Crops

Crop production, $PRD_{i,r,t}$, is defined as the *product* of acreage harvested, $AHV_{i,r,t}$, and yield, $YLD_{i,r,t}$, while producer prices are allowed to adjust.

Area harvested of crop i in country r and year t is a function of area and the crop's own producer price and producer prices of other crops, which may be complementary or competing for acreage as follows:

$$(1) \quad AHV_{i,r,t} = a_{i,r,t|(1)} AHV_{i,r,t-1}^{\lambda_{i,r}} \left[\prod_{i,j}^{n} PPR_{i,j,r,t-1}^{\varepsilon_{ij}} \right]$$

where $\alpha_{i,r,t|(1)}$ is a measure that captures the past interaction between the producer price and crop area,[2] $AHV_{i,r,t-1}$ is lagged area of crop i, $\lambda_{i,r}$ is a partial adjustment parameter, $PPR_{i,r,t-1}$, is own lagged producer price, $PPR_{j,r,t-1}$ is lagged producer prices of other crops j, and $\varepsilon_{i,j}$ is cross price elasticities for crop area.

As explained earlier, the reduced form yield equation is then specified as a function of the *ith* crop's lagged yield and lagged producer price of crop $PPR_{i,r,t-1}$ as follows:

$$(2) \quad YLD_{i,r,t} = a_{i,r,t|(2)} YLD_{i,r,t-1}^{\mu_{i,r}} PPR_{i,r,t-1}^{\eta_{i,r}}$$

[2]This modeling is appropriate when, for example, the production technology is described by a Cobb-Douglas function, such as: $Y(L,K) = AL^b K^{1-b}$ where Y represents total production, L, represents labor, K represents capital, and A represents total factor productivity, which includes effects from technology, efficiency, and scale. Given that A varies over time, we can use it as a variable to help calibrate the long-term projections.

where $\alpha_{i,r,tl(2)}$ is a measure that captures the past interaction between the producer price and yield, $\mu_{i,r}$ is a partial adjustment parameter, $\eta_{i,r}$ is the price elasticity of the yield for crop i, and $PPR_{i,r,t-1}$ is own lagged producer price.

Oilseed products

As stated earlier, the model includes 10 oilseed, oil, and meal products. The "other oilseed oil" aggregate includes canola oil, flax seed oil, and tropical oils (palm oil, olive oil, coconut oil, and other oil).

Production of oilseed products i in country r at time $t(PRD_{i,r,t})$ is determined by the quantity of jth oilseed crushed and by an exogenous extraction rate as follows:

$$(3) \quad PRD_{i,r,t} = \sum_i CRU_{j,r,t} ERT_{i,j,r,t}$$

where $CRU_{j,r,t}$ is the crush of the associated oilseed and $ERT_{i,j,r,t}$ is the extraction rate assuming fixed proportion in the technology of meal and oil production. The determinants of oilseed crushing are discussed later in equation 15.

Livestock products

Production of livestock product i in country at r year t, $PRD_{i,r,t}$, is a function of its own producer price and the producer prices of the other livestock products, a feed cost index for that product, and production of that product in the previous year as:

$$(4) \quad PRD_{i,r,t} = \beta_{i,r,t} \, PRD_{i,r,t-1}^{\lambda_{i,r}} \prod_{j \in livestock}^{n} PPR_{i,j,r,t-1}^{\sigma_{i,j,r}} \, FeCost_{i,r,t}^{\eta_{i,r}}$$

where $\beta_{i,r,t}$ is a measure that captures the past interaction between (1) the producer price and feed costs and (2) production, $PRD_{i,r,t-1}$ is production of ith livestock product the previous year, $\lambda_{i,r}$ is a partial adjustment parameter, $PPR_{i,j,r,t}$ is the producer price of i livestock products (own) and the producer prices of other livestock products j (cross price) in country r, $\sigma_{i,j,r}$ is the price elasticity of production, $FeCost_{i,r,t}$ is the feed cost index for each livestock product i, and $\eta_{i,r}$ is the elasticity of production with respect to input prices.

The feed cost index, $FeCost_{i,r,t,}$ is a function of feed use and feed prices and is specified as:

$$(5) \quad FeCost_{i,r,t} = \sum_{j=feeds} FeedUse_{i,j,r,t} \, PPR_{j,r,t}$$

where $FeedUse_{i,j,r,t}$ is the jth feed used in the production of livestock product i, in country r at time t, and $PPR_{j,r,t}$, is the price of feed j in country r at time t. There are nine commodities in the model that can potentially be used as livestock feed: wheat, corn, other coarse grains, DDGs, and all meals (byproducts of soybeans, sunflower seeds, and rapeseeds).

Dairy products

As mentioned earlier, the model identifies six dairy products. The "other dairy products" aggregate includes ice cream, yogurt, and whey. Dairy products are processed from raw milk, one of the four livestock products in the model. In some regions, production of one or more dairy products is zero or negligible, in which case production is set equal to zero in the model.

Production of dairy products i at year t is modeled as proportional to the total quantity of raw milk processed $PRD_{i,r,t} \big/ PRD_{'processed\,milk',r,t}$ (because all processed dairy products are derived from raw milk) and the price of dairy products.[3] With this specification, a change in the price of one processed dairy product relative to another leads to changes in the mix of processed dairy products made from raw processed milk. The equation is as follows:

$$(6) \quad \frac{PRD_{i,r,t}}{PRD_{'processed\,milk',r,t}} = \beta_{i,r,t} \left[\frac{PRD_{i,r,t-1}}{PRD_{'processed\,milk',r,t-1}} \right]^{\lambda_{i,r}} \left[\prod_{j \in dairyproducts} PPR_{j,r,t}^{\sigma_{i,j,r}} \right]$$

where $PRD_{i,r,t}$ is production of the *ith* dairy product in country r at time t, $PRD_{'processed\,milk',r}$ is total production of raw processed milk, $PRD_{i,r,t} \big/ PRD_{'processed\,milk',r,t}$ is the proportionality, which indicates that the production of the *ith* dairy product varies in direct proportion with the total production of raw processed milk, $PRD_{i,r,t-1} \big/ PRD_{'processed\,milk',r,t-1}$ is the proportionality lagged one year, $PPR_{j,r,t}$ is the producer price of dairy product j, $\beta_{i,r,t}$ is a technology parameter that determines the production of dairy products over time, $\lambda_{i,r}$ represents the rate of adjustment, and $\sigma_{i,j,r}$ is the own and cross-price elasticity of supply for dairy products.

Biofuels

Production of biofuels, $PRD_{i,r,t}$, is expressed as the summation of biofuels from the various feedstocks, $BioPRD_{i,j,r,t}$, and production of biofuels from nonfeedstock, $OthPRD_{i,j,r,t}$, as follows:

$$(7) \quad PRD_{i,r,t} = \sum_{j} BioPRD_{i,j,r,t} + OthBioPRD_{i,j,r,t}$$

where i indicates the type of biofuels (*ethanol, biodiesel*), j indicates feedstocks, r indicates country, t indicates time, and $OthBioPRD_{i,j,r,t}$ denotes "other" agricultural and nonagricultural related sources, such as corn residues (stover), fermentation of molasses, wheat byproducts (i.e., Australia), or forest byproducts. Note that $OthBioPRD_{i,j,r,t}$ depicts biofuel production from other sources, such as corn stover, wheat straw, and sugarcane residue, driven by *data availability* in the database and is expressed as the summation of biofuels from other resources.

[3]The supply of dairy products is modeled as proportional to the total quantity of raw milk (proportionality) and the previous year's proportionality. Although dairy products and raw milk are perishable, the supply of dairy products *at the first stage of processing* (i.e., milk processing plants) can be subject to the longrun equilibrium specification.

The production of biofuels, both ethanol and biodiesel, from *all* categories of feedstocks, $BioPRD_{i,j,r,t}$, is an intermediate production process and depicts the interindustry technological relation of biofuels and feedstocks in each country in the model. This production process captures the technology balance of converting feedstocks to biofuels. The process is characterized by each country's technological advances depending on the type and availability of alternative feedstocks and can be represented as follows:

(8) $\quad BioPRD_{i,j,r,t} = Conv_{j,r,t|i=biofuels} \ Fue_{j,r,t|i=biofuels}$

where i indicates type of biofuel (*ethanol, biodiesel*), j indicates feedstocks, r indicates country, t indicates time, $Fue_{j,r,t|i=biofuels}$ indicates alternative feedstocks for biofuels, and $Conv_{j,r,t|i=biofuels}$ is the technical coefficient for converting feedstocks to biofuels. In other words, the process captures the technological interindustry relation of converting and producing biofuels from alternative feedstocks in each country in the model.

The main byproducts of corn ethanol, for example, in the case of the U.S. dry-corn milling industry, are DDGs. DDGs are a substitute for corn (as feed) and are used as a protein source for animals in feedlots, particularly finishing cattle and poultry. The production of DDGs, or $PDR_{'DDGs', r,t}$, in the corn ethanol process is proportional to a technical coefficient that captures the technological relation of corn-ethanol production and DDGs. This complementarity relationship between primary product (corn ethanol) and byproduct is linear and can be stated as follows:

(9) $\quad PRD_{'DDGs',r,t} = ddgsConv_{corn',r,t} \ Fue_{corn',r,t}$

where r indicates country, t indicates time, $ddgsConv$ is the technical coefficient that depicts the conversion of corn feedstocks to DDGs in the production of DDGs, and $Fue_{corn',r,t}$ is corn feedstocks in the production of corn ethanol.

Sugar

Sugar production, $PRD_{sugar',r,t}$, from sugarcane and sugar beet feedstocks is defined as follows:

(10) $\quad PRD_{sugar',r,t} = \sum_i sugCon_{i,r,t|i=sugar} \ Foo_{i,r,t}$

where i indicates the type of sugar-producing crop, r indicates country, t indicates time, and $sugCon_{i,r,t\backslash i=sugar}$ denotes a technology-related concept in the sugar-processing sector determining the technological relationship between processed sugar (output) and sugarcane and sugar beets (inputs) in each country in the model. Finally, $Foo_{i,r,t}$ denotes sugarcane and sugar beets used (or demanded) by the refined sugar processing sector (an intermediate processing sector), where i refers to sugarcane and sugar beets for refined sugar-processing products.

Demand

The model includes various types of consumption activities, such as food, feed, and fuel demand. The model is specified to maintain consistency between consumption and its subcomponents.

Food demand

Food demand exists for all commodities in the model except raw milk and the three oilseed meals. Food demand is specified as per capita and aggregate. Per capita food demand $PcFOO_{i,r,t}$ for commodity i in country r at time t is a function of consumer price, $PCN_{i,r,t}$, and per capita income in real terms, $PcRGDP_{r,t}$, as follows:

$$(11) \quad PcFOO_{i,r,t} = \beta_{i,r,t} \prod_i PCN_{i,r,t}^{\sigma_{i,j,r}} \ PcRGDP_{r,t}^{\varepsilon_{i,r}}$$

where $\beta_{i,r,t}$ is a measure that captures the interaction between the consumer price, the per capita demand, and per capita income; $\sigma_{i,j,r}$ is the own- and cross-price elasticity of demand; and $\varepsilon_{i,r}$ denotes the income elasticity of food demand for commodity i in country r at time t.

Aggregate food demand for all commodities, $FOO_{i,r,t}$, is specified as a function of per capita commodity demand and population:

$$(12) \quad FOO_{i,r,t} = PcFOO_{i,r,t} \ POP_{r,t}$$

where $PcFOO_{i,r,t}$ is per capita food demand and $POP_{r,j}$ denotes population in country r at time t.

Feed demand

Each region in the model has four feed demand equations, one for each of its four livestock products. As noted earlier, there are seven commodities in the model that can be used as livestock feed: wheat, corn, other coarse grains, soybean meal, sunflowerseed meal, rapeseed meal, and DDGs. In some regions, the use of a particular feed for one or more livestock products is zero or negligible, in which case feed demand is set to zero in the model.

Feed demand, $FEES_{i,k,r,t}$, is specified as a function of livestock production ($PRD_{k,r,t}$), feed conversion ratios ($FR_{i,k,r,t}$), and feed prices ($PFE_{i,r,t}$) as follows:

$$(13) \quad FEES_{i,k,r,t} = \beta_{i,k,r,t} \ PRD_{k,r,t} \ FR_{i,k,r,t} \prod_{i \in feed} PFE_{i,r,t}^{\sigma_{i,j,k,t}}$$

where i depicts feed category, k depicts livestock/meat category, r indicates country, t indicates time, $\beta_{i,k,r,t}$ is a measure that captures the interaction between the feed price and feed demand, $PRD_{k,r,t}$ depicts production of livestock/meat, and $FR_{i,k,r,t}$ indicates feed used by livestock/meat category. Feed prices are depicted as $PFE_{i,r,t}$, while $\sigma_{i,j,k,r}$ is the own feed price elasticity of demand for $i=j$ and the cross-price elasticity of feed demand for $i \neq j$ for meat and milk k.

Aggregate feed demand across all livestock, $FEE_{i,r,t}$, is specified as the summation of feed demand by each livestock type k as follows:

$$(14) \quad FEE_{i,r,t} = \sum_k FEES_{i,k,r,t}$$

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Crush

Oilseed crush, $CRU_{i,r,t}$, is specified as a function of lagged crush, $CRU_{i,r,t\text{-}1}$, and crush margins, $MGN_{i,r,t}$, as follows:

$$(15) \quad CRU_{i,r,t} = \beta_{i,r,t}\, CRU_{i,r,t-1}^{\lambda_{i,r}} \prod_{j\in\,oilseeds} MGN_{i,r,t}^{\sigma_{i,j,r}}$$

where i depicts oilseed category, r depicts country, t indicates time, and $\beta_{i,r,t}$ is a measure that captures the past interaction between the crush margin and crushing. In this specification, the demand for crush increases with increases in the processing/crushing margins and vice versa. In other words, as the crush margin increases, there will be a greater demand for oilseeds for crushing, resulting in a gradual rise in oilseeds prices. Conversely, when the margin starts falling, one can expect weaker demand for oilseeds and falling oilseeds prices, $\sigma_{i,j,r}$ is the crush elasticity with respect to own price (i) and cross price (j) of oilseeds in country r, and $\lambda_{i,r}$ is a partial adjustment parameter.

The crushing margin, $MGN_{i,r,t}$, is specified as a function of the extraction rate of crush products, the prices of crush products (meal and oil), and the consumer prices for oilseeds:

$$(16) \quad MGN_{i,r,t} = \frac{\sum_{j} ERT_{i,j,r,t}\; PPR_{j,r,t}}{PCN_{i,r,t}}$$

where i indicates oilseed category, such as soybeans, rapeseeds, and sunflower seeds, j indicates crush products (meal and oil), r indicates country, t indicates time, $ERT_{i,j,r,t}$ is the extraction rate of oilseed crush (meal and oil), $PPR_{j,r,t}$ is the price of crush products, and $PCN_{i,r,t}$ is the consumer price of oilseeds.

Raw milk processing

Raw milk processing demand, $CON_{i,r,t}$, for j dairy products, such as fluid milk, cheese, butter, nonfat dry milk, whole dry milk, and other dairy products, is specified as a function of lagged raw milk demand, $CON_{i,r,t\text{-}1}$, and the ratio of the value of the processed dairy product to the value of raw milk used (needed) in processing:

$$(17) \quad CON_{i,r,t} = \beta_{i,r,t}\, CON_{i,r,t-1}^{\lambda_{i,r}} \left[\sum_{j} PRD_{j,r,t}\; PPR_{j,r,t} \right]^{\sigma_{i,r}}$$

where i indicates raw milk, j indicates dairy processed products, r indicates country, t depicts time, $\beta_{i,r,t}$ is a measure that captures the past interaction between processed dairy products and the demand for raw milk to be processed (as projected in the AGLINK-COSIMO (OECD and FAO model/ baseline). $PPR_{j,r,t}$ is the producer price of processed dairy products (j = fluid milk, cheese, butter, nonfat dry milk, etc.), $PCN_{i,r,t}$ is the price of raw milk, $\lambda_{i,r}$ is the partial adjustment parameter, and $\sigma_{i,r}$ is the price elasticity of demand for raw milk in the processing of dairy products. In this specification,

$PRD_{j,r,t}$ depicts (over time t) demand of dairy products, while $CON_{i,r,t}$ depicts demand of raw milk for processing.

Biofuels-Ethanol

The demand for ethanol $FUE_{'ethanol',r,t}$ is specified as a function of ethanol price, PCN, the price of gasoline, $pGAS$, and real income, $RGDP$, as follows:

$$(18) \quad FUE_{'ethanol',r,t} = a_{'ethanol',r,t} \ PCN^{\sigma_r}_{'ethanol',r,t} \ pGAS^{\varepsilon_r}_{'gasoline',r,t,} \ RGDP^{\phi_r}_{r,t}$$

where r denotes country at time t, $\alpha_{'ethanol',r,t}$ is a measure that captures the interaction between the ethanol price and ethanol demand, σ_r is the ethanol price elasticity of demand, ε_r is the gasoline price elasticity of demand for ethanol, and ϕ_r is the income elasticity of demand for ethanol.

Biofuels-Biodiesel

The demand for biodiesel, $FUE_{'biodiesel',r,t}$, is specified as a function of the biodiesel price, PCN, the price of diesel, $pDSL$, and real income, $RGDP$, as follows:

$$(19) \quad FUE_{'biodiesel',r,t} = a_{'biodiesel',r,t} \ PCN^{\rho_r}_{'biodiesel',r,t} \ pDSL^{e_r}_{'diesel',r,t,} \ RGDP^{\eta_r}_{r,t}$$

where r denotes country, t indicates time, $\alpha_{'biodiesel',r,t}$ is a measure that captures the interaction between the biodiesel price and biodiesel demand, ρ_r is the biodiesel price elasticity of demand, e_r is the diesel price elasticity of demand for biodiesel, and η_r is the income elasticity of demand for biodiesel.

Biofuels-Feedstock

The demand for feedstocks, $Fue_{j,r,t|biofuels}$, is an intermediate production activity and thus depends on the prices of feedstocks (upstream inputs) and biofuels (downstream outputs). The allocation (demand) of feedstocks for alternative biofuels, $Fue_{j,r,t|biofuels}$, is specified as a function of the price of biofuels and the price of feedstock as follows:

$$(20) \quad Fue_{j,r,t|biofuels} = \alpha_{j,r,t|biofuels} \ (\sum_{i=biofuels} PPR^{\varepsilon_{i,j,r}}_{i,r,t} \ ) \ PCN^{e_{j,r}}_{j,r,t}$$

where i denotes biofuels (ethanol and biodiesel), j denotes feedstocks (such as corn, soybean oil, sugarcane, sugar beets, and rapeseed oil), r denotes country, t denotes time, $\alpha_{j,r,t|biofuels}$ is a measure that captures the interaction between biofuel prices and the demand for feedstock for alternative fuels, $PPR_{i,r,t}$ denotes the producer price of biofuels i, $\varepsilon_{i,j,r}$ is the price elasticity of demand for feedstocks in production of biofuel type/category, $PCN_{j,r,t}$ is the consumer price of feedstocks j, and $e_{j,r}$ represents the feedstock price elasticity of demand for biofuels.

Demand for sugarcane and sugar beets

The demand for sugarcane and/or sugar beets, $FOO_{i,r,t\setminus i=sugarcane/sugar\text{-}beets}$, is defined as follows:

$$(21) \quad FOO_{i,r,t|i=sugarcane/sugar-beets} = \alpha_{i,r,t} \; PPR_{'sugar',r,t}^{\varphi_{i,r}} \; PCN_{i,r,t}^{\theta_{i,r}}$$

where i denotes sugarcane/sugar beets, $\alpha_{i,r,t}$ is a measure that captures the interaction between (1) the prices of both sugar feedstock and refined sugar and (2) the demand for sugarcane or sugar beets in country r and year t, $PPR_{'sugar',r,t}$ is the producer price of refined sugar, $\phi_{i,r}$ is the price elasticity of demand for sugarcane/sugar beets for the production of refined sugar, PCN is the consumer price of sugarcane/sugar beets, and $\theta_{i,r}$ represents the price elasticity of demand for sugarcane/sugar beets in country r at time t.

Other demand

Other use demand is generally small, and it is assumed to change over time by the same proportion as the change in the sum of food, fuel, feed, and crush, as follows:

$$(22) \quad \frac{Oth_{i,r,t}}{Oth_{i,r,'baseline'}} = \frac{FUE_{i,r,t} + FOO_{i,r,t} + FEE_{i,r,t} + CRU_{i,r,t}}{FUE_{i,r,'baseline'} + FOO_{i,r,'baseline'} + FEE_{i,r,'baseline'} + CRU_{i,r,'baseline'}}$$

Total

Total demand of all commodities i in country r at time t except raw milk is specified as the summation of the components food, fuel, feed, crush, and other, as follows:

$$(23) \quad CON_{i,r,t} = FEE_{i,r,t} + FOO_{i,r,t} + FUE_{i,r,t} + CRU_{i,r,t} + OTH_{i,r,t}$$

Stocks

Ending stocks for crops $EST_{i,r,t}$ are specified as a function of ending stocks in the previous period and the ratio of commodity world reference prices over lagged world reference prices for crops:

$$(24) \quad EST_{i,r,t} = \alpha_{i,r,t} \; EST_{i,r,t-1} \left[\frac{PRFC_{i,r,t}}{PRFC_{i,r,t-1}} \right]^{\varepsilon_{i,r,t}}$$

where $\alpha_{i,r,t}$ is a measure that captures that past interaction between world references prices and ending stocks for crops, i indicates commodity, r indicates country, t indicates time, $PRFC$ denotes transmitted world price of a commodity i in country r at time t, and $\varepsilon_{i,r,t}$ is the ending stocks price elasticity of demand. The proportionality of stocks at time "t" over stocks at time "t-1" ($EST_{i,r,t} / EST_{i,r,t-1}$) for a commodity depends on the proportionality of the transmitted world price of commodity i in country r at time "t" over price at time "t-1" ($PEFC_{i,r,t} / PRFC_{i,r,t-1}$). If this were not the case, the

demand for commercial stocks would be a function of the good's expected price, that is, the demand for stocks would be speculative (Meilke, 1999; Gadson et al., 1982).

Prices

Domestic prices are endogenously determined in the model. World prices are in U.S. dollars, and all domestic prices and policies are expressed in local currency. Real exchange rates are treated as exogenous. In each region, domestic prices for all traded commodities (except raw milk, fluid milk, and other dairy products) depend on world prices, exchange rates, transportation costs, and country-specific policies that affect prices.

World prices

$PRF_{i,t}$ depicts the *world reference price* of commodity i, while $PRFC_{i,r,t}$ depicts the *transmitted* world price of commodity i to country r at time t. The *transmitted world price* to country r is defined as a function of the world reference price, $PRF_{i,t}$, (see equation 40) expressed in U.S. dollars as follows:

(25) $$PRFC_{i,r,t} = TRANSM_{i,r,t} PRF_{i,t} REXR_{r,t}$$

where $TRANSM_{i,r,t}$ represents a price transmission mechanism used in the model to capture the effects of world reference price on a country's price, and $REXR_{r,t}$ is a real exchange rate for a tradable commodity i in country r at time t. Note that the world reference price is indexed over commodity i and time t while the transmitted world price commodity to a country is indexed over commodity i in country r at time t. Real exchange rates are exogenous in the model, while the transmission parameters are assigned or have the value of 1 for all tradable commodities.[4]

Domestic prices

For most region/commodity pairs, the domestic price $PDOM_{i,r,t}$ is defined as a weighted average of export prices, $PEX_{i,r,t}$, and import prices, $PIM_{i,r,t}$. The model assumes homogenous products, such that one domestic price is specified as:

(26) $$PDOM_{i,r,t} = \theta_{i,r,t} PEX_{i,r,t} + (1 - \theta_{i,r,t}) PIM_{i,r,t}$$

where $\theta_{i,r,t}$ is the weight ($0 \leq \theta_{i,r,t} \leq 1$)and is equal to the baseline-years exports divided by the sum of baseline exports and baseline imports,

$$EXP_{i,r,t} \Big/ (EXP_{i,r,t} + IMP_{i,r,t})$$

Producer prices

Producer prices, $PPR_{i,r,t}$, are specified as a function of the domestic prices adjusted by producer subsidies or taxes for commodity i in country r at time t. Producer subsidies in the model can be either exogenous or endogenous

[4]Because of either policies (such as domestic price support or consumer subsidies) or weak market infrastructure, the transmission of changes in agricultural border prices to domestic prices within countries might be incomplete (less than one). This could especially be the case for many developing countries. Given that country coverage was expanded in this version of the model, particularly to include more emerging market and developing economies, future efforts will be devoted to determining more accurate price transmission parameters.

(e.g., subsidies that vary depending on the domestic price or other variables in the model):

(27) $\quad PPR_{i,r,t} = PDOM_{i,r,t} + TW_{i,r,t}$

where $TW_{i,r,t}$ represents variable production subsidies (if any) associated with target price policies.

Consumer prices

Consumer prices, $PCN_{i,r,t}$, for commodity i in country r at time t are specified as a function of domestic prices adjusted by consumer subsidies (by subtracting) or taxes (by adding) as follows:

(28) $\quad PCN_{i,r,t} = PDOM_{i,r,t} \, (1.0 \pm TC_{i,r,t})$

where $TC_{i,r,t}$ represents consumer subsidies/taxes. Consumer subsidies/taxes can be either exogenous or endogenous (i.e., subsidies/taxes that vary depending on other variables in the model).

Feed prices

Feed prices for feed category i in country r at time t, $PFE_{i,r,t}$, are specified as a function of consumer prices (used as a proxy for the feed price at the retail level) adjusted for any difference between country/region consumer prices and the transmitted world price to the country r:

(29) $\quad PFE_{i,r,t} = PCN_{i,r,t} + FeedPRFAC_{i,r,t} \, (PRFC_{i,r,t} - PCN_{i,r,t})$

where the feed price, $FeedPRFAC_{i,r,t}$, captures the difference (if any) between domestic consumer prices and world prices. Feed prices within the model can differ from domestic consumer prices by any endogenous or exogenous amount, added or subtracted, on an ad valorem or specific basis. However, as of this time, no policies have been introduced into the model for any countries that could cause feed prices to deviate from domestic consumer prices. Consequently, $FeedPRFAC_{i,r,t}$ is set equal to *zero* due to lack of data availability, except for wheat and other coarse grains in Japan.

Import prices

Import prices, $PIM_{i,r,t}$, are specified as a function of world prices adjusted for an ad valorem tariff (i.e., first tier, or in-quota rate, $Tm1_{i,r,t}$, and second tier, or over-quota rate, $Tm2_{i,r,t}$) and transportation costs, $Trans_{i,r,t}$, as follows:

(30) $\quad PIM_{i,r,t} = PRFC_{i,r,t} \, (1.0 + Tm1_{i,r,t} + Z_{i,r,t} * Tm2_{i,r,t}) + Trans_{i,r,t}$

where $PRFC_{i,r,t}$ denotes the transmitted world price to country r of commodity i at time t while $Z_{i,r,t}$ relates to the TRQ. $Z_{i,r,t}$ is bounded with values ranging from 0 to 1 [*0,1*] and solves endogenously for the level where the quota operates. If the quota is not binding, it takes the value of 0;

otherwise, it takes the value of 1 (see also equation 33). Instead of "tariffica-tion" of TRQs or linear approximations, TRQs are specified as functions and are solved explicitly in the model, taking account of the discontinuity in the tariff rate using the MCP formulation.[5]

Export prices

Export prices, $PEX_{i,r,t}$, are specified as a function of the transmitted world prices to country r, $PRFC_{i,r,t}$, adjusted for export subsidies and/or taxes as follows:

$$(31) \quad PEX_{i,r,t} = (1.0 + \exp Sub_{i,r,t})\, PRFC_{i,r,t}$$

where i denotes commodity, r denotes country, t denotes time, $PRFC_{i,r,t}$ denotes the transmitted world price to country r, and $\exp Sub_{i,r,t}$ denotes ad valorem export subsidies or taxes. Note that the transmitted world price to a country r depends on the world reference price (see equation 25).

Producer price for nontraded commodities

The domestic prices for nontradable commodities (raw milk, fluid milk, other dairy products, sugarcane, and sugar beets) are determined by domestic supply-demand equilibria, or the material balance equation holds:

$$(32) \quad PRD_{i,r,t} + EST_{i,r,t-1} - CON_{i,r,t} - EST_{i,r,t} = 0$$

where i denotes nontraded commodity, and r denotes country at time t.

Policies

The core set of policies includes both specific and ad valorem import and export taxes/subsidies, TRQs, and producer and consumer subsidies. In addi-tion, the model includes other policies that constitute important aspects of agricultural policy in particular countries.

In particular, for the United States, the model includes loan rates with marketing loan benefits for crops and export subsidies for dairy products. For the EU, the model includes export subsidies and production quotas for raw milk and sugar. For Canada, the model includes production quotas for milk that target producer prices.

Tariff-rate quotas

A TRQ is a policy in which imports are subject to one (low) tariff below a specified quantity of imports and a second higher tariff on imports in excess of this limit. To model a TRQ directly, we use complementarity to capture this switching from one regime to another.

There are two issues in modeling TRQs: how to model a switch from one regime to another and how to handle the boundary case where imports are exactly at the quota limit. In the latter case, we have a range of possible supply

[5]The MCP formulation allows in the absence of TRQs the actual applied tariff rates to be used. The model uses actual applied tariff rates rather than World Trade Organization bound rates whenever such data are available. For most of the agricultural products imported by the United States, Canada, and Mexico, tariffs under the North American Free Trade Agreement (NAFTA) are more important than Most Favored Nation tariffs because more agricultural imports are from within NAFTA than outside NAFTA.

prices whose bounds are determined by the below- and above-quota tariffs. The model determines a supply price in this range such that markets clear.

Our TRQ policy for an imported commodity i in country r at time t is defined by a lower (tier 1) tariff, $Tm1_{i,r,t}$, an upper (tier 2) tariff, $Tm2_{i,r,t}$, a TRQ quota limit, $Tarqta_{i,r,t}$, and the import quantity $q_{i,r,t}$ (aka $IMP_{i,r,t}$). Note that the over-quota tariff is the sum $Tm1_{i,r,t} + Tm2_{i,r,t}$, and both $Tm1_{i,r,t}$ and $Tm2_{i,r,t}$ are positive—if not, we have one regime (i.e., a normal tariff). If we introduce a variable $Z_{i,r,t}$ in [0,1] complementarity to a function:

(33) $F_{i,r,t}(\bullet) := Tarqta_{i,r,t} - q_{i,r,t}$ $\perp$ $0 \le Z_{i,r,t} \le 1$

then by definition (see appendix), below-quota imports imply that $Z_{i,r,t} = 0$. To see this, observe that below-quota imports imply that $F_{i,r,t} > 0$, so by definition, $Z_{i,r,t}$ must be at its lower bound. Similarly, above-quota imports imply that $Z_{i,r,t} = 1$. At-quota imports imply that $F_{i,r,t} = 0$, so that $Z_{i,r,t}$ is allowed to float between 0 and 1. Note that in this specification, $F_{i,r,t}$ is perpendicular to $Z_{i,r,t}$ even though $F_{i,r,t}$ does not depend on $Z_{i,r,t}$.

Given this behavior for $Z_{i,r,t}$, we can write the function determining the supply price for this import as in equation 30 or:

$$PIM_{i,r,t} = PFRC_{i,r,t} (1.0 + Tm1_{i,r,t} + Z_{i,r,t} * Tm2_{i,r,t}) + Trans_{i,r,t}$$

where we would have used only Tm (or tariff rate) for a "simple" tariff case. With this, we get the (tier 1) tariff $Tm1_{i,r,t}$ below the quota, the (tier 2) tariff $Tm1_{i,r,t} + Tm2_{i,r,t}$ above the quota, and the possibility to choose any value in between these two when we are at the quota. The at-quota behavior allows the model to choose $Z_{i,r,t}$ in [0,1] so that the supply price is equal to the demand price at equilibrium.

Variable production subsidies

In the presence of variable production subsidies for certain agricultural commodities, producers receive government payments when the domestic price falls below a certain level known as the target price. The payment amount is equal to some fixed percentage of the price shortfall. We assume that the payment is zero when the domestic price is at or above the target price. To implement these subsidies, we introduce a price wedge $TW_{i,r,t}$ (variable production subsidy) equal to the maximum of and a percentage of the price shortfall. To compute, we utilize the complementarity framework as follows:

(34) $TW_{i,r,t} \ge PtargetFac_{i,r,t}\left(Ptarget_{i,r,t} - PDOM_{i,r,t}\right)$ $\perp$ $TW_{i,r,t} \ge 0$

where $Ptarget_{i,r,t}$ is the target price, $PtargetFac_{i,r,t}$ is the percentage, and $PDOM_{i,r,t}$ is the domestic price for a commodity i in country r at time t. Complementarity implies that $TW_{i,r,t}$ takes on the desired maximum (*max*) value:

(35) $TW_{i,r,t} = \max\left[0, PtargetFac_{i,r,t}\left(Ptarget_{i,r,t} - PDOM_{i,r,t}\right)\right]$

Production quotas

The model incorporates production quotas for policies that limit production by placing explicit upper bounds on the quantity produced. When such a quota for a commodity is binding, the producer price for that commodity in the production equation (called the area harvested equation in the case of crops) is adjusted endogenously by adjustment factor ($ShadSlk_{i,r,t}$).[6]

Complementarity allows this endogenous adjustment factor $ShadSlk_{i,r,t}$ to be nonzero only if production is at the quota level for a commodity i in country r at time t. To implement this, we replace the producer price $PPR_{i,r,t}$ in production equations with the adjusted price and set:

(36) $ProdLim_{i,r,t} \geq PRD_{i,r,t} \quad \perp \quad ShadSlk_{i,r,t} \geq 0$

Price supports

In the absence of price supports, we have at equilibrium that the producer price $PPR_{i,r,t}$ is equal to the domestic price $PDOM_{i,r,t}$. Note that the consumer price in the absence of policy-distorted taxes and or subsidies is equal to the domestic price. If the government has a price support policy (i.e., it pays producers the shortfall between the producer price and the announced support price), then the equality will not hold when the price drops below the support price. In this case, the producer sets output at a level consistent with the higher support price and the consumer sets demand/consumption consistent with the lower consumer price to clear the market. This happens when we set a lower bound on the producer price and equate producer and consumer prices as follows:

(37) $PPR_{i,r,t} \geq PDOM_{i,r,t} \quad \perp \quad PPR_{i,r,t} \geq PPR\_Support_{i,r,t}$

Trade and Model Closure

Global net trade in each commodity must be zero for international markets to clear. The model is nonspatial in the sense that a region's imports and exports are not distinguished by their source or destination, respectively. It is a *gross trade model* that accounts for total exports and total imports of each commodity in every region. This is accomplished in most cases by distinguishing gross exports and gross imports as follows: the smaller of the two (exports or imports) in a region is governed by a behavior equation that is consistent with historical trade, while the larger of two (exports or imports) adjusts to clear global agricultural markets. For the nontraded commodities, supply and demand in each region must be equal. The revamped model was extended and includes a separate module to handle bilateral trade flows.

Internationally traded commodities

The model balances supply and demand for each tradable commodity i in region r at time t as follows:

(38) $NET_{i,r,t} = PRD_{i,r,t} + EST_{i,r,t-1} - CON_{i,r,t} - EST_{i,r,t}$

[6]In the previous (static) version of the model (ERS/Penn State Trade model), when a quota for a commodity was binding, the producer price for that commodity in the production equation (called the area harvested equation in the case of crops) was replaced by an endogenous shadow price that was assumed to be equal to marginal cost. In that specification, the shadow price was equal to the producer price when the quota was not binding. In the latest version of PEATSim, when a quota for a commodity is binding, the producer price for that commodity in the production equation (called the area harvested equation in the case of crops) is adjusted endogenously by an adjustment factor ($ShadSlk_{i,r,t}$). *Complementarity* allows this endogenous adjustment factor to be nonzero only if production is at the quota level. The complementarity condition holds for current production. Note that the adjustment factor is not the shadow price.

where $PRD_{i,r,t}$ is production, $EST_{i,r,t-1}$ is lagged ending stocks (or beginning stocks for time t), $CON_{i,r,t}$ is consumption, and $EST_{i,r,t}$ is ending stocks.

Equilibrium conditions for world markets require that the sum of exports, $EXP_{i,r,t}$, be equal to the sum of imports, $IMP_{i,r,t}$, across regions for tradable commodity i in region r and year t:

$$(39) \quad \sum_r EXP_{i,r,t} = \sum_r IMP_{i,r,t}$$

and under the complementarity condition, the world price, $PFR_{i,r,t}$ is the equilibrating variable in the world balance equation, or

$$(40) \quad \sum_r EXP_{i,r,t} = \sum_r IMP_{i,r,t} \quad \perp \quad PRF_{i,r,t} \geq 0$$

Trade

For any tradable commodity i in region r and time t, one of the export/import pairs is specified as a function of price while the other is left relatively free to allow the market to clear. In the description that follows, we consider the case where export quantities are determined as a function of export price and imports float to clear the market. The case where the roles are reversed is treated similarly.

The complementarity conditions determining the import and export quantities when $\alpha_{i,r,t} PEX_{i,r,t}^{\varepsilon_{i,r}}$ is positive are:

$$(41) \quad EXP_{i,r,t} \geq \alpha_{i,r,t} PEX_{i,r,t}^{\varepsilon_{i,r}} + Neterr_{i,r,t} \quad \perp \quad EXP_{i,r,t} \geq 0$$

$$(42) \quad \alpha_{i,r,t} PEX_{i,r,t}^{\varepsilon_{i,r}} + Neterr_{i,r,t} \geq NET_{i,r,t} \quad \perp \quad Neterr_{i,r,t} \geq 0$$

$$(43) \quad IMP_{i,r,t} \geq EXP_{i,r,t} - NET_{i,r,t} \quad \perp \quad IMP_{i,r,t} \geq 0$$

where $\alpha_{i,r,t}$ represents a technology-related attribute, $PEX_{i,r,t}$ is the price of *export-bounded commodity*, $\varepsilon_{i,r}$ is an export elasticity, and $Neterr_{i,r,t}$ is an endogenous variable used as a correction factor to ensure that $EXP_{i,r,t} \geq NET_{i,r,t}$. When $\alpha_{i,r,t} PEX_{i,r,t}^{\varepsilon_{i,r}}$ is not positive, we use these conditions:

$$(44) \quad EXP_{i,r,t} \geq NET_{i,r,t} \quad \perp \quad EXP_{i,r,t} \geq 0$$

$$(45) \quad IMP_{i,r,t} \geq EXP_{i,r,t} - NET_{i,r,t} \quad \perp \quad IMP_{i,r,t} \geq 0$$

Import quantity, in this case, is the *equilibrating* variable.

Nontraded Commodities

For commodity i in region r at time t that is not traded internationally, domestic supply must be equal to domestic demand (i.e., $NET_{i,j,t}$ must be zero):

$$(46) \quad NET_{i,j,t} = 0, for\ i,j \in nontraded\ commodities$$

Data Sources and Calibration

Currently, the model utilizes the domestic USDA baseline, the Country-Commodity Linked System, and the AGLINK-COSIMO model of OECD and FAO. The Linked System joins ERS foreign country models and is used to perform the first rounds of projections for the international baseline. PEATSim uses the AGLINK-COSIMO database for all dairy products, sugar, sugarcane and sugar beets, and biofuels. The data in PEATSim calibrate to *USDA Agricultural Projections to 2019* and 2019 AGLINK-COSIMO (OECD-FAO, 2010) projections.

We develop a suite of programs in GAMS for accessing, calibrating, and performing consistency tests: (1) the OECD AGLINK baseline database, and (2) the USDA Baseline with the Linked System's foreign country models in conjunction with the AGLINK databases. This involves reading the databases from various formats and platforms (i.e., Microsoft Excel and Microsoft Access) into GAMS and, using an Information-Theoretic estimation procedure (cross entropy), we estimate a new set of information (or data in a general context) close to the prior (or available data). In other words, the objective of this approach, which aims to use all available information, is to minimize the entropy between the probabilities that are consistent with the information in the data and the priors (see Judge et al., 1985; Golan et al., 1996; and Golan, 2002). In PEATSim, we ensure the basic *identities* of supply equal use (*material balanced*):

Σ(*Production, Imports, Beginning Stocks*)= Σ(*Consumption, Exports, Beginning Stocks*)

and similarly,

Consumption=Σ(of sub-consumption component)=Σ(Food, Feed, Fuel, Crush, Other)

This way, we ensure the base data clear the markets and, hence, the model's price solutions are consistent with market-clearing conditions. We develop an approach in GAMS for calibrating the rest of the world as a region along with the specific countries and the world, while maintaining the integrity of the U.S. baseline, by the use of maximum-entropy econometrics (though without using a penalty approach). Maximum entropy econometrics provides a procedure for economic and statistical models that may be nonregular in the sense that they are ill-posed or underdetermined and the data are partial or incomplete (Golan et al., 1996; and Golan, 2002). By using the maximum entropy formalisms and techniques used in the physical sciences, we are able to recover information about economic systems such as the USDA Baseline. Since entropy is a measure of uncertainty for a single random variable, we use entropy econometrics as a measure of uniformity (Golan and Gzyl, 2003).

Values for **elasticities** and other parameters in the model are drawn from studies; reviews of the literature; other trade models, such as Abler (2001), Dyck (1988), Hahn (1996), Hertel et al. (1989), Huang (1993), and Regmi (2001); and other models, such as European Simulation Model (ESIM), ERS Baseline Projections Model, FAPSIM, and the IMPACT Model – International Food Policy Research Institute. In the past, certain models'

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elasticities that account/depend on the *base data* were derived in spreadsheets. At present, we developed a process that integrates the derivation of these elasticities in GAMS. This way, the elasticities are revised/updated when the model is calibrated to different base years.

We also are in the process of updating trade policies using the World Integrated Trade Solution (WITS), a software developed by the World Bank in close collaboration with various international organizations, including United Nations Conference on Trade and Development (UNCTAD), International Trade Center (ITC), United Nations Statistical Division (UNSD), and the World Trade Organization. WITS gives access to major international trade, tariff, and nontariff data (http://wits.worldbank.org/wits).

Application

The global biofuels sector is growing rapidly, and the expansion of biofuel production is changing agricultural markets worldwide. The revamped version of PEATSim includes a biofuels-feedstock module that allows an indepth examination of the effects of global biofuels expansion on agricultural production and trade, accounting for links between relevant upstream (inputs) and downstream (outputs) industries. Our empirical application examines the impact of alternative petroleum prices and macroeconomic conditions on the demand for biofuels and their feedstocks on global agricultural markets.

As different countries use different feedstock sources for the production of biofuels, the challenge is to capture and properly model both the demand for biofuels and the supply response specific to each country. A "stylized" representation of biofuels production would fail to capture the complexity and interaction of biofuel/feedstock production. For this reason, it is important that each country's production of biofuels be explicitly represented in the model to consistently account for links between biofuels and feedstock needed/used and their interaction with other sectors. This calls for an innovative way to capture the impacts of biofuels expansion from the producer level all the way to the global agricultural markets and trade. We focus on accessing variability of the simulation under alternative levels of real gross domestic product (GDP) and prices of crude oil in the long run.

The expansion of biofuel production and consumption is not limited to the United States. For example, over the last several decades, production of crop-based biofuels increased in Brazil as it used sugarcane as a feedstock to produce ethanol and then used ethanol on a large scale to fuel vehicles. The EU has used rapeseed oil to produce biodiesel for fuel use in relatively large quantities over the last decade. Government policies are also influencing biofuel industries in Canada, Argentina, China, countries of the former Soviet Union, India, and Indonesia. A number of developed and developing countries have instituted programs to promote biofuel production and consumption and have set targets for increasing the use of biofuels.

Scenarios

We consider alternative scenarios that focus on the variability of ethanol and biodiesel production worldwide under various crude petroleum price levels and macroeconomic conditions in the long run. In particular, we utilize the long-term 30-year projections for the United States from the Information Handling Services (IHS) Global Insights as posted on www.ihsglobalinsight. com. Following Global Insights, we adopt three projections characterized by different long-term outlooks: optimistic, pessimistic, and the "middle" projection, or baseline. According to Global Insights, under the optimistic projection, economic growth proceeds smoothly but more rapidly than under the baseline. Under the pessimistic projection, economic growth proceeds smoothly but more slowly than under the baseline. The assumptions on the long-term optimistic and pessimistic projections form a bandwidth around the baseline. For this reason, the baseline's results are omitted. We adopt the same assumptions of economic growth for all countries in the model.

In the long term, scarcity of energy supplies and/or sources tends to bid energy prices up, while new technologies tend to hold them down. In the end, according to Global Insights projections, these two forces are more likely to balance out, and real crude oil prices are projected to remain flat over the baseline projection, especially over the long-term forecast period. For each of the macroeconomic scenarios, we consider three alternative crude petroleum prices: low ($70/barrel), medium ($90/barrel), and high ($120/barrel). A crude oil price outcome higher than the baseline projection could result from stronger demand growth (perhaps notably in China) and/or weaker supply. Although crude oil prices and growth tend to move together, a crude oil price outcome lower than the baseline projection could result from higher efficiency standards, recession, and/or better supply prospects (http://myinsight. ihsglobalinsight.com/servlet/cats?filterID=876&serviceID=1784&typeID=4 410&pageContent=report).

The macroeconomic conditions and the price of crude oil are exogenous to the model, and the scenario analyses account for demand and supply responses for both upstream and downstream sectors/industries and their interdependence on the global modeling framework. Given the structure of the model, producers and consumers are allowed to respond to these exogenous changes by making adjustments that affect world prices and production of each commodity as well as prices and production for each country/region, but the impacts vary among sectors and countries/regions. In turn, these price and production changes that are generated by links between, for example, producers and consumers or between imports and exports depict new long-term solutions.

We employ the "bootstrap" technique (see Efron and Tibshirani, 1994; Varian, 1996) in assessing the variability of the scenario results generated by exogenous variables in the model. In particular, we draw 5,000 bootstrap samplings of real gross domestic product for each macroeconomic projection and 5,000 bootstrap samplings of crude oil prices for each alternative crude oil petroleum price (high, medium, and low). The model is solved 5 times 5,000 (or 25,000 simulations were performed in all), and estimates of the mean, variability, and confidence intervals of the simulations outcomes are obtained.

Since the purpose of this report is to *illustrate* how PEATSim can be used for scenario analyses, only selective model results are reported. The source of variability also illustrates the purpose, since many other sources of variability, such as yield and exchange rate, are not considered.

Results

Overall, the model simulations show that the price of crude petroleum oil, leaving other factors unchanged, has a larger effect on feedstock use for biofuel production than do long-term macroeconomic projections of the world's economy (tables 1-6 and figures 2-5). Biofuel production from various feedstocks is affected more by alternative crude oil prices than by alternative economic conditions.

Table 1

Global ethanol production (in million metric tons) from various feedstocks, under alternative macroeconomic projections and crude oil prices, 2010-49

		Optimistic projection				Pessimistic projection			
		Mean	Standard deviation	Lower bound	Upper bound	Mean	Standard deviation	Lower bound	Upper bound
		High oil price				High oil price			
EU27	Corn	2.320	0.201	2.260	2.369	2.275	0.190	2.222	2.326
	Wheat	0.681	0.299	0.602	0.751	0.744	0.274	0.676	0.808
	Sugar beets	1.802	0.190	1.754	1.852	1.793	0.193	1.747	1.844
China	Corn	1.375	0.258	1.307	1.440	1.383	0.247	1.329	1.441
Argentina	Sugarcane	0.562	0.118	0.531	0.593	0.531	0.096	0.506	0.555
Brazil	Sugarcane	43.470	7.530	41.334	45.720	40.716	5.879	38.976	42.492
U.S.	Corn	50.732	3.065	49.783	51.477	49.881	2.977	49.048	50.619
Canada	Wheat	0.166	0.070	0.146	0.188	0.184	0.064	0.165	0.200
	Corn	0.705	0.044	0.692	0.718	0.693	0.044	0.678	0.705
India	Sugarcane	0.237	0.050	0.220	0.252	0.224	0.041	0.212	0.236
		Medium oil price				Medium oil price			
EU27	Corn	1.947	0.132	1.914	1.979	1.906	0.115	1.875	1.933
	Wheat	0.685	0.299	0.606	0.764	0.755	0.278	0.682	0.827
	Sugar beets	1.563	0.157	1.521	1.601	1.548	0.163	1.509	1.592
China	Corn	1.160	0.216	1.096	1.206	1.153	0.216	1.095	1.204
Argentina	Sugarcane	0.490	0.102	0.465	0.514	0.460	0.081	0.440	0.480
Brazil	Sugarcane	37.553	6.307	35.634	39.488	35.015	4.725	33.554	36.393
U.S.	Corn	42.814	1.500	42.407	43.186	41.990	1.364	41.611	42.315
Canada	Wheat	0.168	0.070	0.146	0.189	0.185	0.065	0.167	0.206
	Corn	0.595	0.025	0.588	0.603	0.582	0.027	0.574	0.591
India	Sugarcane	0.198	0.041	0.186	0.210	0.186	0.033	0.176	0.196
		Low oil price				Low oil price			
EU27	Corn	1.057	0.117	1.029	1.091	1.016	0.121	0.988	1.055
	Wheat	0.704	0.309	0.626	0.784	0.776	0.286	0.704	0.842
	Sugar beets	0.923	0.171	0.882	0.976	0.901	0.183	0.860	0.949
China	Corn	0.633	0.188	0.589	0.685	0.615	0.194	0.569	0.671
Argentina	Sugarcane	0.294	0.066	0.278	0.311	0.271	0.051	0.259	0.284
Brazil	Sugarcane	22.010	3.730	20.923	23.189	20.235	2.682	19.485	21.085
U.S.	Corn	23.559	2.772	22.888	24.423	22.760	2.775	22.073	23.578
Canada	Wheat	0.172	0.072	0.151	0.191	0.191	0.066	0.170	0.208
	Corn	0.328	0.049	0.315	0.347	0.317	0.054	0.302	0.337
India	Sugarcane	0.107	0.024	0.100	0.114	0.098	0.018	0.094	0.104

Table 2

Global biodiesel production (in million metric tons) from various feedstocks, under alternative macroeconomic projections and crude oil prices, 2010-49

		Optimistic projection				Pessimistic projection			
		Mean	Standard deviation	Lower bound	Upper bound	Mean	Standard deviation	Lower bound	Upper bound
		High oil price				High oil price			
EU27	Rapeseed oil	5.611	0.417	5.492	5.717	5.615	0.409	5.493	5.715
	Other oil	0.426	0.118	0.395	0.456	0.427	0.119	0.397	0.456
	Soybean oil	0.465	0.118	0.437	0.496	0.465	0.118	0.437	0.495
Argentina	Soybean oil	1.909	0.200	1.859	1.963	1.909	0.199	1.859	1.959
Brazil	Soybean oil	1.027	0.234	0.956	1.098	1.027	0.234	0.967	1.096
U.S.	Soybean oil	2.656	0.221	2.588	2.711	2.656	0.223	2.587	2.709
Canada	Rapeseed oil	0.055	0.018	0.049	0.060	0.055	0.018	0.049	0.059
India	Rapeseed oil	0.146	0.025	0.139	0.155	0.146	0.025	0.139	0.153
		Medium oil price				Medium oil price			
EU27	Rapeseed oil	4.461	0.214	4.399	4.512	4.480	0.201	4.423	4.535
	Other oil	0.342	0.095	0.317	0.365	0.357	0.087	0.334	0.379
	Soybean oil	0.390	0.108	0.361	0.414	0.415	0.099	0.389	0.443
Argentina	Soybean oil	1.515	0.158	1.479	1.559	1.598	0.134	1.562	1.631
Brazil	Soybean oil	0.857	0.204	0.794	0.910	0.919	0.188	0.862	0.974
U.S.	Soybean oil	2.110	0.111	2.077	2.136	2.113	0.108	2.076	2.141
Canada	Rapeseed oil	0.047	0.016	0.042	0.052	0.051	0.015	0.047	0.055
India	Rapeseed oil	0.126	0.024	0.119	0.132	0.137	0.018	0.132	0.142
		Low oil price				Low oil price			
EU27	Rapeseed oil	1.746	0.457	1.634	1.901	1.749	0.471	1.635	1.893
	Other oil	0.148	0.072	0.129	0.167	0.152	0.072	0.134	0.171
	Soybean oil	0.188	0.097	0.163	0.214	0.199	0.094	0.176	0.224
Argentina	Soybean oil	0.603	0.208	0.552	0.670	0.632	0.204	0.585	0.693
Brazil	Soybean oil	0.402	0.151	0.357	0.451	0.431	0.142	0.393	0.477
U.S.	Soybean oil	0.849	0.176	0.806	0.909	0.847	0.176	0.804	0.899
Canada	Rapeseed oil	0.024	0.011	0.020	0.027	0.025	0.010	0.023	0.029
India	Rapeseed oil	0.064	0.024	0.058	0.071	0.069	0.021	0.063	0.077

Table 3
Global ethanol production (in million metric tons) from various feedstocks, optimistic projection

		High oil price		Medium oil price		Low oil price	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
		2010-19					
EU27	Corn	2.319	0.209	1.950	0.128	1.058	0.122
	Wheat	0.683	0.300	0.685	0.303	0.705	0.314
	Sugar beets	1.805	0.190	1.561	0.157	0.924	0.184
China	Corn	1.386	0.261	1.157	0.215	0.634	0.195
Argentina	Sugarcane	0.562	0.118	0.493	0.102	0.293	0.065
Brazil	Sugarcane	43.528	7.638	37.448	6.338	22.087	3.771
U.S.	Corn	50.721	3.328	42.828	1.571	23.594	3.042
Canada	Wheat	0.165	0.070	0.168	0.071	0.173	0.073
	Corn	0.705	0.045	0.596	0.025	0.328	0.055
India	Sugarcane	0.238	0.049	0.198	0.042	0.107	0.024
		2020-29					
EU27	Corn	2.323	0.206	1.951	0.127	1.058	0.124
	Wheat	0.690	0.305	0.689	0.302	0.709	0.308
	Sugar beets	1.796	0.191	1.561	0.157	0.928	0.189
China	Corn	1.378	0.261	1.162	0.218	0.632	0.190
Argentina	Sugarcane	0.561	0.118	0.488	0.101	0.294	0.066
Brazil	Sugarcane	43.384	7.586	37.502	6.337	22.040	3.735
U.S.	Corn	50.720	3.345	42.813	1.626	23.639	3.180
Canada	Wheat	0.166	0.070	0.168	0.071	0.173	0.073
	Corn	0.705	0.044	0.595	0.025	0.328	0.055
India	Sugarcane	0.237	0.049	0.199	0.042	0.107	0.024
		2030-39					
EU27	Corn	2.323	0.203	1.949	0.134	1.056	0.123
	Wheat	0.673	0.299	0.685	0.302	0.707	0.309
	Sugar beets	1.800	0.191	1.563	0.157	0.925	0.188
China	Corn	1.375	0.256	1.154	0.214	0.635	0.197
Argentina	Sugarcane	0.563	0.119	0.488	0.100	0.293	0.065
Brazil	Sugarcane	43.426	7.408	37.502	6.225	22.013	3.710
U.S.	Corn	50.734	3.281	42.812	1.612	23.576	3.023
Canada	Wheat	0.166	0.070	0.167	0.071	0.174	0.073
	Corn	0.704	0.046	0.595	0.025	0.327	0.055
India	Sugarcane	0.238	0.049	0.199	0.040	0.107	0.024
		2040-49					
EU27	Corn	2.319	0.207	1.948	0.131	1.057	0.124
	Wheat	0.675	0.300	0.679	0.299	0.706	0.313
	Sugar beets	1.804	0.192	1.563	0.157	0.924	0.186
China	Corn	1.377	0.257	1.153	0.212	0.629	0.193
Argentina	Sugarcane	0.561	0.117	0.490	0.102	0.295	0.065
Brazil	Sugarcane	43.225	7.706	37.564	6.331	22.013	3.765
U.S.	Corn	50.759	3.225	42.807	1.621	23.554	2.995
Canada	Wheat	0.166	0.071	0.169	0.071	0.172	0.073
	Corn	0.706	0.045	0.595	0.025	0.327	0.053
India	Sugarcane	0.237	0.050	0.197	0.042	0.107	0.024

Table 4

Global biodiesel production (in million metric tons) from various feedstocks, optimistic projection

		High oil price		Medium oil price		Low oil price	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
		2010-19					
EU27	Rapeseed oil	5.611	0.425	4.464	0.219	1.747	0.502
	Other oil	0.424	0.118	0.344	0.096	0.148	0.076
	Soybean oil	0.463	0.117	0.390	0.108	0.188	0.099
Argentina	Soybean oil	1.911	0.202	1.516	0.160	0.606	0.229
Brazil	Soybean oil	1.028	0.234	0.855	0.203	0.403	0.157
U.S.	Soybean oil	2.655	0.238	2.108	0.119	0.847	0.187
Canada	Rapeseed oil	0.055	0.019	0.047	0.016	0.024	0.012
India	Rapeseed oil	0.146	0.025	0.125	0.024	0.064	0.025
		2020-29					
EU27	Rapeseed oil	5.611	0.431	4.464	0.212	1.749	0.499
	Other oil	0.429	0.119	0.341	0.094	0.146	0.073
	Soybean oil	0.468	0.119	0.389	0.108	0.188	0.099
Argentina	Soybean oil	1.909	0.203	1.512	0.157	0.597	0.207
Brazil	Soybean oil	1.025	0.234	0.859	0.206	0.404	0.158
U.S.	Soybean oil	2.658	0.227	2.109	0.116	0.851	0.196
Canada	Rapeseed oil	0.055	0.018	0.047	0.016	0.024	0.012
India	Rapeseed oil	0.146	0.025	0.126	0.024	0.064	0.025
		2030-39					
EU27	Rapeseed oil	5.611	0.441	4.459	0.220	1.747	0.499
	Other oil	0.427	0.118	0.341	0.094	0.147	0.075
	Soybean oil	0.466	0.117	0.387	0.109	0.191	0.102
Argentina	Soybean oil	1.908	0.198	1.517	0.160	0.605	0.224
Brazil	Soybean oil	1.027	0.234	0.857	0.205	0.400	0.152
U.S.	Soybean oil	2.656	0.236	2.111	0.113	0.851	0.195
Canada	Rapeseed oil	0.055	0.018	0.047	0.016	0.023	0.011
India	Rapeseed oil	0.146	0.025	0.126	0.024	0.064	0.026
		2040-49					
EU27	Rapeseed oil	5.609	0.437	4.457	0.221	1.738	0.483
	Other oil	0.425	0.119	0.342	0.095	0.149	0.076
	Soybean oil	0.462	0.118	0.392	0.109	0.187	0.098
Argentina	Soybean oil	1.908	0.202	1.514	0.159	0.603	0.221
Brazil	Soybean oil	1.028	0.237	0.859	0.203	0.402	0.155
U.S.	Soybean oil	2.655	0.238	2.110	0.113	0.849	0.191
Canada	Rapeseed oil	0.054	0.018	0.047	0.016	0.023	0.011
India	Rapeseed oil	0.145	0.025	0.126	0.024	0.064	0.025

Table 5
Global ethanol production (in million metric tons) from various feedstocks, pessimistic projection

		High oil price		Medium oil price		Low oil price	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
		2010-19					
EU27	Corn	2.274	0.194	1.903	0.119	1.018	0.131
	Wheat	0.750	0.277	0.752	0.278	0.773	0.282
	Sugar beets	1.790	0.192	1.549	0.162	0.903	0.191
China	Corn	1.385	0.251	1.152	0.217	0.614	0.199
Argentina	Sugarcane	0.533	0.097	0.462	0.083	0.271	0.051
Brazil	Sugarcane	40.715	6.050	34.920	4.766	20.211	2.679
U.S.	Corn	49.868	3.208	41.971	1.505	22.792	3.119
Canada	Wheat	0.183	0.064	0.186	0.064	0.190	0.067
	Corn	0.692	0.044	0.582	0.027	0.315	0.057
India	Sugarcane	0.224	0.042	0.187	0.034	0.099	0.018
		2020-29					
EU27	Corn	2.277	0.196	1.905	0.120	1.022	0.139
	Wheat	0.749	0.274	0.752	0.275	0.774	0.286
	Sugar beets	1.790	0.192	1.548	0.164	0.903	0.199
China	Corn	1.377	0.245	1.145	0.216	0.617	0.205
Argentina	Sugarcane	0.533	0.097	0.461	0.082	0.272	0.051
Brazil	Sugarcane	40.655	6.046	35.037	4.779	20.255	2.707
U.S.	Corn	49.879	3.123	41.993	1.438	22.794	3.146
Canada	Wheat	0.183	0.063	0.184	0.064	0.190	0.066
	Corn	0.693	0.044	0.582	0.027	0.317	0.059
India	Sugarcane	0.224	0.041	0.187	0.033	0.098	0.018
		2030-39					
EU27	Corn	2.276	0.194	1.905	0.121	1.020	0.136
	Wheat	0.746	0.277	0.758	0.281	0.776	0.289
	Sugar beets	1.797	0.194	1.545	0.162	0.899	0.188
China	Corn	1.379	0.247	1.151	0.216	0.609	0.192
Argentina	Sugarcane	0.532	0.097	0.462	0.082	0.270	0.050
Brazil	Sugarcane	40.582	5.924	34.918	4.749	20.265	2.674
U.S.	Corn	49.925	3.089	41.972	1.483	22.769	3.118
Canada	Wheat	0.182	0.063	0.185	0.066	0.190	0.066
	Corn	0.693	0.045	0.582	0.027	0.316	0.059
India	Sugarcane	0.223	0.041	0.186	0.034	0.099	0.019
		2040-49					
EU27	Corn	2.277	0.200	1.906	0.118	1.017	0.127
	Wheat	0.742	0.276	0.750	0.279	0.775	0.283
	Sugar beets	1.790	0.191	1.544	0.164	0.902	0.192
China	Corn	1.387	0.248	1.154	0.219	0.612	0.196
Argentina	Sugarcane	0.532	0.097	0.458	0.081	0.273	0.052
Brazil	Sugarcane	40.748	5.932	35.062	4.734	20.237	2.682
U.S.	Corn	49.957	3.031	42.007	1.410	22.796	3.123
Canada	Wheat	0.184	0.064	0.184	0.064	0.191	0.067
	Corn	0.692	0.045	0.582	0.027	0.316	0.059
India	Sugarcane	0.224	0.040	0.186	0.033	0.099	0.018

Table 6

Global biodiesel production (in million metric tons) from various feedstocks, pessimistic projection

		High oil price		Medium oil price		Low oil price	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
		2010-19					
EU27	Rapeseed oil	5.606	0.435	4.483	0.207	1.735	0.484
	Other oil	0.427	0.119	0.357	0.087	0.150	0.071
	Soybean oil	0.464	0.118	0.415	0.099	0.200	0.097
Argentina	Soybean oil	1.905	0.198	1.597	0.134	0.637	0.222
Brazil	Soybean oil	1.026	0.235	0.919	0.188	0.434	0.156
U.S.	Soybean oil	2.663	0.221	2.111	0.119	0.848	0.191
Canada	Rapeseed oil	0.055	0.018	0.051	0.015	0.025	0.011
India	Rapeseed oil	0.146	0.025	0.137	0.018	0.070	0.023
		2020-29					
EU27	Rapeseed oil	5.609	0.429	4.483	0.207	1.739	0.490
	Other oil	0.426	0.118	0.355	0.086	0.151	0.071
	Soybean oil	0.465	0.118	0.414	0.098	0.200	0.096
Argentina	Soybean oil	1.911	0.202	1.597	0.134	0.637	0.222
Brazil	Soybean oil	1.034	0.239	0.915	0.187	0.430	0.150
U.S.	Soybean oil	2.657	0.233	2.111	0.118	0.852	0.200
Canada	Rapeseed oil	0.055	0.019	0.051	0.015	0.025	0.011
India	Rapeseed oil	0.146	0.025	0.137	0.018	0.069	0.023
		2030-39					
EU27	Rapeseed oil	5.615	0.431	4.479	0.207	1.760	0.536
	Other oil	0.427	0.118	0.357	0.087	0.152	0.074
	Soybean oil	0.465	0.118	0.417	0.099	0.200	0.097
Argentina	Soybean oil	1.912	0.204	1.595	0.134	0.633	0.217
Brazil	Soybean oil	1.028	0.236	0.917	0.186	0.431	0.149
U.S.	Soybean oil	2.658	0.233	2.111	0.121	0.846	0.189
Canada	Rapeseed oil	0.055	0.018	0.051	0.015	0.026	0.011
India	Rapeseed oil	0.146	0.025	0.137	0.018	0.069	0.023
		2040-49					
EU27	Rapeseed oil	5.619	0.426	4.486	0.201	1.745	0.500
	Other oil	0.425	0.117	0.357	0.087	0.152	0.075
	Soybean oil	0.464	0.118	0.415	0.100	0.198	0.093
Argentina	Soybean oil	1.907	0.201	1.598	0.134	0.636	0.221
Brazil	Soybean oil	1.029	0.234	0.916	0.188	0.430	0.149
U.S.	Soybean oil	2.654	0.237	2.115	0.113	0.845	0.187
Canada	Rapeseed oil	0.055	0.018	0.051	0.015	0.026	0.011
India	Rapeseed oil	0.146	0.025	0.137	0.018	0.069	0.023

The Global Perspective: Alternative Macroeconomic Conditions Versus Crude Oil Prices

Under alternative prices of crude oil, the demand for feedstock for the production of biofuels varies considerably. Figures 2 through 5 and tables 1 and 2 capture the demand/consumption of feedstock (mean, variance, and confidence intervals) for the production of ethanol and biodiesel under high, medium, and low crude oil prices and alternative long-term macroeconomic projections at the global level (comparing left and right panels).

The long-term macroeconomic conditions under alternative economic growth perspectives have a rather small impact on the demand of feedstock used for the production of ethanol, both at the mean and variance (figs. 2 and 3 and table 1). Unlike the effect of economic growth projections (i.e., optimistic vs. pessimistic), the level of the crude oil price significantly affected demand (both the mean and variance) for the relevant feedstocks in the production of ethanol. Interestingly, the effect of crude oil prices on feedstock demand is not symmetric. In this sense, the mean values of feedstock demand under high and medium crude oil prices are not quite the same as feedstock demand under low crude oil prices. On the other hand, the variability measured by the standard deviation of the feedstock demand in the case of medium and low crude oil prices is much smaller than in the case of the higher crude oil prices (fig. 3 and table 1).

At the mean, the feedstock demand/consumption for biodiesel production is much smaller than the demand of feedstock for the production of ethanol worldwide (figs. 4 and 5, table 2). Again, the impact of long-term macroeconomic projections, such as optimistic vs. pessimistic, on global demand for feedstock for biodiesel production is small compared with the impact of crude oil prices. The variability measured by the standard deviation of the demand for feedstock is larger for the case of high crude oil prices and almost the same for the medium and low crude oil prices. Under the pessimistic outlook projection, the variability of global demand of feedstock is smaller than under the optimistic outlook projection (fig. 5 and table 2). The 90-percent confidence interval of the mean measured by the lower and upper bound indicates the probability that the confidence interval contains the true population mean.

In sum, crude oil prices have a larger effect on the demand for feedstock for ethanol and biodiesel production than macroeconomic growth projections. At the global level, both the mean and variance are affected considerably by the level of crude oil, but the effect is asymmetric. The variability depends on feedstock demand and is different for the demand/consumption of feedstock for ethanol than for biodiesel. Note that we do not conduct any formal testing on the causal relationship of crude oil prices and the first and second moments (mean and variance) of the distribution of the feedstock productions. Such an effort is beyond of the scope of this report.

The Country Perspective: Diverge Variability of Production of Feedstock for Ethanol

A variety of feedstock is used for the production of ethanol worldwide. Tables 3 and 4 present the model results (mean and standard deviation) of feedstock demand for the production of ethanol by country under alternative
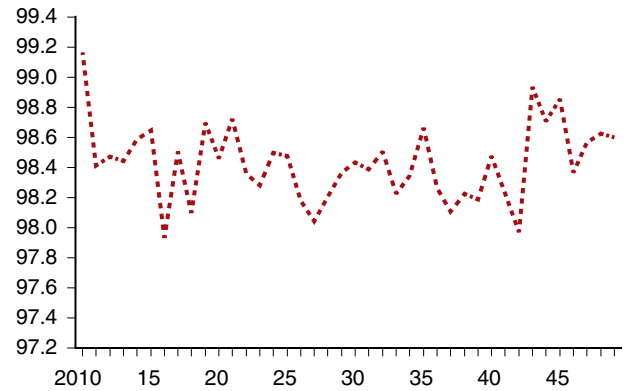
Figure 2

Global ethanol production (mean) under alternative crude oil prices and macroeconomic projections
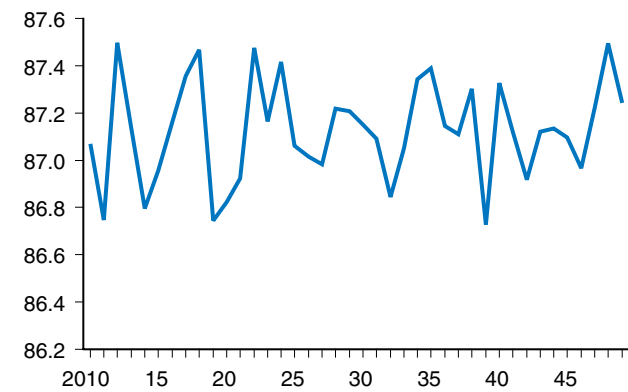
High crude oil price, optimistic projection
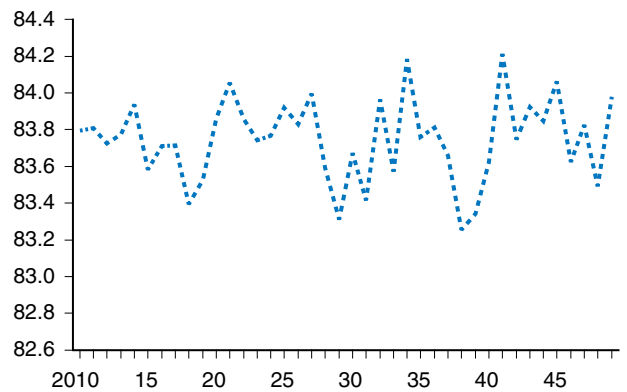Mil. metric tons
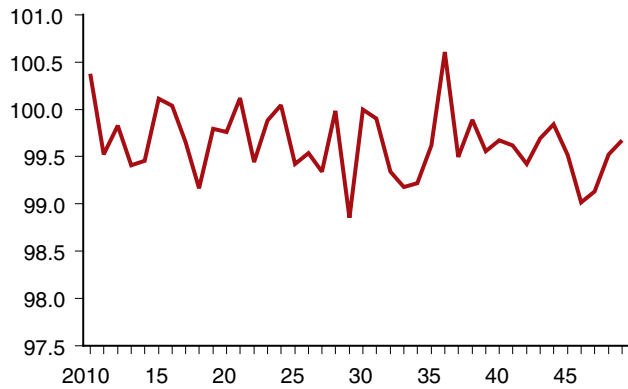
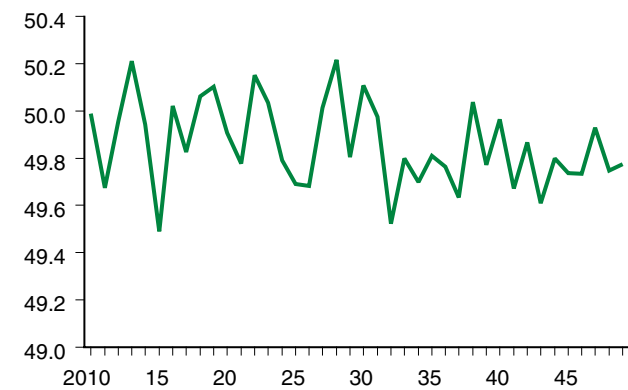High crude oil price, pessimistic projection
Mil. metric tons

Medium crude oil price, optimistic projection
Mil. metric tons

Medium crude oil price, pessimistic projection
Mil. metric tons

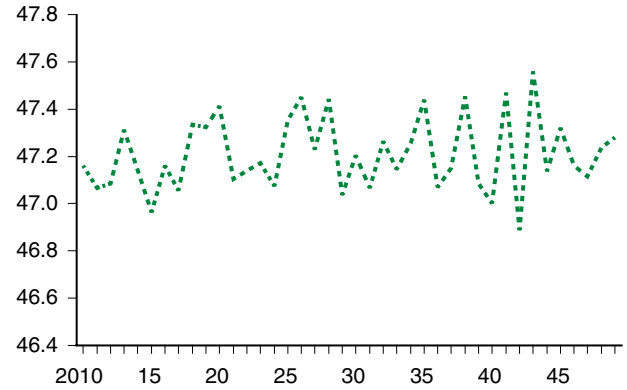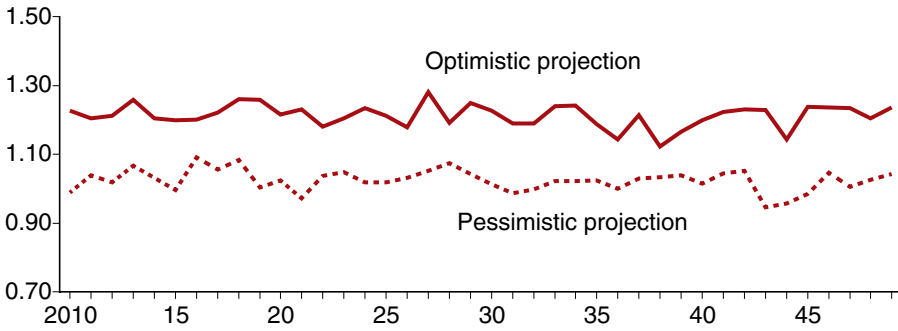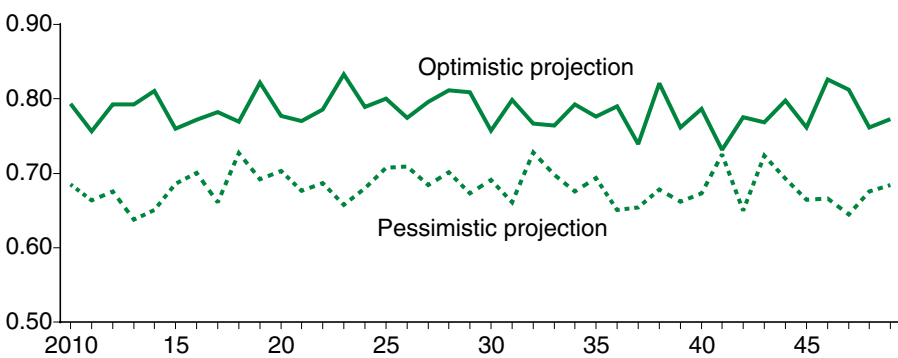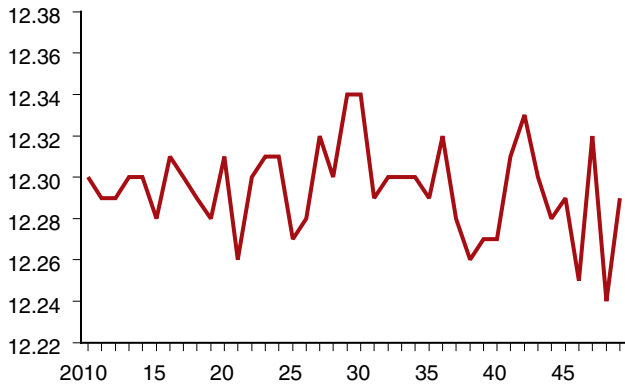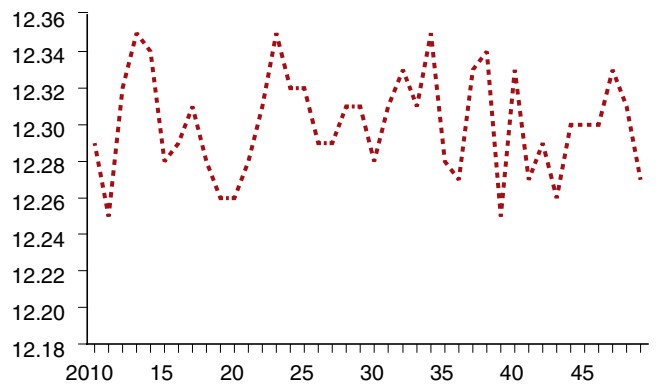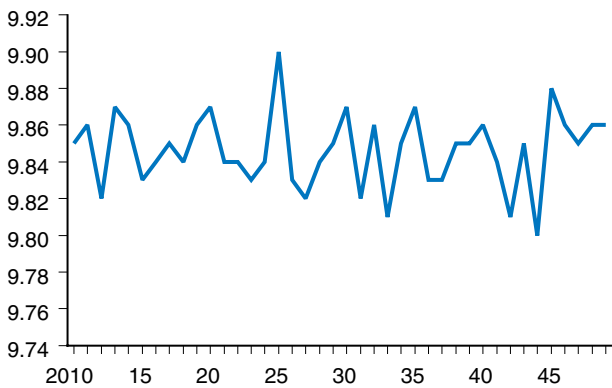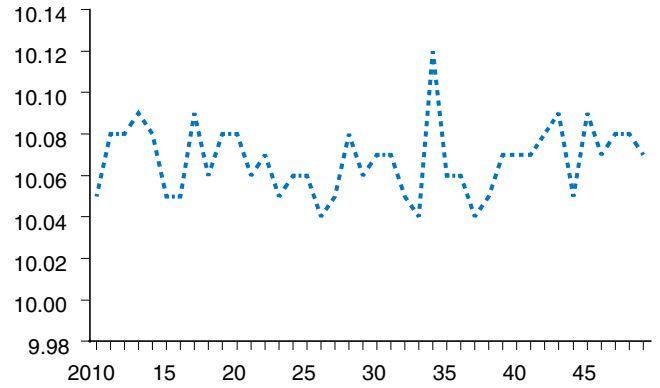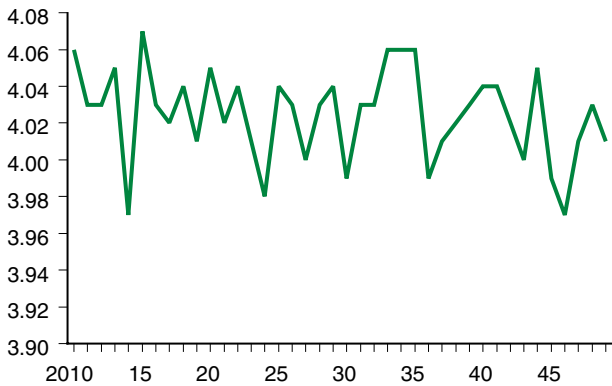Low crude oil price, optimistic projection
Mil. metric tons

Low crude oil price, pessimistic projection
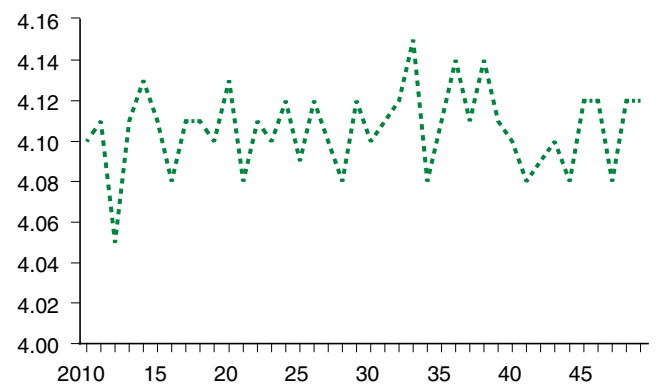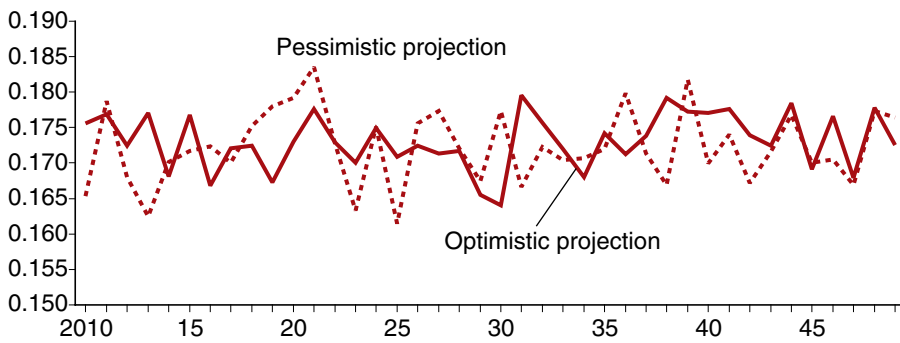Mil. metric tons

Figure 3

Global ethanol production variability (standard deviation) under alternative crude oil prices and macroeconomic projections

High crude oil price, standard deviation
Mil. metric tons

Medium crude oil price, standard deviation
Mil. metric tons
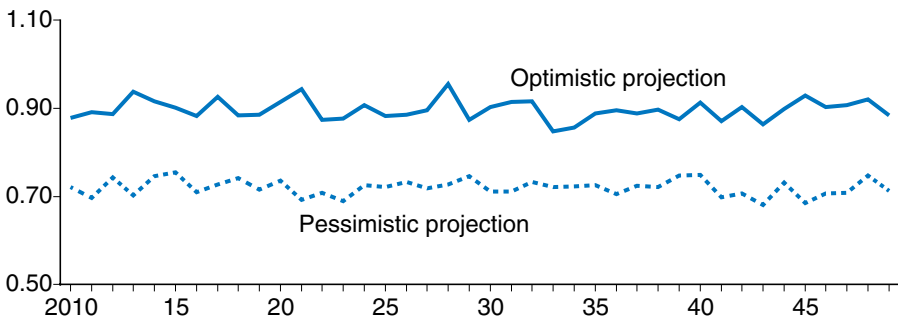
Low crude oil price, standard deviation
Mil. metric tons

Figure 4

Global biodiesel production (mean) under alternative crude oil prices and macroeconomic projections

High crude oil price, optimistic projection
Mil. metric tons

High crude oil price, pessimistic projection
Mil. metric tons

Medium crude oil price, optimistic projection
Mil. metric tons

Medium crude oil price, pessimistic projection
Mil. metric tons

Low crude oil price, optimistic projection
Mil. metric tons

Low crude oil price, pessimistic projection
Mil. metric tons

Figure 5
Global biodiesel production variability (standard deviation) under alternative crude oil prices and macroeconomic projections
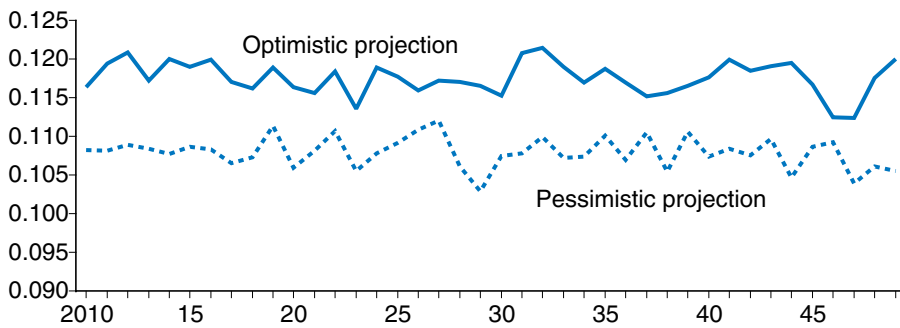
High crude oil price, standard deviation
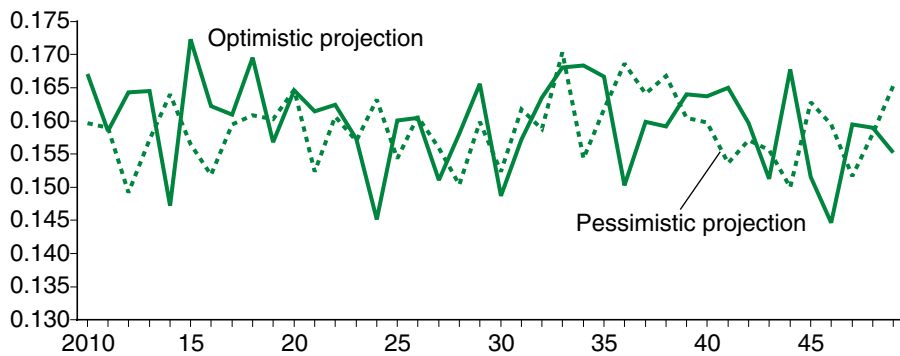Mil. metric tons

Medium crude oil price, standard deviation
Mil. metric tons

Low crude oil price, standard deviation
Mil. metric tons

macroeconomic conditions and crude oil prices. Again, the results clearly indicate that at the country level, the price of crude oil has a much larger effect on feedstock demand/production, both at the mean and standard deviation, than the long-term macroeconomic projections/conditions. Furthermore, the price of crude oil affects the demand for feedstock for ethanol asymmetrically while the variability of feedstock in the production of ethanol depends considerably on the type of the feedstock.

For example, under the high crude oil price scenario, Brazil's sugarcane feedstock demand (at the mean) is 15 percentage points higher than under the medium crude oil price (scenario). The demand/production of feedstock (at the mean) for ethanol in all countries in the model exhibits the same pattern as that in Brazil (tables 3 and 4). In other words, the higher the crude oil price, the larger the demand/production of feedstock for ethanol.

Under the low crude oil price scenario, Brazil's sugarcane feedstock demand (at the mean) is about 40 percentage points lower than under medium prices during the period under study (2010-49) (tables 3 and 4). The model results indicate that under the low vs. medium crude oil price scenario, the demand/production of sugarcane (India and Argentina), sugar beets (EU), and corn (United States, Canada, and the EU) follow the same pattern as that of Brazil over the long-term 40-year projections. However, under the low crude oil price level, the feedstock demand/production for ethanol from corn in China and wheat from Canada and the EU is only 11 percentage points different than that under medium price level (tables 3 and 4). Furthermore, the production/demand for feedstock (at the mean) for ethanol follows the similar pattern under the alternative long-term scenarios (optimistic vs. pessimistic projection).

In conducting the scenarios, we treat all commodities and/or countries similarly under the same specifications of the long-term projections (www.ihsglobalinsight.com). Note that by only employing the bootstrapping technique, we are able to access the *variability* of the simulation results and derive estimates of the underlying statistics (mean, standard deviation, and confidence intervals) as the outcomes of simulated scenarios completely lack any statistical property. Since the bootstrapping procedure is distribution independent, it provides us with an indirect method to assess the distributional properties of the underlying outcomes/sample. Using the bootstrapping technique along with simulation matters as the outcomes are associated with stochastic properties that also matter. A Monte-Carlo type of stochastic simulation that accounts for cross-commodity correlations would greatly enhance the procedure. However, such a simulation is beyond the scope of this report.

The variability of feedstock production depends on the kind of feedstock used for the production of ethanol. Under the optimistic projection, Brazil's sugarcane variability (standard deviation) is higher under high crude oil prices than under medium or low crude prices. In this sense, it is almost 20 percentage points higher when under high vs. medium prices and 40 percentage points lower under low vs. medium crude oil prices. U.S. corn, EU sugar beets, and China corn feedstock demand/production variability (standard deviation) is much larger under high and low crude oil prices than under medium crude oil prices. The use of bootstrapping indicates that if there are policies in place in a certain country that are targeting certain levels of production of ethanol, which

is associated with certain levels of feedstock production, the level and the variability of the relevant feedstock must be considered.

Under the pessimistic projection and for all levels of crude oil prices, variability follows the same pattern as that of the optimistic projection but at a smaller magnitude (tables 3 and 4).

The Country Perspective: Diverge Variability of Production of Feedstock for Biodiesel

Overall feedstock demand/production for biodiesel is much smaller than feedstock demand/production for ethanol. The simulation results again indicate that crude oil prices have a much larger asymmetric effect on feedstock demand (mean and standard deviation) for the production of biodiesel than do the long-term macroeconomic projections/conditions.

The EU is the largest biodiesel producer followed by the United States, Argentina, and Brazil. While the EU uses a variety of feedstocks for the production of biodiesel, the United States, Argentina, and Brazil use soybean oil (tables 5 and 6). In the case of the EU, rapeseed oil feedstock demand (at the mean) is 25 percentage points higher than under medium crude oil prices (high vs. medium crude oil price) and declines by 60 percentage points under low crude oil prices (low versus medium crude oil price). For the United States, the price of crude oil affects the demand for soybean oil for the production of biodiesel (at the mean) considerably, but its fluctuation increases when crude oil prices decline. The same is observed with Argentina's and Brazil's soybean production and India's rapeseed production for biodiesel (tables 5 and 6).

Conclusion

This report documents the latest version of the PEATSim model and illustrates global biofuel production/demand under various crude oil prices and economic growth (macroeconomic conditions) scenarios.

PEATSim is a partial equilibrium, multiple-commodity, multiple-region dynamic model of agricultural policy and trade for policy analysis and reforms on a multilateral and bilateral basis.

Policy specification is treated on a policy-by-policy basis rather than being aggregated and introduced as price "wedges," as is done in several Computable General Equilibrium models.

The partial equilibrium approach allows the use of commodity-specific elasticity specifications. These features can reflect own- and cross-price elasticities much better than could ever be achieved in a general equilibrium model that has far less flexibility in the assignment of parameters that determine the production/consumption response of each particular commodity. The specification of PEATSim is consistent with economic theory due to the imposition of symmetry and homogeneity conditions on elasticity assignments in the model wherever appropriate.

The report also examines an application of the model to illustrate its dynamic structure and to demonstrate the differential behavior of global commodity markets. In particular, the application examines the impact of macroeconomic conditions and crude oil prices on the feedstock demand for ethanol and biodiesel. Based on a comparison of the outcomes of alternative scenarios, the application finds that unlike macroeconomic conditions, crude oil prices have a profound effect on feedstock demand for ethanol and biodiesel production. At the global level, both the mean and variance are affected considerably by the level of crude oil prices, but the effect is asymmetric. Overall, the scenario simulations show that the effects on feedstock demand/production for biodiesel are much smaller than those on demand/production of feedstock for ethanol.

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Appendix: Mixed Complementarity Problem

Complementarity problems are used extensively in economics because the concept of complementarity is intimately linked with the notion of system equilibrium. The balance of supply and demand in economics is often described by a complementary relationship between price and excess supply. In this case, the complementarity condition requires that the excess supply of a commodity must be zero if its price is positive, and the price of the commodity must be zero if there is positive excess supply.

Nonlinear complementarity problems, $NCP(F)$, seek to find an $x \in \Re^n$ given a *nonlinear function* defined as $F: \Re^n \rightarrow \Re^n$, such that

$$NCP(F): \qquad 0 \leq x \perp F(x) \geq 0$$

where the perpendicular notation "$\perp$" indicates that in addition to the above inequalities, the equation $x^T F(x) = 0$ also holds. This equation can be stated as:

$$0 \leq x, \quad F(x) \geq 0, \quad x_i F_i(x) = 0, \quad i = 1,2,\ldots,n$$

In other words, complementarity implies that for each variable, either the variable or its complement (i.e., either x_i or $F_i(x)$) must be zero for each $i = 1,2,\ldots,n$.

Complementarity problems of this form are associated with the Karush-Kuhn-Tucker conditions of a constrained nonlinear program as follows:

minimize $\theta(x)$

subject to $g(x) \leq 0, \quad x \geq 0$

where $\theta: \Re^n \rightarrow \Re$ is a continuously differentiable *real-valued function* and $g: \Re^n \rightarrow \Re^m$ is a continuously differentiable *vector-valued function*.

The $NCP(F)$ can be generalized by the introduction of (possibly infinite) lower and upper bounds on the variable x. This results in the $MCP(F,L,U)$ defined as follows:

given $F: \Re^n \rightarrow \Re^n$ and bounds L,U where $L \in \Re^n \bigcup -\infty$, and $U \in \Re^n \bigcup +\infty$ find:

$x \in [L,U]$

s.t. $\forall\ i$, one of the following conditions holds:

i) $F_i(x) = 0$
ii) $F_i(x) > 0$ and $x_i = L_i$
iii) $F_i(x) < 0$ and $x_i = U_i$.

The $NCP(F)$ is a special case of $MCP(F,L,U)$ obtained by setting $L = 0$ and $U = -\infty$.

Another special case occurs if we set $L = -\infty$ and $U = +\infty$; then alternatives (ii) and (iii) above are impossible and we have $F_i(x) = 0$ (i.e., a smooth system of nonlinear equations). This concept is applied to discontinuous functions in PEATSim, such as production quotas, tariff-rate quotas, and discontinuous demand issues created by mandates or targets.