University of New England

Graduate School of Agricultural and Resource Economics

Does the Separation Theorem explain why farmers have so little interest in futures markets?

by

Phil Simmons

No. 99-1 – March 1999

Working Paper Series in

Agricultural and Resource Economics

ISSN 1442 1909

 ISBN 1 86389 564 7

Does the Separation Theorem explain why farmers have so little interest in futures markets?[*]

Phil Simmons[**]

Abstract

A farm financial model with leverage and investment in two farm enterprises is specified. The model is extended to incorporate futures hedging and the Separation Theorem is used to show that optimal hedging is zero. The assumption of a risk-free asset is relaxed and, while this leads to a violation of the Separation Theorem, the result that optimal hedging is zero is maintained providing that futures markets are efficient. It is concluded that if capital markets are efficient then farmers will have little interest in futures markets except to speculate.

Key Words: separation theorem, futures trading, hedging rules

---

[*] An earlier version of this paper was presented at the 1997 Conference of the Australian Agricultural and Resource Economics Society in Armidale, New South Wales.

[**] The author is a Senior Lecturer in the School of Economic Studies, and the Director of the Graduate School of Agricultural and Resource Economics at the University of New England. Contact information: Graduate School of Agricultural and Resource Economics, University of New England, Armidale, NSW 2351, Australia. Email: psimmons@metz.une.edu.au.

Does the Separation Theorem explain why farmers have so little interest in futures markets?

1. Introduction

The non-farming community has long associated futures markets with farmers and the connection has been promoted by futures exchanges. The Chicago Board of Trade likes to tell of how two centuries ago farmers sold forward into the Chicago market and the Sydney Futures Exchange (SFE) takes pride in its connection with the earlier Sydney Greasy Wool Futures Exchange. This connection between farmers and futures markets is reflected in the agricultural economics literature, with 'optimal hedge' models being derived under a broad range of assumptions. Early papers include Telser (1955), who found optimal hedge ratios under mean-variance and safety first assumptions, and Heifner (1972), Peck (1975) and others who measured optimal hedge ratios using farm data. Recently, Lapan and Moschini (1994) found exact solutions for an optimal hedge under Constant Absolute Risk Aversion (CARA) assumptions, and Myers and Hanson (1996) found dynamic solutions. All of these studies predict that, under plausible assumptions about risk aversion and transaction costs, farmers will hedge on futures markets.

However, farmers appear to have little interest in futures markets. Lubulwa, Lim-Applegate and Martin (1997) surveyed Australian wheat farmers and found that only four per cent used futures contracts in the 1996 season. Berck (1981) reports that less than five per cent of American farmers used futures. In addition, there is hearsay evidence that Australian farmers rarely hedged wool even in the heyday of the wool futures market prior to the Reserve Price Scheme.

Two reasons are commonly given for futures contracts not being popular with farmers. First, it is argued that futures markets are too complex for farmers to understand. While the complexity of futures hedging may be discouraging, 'farmer ignorance' arguments lose much of their strength when the relatively complicated marketing arrangements entered into by farmers outside of futures markets are considered. The second reason given for lack of interest by farmers in futures markets is that basis risk makes hedging ineffective (Bond, Thompson and Geldard 1985; Bond and Thompson 1985; Lubulwa, Beare, Bui-Lan and Foster 1997a). Farmers can lose from hedging on futures markets if their production differs from the futures contract specification. Lubulwa et al. (1997a) show that Australian farmers producing wool only a few microns different to contract specification face considerable financial risks in hedging wool. However, again, the evidence does not provide a satisfactory explanation for lack of interest by farmers in futures contracts. The same authors argue that around 17 per cent of Australian wool producers could reduce price risk by 90 per cent using the SFE wool contract. Also, since the SFE wheat contract is designed specifically for Australian producers, most wheat produced is close to futures contract specification (Simmons and Rambaldi, 1997).

In this paper, the Separation Theorem provides a starting point for discussion of the reason farmers have little interest in futures markets. In Section 2, a version of the Separation Theorem is developed in the context of a farm financial model and, in Sections 3 and 4, the model is extended to incorporate hedging and more general assumptions. In Section 5 there is a discussion and conclusions are drawn.

2. Separation Theorem

Assume that farmer utility can be expressed in the form of a mean-variance function:

$$u = y - \frac{k}{2} vy \tag{1}$$

where u is subjective expected utility, k is a risk coefficient, y is subjective expected income and vy is the variance of income. The mean-variance utility assumption has several limitations. First, it is inconsistent unless income is normally distributed (Tsiang 1972). Newbery (1988) has shown that errors may occur in calculating hedge ratios from mean-variance utility models if the income distribution is erroneously assumed to be normal. This problem has led to use of CARA utility assumptions which, however, make exact solutions very difficult to obtain. Lapan and Moschini (1994) were able to obtain exact solutions for a CARA specification of the hedge problem however only with normally distributed prices, an undesirable assumption. A second limitation of mean-variance utility is that it is Increasing Absolute Risk Aversion (IARA) which is inconsistent with econometric evidence for DARA utility in farm populations (Pope and Just 1991; Chavas and Holt 1996). While acknowledging these limitations, the mean-variance assumption allows tractable, exact solutions and, hence, we adopt it in this study. In defence of this strong assumption, many important qualitative results from portfolio theory were initially derived in the mean-variance framework and have usually withstood later generalisation.

It is further assumed that the farm has constant returns to scale and produces two crops, s and w. The expected return from farm capital is ra:

$$ra = ps\, rs + pw\, rw \tag{2}$$

where *rs* and *rw* are the expected returns from *s* and *w* respectively and *ps* and *pw* are the proportions of farm capital allocated to *s* and *w.* Some additional restrictions are applied with *ps* and *pw* non-negative, an additivity restriction *pw* = 1-*ps,* and *rs* and *rw* independent so that Cov(*rs*, *rw*) = 0.

The farmer borrows or lends at a 'risk-free' rate *rb* hence the expected return to equity is *re*:

$$re = pa\,ra + pb\,rb \tag{3}$$

where *rb* is less than *ra* or its components *rs* and *rw*, *pa* is the ratio of farm value to equity, *pb* is the ratio of borrowing or lending to equity, *pa* is non-negative to prevent 'shorting' of farm capital and the additivity restriction is *pa* = 1-*pb*.

Expected utility is obtained in terms of capital returns by assuming there is one unit of equity capital and substituting the two additivity restrictions, (3) and (2) into (1):

$$u = pb\,rb + (1 - pb)(ps\,rs + (1 - ps)rw) - \frac{k}{2}(1 - pb)^2(ps^2\,vrs + (1 - ps)^2\,vrw) \tag{4}$$

where *vrs* and *vrw* are the variances of *rs* and *rw* respectively.

The farmer chooses the crop mix and amount of borrowing or lending hence the decision variables are *pb* and *ps*. First-order conditions to maximise *u* are:

$$\frac{\partial u}{\partial pb} = rb - ps\,rs - (1 - ps)\,rw + k\,(1 - pb)\,((1 - ps)^2\,vrw + ps^2\,vrs) = 0 \tag{5.1}$$

$$\frac{\partial u}{\partial ps} = (1 - pb)(rs - rw - k(1 - pb)(ps\,vrs - (1 - ps)\,vrw)) = 0 \qquad (5.2)$$

(Second-order conditions are reported in Section 4 for a more general version of the model.) (5.1) and (5.2) are solved simultaneously for *pb* and *ps*[1]:

$$pb = 1 - \frac{(rs - rb)\,vrw + (rw - rb)\,vrs}{k\,vrs\,vrw} \qquad (6.1)$$

$$ps = \frac{(rs - rb)\,vrw}{(rw - rb)\,vrs + (rs - rb)vrw} \qquad (6.2)$$

An increase in *pb* corresponds to reduced borrowing or increased lending thus, from (6.1), an increase in *rb* (or reduction in *rs* or *rw*) reduces borrowing or increases lending. An increase in risk aversion, *k*, reduces borrowing and increases lending. When *k* is zero, (6.1) has no solution because of the assumption of constant returns and the 'replication argument' (Varian 1992). From (6.1), if *rs* increases then investment in *s* increases and, as expected, if *rw* increases, *ps* falls. There is no solution if both *vrs* and *vrw* are zero since only the crop with the highest return is produced under these circumstances.

From the standpoint of this study, the most important result, from (6.2), is that *k* does not influence on-farm allocation of capital. This reflects the Separation Theorem that states, in the context of farming, that if capital markets are efficient then the crop mix is not influenced by risk preferences. Jones (1996) provides a general discussion of the Separation Theorem.

---

[1] Computations here and elsewhere were undertaken using Mathematica 3.0 (Wolfram, 1996)

3. Separation Theorem with Hedging

In this section, the model is extended to incorporate a futures market for *w*. To limit the number of new variables, the simplifying assumption is made that the futures market is in *rw*, the return to capital in crop *w*. This is equivalent to having a futures market for *w* where the price of *w* is the only source of variation in *rw*. This assumption, which amounts to suppressing production risk, is benign so long as farmers, taken as a group, are not price makers with production risk correlated across their industry (Grant 1985).

The extension is undertaken by modifying (2) so that:

$$ra = ps\,rs + pw\,(rw + h\,(fp - rw)) \qquad (7)$$

where *h* is the hedge ratio and *fp* is the current futures price for a contract maturing at harvest expressed as a rate of return on investment in *w*. Hence, if *rw* falls after hedging, the farmer receives *pa pw h*(*fp* - *rw*) dollars and, if *rw* increases, margin calls of *pa pw h*(*fp* - *rw*) are paid. Speculative gains are possible if the market is inefficient and *fp* is biased so that *fp* - *rw*0 (Newbery and Stiglitz 1981).

The same substitutions as in Section 2 are repeated with (7) replacing (2). Expected utility becomes:

$$\begin{aligned} u = {} & pb\,rb + (1 - pb)\,(ps\,rs + (1 - ps)\,(rw + h(fp - rw))) \\ & - \frac{k}{2}(1 - pb)^2\,(ps^2\,vrs + (1 + h^2)(1 - ps)^2\,vrw) \end{aligned} \qquad (8)$$

There are now three decision variables *pb*, *ps* and *h* and hence three first-order conditions:

$$\frac{\partial u}{\partial pb} = rb - ps\,rs - (1 - ps)(rw + h(fp - rw))$$
$$+ k(1 - pb)(ps^2\, vrs - (1 - ps)^2 (1 + h^2) vrw) = 0 \qquad (9.1)$$

$$\frac{\partial u}{\partial ps} = (1 - pb)(rs - rw - h(fp - rw)$$
$$- k(1 - pb)(ps\, vrs - (1 - ps)(1 + h^2) vrw)) = 0 \qquad (9.2)$$

$$\frac{\partial u}{\partial h} = (1 - pb)(1 - ps)(fp - rw - h\,k(1 - pb)(1 - ps) vrw) = 0 \qquad (9.3)$$

Following Kahl (1983), the first-order conditions are solved simultaneously to provide equilibrium values for *pb*, *ps* and *h*:

$$pb = 1 - \frac{(rs - rb)\, vrw + (rw - rb)\, vrs}{k\, vrs\, vrw} \qquad (10.1)$$

$$ps = \frac{(rs - rb)\, vrw}{(rw - rb)\, vrs + (rs - rb) vrw} \qquad (10.2)$$

$$h = \frac{fp - rw}{rw - rb} \qquad (10.3)$$

Comparing (10.1) and (10.2) with (6.1) and (6.2), the equilibrium values for *pb* and *ps* are the same as without a futures market. Most importantly, *k* does not enter (10.3) indicating that the Separation Theorem can be extended to include hedging. From (10.3), the only interest in futures markets is speculative.

4. Variable Borrowing and Lending Rate

An assumption of the Separation Theorem is that the borrowing and lending rate, *rb*, has zero variance and the theorem is violated if this assumption is relaxed. However, the implications for hedging of relaxing this assumption are not clear, hence, expected utility is extended to incorporate the variance of *rb*, *vrb* > 0 where Cov(*rb*, *rs*) = Cov(*rb*, *rw*) = 0:

$$\begin{aligned} u = {} & pb\, rb + (1 - pb)\,(ps\, rs + (1 - ps)\,(rw + h(fp - rw))) \\ & - \frac{k}{2}\,(pb^2\; vrb + (1 - pb)^2\,(ps^2\; vrs + (1 + h^2)\,(1 - ps)^2\; vrw)) \end{aligned} \tag{11}$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial u}{\partial pb} = {} & rb - ps\, rs - (1 - ps)(rw + h(fp - rw)) \\ & - k\,(pb\; vrb - (1 - pb)\,(ps^2\; vrs - (1 - ps)^2\;(1 + h^2)vrw)) = 0 \end{aligned} \tag{12.1}$$

$$\begin{aligned} \frac{\partial u}{\partial ps} = {} & (1 - pb)(rs - rw - h(fp - rw) \\ & - k(1 - pb)\,(ps\; vrs - (1 - ps)(1 + h^2)vrw)) = 0 \end{aligned} \tag{12.2}$$

$$\frac{\partial u}{\partial h} = (1 - pb)(1 - ps)(fp - rw - h\,k(1 - pb)(1 - ps)vrw) = 0 \tag{12.3}$$

and second-order conditions to ensure a maximum are:

$$\frac{\partial^2 u}{\partial pb^2} = -k(vrb + ps^2 vrs + (1 + h^2)(1 - ps)^2 vrw < 0 \tag{13.1}$$

$$\frac{\partial^2 u}{\partial ps^2} = -k(1 - pb)^2 (vrs + (1 + h^2)vrw) < 0 \tag{13.2}$$

$$\frac{\partial^2 u}{\partial h^2} = -k(1-pb)^2(1-ps)^2 vrw < 0 \qquad (13.3)$$

Equilibrium values for *pb*, *ps* and *h* are:

$$pb = \frac{k\, vrs\, vrw - (rs - rb)\, vrw - (rw - rb)\, vrs}{k(vrs\, vrw + vrb(vrs + vrw))} \qquad (13.1)$$

$$ps = \frac{(rs - rb)\, vrw - k\, vrb\, vrw}{(rw - rb)\, vrs + (rs - rb) vrw - k\, vrb(vrs + vrw)} \qquad (13.2)$$

$$h = \frac{(fp - rw)(vrs\, vrw + vrb(vrs + vrw))}{vrw(vrs(rw - rb) + vrb(rw - rs) + k\, vrb\, vrs)} \qquad (13.3)$$

From (13.2), relaxing the 'risk-free asset' assumption violates the Separation Theorem since *k* now influences the crop mix. While the same appears to be true of the hedging decision, (13.3) warrants further examination. If *rw*, the subjective expected return to investment in *w*, differs from *fp* (in the first bracketed term in the numerator) then speculation occurs with *k* influencing the level of speculative activity. However, if the farmer believes that the futures market is efficient, so that *fp*-*rw* = 0, then *h* is zero. There will be no hedging by farmers if futures markets are efficient.

5. Discussion and Conclusions

The analysis provides a possible explanation of the reason that most farmers do not hedge on futures markets and, through its speculative component, a possible explanation of why some do. A perspective is also provided on commodity futures markets since, under our assumptions, these are simply clearing houses for market information and are not used for hedging.

A key assumption in the analysis concerns the treatment of transaction costs. Transaction costs in futures markets were assumed to be zero and in capital markets they entered (3) implicitly through the choice of borrowing and lending rate.

Direct costs for farmers dealing in Australian futures markets are around two per cent of the value of contracts. At this level, costs should not strongly influence the decision to hedge unless risk premiums are very small, in which case farm hedging may not be an issue anyway. In addition, inclusion of positive futures transaction costs would presumably discourage hedging and strengthen our thesis that futures hedging is not attractive to farmers.

Transaction costs in capital markets were implicitly included in the interest rate and an important simplifying assumption here was that borrowing rates are constant which is unlikely to be true. Brorsen (1995) has shown that risk neutral agents will hedge if borrowing costs are 'non-linear' and that such hedging increases with debt levels. In the context of the Separation Theorem, Brorsen's result might be explained with the traditional textbook figure where non-linear borrowing costs cause the risk-efficient frontier to 'bend' around, and hence incorporate, part of the farm E-V frontier (Levy and Sarnat 1984, p. 409).

It seems likely that high lending margins associated with relatively high debt explain some of the observed futures activity by farmers. Since the results of our analysis could change if these margins were explicitly incorporated into the model, this could be an area for further investigation.

References

Berck, P., 1981. Portfolio theory and the demand for futures' *Am. J. Agric. Econ*., 64, 466-74.

Bond, G.E. and Thompson, S.R., 1985. Risk aversion and the recommended hedge ., 67, 870-2.

Bond, G.E., Thompson, S.R. and Geldard, J.M., 1985. Basis risk and hedging strategies for Australian wheat exports' *Aust. J. Agric. Econ*., 29, 199-209.

Brorsen, B.W., 1995. Optimal hedge ratios with risk neutral producers and non-linear *Am. J. Agric. Econ*., 77, 174-81.

Chavas, J.P. and Holt, M.T., 1996. Economic behaviour under uncertainty: a joint analysis of risk preferences and technology' *Rev. of Econ. and Stat*., 78, 329-35.

Grant, D., 1985. Theory of the firm with joint price and output risk and a forward *Am. J. Agric. Econ*., 67, 630-5.

Heifner, R.J., 1972. Optimal hedging levels and hedging effectiveness in cattle feeding' *Agric. Econ. Res*., 25, 25-36.

Jones, C.P., 1996. *Investments, analysis and management*, 3rd edn. John Wiley & Sons Inc, New York.

Kahl, K.H., 1983. Determination of the recommended hedging ratio' *Am. J. Agric. Econ*., 65, 603-5.

Lapan, H. and Moschini, G., 1994. Futures hedging under price, basis, and production *Am. J. Agric. Econ*., 76, 465-77.

Levy, H. and Sarnat, M., 1984. *Portfolio and investment selection: theory and practice*, Prentice Hall, New York.

Lubulwa, M., Lim-Applegate, H. and Martin, P., 1997. Managing price risk' *Australian Farm Surveys Report*, Australian Bureau of Agricultural and Resource Economics.

Lubulwa, M., Beare, S., Bui-Lan, A, and Foster M., 1997. Wool futures: price risk management for Australian wool growers' *Res. Report 97.1*, Australian Bureau of Agricultural and Resource Economics.

Myers, R.J. and Hanson, S.D., 1996. Optimal dynamic hedging in unbiased futures *Am. J. Agric. Econ*., 78, 13-20.

Newbery, D.M.G. and Stiglitz, J.E., 1981. *The theory of commodity price stabilization: a study in the economics of risk*, Clarendon Press, Oxford.

Newbery, D.M.G., 1988. On the accuracy of the mean-variance approximation for *Econ. Let*. 28(1) 63-68.

Peck, A.E., 1975. Hedging and income stability: concepts, implications, and an *Am. J. Agric. Econ*., 57, 410-19.

Pope, R.D. and Just, R.E., 1991 On testing the structure of risk preferences in *Am. J. Agric. Econ*., 73, 741-8.

Simmons, P.R. and A.N. Rambaldi 1997, 'Potential demand for hedging by Australian wheat producers', *Australian Journal of Agricultural and Resource Economics* 41(2), 157-168.

Telser, L.G., 1955. Safety first and hedging,' *Rev. of Econ. Stud*., 6, 410-9.

Tsiang, S.C., 1972. The rationale of the mean-standard deviation analysis, skewness preference, and demand for money,' *Am. Econ. Rev*., 62, 354-71.

Varian, H.R., 1992. *Microeconomic analysis*, 3rd edn. W.W. Norton & Co., New York.

Wolfram, S., 1996. *The Mathematica book*, 3rd edn. Cambridge University Press, Cambridge, Mass.