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CENTRAL PLACE MODELS, REGIONAL TYPOLOGIES, AND PLANNING

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Introduction

Central place models provide a locational norm for the numbers and population sizes of towns and regions in a hierarchical system, but have not emphasized the characterization of alternative regional structures from which planning inferences may be made.[1] Population-based versions of central place theory are particularly attractive due to the availability of population data for the smallest rural towns and counties. Persistent deviations from central place regularity may indicate problems that influence regional development, but the pattern of such deviations requires a regional typology based on an estimation methodology for a single region. The methodology should include estimation of all central place parameters and their use in calculating prediction error patterns for urban areas and regions at all hierarchical levels. The typology suggested here is based on the degree of population centrality, intraregional specialization and central convergence (or divergence) over time.

The central place literature is reviewed comprehensively in Mulligan (1984). Theoretical studies include Beckmann (1958), Parr (1969), and Beckmann and McPherson (1970). Parr's model assumes a constant productivity coefficient. Tests for both Parr and Beckmann and McPherson are discussed in Abu-Hijleh (1989, pp. 37-44); for brevity, the analysis here is confined to the generally better results attained with the Beckmann-McPherson model. (The Parr prediction errors are substantially larger than the errors calculated for the Beckmann-McPherson model.) Horn-Prescott (1978) and Haining (1980) provide examples of regression analyses for empirical estimation, and Suarez-Villa (1980, 1982) utilizes rank-size distribution methods. The empirical articles do not attempt to estimate prediction errors or establish regional typologies based on population deviations from central place regularity.

The main purpose of this article is to provide a central place estimation method for a rural labor market and analyze a regional typology based on the patterns of prediction errors for the urban and regional populations. The Beckmann-McPherson equations are discussed, as are characteristics of the central place parameters that establish the

[1] Mulligan (1984) provides an excellent literature review and recognizes, among other authors, the limited relevance of central place models for planning and policy. Versions of central place theory that directly deal with goods and services (not populations) involve numerous restrictive economic assumptions before locational efficiency can be established.

typology. The subject region's database and statistical estimates of the parameters are discussed; calculations for the prediction equations also are presented and analyzed. A final section summarizes the planning implications of the typology and provides concluding remarks.

Central Place Models and the Regional Typology

The parameters in central place theory involve the populations of the urban centers (or servicing units), the populations outside the urban centers (or complementary populations), and the number of subregions at the next smallest level (or satellite number). The population in complementary regions purchase goods and services in cities of various sizes where the variety of goods and services hierarchically increases with the size of urban center. Rural populations produce primarily for export outside the immediate region and rely on the various sized urban centers for residentiary and business-related goods and services. These parameters (defined in equations (1) and (2) below) can be used to predict the populations of larger cities and regions from empirical estimates calculated for the smallest towns and their complementary regions. In this paper, first order (or level) refers to the smallest towns and their rural areas, and third order refers to the entire region with the largest city termed the *labor market capital.*

Beckmann-McPherson Equations

The respective urban and regional predicting equations from Beckmann-McPherson (1970) are given as follows:

$$(1)\ c_n = \sum_{i=1}^{n} \left[\frac{k_i}{1 - \sum_{j=1}^{n} k_j} \right] s_i$$

$$(2)\ P_n = \frac{s_1}{1 - k_1} \prod_{i=1}^{n-1} \frac{(1 + m_i) + (1 - K_i)}{1 - K_{i+1}}$$

where:

- c_n = Population of the central place at the n^{th} order;
- s_i = Complementary population at the i^{th} order;
- k_i = Ratio of urban servicing population to itself plus the complementary population at the i^{th} order;
- P_n = Population of the total region at the n^{th} level;
- m_i = Satellite number at the $(n\text{-}1)^{th}$ level; and

$$K_i = \sum_{i=1}^{n} k_i.$$

Several characteristics of these prediction equations should be noted:

- Estimates of the higher order populations depend on calculations at the lowest order which, for our three level rural labor market, includes c_1 (the smallest rural community) and s_1 (the rural population serviced by c_1).[2] Satellite numbers (m_i) are independent estimates at the second and third order levels, but the k_i at these levels are residuals after estimating lower order k's;
- The k_i reflect goods and services demand and delivery efficiencies for the hierarchical good, and their summation to any level will estimate the degree of urbanization. Factor prices should not vary much within a labor market. A similarly minor variation is expected for delivery technologies for low order goods and services;
- Predicted populations will increase with increases in k_i, s_i, and m_i and converge on central place norms if regions previously were underpopulated. For U.S. rural labor markets the rural population (s_1) declines on trend, and m_i is expected to be stable as town births and incorporation dissolutions are infrequent.

The following calculations from equations (1) and (2) are used in characterizing alternative regional structures.

Net prediction errors are $[(\hat{c}_n - c_n)/c_n]$ and $[(\hat{P}_n - P_n)/P_n]$ which are averaged over urban and regional units at a particular central place level. Their sign and magnitude across central place levels indicate the extent of decentralization or centralization within the labor market. Gross prediction errors utilize $|(\hat{c}_n - c_n|$ and $|\hat{P}_n - P_n|$ in the numerator, weighting negative and positive errors equally to indicate how well the model fits observed values; these also are averaged over urban and regional units at a given central place level.

Estimates of the productivity coefficients ($\hat{k}_i$) are made at each level. Var ($\hat{k}_i$) for a particular level indicates the intralevel heterogeneity in the provision of higher order goods and services. The variance in urbanization ratios is a broader indicator of this heterogeneity. Because our region has a single labor market capital, these measures are estimated for the first two central place levels.

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[2] The next highest orders are the county and regional capital with their respective complementary regions. Note that some averaging occurs here with bedroom towns offset by employment centers. The farm wife who is a school teacher may be balanced by the retired town resident who works part time on a nearby farm. An exact allocation to the appropriate urban and rural categories would require unavailable detailed occupational data for small towns and rural areas. The towns in the region include all incorporated and unincorporated communities.

The measures in both equations (1) and (2) above may converge or diverge over time, indicating the pattern of temporal adjustment to the predictions of the Beckmann-McPherson model. As central place models predict population levels (and not variances in key parameters), the emphasis here is on the measures in equation (1) above.

Regional Typologies

For the first characteristic of the typology, regions are termed *strongly centralized* if negative net prediction errors consistently increase for both equations (1) and (2) from the lowest to highest order urban and regional units. Both the urban and complementary populations are increasingly larger than predicted, as the size of region increases and *strongly decentralized* labor markets would have the opposite sign pattern. Strongly centralized regions are likely to have communications and transit flows oriented toward the labor market capital with enough suppliers of the marginally hierarchical goods bundle at every central place level to satisfy intraregional demand adequately.[3] Strongly centralized regions also may be relatively isolated from competition with higher order urban centers in contiguous regions that otherwise might attract customers from the subject labor market region.

Mixed cases also may occur for this first characteristic as, for example, where urban populations [equation (1)] are centralizing but the P_n [equation (2)] are not. Low rural densities contribute to weak complementary regions and possibly fewer satellites around a second order urban center. Large average farm sizes may be due to technological mechanization resulting in smaller average household sizes and lower goods and services demand. Residentiary goods sales will be more self-contained within the variously sized urban centers along the hierarchy with more significant inter-urban transit and communications flows.

The second attribute of the typology is the degree of intraregional specialization where unspecialized regions have minor variations in k_i among subregions at a given central place level. If real incomes per household and preferences vary little among subregions at the i^{th} level, then the private component of k_i also should have a low variance. In unspecialized regions, goods demand is likely to be met locally with fewer shopping trips to alternative destinations within the same central place level. Urban populations also will have a public sector component that will increase the variance in k_i if demands for public services are not

[3] Reduced transit costs usually favor larger market areas, and centralized radial transit systems usually encourage higher order establishments in the regional capital. This pattern also encourages land price differentials between the regional capital and peripheral subregions that would be smaller with decentralized transit networks. Economies of scale also may favor centralized locations within the region.

uniform at a particular central place level.[4] Some of these services will be provided solely to residents of a particular central place and cannot be satisfied by trips taken within the region.

Mixed cases also may occur for this characteristic where the VAR (k_i) are different by central place level. If the lowest order variance [i.e., VAR (k_1)] is zero but VAR (k_2) is large, then higher order goods demand may be satisfied only by trips across second order regional levels. Shorter trips among the lowest order communities are likely if the variance among levels is reversed. As the lowest order goods and services are primarily residential and business-related necessities, the VAR (k_i) may increase with central place level, if the k's accurately measure the private sector component only. The public services component may result in similar transit patterns for the two cases noted above, subject to the qualification regarding service provision only to residents.

The last attribute is temporal change in the central place system which may be due to numerous factors both internal and external to the labor market. The successful substitution of labor-intensive secondary employment for a declining agricultural sector may encourage convergence in predicted central place populations in underpopulated regions. The addition of a high income employment component that may result from this transition may encourage higher order goods and services establishments in the larger communities of the region. Various locational patterns affecting the variance in urban populations (ranging from a single large manufacturing plant to a more even spatial growth of small establishments) are possible. Agricultural decline in locationally disadvantaged contiguous regions is an external factor that may contribute to this change in employment composition within the labor market.

This typology can be generalized to a substantial number of alternative regional structures, each with its own set of development strategies. The sign patterns of prediction errors in equations (1) and (2) have numerous possible combinations. For a given date, the relative levels of urban and regional prediction errors also can be categorized (i.e., whether the errors are increasing or declining individually for both the urban and regional populations as between orders (2) and (3)). Various degrees of variation in the k_1 and k_2 subregional parameters and extensions beyond the three level labor market considered here will add further to the number of regional structures. Though the alternative possibilities are numerous, the discussion below emphasizes methodological issues and planning inferences for the three level example region.

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[4] Note that specialized regions may have low variances in k_1 and k_2 if the public and private employment components tend to offset each other. Detailed primary occupational surveys at each central place level would be necessary to determine this possibility.

Empirical Estimates

The empirical tests are conducted on a detailed data set for a nine county rural labor market in southwest Iowa for the years 1970 and 1984. The data include the population size distribution of towns for each year supplemented with county totals for the rural and urban populations. Sectoral employment and occupational data are not available for towns this small. In any case, central place models are based on population distributions, and empirical tests should be conducted on demographic information. Creston in Union County is the labor market capital, with the county seat towns and the smallest rural communities comprising the lower two levels of the central place hierarchy.

Several general population characteristics influence the estimates of the central place parameters.

- The total regional population (P_3) increased slightly over the period, with a 1984 population of 86,652 and a decline in percent rural from 42.6 percent (1970) to 40.1 percent (1984);[5]
- Counties have variant total populations and urbanization ratios at the P_2 level. (The 1970 urbanization ratios, for example, ranged from 73.7 percent (Union County) to 42.8 percent (Adams County)). Though urbanization ratios among counties were fairly stable, percent changes in total county populations showed some variation over time;
- The county seats (at the P_2 level) also had a wide population range but fairly stable total population rankings over the 1970-1984 period. See Abu-Hijleh (1989, pp. 12-19) for a more detailed analysis of these population characteristics.

Averaged Parameter Estimates

Table 1 summarizes characteristics of the first method of estimating the central place parameters. The rural complementary populations (s_1) are the county rural populations divided by the number of towns providing first order goods. k_1 is the ratio of the average population of first order towns to itself plus s_1. The k_2 and k_3 are obtained as residuals for second and third order goods after subtracting the populations needed to provide lower order goods. Thus, the county's k_1 is multiplied by (s_1 plus the county seat population) to estimate the persons in the county needed to provide first order goods. The balance of the county seat population provides second order goods; k_2 is the latter divided by

[5] The year 1980 also was included in the original analysis but was excluded for brevity in this paper. The 1970-1980 decade was favorable for agriculture but the 1960-1970 decade was characterized by a 10,000 population loss for the entire region. Thus, the 1970-1984 period is characterized as a stable growth period for the entire region; the 1980 data are included in the analysis in Abu-Hijleh (1989). This region is chosen due to long-term development problems. The database is keyed to two census dates, with 1984 included because of special census tabulations for the populations of the towns and rural areas at that date.

the county population. Average k_2 values have been applied to the labor market capital (Creston in Union County) along with Union's k_1 to calculate the k_3 estimate for the third order city.

Several characteristics should be noted from Table 1. As expected the k_i declines as central level increases. Over the 1970-1984 period k_1 rises and k_2 and k_3 decline. The decline in s_1 is consistent with the increase in urbanization noted above. The specialized nature of this region is indicated by the wide among-county range of the central place estimates. The high-low figures are shown in Table 1 along with the ratio of the range to mean values in the last row. For s_1, k_1 and k_2 two estimates have range values exceeding 100 percent of the mean, with the rest ranging from 75 percent to 96 percent of their averages. The relative variations for s_1 and k_2 increase from 1970 to 1984, while the variation for k_1 declines. This intraregional diversity suggests a heterogeneous rural labor market at the lower levels of the central place hierarchy.

Regional Estimates

As a check on the averaging method, various regression specifications of equation (1) are used to estimate the k_i.[6] These regressions use average data for the nine counties based on an average of the 46 lowest order satellites for the 1970 and 1984 years. The k_1 parameter is from a homogeneous regression of the form, $c_1 = b_1 s_1$ where the coefficient b_1, equals $k_1/(1 - k_1)$ and there are eight degrees of freedom. The k_2 parameter is a direct regression estimate using $(c_2 - k_1 c_2)$ as the dependent variable and the total county population as the independent variable with eight degrees of freedom. These separate regressions are used as joint estimations of the first two right side terms in equation (1) do not produce satisfactory statistical results. Nonhomogeneous linear versions of these regressions produce less satisfactory statistical results and are, in any case, inconsistent with an economic base interpretation of central place models. (If c_1 is positive with $s_1 = 0$, then $k_1 = 1.0$ and there is no sector earning income from sales external to the region.) Quadratic and double log versions of these regressions also produce no evidence of statistically significant nonlinearities. All regression estimates utilize ordinary least squares techniques with no evidence of heteroscedasticity. See Abu-Hijleh (1989, pp. 24-35) for a more detailed discussion of these regressions.

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[6] Haining (1980), Horn and Prescott (1978), and Suarez-Villa (1980, 1982) provide methods of estimating k_i. The calculations here come closest to Haining's intraregional regression specification.

Table 2 shows characteristics of these regressions. The k_1 regressions have highly significant evaluative statistics (t-ratios, F and $\bar{R}^2$) and the estimated k_1 rises slightly over the 1970-1984 period as with the estimates in Table 1. The k_1 estimates are only slightly lower than the coefficients in Table 1. The k_2 regressions have even better evaluative statistics with slightly higher values than in Table 1. (The regression coefficients in Table 2 are direct estimates of k_2.) As with the k_2 estimates in Table 1, the regression k_2 coefficients decline over the 1970-1984 period. See Abu-Hijleh (1989, pp. 24-31) for a discussion of similar results attained for regression estimates with pooled data.

Rank size distributional regressions for the 1970 and 1984 data sets show a slightly declining conformity to central place regularity. These estimates are provided at the bottom of Table 2 and are based on data for the urban populations at all three levels. Evaluative statistics (t ratios parenthesized in Table 2, F and $\bar{R}^2$) decline over the 1970-1984 period, with a similar decline in the slope coefficient. With the log of the town's population as the dependent variable this coefficient change suggests an increasing dominance of the regional capital (Creston), but the latter's negative residual actually rose over the 1970-1984 period. The slope decline is due to an increase in the standard deviation of the 55 town population observations from 1,506 (1970) to 1,550 (1984), with an average size increase from 914 to 941 persons. The estimated urban population rose about ((941-914) (55) = 1,485), consistent with a relatively stable total regional population and the slight decline in the percent rural noted above. The rank-size distributional calculations are discussed in more detail in Abu-Hijleh (1989, pp. 45-53).

Prediction Errors for the Central Place Models

Table 3 shows the prediction errors for the Beckmann-McPherson model with both methods of k_i estimation for calculating errors.[7] The averaged k_i's account for the variability among regional subunits and provide evidence regarding the second characteristic of the typology. The regressions k_i also provide a check on the averaged k_i, but in the present case they are comparable due to the linear homogeneous regression specification.

The average prediction errors in Table 3 are all positive and increase from the county to regional levels. The model consistently predicts larger populations than are observed for both dates. For

[7] The substantial number of calculations embodied in Table 3 is discussed in Abu-Hijleh (1989, pp. 37-44). The comparison to the Parr model predictions indicate the latter are particularly high for the regional capital. Also analyzed in this reference is an index of the ratio of net error to absolute error (ranging from +1.0 to −1.0) which assesses the significance of the mix of positive and negative errors in the predictions.

example, the county seat (c_2) predictions vary from 22.38 percent to 32.63 percent across the two methods of parameter estimation. For the regional capital (c_3), the range of these errors increases from 39.50 percent to 126.75 percent. A similar pattern occurs for the county (P_2) and the entire region (P_3)—population deficits become increasingly more serious as the regional unit size increases.

The average prediction errors also indicate a mixed pattern of relative errors for the urban centers and their complementary areas. The four errors for P_2 are respectively lower than comparable estimates for c_2, while the reverse pattern occurs when comparing P_3 and c_3. Underpopulation in the small towns and rural farms at the county level is significantly less serious than for all regional units outside of the labor market capital. The county seats appear to be a source of low complementary populations for c_3, as the first order towns and farms are included in the P_2 estimates. An important contributing factor to the small size of c_3 are the relatively lower populations outside c_3 but within the entire region.

The convergence/divergence characteristics of Table 3 suggest a mixed pattern. Three of four comparisons at the county seat and county levels from 1970 to 1984 suggest increasing errors or divergence, but the changes are not substantial. The patterns are reversed at the c_3 and P_3 levels, with the regional capital converging and the entire region diverging from population predictions from the model. Thus, the county changes can be typified as stable while only the regional capital appears to be catching up over the 1970-1984 period.

The absolute prediction errors per resident show a somewhat similar pattern and indicate how well the model fits the regional data set. The c_2 errors are slightly larger than the errors for P_2, and all errors increase slightly from 1970 to 1984 at the county level. The errors for the regional capital are generally larger than for c_2 and P_2 and show a convergent pattern over the period. The average k_i errors for P_3 are larger than the c_3 errors but show only a slightly divergent pattern. The regression k_i estimates are low.

The following central place structural patterns are relevant to the regional typology.

- Compared to central place norms, population deficits occur at all place levels but are increasingly more serious as the place level increases. Complementary area populations are particularly weak for the regional capital and relatively stronger at the county seat level;
- From Table 1, the k_1 and k_2 estimates have a high intraregional variance with the variance in k_2 rising over time and the variance in k_1 declining;
- There is a mixed pattern of divergence and convergence to the model's population predictions over time, but the regional capital's population deficit significantly declines over the 1970-1984 period.

Regional Planning and Concluding Remarks

The empirical tests suggest that regional planning strategies should be based on a decentralized, specialized structure with convergence to central place predictions exhibited mainly at the regional capital level. The following planning strategies are suggested from the regional characteristics exhibited by our example labor market.

Regional Planning Strategies

Development strategies that rely on strong leadership from the labor market capital should be questioned despite the convergence of the region's population to central place predictions from 1970 to 1984. The differences between prediction errors at levels two and three suggest a strongly decentralized region. Multicounty planning agencies probably will require a broad representation of different interests throughout the region with an emphasis on decentralized planning. Center-periphery relationships will be relatively less significant than in a region with a dominant labor market capital. A similar conclusion will be applicable at the county level. (The c_2 prediction errors in Table 3 do not suggest strong county seat leadership at the second order regional level).

Transit and communication flows also will be more decentralized and probably less predictable than in regions with dominant labor market capitals. With decentralized and somewhat erratic population change in outlying counties, large fixed investments are risky and resources need to be geographically flexible. Hinterland counties may have more significant linkages within their own counties or with other outlying counties than with the central county. Transit flows external from this region are likely due to two MSAs (Des Moines and Omaha-Council Bluffs) located to the east and west with counties contiguous with this rural labor market. Planning organizations that can promote cooperation among outlying counties will be most successful in fulfilling needs in these areas.

Wide variations in k_i and urbanization ratios may reflect heterogeneous preferences for public and private goods and services. (More detailed primary surveys would be advisable to estimate the private and public employment components within the urban communities.) Public service programs in specialized regions may be more varied with a smaller common basis for administrative coordination than in more homogeneous regions. For example, school districts vary substantially in size and emphasis within this region. This leads to problems with voluntary efforts at consolidation and administrative cooperation. Long-run population stability or decline warrants cost-cutting efforts in services consolidation at all governmental levels, but this may be less easy to attain in specialized, heterogeneous regions.

Heterogeneity, however, suggests the possibility of among-region complementarities not found in regions with a similar distribution of programs and resources. Rural counties with natural resources can provide parks and recreational facilities to residents of urban counties. Urban educational programs in the larger school districts can be extended to smaller rural school districts with limited resources. Higher order cultural programs usually found in larger cities similarly may be extended to smaller communities. Identifying these complementarities could be a major focus for planning in spatially specialized regions.

Patterns of steady, locationally stable population growth would minimize risks for large, locationally fixed public and private sector investments. The entire region is stable, but there are substantial variations in county growth rates; therefore, large population-dependent investments may be risky. Unless the relative population improvement of the labor market capital constitutes a long-run trend, a growth-centers approach to centralize publicly subsidized investments seems particularly ill-advised. Under conditions of population instability, public structural investments should have multipurpose designs, minimum serviceable lives, and little (if any) excess capacity to accommodate future growth.

Uniform and mixed patterns of convergence and divergence among regional units may occur over time. Uniform convergence suggests a balanced pattern of population change, with prediction errors for all regional units moving toward central place norms. The total regional population may be declining or increasing, but distinctions among urban centers and their complementary areas at all central place levels will be difficult to make. The convergent–mixed pattern suggests relative strengths and weaknesses in the population change between urban centers and their complementary regions. Planners should be able to identify these patterns and determine whether strategic planning actions are consistent with these changes. In the present case, investments in the regional capital may be complementary with its convergence, but it is likely that substantial investments would have to be made to strengthen the regional capital's complementary region. (Such decentralized investments, of course, may meet other development objectives at the lower order level.)

In summary, the typology estimated for the example labor market suggest difficulties for regional planners at this spatial level. Though labor markets are particularly attractive regions for planning purposes (as they internalize the place-of-work and residence of the regional population), the somewhat weak and variant patterns of population change among urban communities suggest difficulties in anticipating locational needs for public and private investments within the region. This does not preclude, however, other intraregional types of planning efforts that may reduce tax levels and public service costs within this labor market. State legislation, for example, authorizes city–city and city–county governmental consolidation and other public service sharing (e.g., police and fire protection) among communities. Another planning effort could be focused on attaining external program funding for specialized populations such as the elderly that have become an increasing proportion of households in Iowa's rural labor markets. The typology suggested here can characterize the central place general population structure of a region, but not all regional policies and programs are oriented primarily to aggregate population characteristics.

Concluding Remarks

This paper uses population-based central place theory as a norm for assessing the under- or overachieving status of urban areas and their complementary regions along the place hierarchy. Such theoretical norms are necessary to make any sense from raw population data on the distribution of central places. Also, many variants of the structural typology suggested here are possible that would extend planning inferences based on more simplistic core–periphery models. Additional applications would suggest the extent to which regional structures deviate from the typology developed here and, hence, the variety of planning considerations that would be applicable at this regional level.

Two final comments conclude this paper. The large prediction errors in Table 3 may be due to unique characteristics of the example region. Agricultural productivity historically has been lower in the southern tier of Iowa's labor markets, so the rural economic base is relatively weak. The regional capital also has been an underachiever historically in public and private political and business leadership compared to other rural labor market capitals in Iowa. The planning inferences suggested above are general, of necessity, but are indicators of more detailed surveys that might be conducted on specific private and public sectors. Aggregate population-based applications cannot be the sole database for detailed sectoral planning. But if central place theory is to have planning and policy implications, models either must incorporate specific policy variables in their structure or allow for regional structural typologies on which planning inferences can be made. As the former has not been forthcoming, the latter approach has been adopted here.

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Table 1—Average Central Place Estimates 1970–1984

	s_1 1970	s_1 1984	k_1 1970	k_1 1984	k_2 1970	k_2 1984	k_3 1970	k_3 1984
Mean	835	782	.325	.345	.213	.205	.070	.041
High	1,217	1,328	.453	.492	.313	.324		
Low	450	473	.185	.232	.108	.114		
Range/Mean	.918	1.093	.824	.753	.962	1.024		

Table 2—Central Place Parameters and Rank–Size Statistics From Ordinary Least Squares Regressions

k_1	b (t-ratio)	F	$\bar{R}^2$	k_1
1970	.450 (7.81)	60.9	.88	.31
1984	.496 (6.81)	46.4	.85	.33

k_2	b (t-ratio)	F	$\bar{R}^2$
1970	.223 (9.21)	84.8	.93
1984	.214 (9.42)	88.0	.93

Rank size regression log (population) = f log (rank)	Intercept	Slope Coefficient	F	$\bar{R}^2$
1970	9.91 (86.19)	-1.273 (-35.0)	1,231	.954
1984	10.02 (75.47)	-1.298 (-31.1)	969	.948

Table 3—Prediction Errors for the Beckmann-McPherson Model

	c_2 1970	c_2 1984	P_2 1970	P_2 1984	c_3 1970	c_3 1984	P_3 1970	P_3 1984
Average Prediction Error (%)								
Average k_i	22.38	22.50	17.63	20.00	126.75	50.50	141.88	162.13
Regression k_i	34.00	32.63	16.50	19.38	121.88	39.50	122.82	133.58
Absolute Prediction Error per Resident								
Average k_i	1.72	1.73	1.48	1.62	10.15	4.05	11.37	11.98
Regression k_i	3.39	3.42	2.51	2.97	9.76	3.17	1.23	1.14