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Economic Surplus Measurement in Multi-Market Models *

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Abstract

Despite continuing controversy, economic surplus concepts have continued to be used in empirical cost-benefit analyses as measures of welfare to producers and consumers. In this paper, the issue of measuring changes in producer and consumer surplus resulting from exogenous supply or demand shifts in multi-market models is examined using a two-input and two-output equilibrium displacement model. When markets are related through both demand and supply, it is shown that significant errors are possible when conventional economic surplus areas are used incorrectly. The economic surplus change to producers or consumers should be measured sequentially in the two markets and then added up.

Key Words: equilibrium displacement model, multi-market, economic surplus, R&D evaluation

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Introduction

Exogenous changes in one sector of an industry have spill-over effects in other vertically- and horizontally-related markets. Estimating these spill-over or general equilibrium effects has been important in the evaluation of the impacts of government interventions and R&D and promotion investments. These impacts have generally been measured in terms of economic welfare changes to individual industry sectors and consumer groups. Producer and consumer surpluses are typically used as measures of welfare changes (Just, Hueth and Schmitz 1982; Alston, Norton and Pardey 1995).

Just, Hueth and Schmitz (1982) presented a complete and rigorous analysis of general equilibrium welfare effects within vertical and horizontal market structures. They described two approaches that can be used to analyse multi-market equilibrium welfare effects (pp.469-470). One way is to estimate the total economic welfare change in the single market where the exogenous change is introduced, as the sum of producer and consumer surplus changes measured off the *general equilibrium* (GE) supply and demand curves in that market. This approach gives the overall welfare effect but not its distribution among individual market sectors. The alternative approach that can provide the often-needed information about the distribution of welfare effects, is to *sequentially* evaluate the welfare effects off the *partial equilibrium* (PE) supply or demand curves in all individual markets and then add them up to give total welfare effects. The two approaches provide consistent measures as long as the demand and supply functions in all markets are integrable; that is, as long as they are consistently derived from a set of underlying decision functions.

Thurman (1991a, b) concentrated on the issue of measuring multi-market welfare effects in a single market. In particular, he studied the welfare significance and non-significance of general equilibrium demand and supply curves in a single market. He pointed out that when the equilibrium feedback comes from both demand and supply channels in the intervened-in market (or, in other words, when there is more than one source of equilibrium feedback), although the sum of consumer and producers surpluses from the general equilibrium demand and supply curves still measures the total welfare change, the individual measures no longer have welfare significance.

While it is valuable to be able to measure the general equilibrium welfare effect of an exogenous change in a single market, especially when it is difficult to obtain data from all related markets, partial equilibrium analysis in individual markets is also desirable for the information it provides on the distribution of welfare changes between market sectors. For example, in many applied studies where data are available, multi-market equilibrium models involving several industry sectors are often specified. It is important to measure the welfare effects to individual industry groups resulting from policy interventions or research or promotional investments (for example, Zhao 1999; Piggott, Piggott and Wright 1995; Mullen, Wohlgenant and Farris 1988). While the literature acknowledges that changes in welfare of different sectors can be measured off partial equilibrium curves in different markets even when there are multiple sources of feedback, the procedures for doing this are not intuitive and are rarely explained.

In this paper, a hypothetical two-input, two-output model is used to empirically demonstrate the meaning and significance of the various economic surplus measures off both partial equilibrium and general equilibrium curves in multiple markets. The analytical approach in

Thurman (1991b) is used to examine the relationship between the analytical welfare integrals of underlying decision functions and the conventional 'off-the-curve' economic surplus areas. It is shown with a numerical example that significant errors are possible when failing to measure the partial equilibrium effects in a *sequential* manner, as evident in some existing studies.

It should be pointed out that economic surplus changes measured along the ordinary/uncompensated demand and supply curves are the focus in this exercise. As a result, the surplus change measures approximate the exact willingness to pay measures to the extent that noncompensated equilibrium approximate compensated equilibrium. The well-known result by Willig (1976) and Hausman (1981) for a single market and the multi-market result by LaFrance (1991) suggest that the errors of this approximation are likely to be small if the focus is the trapezoid areas of welfare changes rather than the triangular 'deadweight loss'. The derivation in this paper (see also Zhao 1999, Chapter 6) also suggests that integrability conditions may only affect the second-order terms when the considered exogenous shifts are small, and thus the errors are likely to be small when integrability is not satisfied.

A Hypothetical Model

Consider an example where two inputs, X_1 and X_2, are used to produce two outputs, Q_1 and Q_2. Suppose that the supplies of X_1 and X_2 are not related but Q_1 and Q_2 are substitutes in demand for final consumers. For example, pigs (X_1) and other processing inputs (X_2) are used to produce pork (Q_1) and bacon and ham (Q_2). In this case, the two inputs are supplied by separate decision makers, i.e. pig producers and processors, but consumers will adjust their consumption of the two products according to the relative prices of Q_1 and Q_2. Suppose that the purpose of the study is to evaluate the individual welfare implications to farmers, processors and consumers resulting from two exogenous shift scenarios: (1) an exogenous supply shift in the X_1 market, say, due to a research-induced productivity gain in pig production, and (2) an exogenous demand shift in the Q_1 market, due to pork promotion that increases the consumers' willingness to pay.

As discussed in the Introduction, total welfare gains can be obtained from the general equilibrium curves in a single market (the X_1 and Q_1 markets respectively for the two scenarios). To evaluate the welfare gains to individual industry groups, welfare measures from the partial equilibrium curves in individual markets are required.

Because X_1 and X_2 are not related in supply, their supply functions are determined exogenously and do not shift endogenously. The only source of equilibrium feedback in each factor market comes from the demand side where X_1 and X_2 are related because of the possibility of input substitution. Consequently, the producer surplus changes measured off the supply curves in the X_1 and X_2 markets are estimates of welfare implications to pig producers and processors, respectively.

However, the two products Q_1 and Q_2 are related in both demand and supply, and both demand and supply curves shift endogenously. On the supply side this relationship arises because pigs and processing inputs are used to jointly produce either pork or bacon. This is the case that Thurman (1991a, b) identified as having two sources of equilibrium feedback where individual general equilibrium supply and demand curves have no welfare significance. Hence, in order to measure the welfare distribution among industry groups, surplus changes must be measured off ordinary partial equilibrium curves.

In the following, general equilibrium and partial equilibrium surplus measures in Q_1 and Q_2 markets are studied. Two alternative approaches to estimating welfare changes to individual industry groups are demonstrated with a numerical example. It is evident from the example that significant errors are possible when failing to measure the partial surplus areas in a sequential manner. Some examples from the literature where incorrect processes have been followed are identified. Details of the technical specification and parameter values for the numerical example are given in the Appendix.

The two scenarios examined in this paper are first an exogenous shift in supply in a factor market (new technology in producing pigs for example), and second, an exogenous shift in final demand (through the promotion of fresh pork, for example).

An Exogenous Shift in Factor Supply

Suppose that a new technology is adopted in pig production and consequently the unit cost of producing pigs is reduced by $|K|$ for all output levels where $K<0$ is a constant (i.e. the supply shift is parallel).

Consider first the welfare changes to the two producer groups, pig producers and processors. Assume that the industry level profit function for pig producers is $\pi(w_1, \tilde{w})$, where w_1 is the price of X_1 and $\tilde{w}$ is the vector of other prices affecting the profit function, which is exogenous to the model. $\tilde{w}$ is assumed constant during the displacement and is therefore suppressed in the following discussion. Consequently, the profit function is shifted from $\pi(w_1)$ to $\pi(w_1-K)$ and its partial derivative, the supply curve, from $S(w_1)$ to $S(w_1-K)$. Referring to Figure 1, in the first instance, the initial downward shift in the pig supply curve reduces the equilibrium price of pigs. The decrease in the pig price then induces supply shifts in the Q_1 and Q_2 markets and changes their relative prices and quantities. As a feedback effect of the output price and quantity changes, the demand curve for pigs is also shifted up endogenously. A new set of equilibrium prices and quantities is eventually reached in all markets.

Suppose the initial price and the new price for pigs are $w_1^{(1)}$ and $w_1^{(2)}$ respectively. The change in pig producers' welfare is the change in their profit before and after the displacement:

$$\begin{aligned}\Delta\pi &= \pi(w_1-K)\big|_{w_1=w_1^{(2)}} - \pi(w_1)\big|_{w_1=w_1^{(1)}} = \pi(w_1^{(2)}-K)-\pi(w_1^{(1)}) \\ &= \int_{w_1^{(1)}}^{w_1^{(2)}-K} \frac{\partial\pi(w_1)}{\partial w_1}\,\mathrm{d}w_1 = \int_{w_1^{(1)}}^{w_1^{(2)}-K} S(w_1)\,\mathrm{d}w_1 = \int_{w_1^{(1)}+K}^{w_1^{(2)}} S(w_1-K)\,dw_1 .\end{aligned} \tag{1}$$

In Figure 1, the second last expression relates to the producer surplus area measured off the original supply curve from $w_1^{(1)}$ to $w_1^{(2)}$-K, and the last expression to that off the new supply curve; that is, the dotted trapezoid area $ABCE^{(2)}$ for the last expression.

If the amount of shift K is represented as a percentage of initial price $w_1^{(1)}$, i.e. $t_{x1} = K/w_1^{(1)}$, and the proportional changes in price and quantity are represented as $Ew_1=(w_1^{(2)}- w_1^{(1)})/ w_1^{(1)}$ and $EX_1=(X_1^{(2)}- X_1^{(1)})/ X_1^{(1)}$, respectively, it can be shown that the welfare change to pig producers, that is, the last integral in Equation (1) given by area $ABCE^{(2)}$, can be calculated as

(2) $\Delta PS_{X1} = w_1^{(1)}X_1^{(1)}(\mathrm{E}w_1 - t_{X1})(1+0.5\mathrm{E}X_1)$ *pig producers surplus*

The supply curve for processing inputs X_2 is not shifted, and the welfare change for processors can be measured similarly as the producer surplus change off the stationary supply curve for X_2 as

(3) $\Delta PS_{X2} = w_2^{(1)}X_2^{(1)}\mathrm{E}w_2(1+0.5\mathrm{E}X_2)$ *processors surplus*

Now consider the welfare implications for the final consumers and the significance of the various GE and PE surplus areas in the Q_1 and Q_2 markets. The GE feedback comes from both supply and demand channels. Consider two alternative approaches to measure consumer welfare change.

Measuring from the GE curve in a single market

Based on Just, Hueth and Schmitz (1982) and Thurman[1] (1991b), when the X_1 supply curve is exogenously shifted down by a percentage, t_{X1}, the total welfare gain to the 'whole society' (ΔTS) can be measured in the X_1 market alone. In particular, ΔTS is the sum of the producer surplus change measured off the exogenously determined supply curve of X_1 and the consumer surplus change measured off the *general equilibrium* demand curve for X_1.

Figure 1 illustrates the X_1 market for Scenario 1. As discussed earlier, the producer surplus change to pig producers is given by area ABCE$^{(2)}$ and Equation (2). The partial equilibrium (or *conditional*) demand curve for X_1 has been shifted up endogenously from D$^{(1)}$: D(w_1| $P^{(1)}$) to D$^{(2)}$: D(w_1| $P^{(2)}$), where $P^{(1)}$ and $P^{(2)}$ are the levels of all other prices in the model before and after the equilibrium displacement. E$^{(1)}$ and E$^{(2)}$ are the old and new equilibrium points. The line connecting E$^{(1)}$ and E$^{(2)}$, denoted D*, is the general equilibrium demand curve for X_1 that traces the demand-price relationship for different levels of t_{X1} and P. The change in consumer surplus area measured off D* is given by

(4)
$$\Delta CS^*_{X1} = \int_{w_1^{(2)}}^{w_1^{(1)}} D^*(w_1)dw_1 = Area(CDE^{(1)}E^{(2)})$$
$$= -w_1^{(1)}X_1^{(1)}\mathrm{E}w_1(1+0.5\mathrm{E}X_1).$$

In this case, ΔCS_{X1}^* measures the benefits to pig processors and final consumers. Using the expression for ΔPS_{X1} in Equation (1), the total welfare change is given by

[1] For the case when the two products are related in demand but not in supply, the derivation of this result via integrals is given in Thurman (1991b, pp.2-7), and is not repeated here.

$$\Delta TS = \Delta PS_{X1} + \Delta CS^{*}_{X1} = Area(ABDE^{(1)}E^{(2)})$$

$$= \int_{w_1^{(1)}+K}^{w_1^{(2)}} S(w_1 - K)dw_1 + \int_{w_1^{(2)}}^{w_1^{(1)}} D^{*}(w_1)dw_1$$

(5) $$= w_1^{(1)}X_1^{(1)}(Ew_1 - t_{X1})(1+0.5EX_1) - w_1^{(1)}X_1^{(1)}Ew_1(1+0.5EX_1)$$

$$= -w_1^{(1)}X_1^{(1)}t_{X1}(1+0.5EX_1).$$

Thus, the benefit to consumers can be obtained as the residual as

(6) $$\Delta CS_Q = \Delta TS - \Delta PS_{X1} - \Delta PS_{X2},$$

where ΔPS_{Xi} (i = 1 and 2) are given in Equations (2) and (3).

The formulas for estimating consumer welfare changes via GE curves are summarised in the first column of Table 1.

Welfare impacts from a productivity gain in the processing sector, which can be modelled as an exogenous supply shift in X_2 market, can be obtained similarly.

Measuring Directly from PE Curves in Individual Markets

Alternatively, the welfare change to consumers can be measured directly as the consumer surplus areas off the partial equilibrium demand curves in the Q_1 and Q_2 markets.

Consider the two output markets in Figure 2 when the cost of pig production is reduced. The expenditure function for Q_1 and Q_2 consumers and its derived demand functions are not changed by the exogenous shift in the supply of pigs. They are denoted as e(p_1, p_2, $\tilde{p}$) and $D_1^h(p_1, p_2, \tilde{p})$ and $D_2^h(p_1, p_2, \tilde{p})$, for both before and after the displacement, where $\tilde{p}$ is the vector of other prices outside the model that affect the consumers expenditure. $\tilde{p}$ is suppressed below without losing generality. The profit function and the derived supply functions for Q_1 and Q_2 *are* changed as a direct result of the initial shift in pig supply. In particular, in the first instance, both supply curves for Q_1 and Q_2 are shifted down. Because the two products are related in both demand and supply, as second round effects, both the conditional demand and supply curves are shifted further as the result of relative price changes between the two meat products. The situation is illustrated in Figure 2.

Note that if it is assumed that all profit and utility functions in the model are quadratic and all demand and supply functions are linear around the local areas of the initial equilibrium, a parallel initial shift in the supply of pigs (X_1) implies that all induced shifts in other markets are also parallel around the local areas.[2]

[2] In this example, the initial shift K in the pig market changes the profit function of Q_1 and Q_2 producers from $\pi(p_1, p_2, w_1, W)$ to $\pi(p_1, p_2, w_1\text{-}K, W)$, where w_1 is the price of pigs and W is the price for all other prices in the model. The conditional demand curves before and after the shift for Q_1 are $S_1(p_1 | p_2^{(1)}, w_1^{(1)}, W^{(1)})$ and $S_1(p_1 | p_2^{(2)}, w_1^{(2)}\text{-}K, W^{(2)})$. If $\pi(.)$ is quadratic, changing the values of other prices or subtracting another price variable with a constant only changes the intercept of the conditional supply curve, which implies a parallel shift of the linear supply curve.

Following the approach in Thurman (1991b), as the expenditure function is unchanged, the changes in the Q_1 and Q_2 consumers' welfare can be measured as

$$
\begin{aligned}
-\Delta e &= -(e(p_1^{(2)}, p_2^{(2)}) - e(p_1^{(1)}, p_2^{(1)})) \\
&= -(e(p_1^{(2)}, p_2^{(1)}) - e(p_1^{(1)}, p_2^{(1)}) + e(p_1^{(2)}, p_2^{(2)}) - e(p_1^{(2)}, p_2^{(1)})) \\
(7) \qquad &= -(\int_{p_1^{(1)}}^{p_1^{(2)}} \frac{\partial e(p_1, p_2^{(1)})}{\partial p_1} dp_{nd} + \int_{p_2^{(1)}}^{p_2^{(2)}} \frac{\partial e(p_1^{(2)}, p_2)}{\partial p_2} dp_2) \\
&= \int_{p_1^{(2)}}^{p_1^{(1)}} D_1^h(p_1 | p_2^{(1)}) dp_1 + \int_{p_2^{(2)}}^{p_2^{(1)}} D_2^h(p_2 | p_1^{(2)}) dp_2
\end{aligned}
$$

If the Marshallian demand curves are used in place of the Hicksian demand curves in the above integrals, the welfare change can be approximated, to the extent that the non-compensated equilibrium approximates the compensated equilibrium, by conventional economic surplus areas as

$$(8) \qquad \Delta CS_Q = \int_{p_1^{(2)}}^{p_1^{(1)}} D_1(p_1 | p_2^{(1)}) dp_1 + \int_{p_2^{(2)}}^{p_2^{(1)}} D_2(p_2 | p_1^{(2)}) dp_2$$

That is, the change in the economic surplus of consumers is given by the sum of areas integrated *sequentially* off the partial demand curves in both markets. Note that, in Figure 2, the first integral is area $Ap_1^{(2)}p_1^{(1)}E^{(1)}$ integrated off the *initial* demand curve in the Q_1 market, and the second integral relates to area $BE^{(2)}p_2^{(2)}p_2^{(1)}$ integrated off the *new* demand curve in the Q_2 market. These relate to the dotted areas in Figure 2.

It can be shown that, for local linear demand functions, ΔCS_Q in Equation (8) can be calculated as

$$
\begin{aligned}
(9) \qquad \Delta CS_Q &= \text{Area}(Ap_1^{(2)}p_1^{(1)}\text{E}^{(1)}) + \text{Area}(BE^{(2)}p_2^{(2)}p_2^{(1)}) \\
&= -p_1^{(1)}Q_1^{(1)}\text{E}p_1(1+0.5\eta_{(Q1,p1)}\text{E}p_1) \\
&\quad -p_2^{(1)}Q_2^{(1)}\text{E}p_2(1+\text{E}Q_2 - 0.5\eta_{(Q2,p2)}\text{E}p_2).
\end{aligned}
$$

Two things are worth mentioning at this point. First, the derivation in Equation (7) followed a particular equilibrium path from $\text{E}^{(1)}$ to $\text{E}^{(2)}$; that is, $(p_1^{(1)}, p_2^{(1)})$ to $(p_1^{(2)}, p_2^{(1)})$ first and then $(p_1^{(2)}, p_2^{(1)})$ to $(p_1^{(2)}, p_2^{(2)})$. There is an infinite number of paths for the same displacement from $\text{E}^{(1)}$ to $\text{E}^{(2)}$. For example, considering a path via $(p_1^{(1)}, p_2^{(2)})$ instead of $(p_1^{(2)}, p_2^{(1)})$ in Equation (7), would result in

$$(10) \qquad \Delta CS_Q = \int_{p_1^{(2)}}^{p_1^{(1)}} D_1(p_1 | p_2^{(2)}) dp_1 + \int_{p_2^{(2)}}^{p_2^{(1)}} D_2(p_2 | p_1^{(1)}) dp_2$$

$$= -p_1^{(1)}Q_1^{(1)}\mathrm{E}p_1(1+ \mathrm{E}Q_1 -0.5\eta_{11}\mathrm{E}p_1)$$

$$-p_2^{(1)}Q_2^{(1)}\mathrm{E}p_2(1+ 0.5\eta_{22}\mathrm{E}p_2)$$

As pointed out in Just, Hueth and Schmitz (1982, p470), the demand and supply functions in the model need to satisfy a set of integrability conditions so that they can be consistently derived from a set of underlying decision functions. The implied integrability constraints in terms of the Marshallian elasticities for the current model are discussed in the Appendix. It can be shown that, under the symmetry condition for the Marshallian cross-price elasticities at the equilibrium point (Equation (A-12) and given the specified model in the Appendix, Equations (9) and (10) are exactly the same. In other words, the symmetry condition imposed on the Marshallian elasticities in Equation (A-12) guarantees path independence, or the uniqueness of the partial measure of consumer surplus change.

Second, it can be shown that, under the symmetry condition in Equation (A-12), both the expressions in Equations (9) and (10) can be written as

$$\text{(11)} \qquad \Delta CS_Q = p_1^{(1)}Q_1^{(1)}(n_{Q1}\text{-}\mathrm{E}p_1)(1+0.5\mathrm{E}Q_1)$$

$$+ p_2^{(1)}Q_2^{(1)}(n_{Q2}\text{-}\mathrm{E}p_2)(1+0.5\mathrm{E}Q_2),$$

where n_{Q1} and n_{Q2} are exogenous demand shifts in the Q_1 and Q_2 markets respectively and $n_{Q1}=n_{Q2}=0$ for the case of the exogenous supply shift t_{X1} in Scenario 1.

In Figure 2, the expression in Equation (11) relates to conventional areas of economic surplus changes measured off the curves connecting $\mathrm{E}^{(1)}$ and $\mathrm{E}^{(2)}$ in both markets, that is, area $p_1^{(1)}\mathrm{E}^{(1)}\mathrm{E}^{(2)}p_1^{(2)}$ in the Q_1 market and area $p_2^{(1)}\mathrm{E}^{(1)}\mathrm{E}^{(2)}p_2^{(2)}$ in the Q_2 market. Note that neither of these two areas has significant economic meaning, but the sum of the two areas measures the surplus change to Q_1 and Q_2 consumers.

It is obvious from the above derivation that, without the guarantee of integrability conditions, the measure for economic surplus change in the case of multiple price changes is not unique but path dependent. However, an important insight from this exercise is that integrability conditions may only affect the welfare measures at the second order terms. The first-order term, that is, $p_1^{(1)}Q_1^{(1)}(n_{Q1}\text{-}\mathrm{E}p_1)+p_2^{(1)}Q_2^{(1)}(n_{Q2}\text{-}\mathrm{E}p_2)$ in this example, seems to be the same for alternative paths and does not seem to be affected by the integrability conditions. This may be the reason behind Hausman's (1981) and LaFrance's (1991) empirical results that, as long as the shifts considered are small, the errors from using Marshallian measures or ignoring integrability conditions are insignificant for the trapezoid areas of economic welfare changes, though they could be significant in the measures of triangular areas of 'deadweight loss'. The triangular area is a second order measure ($\mathbf{O}(\lambda^2)$ where λ relates to the amount of the exogenous shift), but the trapezoid area is of first-order in magnitude ($\mathbf{O}(\lambda)$)[3].

Finally, the above derivation for partial measures is also correct if the initial exogenous supply shift occurs in the X_2 market. The formula for the direct measure of consumer surplus changes is summarised in the second column of Table 1.

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[3] In fact, we have run the empirical model specified in the Appendix with a set of Marshallian elasticities that does not satisfy the integrability restrictions. As expected, the consumer surplus changes obtained from Equations (9), (10) and (11) are different, indicating path dependency, but the differences are minor.

An Intuitive (but Incorrect) Partial Measure

Note, however, the sequential partial measures derived in Equation (8) are not the same as the changes between the old and new consumer surplus areas one might intuitively expect. The consumer surplus in the Q_1 market off the initial and new PE demand curves are areas $p_1^{(1)}E^{(1)}C^{(1)}$ and $p_1^{(2)}E^{(2)}C^{(2)}$, giving a difference of area $GHE^{(2)}p_1^{(2)}$. Similarly, the change in consumer surplus areas off the new and old conditional demand curves in the Q_2 market is area $IJE^{(2)}p_2^{(2)}$. It is tempting to use the sum of the areas $GHE^{(2)}p_1^{(2)}$ and $IJE^{(2)}p_2^{(2)}$ as the consumers' welfare measure[4].

It can be shown that these two areas can be calculated as

$$\text{(12)} \qquad \Delta \widetilde{CS}_Q = \text{Area}(GHE^{(2)}p_1^{(2)}) + \text{Area}(IJE^{(2)}p_2^{(2)})$$
$$= p_1^{(1)}Q_1^{(1)}(-\text{E}Q_1/\eta_{11})(1+0.5\text{E}Q_1)$$
$$+ p_2^{(1)}Q_2^{(1)}(-\text{E}Q_2/\eta_{22})(1+0.5\text{E}Q_2),$$

where η_{11} and η_{22} are own-price Marshallian demand elasticities for the two outputs. As indicated in Figure 2, this could seriously underestimate the consumer surplus change. A numerical example of this third approach is given below and the error is shown to be significant.

Comparison of the Alternative Measures with an Numerical Example

The specification of an equilibrium displacement model and its numerical parameters for the two-input, two-output example is given in the Appendix. A 1% downward shift in the supply of X_1 (t_{X1} =-0.01) is simulated. Integrability constraints are imposed to the set of Marshallian elasticities at the initial equilibrium. It is well known that the necessary condition for the equivalence of the two approaches in parts A and B is that of integrability. As discussed in the Appendix and more in Zhao (1999, p35 and p91), since only a small displacement from the initial equilibrium (1% shift) is considered, the errors resulting from not satisfying the integrability conditions globally are expected to be small. As argued in Zhao (1999), although the equilibrium displacement model implicitly assumes local linear functions for all demand and supply functions and although it is impossible for the linear functions to be integrable beyond a single point, the equilibrium displacement model can be viewed as a local linear approximation to a true model which is globally or locally integrable. A more theoretically consistent approach that enables global integribility is through the explicit specification of profit and expenditure functions with integrable functional forms (Martin and Alston 1994), which is not the focus of this paper. Thus, based on the results in Zhao, Mullen and Griffith (1997) and the empirical results of LaFrance (1991), the two approaches in A and B should give very similar answers as long as the exogenous shift is small.

In Table 2, using the data specified in the Appendix, the results of the consumer surplus changes and the total welfare changes calculated from the three approaches are presented.

[4] For example, these were the areas used in Piggott, Piggott and Wright (1995) and Hill, Piggott and Griffith (1996) for producer surplus changes in multi-feedback models. Another problem with these studies is that the producer groups that are related in supply are given separate welfare measures, while only a joint welfare measure for the two producer groups is meaningful when they have a joint profit function.

Table 1. Three Alternative Approaches to Surplus Change Measures for Scenario 1 (t_{X1} =-0.01)

Producer Surpluses:

Pig Producers: $\Delta PS_{X1} = w_1^{(1)}X_1^{(1)}(\mathrm{E}w_1 - t_{X1})(1+0.5\mathrm{E}X_1)$ (dotted Area($ABCE^{(2)}$) in Fig.1)

Processors: $\Delta PS_{X2} = w_2^{(1)}X_2^{(1)}\mathrm{E}w_2(1+0.5\mathrm{E}X_2)$

Consumer Surplus:

Approach A: via GE curve

$\Delta TS = -w_1^{(1)}X_1^{(1)}t_{X1}(1+0.5\mathrm{E}X_1)$ (Area($ABDE^{(1)}E^{(2)}$) in Fig.1)

$$\Delta CS_Q = \Delta TS - \Delta PS_{X1} - \Delta PS_{X2}$$

Approach B: via PE curves

$\Delta CS_Q = p_1^{(1)}Q_1^{(1)}(-\mathrm{E}p_1)(1+0.5\mathrm{E}Q_1) + p_2^{(1)}Q_2^{(1)}(-\mathrm{E}p_2)(1+0.5\mathrm{E}Q_2)$
(striped Area($p_1^{(1)}E^{(1)}E^{(2)}p_1^{(2)}$) + Area($p_2^{(1)}E^{(1)}E^{(2)}p_2^{(2)}$) in Fig.2)

Note: Other expressions of ΔCS_Q via alternative paths are in Equations (9) and (10), with (9) relating to the two dotted areas in Fig.2

$\Delta TS = \Delta PS_{X1} + \Delta PS_{X2} + \Delta CS_Q$

Approach C: an incorrect PE measure

$\Delta \widetilde{CS}_Q = p_1^{(1)}Q_1^{(1)}(-\mathrm{E}Q_1/\eta_{11})(1+0.5\mathrm{E}Q_1) + p_2^{(1)}Q_2^{(1)}(-\mathrm{E}Q_2/\eta_{22})(1+0.5\mathrm{E}Q_2)$.
(double-striped Area($GHE^{(2)}p_1^{(2)}$) + Area($IJE^{(2)}p_2^{(2)}$) in Fig.2)

$\Delta T\widetilde{S} = \Delta PS_{X1} + \Delta PS_{X2} + \Delta \widetilde{CS}_Q$

It is evident in Table 2 that approaches A and B give almost the same result. However, the areas relating to the differences of two economic surplus areas off the old and new PE demand curves in each market in Approach C, which have been used in some published studies, involve significant error. The consumer surplus is underestimated with 21% error in this example.

An Exogenous Shift in Final Demand

Now consider the scenario of an exogenous shift in the Q_1 market due to promotion of Q_1. Again, estimating the changes in the producer surplus for pig producers and processors is straightforward. For example, in the X_1 market, the initial shift in the product market induces a shift in the demand curve of pigs and thus changes the equilibrium price and quantity of pigs. The supply curve is not affected. In other words, the pig producers' profit function $\pi(w_1)$ and the derived supply function $S(w_1)$ remain the same before and after the shift. The producers' welfare change is given by the change in their profit

Table 2. Comparison of Surplus Measures from Three Alternative Approaches for Scenario 1 (t_{X1} =-0.01) (in $m)

Producer Surpluses:

Pig Farmers: $\Delta PS_{X1} = 3.2425$ Processors: $\Delta PS_{X2} = 0.2959$

Approach A (via GE curve):	**Approach B** (Sequentially via same PE curves):	**Approach C** (via different PE curves):
$\Delta TS = \Delta PS_{X1} + \Delta CS_{X1}^{*}$	ΔCS_Q = Area($Ap_1^{(2)}p_1^{(1)}E^{(1)}$)	$\Delta \widetilde{CS}_Q$ =Area($GHE^{(2)}p_1^{(2)}$)
= 3.2425+6.5401	+ Area($BE^{(2)}p_2^{(2)}p_2^{(1)}$)	+ Area($IJE^{(2)}p_2^{(2)}$)
= 9.7826	= 1.6982 + 4.5450	=1.1287+3.8038
	= 6.2432	= 4.9325
$\Delta CS_{Qd} = \Delta TS - \Delta PS_{X1} - \Delta PS_{X2}$	$\Delta TS = \Delta PS_{X1} + \Delta PS_{X2} + \Delta CS_Q$	$\Delta T\widetilde{S} = \Delta PS_{X1} + \Delta PS_{X2} + \Delta \widetilde{CS}_Q$
= 6.2442	= 9.7816	= 8.4708

(13) $$\Delta\pi = \pi(w_1^{(2)}) - \pi(w_1^{(1)}) = \int_{w_1^{(1)}}^{w_1^{(2)}} \frac{\partial \pi(w_1)}{\partial w_1} dw_1 = \int_{w_1^{(1)}}^{w_1^{(2)}} S(w_1) dw_1,$$

which is the integral measured off the fixed supply curve. This is shown in Figure 3. The welfare change to pig producers is given by the trapezoid area $ABE^{(2)}E^{(1)}$. This area can be calculated using the percentage price and quantity changes as

$$\Delta PS_{X1} = w_1^{(1)} X_1^{(1)} \mathrm{E}w_1 (1 + 0.5\mathrm{E}X_1),$$

which is the same as Equation (2) with t_{X1} =0.

Similarly, the surplus change for processors is given by Equation (3).

Now turn to the Q_1 and Q_2 markets and the measure of consumer surplus change. In addition to the initial demand shift, both demand and supply curves are further shifted endogenously. Again, there are two alternatives to measuring the consumers' welfare gains. One complication in comparison to Scenario 1 is to identify the GE demand and supply curves.

Measuring from the GE Curves in a Single Market

Consider the Q_1 and Q_2 markets in Figure 4 for Scenario 2, where there is an initial upward shift in the demand curve for Q_1. Initially, the demand curve for Q_1 is shifted from $D_1^{(1)}$: $D_1(p_1 | p_2^{(1)})$ to $D_1^{(1)'}$: $D_1(p_1 - K \mid p_2^{(1)})$ where K>0 is a constant. Because the two products are related to each other in both demand and supply, the demand and supply curves for both products are subsequently shifted endogenously before reaching a new equilibrium $E^{(2)}$ in both markets.

Based on the derivation in Thurman (1991b) for the situation involving two channels of equilibrium feedback, the total welfare change can be measured as the sum of the surplus areas measured off the GE demand and supply curves D_1^* and S_1^*, although these two areas do not have welfare significance individually. In Figure 4, the GE supply curve S_1^* is given by the curve connecting $E^{(1)}$ and $E^{(2)}$. The GE demand curve D_1^* is given by the connection of $E^{(2)}$ and G, where G relates to the price the consumer is willing to pay for the initial quantity $Q_1^{(1)}$ after the promotion. Thus, the total economic surplus change is given by

$$
\begin{aligned}
\Delta TS &= Area(HGE^{(2)}E^{(1)}C) \\
(14) \qquad &= \int_{p_1^{(2)}}^{p_1^{(1)}+K} D_1^*(p_1)dp_1 + \int_{p_1^{(1)}}^{p_1^{(2)}} S_1^*(p_1)dp_1 \\
&= p_1^{(1)}Q_1^{(1)}n_{Q1}(1+0.5EQ_1).
\end{aligned}
$$

The consumers' surplus change is thus given by

$$(15) \qquad \Delta CS_Q = \Delta TS - \Delta PS_{X1} - \Delta PS_{X2}$$

As an aside, the total surplus change from a bacon and ham promotion (n_{Q2}) can be measured from Q_2 market alone as

$$\Delta TS = p_2^{(1)}Q_2^{(1)}n_{Q2}(1+0.5EQ_2).$$

Measuring from PE Curves in Individual Markets

The consumers' benefits can also be measured directly through the partial equilibrium curves in the Q_1 and Q_2 markets.

Now consider the economic welfare change for domestic consumers when the initial shock to the system is from a 1% exogenous demand shift in the Q_1 market (n_{Q1}=0.01). The expenditure functions before and after the exogenous shift are $e(p_1, p_2)$ and $e(p_1-K, p_2)$, where K (K>0) is the increase in the domestic consumers' willingness to pay per unit of pork. The compensating variation (CV) is given by

$$
\begin{aligned}
-\Delta e &= e(p_1^{(2)}-K, p_2^{(2)}) - e(p_1^{(1)}, p_2^{(1)}) \\
(16) \qquad &= -(e(p_1^{(2)}-K, p_2^{(2)}) - e(p_1^{(1)}, p_2^{(2)}) + e(p_1^{(1)}, p_2^{(2)}) - e(p_1^{(1)}, p_2^{(1)})) \\
&= -(\int_{p_1^{(1)}}^{p_1^{(2)}-K} D_1^h(p_1, p_2^{(2)})dp_1 + \int_{p_2^{(1)}}^{p_2^{(2)}} D_2^h(p_1^{(1)}, p_2)dp_2)
\end{aligned}
$$

Using Marshallian demand curves, the consumer surplus change is given by

$$(17) \qquad \Delta CS_{Qd} = -\int_{p_1^{(1)}}^{p_1^{(2)}-K} D_1(p_1, p_2^{(2)})dp_1 - \int_{p_2^{(1)}}^{p_2^{(2)}} D_2(p_1^{(1)}, p_2)dp_2 .$$

These two integrals relate to areas measured off the *new* demand curve $D_1^{(2)}$ in the Q_1 market and *initial* demand curve $D_2^{(1)}$ in the Q_2 market. In Figure 4, the first integral relates to area ABCD in the Q_1 market and the second integral relates to area $Ap_2^{(2)}p_2^{(1)}E^{(1)}$ in the Q_2 market. They can be calculated as

(18) $\Delta CS_Q = Area(ABCD) + Area(Ap_2^{(2)}p_2^{(1)}E^{(1)})$

$$= p_1^{(1)}Q_1^{(1)}(n_{Q1} - \mathrm{E}p_1)(1 + \mathrm{E}Q_1 - 0.5\,\eta_{(Q1,\,p1)}(\mathrm{E}p_1 - n_{Q1}))$$

$$- p_2^{(1)}Q_2^{(1)}\,\mathrm{E}p_2(1 + 0.5\,\eta_{(Q2,\,p2)}\mathrm{E}p_2).$$

Similar to the analysis for Scenario 1 for Equations (9)-(11)), it can be shown that under the symmetry condition of Marshallian elasticities in the Appendix (Equation (A-12)), ΔCS_Q is uniquely defined and path independent. Using the integrability relationship in the Appendix, it can be shown that Equation (18) can be written as

(19) $\Delta CS_Q =$ Area$(HGE^{(2)}F)$ + Area$(p_2^{(1)}E^{(1)}E^{(2)}p_2^{(2)})$

$$= p_1^{(1)}Q_1^{(1)}(n_{Q1} - Ep_1)(1+0.5\mathrm{E}Q_1) + p_2^{(1)}Q_2^{(1)}(n_{Q2} - \mathrm{E}p_2)(1+0.5EQ_2),$$

which is the same as Equation (11) for Scenario 1 but with n_{Q1}=0.01 and n_{Q2}=0 for Scenario 2. In Figure 4 these relate to the striped areas of $HGE^{(2)}F$ in Q_1 market and $p_2^{(1)}E^{(1)}E^{(2)}p_2^{(2)}$ in Q_2 market.

Similarly, when the initial shift occurs in the Q_2 market, the consumers' welfare change can be calculated as

(20)
$$\Delta CS_Q = -\Delta e = -p_1^{(1)}Q_1^{(1)}\,\mathrm{E}p_1(1 + 0.5\eta_{(Q1,\,p1)}\mathrm{E}p_1)$$
$$+p_2^{(1)}Q_2^{(1)}(n_{Q2} - \mathrm{E}p_2)(1 + \mathrm{E}Q_2 - 0.5\eta_{(Q2,\,p2)}(\mathrm{E}p_2 - n_{Q2})).$$

Also, Equation (20) becomes Equation (19) under integrability restrictions. In other words, the formula for ΔCS_Q in Equation (19) holds for all exogenous shift scenarios under the Marshallian symmetry condition. These formulas are summarised in Table 3.

An Intuitive but Incorrect Partial Measure

As in the case of Scenario 1, it is tempting to use the differences between the old and new consumer surplus areas measured off the old and new partial demand curves. In the case of Scenario 2 in Figure 4, under the assumption of parallel shifts, these relate to the area of $MNE^{(2)}F$ in the Q_1 market and area $IJE^{(2)}p_2^{(2)}$ in the Q_2 market. As in Scenario 1, these can be calculated as

(21) $\Delta \widetilde{CS}_Q =$ Area$(MNE^{(2)}F)$ + Area$(IJE^{(2)}p_2^{(2)})$

$$= p_1^{(1)}Q_1^{(1)}(-\mathrm{E}Q_1/\eta_{11})(1+0.5\mathrm{E}Q_1)$$

$$+ p_2^{(1)}Q_2^{(1)}(-\mathrm{E}Q_2/\eta_{22})(1+0.5\mathrm{E}Q_2).$$

The three approaches are summarised in Table 3.

Table 3. Three Alternative Approaches to Surplus Change Measures for Scenario 2 (n_{Q1} =0.01)

Producer Surpluses:

Pig Producers: $\Delta PS_{X1} = w_1^{(1)} X_1^{(1)} \mathrm{E}w_1(1+0.5\mathrm{E}X_1)$ (dotted Area($ABE^{(2)}E^{(1)}$) in Fig.3)

Processors: $\Delta PS_{X2} = w_2^{(1)} X_2^{(1)} \mathrm{E}w_2(1+0.5\mathrm{E}X_2)$

Consumer Surplus:

Approach A: via GE curve

$\Delta TS = p_1^{(1)} Q_1^{(1)} n_{Q1}(1+0.5EQ_1)$ (dotted Area($HGE^{(2)}E^{(1)}C$) in Q_1 market in Fig.1)

$\Delta CS_Q = \Delta TS - \Delta PS_{X1} - \Delta PS_{X2}$

Approach B: via PE curves

$\Delta CS_Q = p_1^{(1)} Q_1^{(1)} (n_{Q1} - \mathrm{E}p_1)(1+0.5\mathrm{E}Q_1) + p_2^{(1)} Q_2^{(1)} (-\mathrm{E}p_2)(1+0.5\mathrm{E}Q_2)$
(striped Area($HGE^{(2)}F$) +Area($p_2^{(1)}E^{(1)}E^{(2)}p_2^{(2)}$) in Fig.4)

Note: An expression of ΔCS_Q via an alternative path is in Equation (18) which relates to the two sparsely dotted areas ($ABCD$) and ($Ap_2^{(2)}p_2^{(1)}E^{(1)}$) in Fig.4

$\Delta TS = \Delta PS_{X1} + \Delta PS_{X2} + \Delta CS_Q$

Approach C: an incorrect PE measure

$\Delta \widetilde{CS}_Q = p_1^{(1)} Q_1^{(1)} (-\mathrm{E}Q_1/\eta_{11})(1+0.5\mathrm{E}Q_1) + p_2^{(1)} Q_2^{(1)} (-\mathrm{E}Q_2/\eta_{22})(1+0.5\mathrm{E}Q_2)$.
(double-striped Area($NME^{(2)}F$) + Area($IJE^{(2)}p_2^{(2)}$) in Fig.4)

$\Delta \widetilde{TS} = \Delta PS_{X1} + \Delta PS_{X2} + \Delta \widetilde{CS}_Q$

Comparison of the Alternative Measures with an Numerical Example

A 1% upward shift in the demand of Q_1 (n_{Q1} =0.01) is simulated using the data specified in the Appendix, and the results from the three approaches are presented in Table 4.

Again, approaches A and B result in very close values, but the values from the incorrect approach C are rather different. The consumer surplus is underestimated with 26% error in C.

Table 4. Comparison of Surplus Measures from Three Alternative Approaches for Scenario 2 (n_{Q1} =0.01) (in \$m)

Producer Surpluses:

Pig Farmers: $\Delta PS_{X1} = 1.5040$, Processors: $\Delta PS_{X2} = 0.1966$.

Approach A (via GE curve):	**Approach B** (Sequentially via PE curves):	**Approach C** (via different PE curves):
$\Delta TS = \Delta PS_{X1} + \Delta CS_{X1}^{*}$	$\Delta CS_Q = \text{Area}(Ap_1^{(2)}p_1^{(1)}E^{(1)})$	$\Delta \widetilde{CS}_Q = \text{Area}(GHE^{(2)}p_1^{(2)})$
$= 1.5040+3.3866$	$+ \text{Area}(BE^{(2)}p_2^{(2)}p_2^{(1)})$	$+ \text{Area}(IJE^{(2)}p_2^{(2)})$
$= 4.8906$	$= 1.3253 + 1.8619$	$=1.0935+1.2812$
	$= 3.1872$	$= 2.3747$
$\Delta CS_Q = \Delta TS - \Delta PS_{X1} - \Delta PS_{X2}$	$\Delta TS = \Delta PS_{X1}+\Delta PS_{X2}+\Delta CS_Q$	$\Delta T\widetilde{S} = \Delta PS_{X1}+\Delta PS_{X2}+\Delta \widetilde{CS}_Q$
$= 3.1900$	$= 4.8878$	$= 4.0752$

Conclusions

The concern in Thurman (1991b) for the situation of more than two sources of equilibrium feedback in a general equilibrium analysis relates to how the total welfare change from an exogenous intervention can be measured in a 'single' market, and how the individual GE demand or supply curves do not have welfare significance. It is well known that an alternative is to evaluate the partial equilibrium effects in individual markets and then add them up (Just, Hueth and Schmitz 1982, p469; Alston, Norton and Pardey 1995, p232). The latter approach is often desirable when the analysis requires knowledge of the welfare impacts on individual groups but it also requires that market parameters in all individual markets are available.

In this paper, it is shown that when there exists more than one source of GE feedback, not only do GE demand or supply curves have no welfare significance individually but that caution also needs to be taken in measuring welfare via PE demand or supply curves. Significant errors are involved when failing to measure the PE welfare changes in an appropriate sequential manner.

A two-input and two-output equilibrium displacement model is used to demonstrate the significance of various PE and GE surplus measures, where the two outputs are related in both demand and supply. Welfare changes to individual producer and consumer groups are derived analytically by examining the profit and expenditure functions of the relevant industry groups and the associated integrals of PE supply and demand functions. These welfare changes are also illustrated graphically as surplus areas in the relevant markets.

Two alternative approaches to measuring the welfare changes to individual groups have been discussed: first, where the total surplus change is measured off the GE curves in a single market; and second, where the surplus areas are measured off the PE curves in individual markets. Integrability conditions are imposed on Marshallian elasticities at the base equilibrium, and the two approaches give almost identical results when the considered exogenous shifts are small. This is demonstrated with the numerical model.

When two markets are related through both demand and supply, the economic surplus change to producers or consumers should be measured *sequentially* in the two markets and then added up; that is, the surplus change off the (same) *initial* PE curve in the first market plus the surplus change off the (same) *new* PE curve in the second market, holding the price for the substitute product at initial or new price level in each case. It is not correct to calculate the surplus changes of the displacement based on different PE curves in the same market, as perhaps intuition would first suggest and has been used in some past studies. The numerical example indicates significant errors with this latter approach.

An important point to this problem is that while individual demand curves for the two outputs (conditional on each other's price levels) are able to be recognised, consumers will enjoy both products and hence their choice is reflected in one joint expenditure function which has the prices of both outputs as arguments. Any change in expenditure reflects changes in their welfare as consumers of both products and it does not make sense to attempt a disaggregation. Similarly, it does not make sense to measure welfare consequences to individual producer groups when the products they produce are related in supply, as has been done in some existing studies.

There are two sources of change in the expenditure function through the changes of both prices, and thus the issue of path dependency is relevant. In the paper, it is verified analytically that under the integrability conditions imposed on Marshallian demand elasticities, the economic surplus measures are uniquely defined and independent of the path of the displacement.

However, the economic surplus measures are path dependent when integrability conditions are not satisfied. In this case, the PE economic surplus measures in a multi-market model involving multiple sources of GE feedback are not uniquely defined. This has been the main criticism of using economic surplus measures in multi-market models (Slesnick 1998). But, the derivation in this paper implies that the first-order terms ($\mathbf{O}(\lambda)$ where λ is the small exogenous shift) of the economic surplus changes may be path independent and equal to the first-order terms of CV or EV measures. The integrability conditions may only affect the economic surplus measures at the second order terms ($\mathbf{O}(\lambda^2)$). Note that the changes in economic surplus, i.e. the trapezoid area which is often the interest in R&D and promotion evaluations, are of the first-order magnitude of the initial shifts ($\mathbf{O}(\lambda)$). Thus, as long as the considered equilibrium displacements are small (λ is small), not satisfying integrability conditions may not result in significant errors in using the traditional measures of economic surplus changes (trapezoid areas). However, if the second-order measure of triangular 'deadweight loss' is of interest in a policy study, integrability conditions are vital and violation of them could result in significant errors.

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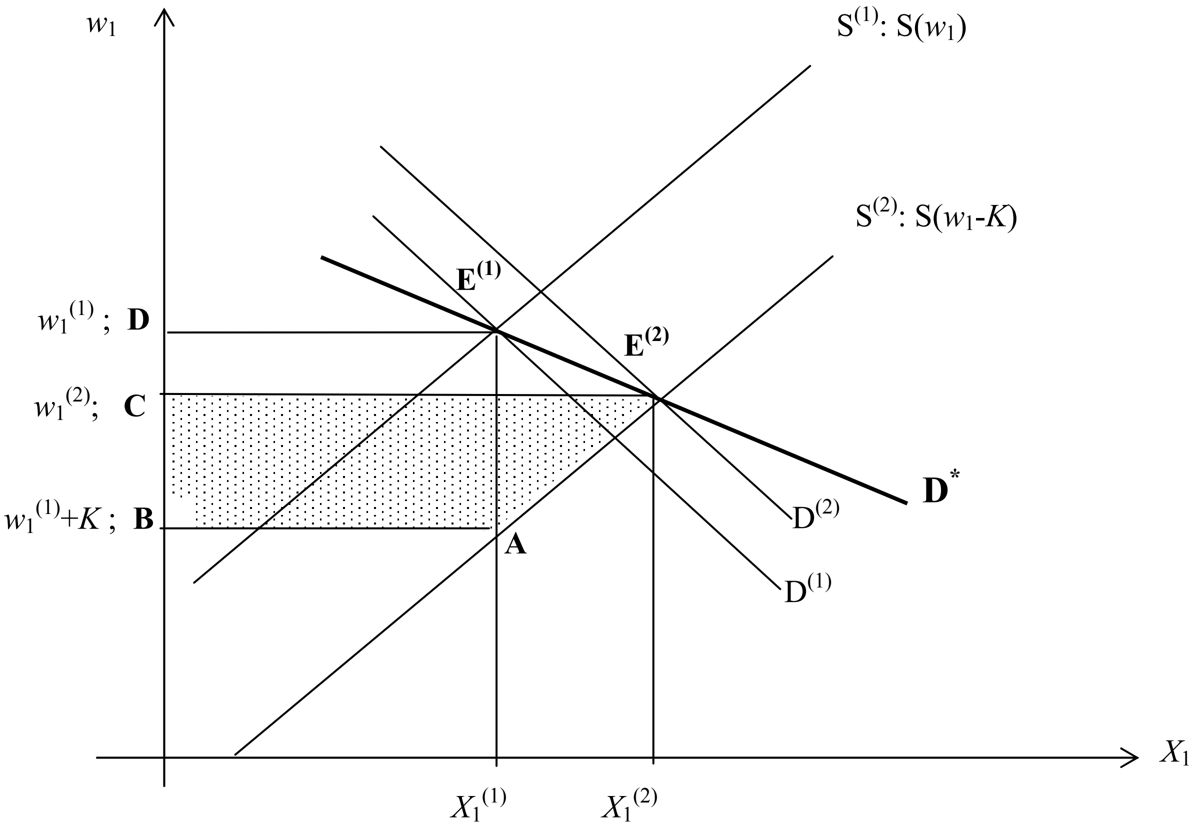

Figure 1. Pig Producers' and Total Surplus Changes for Scenario 1 (t_{X1}=-1%)

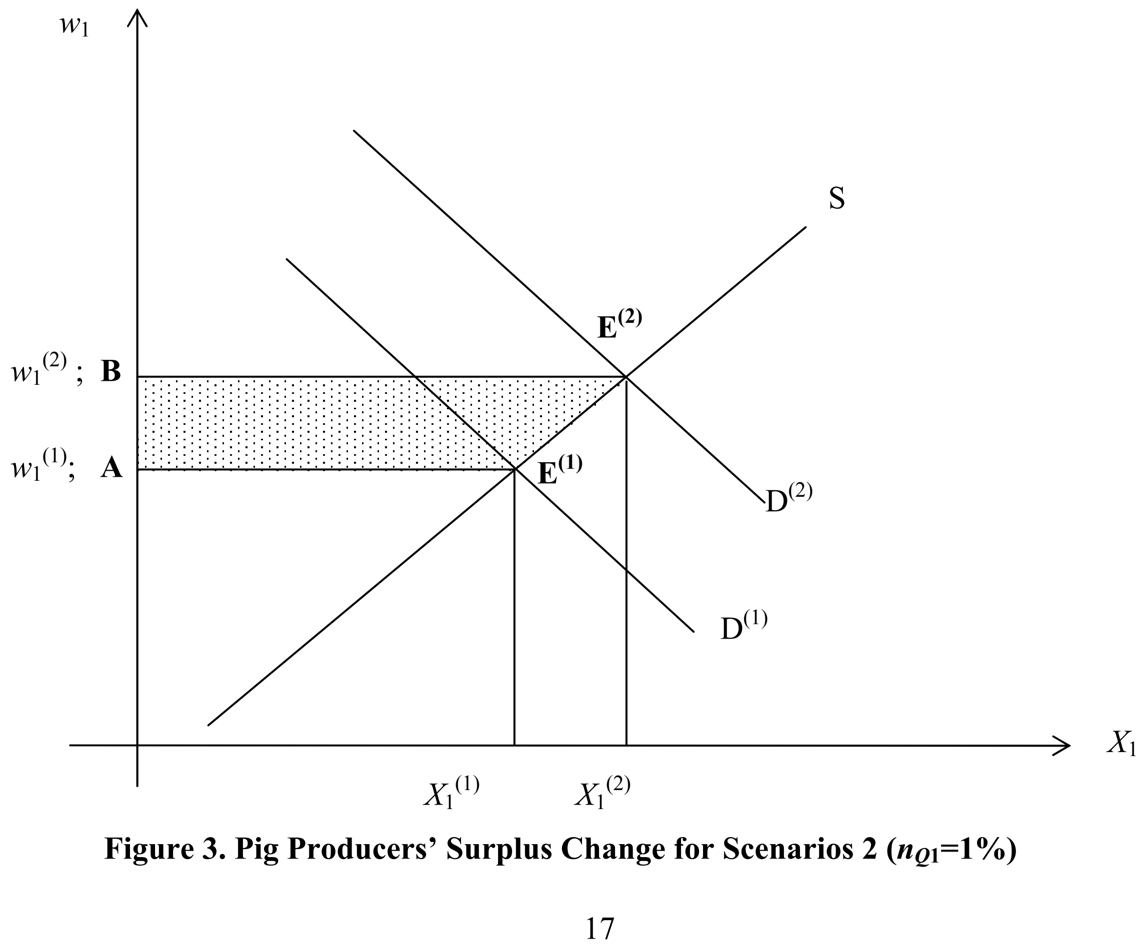

Figure 3. Pig Producers' Surplus Change for Scenarios 2 (n_{Q1}=1%)

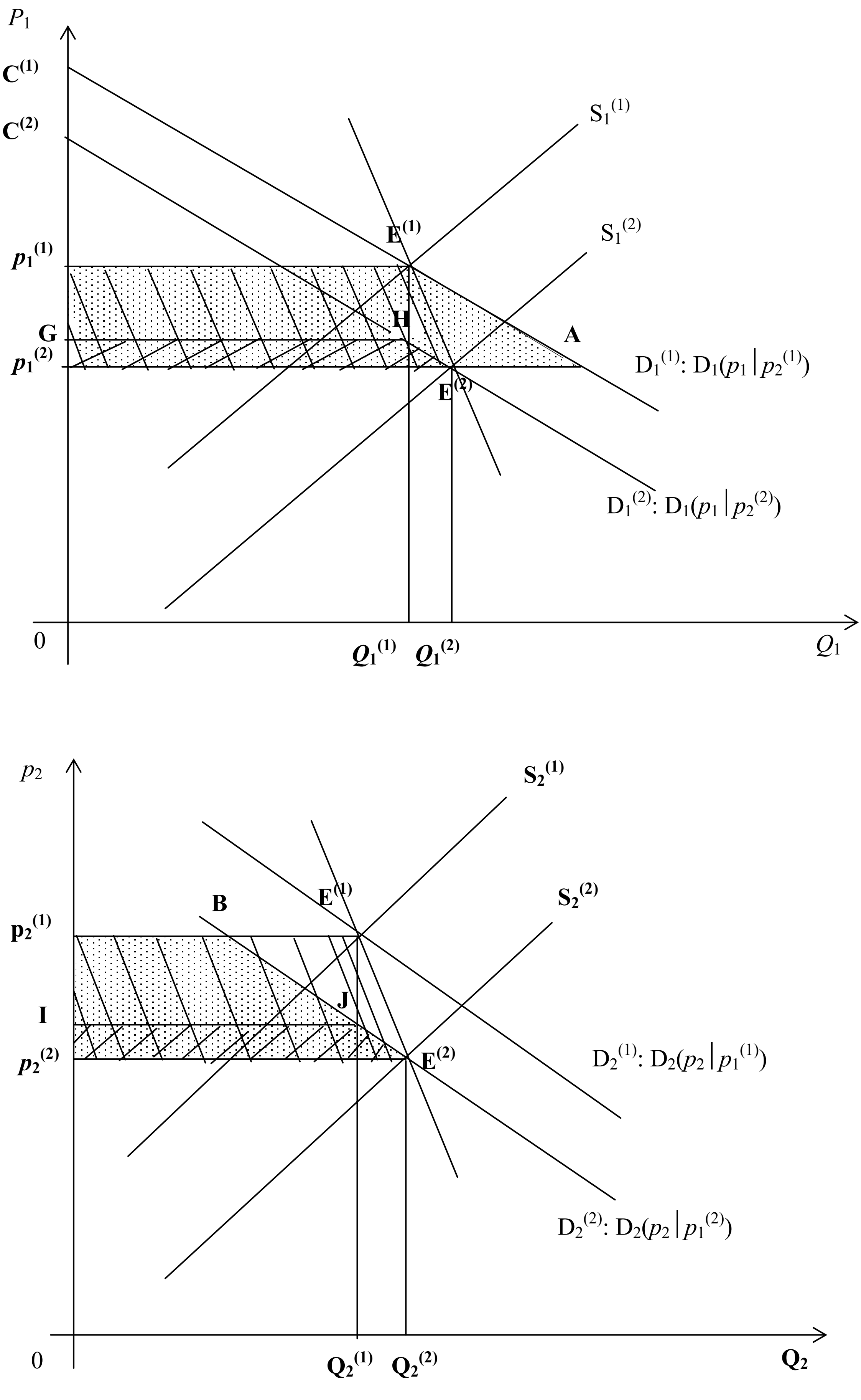

Figure 2. Consumer Welfare Change for Scenario 1 (t_{X1}=-1%)

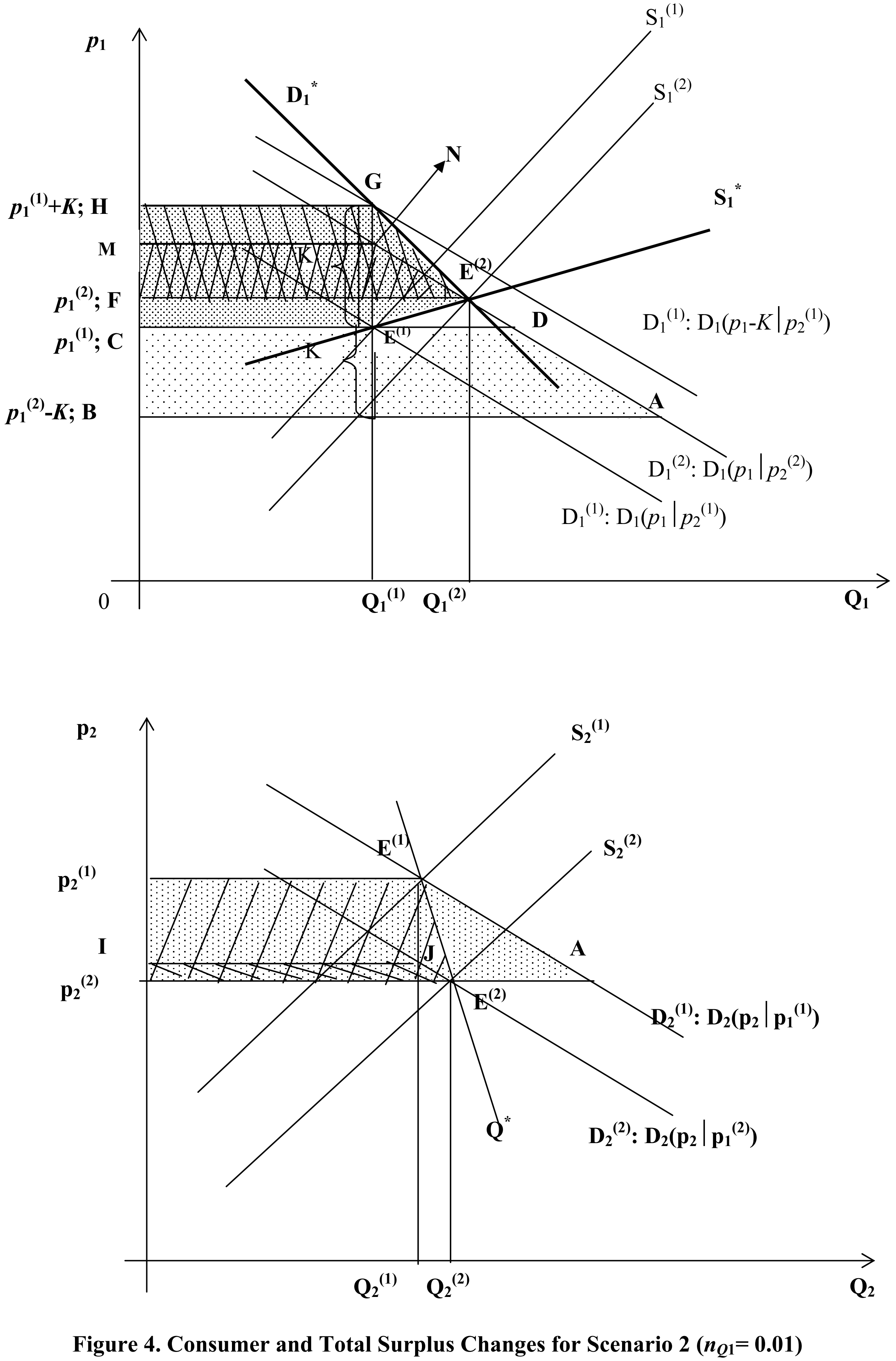

Figure 4. Consumer and Total Surplus Changes for Scenario 2 (n_{Q1}= 0.01)

Appendix. Specification of the Model

Consider a technology that uses two inputs, X_1 (pigs) and X_2 (processing inputs), to produce two outputs, Q_1 (pork) and Q_2 (bacon and ham). Suppose that the supplies of X_1 and X_2 are not related but Q_1 and Q_2 are substitutes in demand for final consumers. Assume that all producer groups are profit maximisers and consumers are utility maximisers. Assume the multi-output production function $X(X_1, X_2) = Q(Q_1, Q_2)$ is separable in inputs and outputs, and has constant returns to scale.

Under these assumptions, the structural model for the demand and supply relationships among all variables can be written in general functional form as

Exogenous Factor Supplies:

(A-1) $X_1 = X_1(w_1, T_{X1})$ *pig supply*

(A-2) $X_2 = X_2(w_2, T_{X2})$ *processing input supply*

Output-Constrained Factor Demand:

(A-3) $X_1 = Q\, c'_{Q,1}(w_1, w_2)$ *demand for pigs*

(A-4) $X_2 = Q\, c'_{Q,2}(w_1, w_2)$ *demand for processing inputs*

Market Equilibrium:

(A-5) $X(X_1, X_2) = Q(Q_1, Q_2)$ *quantity equilibrium*

(A-6) $c_Q(w_1, w_2) = r_X(p_1, p_2)$ *value equilibrium*

Input-Constrained Output Supply:

(A-7) $Q_1 = X\, r'_{X,1}(p_1, p_2)$ *pork supply*

(A-8) $Q_2 = X\, r'_{X,2}(p_1, p_2)$ *bacon and ham supply*

Final Product Demand:

(A-9) $Q_1 = Q_1(p_1, p_2, N_{Q1}, N_{Q2})$ *pork demand*

(A-10) $Q_2 = Q_2(p_1, p_2, N_{Q1}, N_{Q2})$ *bacon and ham demand*

Derivation of the above demand and supply relationship from the underlying decision functions can be found in Zhao (1999) for a more complicated model. In the above, w_i and p_i are prices for X_i and Q_i (i=1,2); $c_Q(.)$ is the unit cost function for producing aggregated output Q; $r_X(.)$ is the unit revenue function earned from aggregated input X; $c'_{Q,i}(.)$ and $r'_{X,i}(.)$ (i=1,2) are partial derivatives of $c_Q(.)$ and $r_X(.)$; and T_{Xi} and N_{Qi} (i=1,2) are exogenous supply and demand shift variables resulting from reductions in production costs and increases in willingness to pay.

(A-1) and (A2) are derived from the underlying profit functions using Hotelling's Lemma; (A-3) and (A-4) from the cost function using Shephard's Lemma; (A-5) is the production function and (A-6) sets unit cost equal to unit revenue under industry equilibrium condition; (A-7) and (A-8) are derived from the revenue function using Samuelson-McFadden Lemma; and (A-9) and (A-10) are derived from the indirect utility function using Roy's identity.

Totally differentiating the above system of equations at the initial equilibrium points and imposing the integrability constraints to the Marshallian elasticities, the equilibrium displacement model for the above system is given by Equations (A-1)'-(A-10)' below. All elasticities relate to the base equilibrium points. As shown in Zhao, Mullen and Griffith (1997), local linear approximation to all demand and supply functions is implied in the comparative static operation, and the errors are small when the true functions are not linear as long as the considered exogenous shift is small.

Exogenous Factor Supplies:

(A-1)' $EX_1 = \varepsilon_1(Ew_1 - t_{X1})$ *pig supply*

(A-2)' $EX_2 = \varepsilon_2 (Ew_2 - t_{X2})$ *processing input supply*

Output-Constrained Factor Demand:

(A-3)' $EX_1 = -\kappa_2\sigma\, Ew_1 + \kappa_2\sigma\, Ew_2 + EQ$ *demand for pigs*

(A-4)' $EX_2 = \kappa_1\sigma\, Ew_1 - \kappa_1\sigma\, Ew_2 + EQ$ *demand for processing inputs*

Market Equilibrium:

(A-5)' $\kappa_1 EX_1 + \kappa_2 EX_2 = \gamma_1 EQ_1 + \gamma_2 EQ_2$ *quantity equilibrium*

(A-6)' $\kappa_1 Ew_1 + \kappa_2 Ew_2 = \gamma_1 Ep_1 + \gamma_2 Ep_2$ *value equilibrium*

Input-Constrained Output Supply:

(A-7)' $EQ_1 = -\gamma_2\, \tau\, Ep_1 + \gamma_2\, \tau\, Ep_2 + EX$ *pork supply*

(A-8)' $EQ_2 = \gamma_1\tau\, Ep_1 - \gamma_1\tau\, Es_2 + EX$ *bacon and ham supply*

Final Product Demand:

(A-9)' $EQ_1 = \eta_{11}(Ep_1 - n_{Q1}) + \eta_{12}(Ep_2 - n_{Q2})$ *pork demand*

(A-10)' $EQ_2 = \eta_{21}(Ep_1 - n_{Q1}) + \eta_{22}(Ep_2 - n_{Q2})$ *bacon and ham demand*

$E(.)=\Delta(.)/(.)$ represents a small finite relative change of variable (.). κ_i is the cost share of X_i (i=1,2) and γ_i is the revenue shares for Q_i (i=1,2). ε_i is the own-price supply elasticity of X_i (i=1,2). η_{ij} is the demand elasticity of Q_i with respect to price changes of Q_j ((i, j =1,2). σ is the input substitution elasticity between X_1 and X_2, and τ is the product transformation elasticity between Q_1 and Q_2. Finally, t_{Xi} is an exogenous parallel shift in the supply of X_i expressed as a percentage of the base price level for X_i (i=1,2), and, n_{Qi} is the parallel shift in the demand of Q_i expressed as a percentage of the base price for Q_i (i=1,2).

The integrability constraints for Marshallian elasticities in terms of output-constrained input demand, input-constrained output supply, and exogenous resource factor supply and final product demand are discussed in detail in Zhao (1999, 4.5.2 and Appendix 2) using Chambers (1991). These can be summarised by symmetry, homogeneity and concavity/convexity conditions, which will not be listed here. Symmetry and homogeneity has been imposed explicitly in Equations (A-3), (A-4), (A-7) and (A-8).

For the exogenous final output demands, as discussed in Zhao (1999, p274-5), the homogeneity and concavity conditions, which become inequality constraints for the incomplete demand system in the model, will be automatically satisfied for 'reasonable' values of η_{ij} (i,j=1,2). Using the Slutsky equation, the Hicksian symmetry condition can be represented in terms of observable Marshallian demand elasticities as

(A-11) $$\eta_{ij} = \frac{\gamma_j}{\gamma_i}\eta_{ji} + s_j(\eta_{jm} - \eta_{im}) \quad (i, j = 1,2) \quad \textit{(symmetry)},$$

where γ_j/γ_i (i =1,2) is the ratio of expenditure shares, which is equal to the ratio of the revenue shares for the processing sector. As the Marshallian economic surplus areas will be used as measures of welfare, it implies that the marginal utility of income is assumed constant and the income effect will be ignored. If assuming a constant marginal utility of income or zero income effect on demand, or even a looser sufficient condition that the income elasticities for pork and processed pigmeat are the same, i.e. $\eta_{1m} = \eta_{2m}$, the slutsky symmetry in (A-11) will become Marshallian symmetry as

(A-12) $$\eta_{ij} = \frac{\gamma_j}{\gamma_i}\eta_{ji} \quad (i, j = 1, 2) \qquad \textit{(symmetry)}.$$

In other words, Equations (A-9)'-(A-10)' need to satisfy (A-12) to guarantee path dependency and consistent measures of economic surplus changes.

In the numerical example in the paper, two scenarios are considered; namely, (1) t_{X1}=-0.01, t_{X2} =n_{Q1}= n_{Q2}=0, and (2) n_{Q1}=0.01, t_{X1} =t_{X2}= n_{Q2}=0. The parameter values are specified as: κ_1=0.6, κ_2=0.4, γ_1=0.3, γ_1=0.7, ε_1=0.9, ε_2=5, η_{11}=-1.2, η_{22}=-0.8, η_{12}=0.1, η_{21}=0.1, σ=0.1, τ=-0.2. Note that these values are chosen for illustration purpose and may not be appropriate for the case of pork and processed pigmeat.

Equations (A-3)' and (A-4)' can be combined to eliminate the unobservable EQ. The same can be done to (A-7)' and (A-8)' to eliminate EX. The resulting relative price and quantity changes, E(.), for all inputs and outputs can be solved from the displacement system in (A-1)'-(A-10)' for the two policy scenarios and then be used for the welfare calculations in the paper.