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**Localized and Incomplete Mutual Insurance**

by

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## **Localized and Incomplete Mutual Insurance**

Rinku Murgai, Paul Winters, Elisabeth Sadoulet and Alain de Janvry\*\*

### **Abstract**

The practice of mutual insurance is conditioned by two types of transaction costs: "association" costs in establishing links with insurance partners and "extraction" costs in using these links to implement insurance transfers. Data on insurance-motivated water exchanges among households along two irrigation canals in Pakistan show that households exchange bilaterally with neighbors and family members but the majority exchange with members of tightly knit clusters. We, therefore, develop a model that endogenizes both cluster formation and the quality of insurance in the chosen cluster as a function of the relative importance of association and extraction costs. Full insurance at the community level, the object of most empirical tests of mutual insurance, is seen to be an extreme case. It is consequently not surprising that tests of the hypothesis of full risk pooling at the community level have led to rejection. The Pakistan data support the proposition that the configuration of insurance clusters and the intensity of exchanges within clusters vary with association and extraction costs. These costs are affected by kinship, distance to neighbors, and exposure to risk. Households with larger kinship groups, closer neighbors, and greater risk exposure insure through larger clusters and more intensive exchange.

Key Words: mutual insurance, transaction costs, clusters

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## **Localized and Incomplete Mutual Insurance**

### **1. Introduction**

When writing about social arrangements which help insulate consumption from fluctuations in income, James Scott (1976) noted that “as soon as a peasant leans on his kin or his patron rather than on his own resources, he gives them a reciprocal claim to his own labor and resources. In fact, they aid him, one might say, because there is a tacit consensus about reciprocity, and their assistance is as good as money in the bank against the time when the situation is reversed.” This raises the question of how effective these informal insurance arrangements are in aiding households cope with risk? Most empirical tests, including research by Deaton (1992), Townsend (1994), Gertler and Gruber (1997), and Jalan and Ravallion (1999) have rejected the hypothesis of complete community-wide insurance against idiosyncratic risk, but have uncovered evidence of some degree of risk sharing.

Limiting factors in the performance of community-based informal insurance arise from the difficulty for insurance groups to observe/monitor members and enforce rules. Incomplete insurance has thus been attributed to failure under those circumstances of designing incentive compatible and implementable contracts that yield full insurance. The purpose of this paper is to develop an alternative plausible explanation for incomplete insurance at the community level. When there are transaction costs of coordinating transfers, monitoring behavior, and enforcing participation, the optimal insurance scheme may only be over a subset of community members and/or for an incomplete level of insurance. Rejection of efficient risk sharing at the community-level may thus be due to selecting the village, rather than the appropriate subset of community members, for applying the test. This suggests that in order to assess the performance of mutual insurance, we need to obtain information not only on how complete transfers are but also on the configuration within which exchanges occur.

The paper is organized as follows. We first review in section 2 the literature on mutual insurance and social exchange, with particular attention to alternative configurations of exchange. Evidence from the pattern of water exchanges along irrigation canals in Pakistan that supports the existence of transaction costs in exchanges and the formation of mutual insurance clusters among farmers is presented in section 3. We, therefore, develop a theory of the role of transaction costs in determining both the *configuration* of insurance-motivated exchanges and the *quality* of the insurance achieved in sections 4 to 7. We show that divergence from community-level efficient insurance can range from partial insurance at the community level, to full insurance in clusters, partial insurance in clusters, and situations in which insurance does not arise at all. While we cannot empirically verify the level of insurance that the clusters provide to their members, in section 8 we confirm with our data that the size of the clusters and the intensity of exchange within clusters decreases with transaction costs and increases with exposure to risk, as predicted by the model. Section 9 provides some concluding remarks.

### **2. Conceptualizing mutual insurance**

Broadly, there are two questions to consider when analyzing the performance of mutual insurance systems. First, what are the mechanisms by which risk is shared between members of an insurance group? And second, how effective are these mechanisms at spreading risk, i.e., is full insurance achieved?

The theoretical and empirical literature on mutual insurance has focussed largely on the second question. Empirical tests of full insurance have been based on the measure of consumption smoothing achieved by any individual relative to the group. Most analysts chose the ‘community’ (i.e., all households in the sample from one village) as the group over which to test for full insurance (Deaton, 1992; Townsend, 1994; Gertler and Gruber, 1997; Ligon, 1998; Ligon, Thomas, and Worall, 1998; Foster and Rosenzweig, 1999; Grimard, 1997<sup>1</sup>). The hypothesis of complete community-wide insurance against idiosyncratic risk was rejected in all studies. Two theories have been used to explain this outcome: contract theory and theories explaining the quality of cooperation.

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<sup>1</sup> Grimard examines risk sharing among ethnic groups, which in Côte d'Ivoire are even larger than villages.

For those using contract theory, rejection of full insurance has been attributed to the need for insurance groups to monitor members and enforce agreements. Studies stressing problems of monitoring argue that households have private information that cannot be obtained by their insurance partners. In this context of moral hazard, inducing truthful revelation and a higher level of effort requires a contract that offers a higher utility for revealed good outcomes than for bad ones, and therefore results in less than full insurance in some states of nature (Ligon, 1998). Studies focusing on enforcement problems show that sustainable risk pooling arrangements that are implementable via repeated interactions are possible, but may lead to only partial risk pooling. When households cannot write binding contracts and are consequently restricted to self-enforcing agreements, risk-sharing is incomplete in states of nature with large shocks: a household provides a transfer only if the discounted expected benefit from participating in the insurance agreement exceeds the one time gain from defection (Kimball, 1988; Coate and Ravallion, 1993).<sup>2</sup>

A parallel literature that provides similar insights for the determinants of the quality of exchange tackles the subject of cooperation within communities, usually for the management of common property resources. In contrast to the contract theory approach, this literature has focused on the ability of the group to define rules, monitor member behavior, and enforce agreed upon rules, because exclusion from the community, which is the main enforcement mechanism in contract theory, is largely not an option and individual incentive mechanisms are difficult to design. Comparison of total costs and benefits of cooperation dictates the community's decision on whether to cooperate or not (Olson, 1965; Wade, 1987; Ostrom, 1992). The quality of cooperation is either characterized by a dichotomous decision whereby the community decides whether or not to cooperate, or as a continuous decision whereby communities balance the cost of enforcing cooperation with benefits derived from cooperating in order to choose an optimal degree of cooperation (McCarthy, de Janvry, and Sadoulet, 1998). Transferring this reasoning to insurance, when faced with transaction costs of monitoring partners and enforcing insurance obligations, community members would decide whether to insure each other or not, and would choose an optimal quality of insurance. The outcome in the case of transaction costs is equivalent to the outcomes predicted in the contract theory literature on insurance.

There is a growing empirical literature that examines the first question mentioned above, namely mechanisms through which insurance functions. In local communities, there is evidence of gift giving, multipurpose rotating savings and credit associations, mutual aid societies, labor groups, and funeral clubs which sometimes incorporate implicit insurance transactions (Bardhan and Udry, 1999). More specifically reciprocal relationships such as might occur between employer and employee, landlord and sharecropper, patron and clients, or a trader and his suppliers also provide channels for sharing risk. In one of the few studies of bilateral exchanges, Udry (1994) shows that credit transactions in rural Nigeria incorporate risk-sharing as the repayment of credit depends on the income shocks of both the borrower and the lender.

Comparatively, the economic literature on the choice of insurance partners and characteristics of groups of individuals who choose to insure each other is much more sparse. The limited success of mutual insurance, which almost always occurs in restricted social spaces, underscores the central importance of good mutual information and effective enforcement mechanisms to support the insurance arrangement. Moreover, mutual insurance can only deal with idiosyncratic risk, and is useless as a means of dealing with covariate risks such as those related to bad weather or macroeconomic shocks. That these factors influence the choice of insurance partners is evident in a study by Rosenzweig and Stark (1989) who find that households in South India selectively choose marriage partners to enhance the quality of insurance between families. Similarly, Fafchamps and Lund (1998) examine the quality of insurance partners for households in the Philippines and find that households who insure with kin achieve better insurance levels.

These studies all use a partial approach as the focus is on insurance mechanisms from the point of view of an individual with one or more insurance partners, with no relationship drawn to the overall structure of exchanges. However, whether or not full insurance within a community is achieved depends on the overall configuration of

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<sup>2</sup> Moving beyond stationary contracts, Ligon, Thomas, and Worrall (1998) show that the optimal achievable insurance contract under limited commitment is not a pure insurance contract but a form of credit contract in which the history of recent transfers influences current transfers.

bilateral exchanges. One approach that addresses the issue of configuration of exchanges is the theory of club-goods (Cernes and Sandler, 1986). In this theory, there is an advantage to belonging to a group within which all individuals exchange with each other over a system in which social exchange occurs in series of direct and indirect links between individuals. This is the case when a cluster, or club, allows individuals to avail of scale economies. The key question is to determine the optimal size of the club and the level of provision of the club good.<sup>3</sup> Transferring this reasoning to mutual insurance, if the village is too large a group and does not necessarily correspond to the best institution for setting up the monitoring and enforcement mechanisms necessary for mutual insurance to work, then one could ask whether the village might be divided into "clusters" within which households extend mutual insurance to each other. The configuration (in terms of size and membership) of clusters would depend on tradeoffs between the benefits of group formation (which increase with group size) and the costs of group maintenance (which are expectedly lower among a set of individuals who interact with each other more frequently). The anthropological literature suggests that groups are likely to coalesce around individuals who are neighbors, kin, from the same caste, etc., depending on the nature of transaction costs (see e.g., Ellsworth, 1989; Fafchamps, 1992; Posner 1980 and citations therein).

As a generalization, the configuration of insurance exchanges between individuals in a community can be characterized by networks of reciprocal exchange (Platteau, 1991; Fafchamps, 1992) that can include clusters of individuals who trade preferentially with each other and individuals that only engage in bilateral exchange. Determining the configuration of an optimal network calls upon network theory which considers a given universe (in this case, a community) as a graph, with all the individuals at the nodes (Jackson and Wolinsky, 1996). A direct link between any two persons can be established at a cost and it brings about a given benefit. Indirect links between individuals confer additional externality benefits but come at no cost. The fundamental issue of interest is to determine networks that are stable such that no individual would prefer to sever a direct link, and no two individuals would prefer to create a direct link that does not exist.

The theory of networks has not been applied to the case of mutual insurance. A fundamental difficulty in identifying the optimal configuration of a mutual insurance network is that it is impossible to specify the benefit of a link between any two individuals (a transfer for insurance) independently of the whole structure of exchanges along the other direct and indirect links for both individuals. Hence, short of specifying exactly the rules of transfers on each direct and indirect link, there is no general property that can be assessed at the level of the network.

However, what can be tested is whether there is evidence of cluster formation (where households within the cluster exchange with each other, and not with other members of the community) within insurance networks or if these are better characterized simply by reciprocal bilateral exchanges (where the quality of exchange depends only on the characteristics of the two insurance partners). Clearly, whether or not households coalesce into clusters depends on the nature of transaction costs and on potential benefits that are context-specific. We consequently start by analyzing, in the next section, the nature of insurance configurations in a case of water trading between farmers along two irrigation canals (watercourses) in Punjab, Pakistan.<sup>4</sup>

### 3. Costly mutual insurance and localized exchange in irrigation water trading

Canal water is distributed according to a rotational water-delivery system, or *warabandi*, which provides turns to use the entire water flow in the canal to farmers at a pre-specified time each week. Access to water is limited to farmers with land in the watercourse command area, and the length of the water turn is proportional to the

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<sup>3</sup> In some cases, clubs need to partition the whole population, i.e., every person from the population has to be member of one and only one club. This is referred to a global economy viewpoint, and its theory is tied to the game theory literature. The key question is then to define a set of clubs that will be stable, i.e., no households can be enticed to split and form another group.

<sup>4</sup> These examples come from field observations during a stay at the International Water Management Institute (IWMI) in Lahore, Pakistan. We are grateful to Pierre Strosser and to the IWMI staff in Lahore and at the field station in Hasilpur for their assistance and for the data used in this analysis.

landholding. Despite the fixed rotation system, during any of their weekly water turns, farmers are at the mercy of day to day fluctuations in canal water flow. Growing problems with operation and maintenance, overuse, and illegal diversions of irrigation channels have meant that water supply fluctuates considerably (the coefficient of variation of water delivery at the head of both watercourses is greater than one). The risk is, to a large extent, independent for the different households since water levels vary within short time spans. These fluctuations are more damaging further down in the watercourse where average water deliveries are lower.

Water shocks endanger crops and risk coping is addressed by voluntary water transfers between households. Data on weekly water exchanges between farmers for the two watercourses during the *kharif* 1994 season show that there are significant transfers of water time within each watercourse and that no payment is provided for the transfers. Some of these transfers may be for barter – water time is transferred with the expectation that a transfer of an equivalent value will be available at a future date. However, the lack of reciprocal transfers (often only one partner receives water in a season) and interviews with farmers along the watercourse indicate that most of these transfers are part of informal insurance arrangements in which households receive water from partners in the event they experience a water shortfall.

The configuration of exchange -- i.e., who exchanges with whom -- within the watercourse is importantly shaped by the presence of transaction costs. Costs arise due to the need for coordination of transfers with partners across different physical and water turn distances. Since flow in the watercourse is continuous, water exchanges require adjustment of the *warabandi* schedule. For example, suppose farmer A receives a low water realization and the next farmer in line, farmer B, as a member of A's insurance group, provides a transfer of water time equal to 10 minutes. Farmer A continues to water his field for 10 extra minutes and farmer B delays receiving water for that amount of time. This is a straightforward transfer of water time. Suppose instead, that farmers A and C are in an informal exchange agreement and farmer C is two turns away from farmer A. If C transfers water to A for 10 minutes, farmer A takes the 10 extra minutes transferred to him, B receives water 10 minutes later but takes water for an extra 10 minutes, and C delays receiving water for 10 minutes. This requires coordinating the activities of three farmers rather than just two as in the first case. Regardless of the size of the transfer (measured in the number of minutes), the same amount of coordination is required. As the water turn distance between partners, and therefore, the number of partners involved in a transfer increases, so do the transaction costs, irrespective of the size of the transfer.

Apart from distance in water turns, there is an added cost proportional to the physical distance between farms. Most often, transfers are partial turn exchanges in which the receiving household, realizing that it needs water for an extra 5-10 minutes, walks over to a partner household to arrange for a turn.<sup>5</sup> The importance of this physical distance is apparent when one considers a water exchange between two households, one located at the head of the watercourse and the other at the tail. Even though these households are neighbors in terms of water turn distance, walking from the tail to the head-end of the watercourse to request a partial turn transfer in the midst of one's *warabandi* turn is impractical.

In this system of water exchange, water deliveries and water needs are readily observable (and verifiable) by all farmers along the watercourse. The problems of information asymmetries within the watercourse, therefore, are relatively unimportant. However, there remain problems of establishing partnerships and enforcing the obligation of lucky households who received sufficient water and are required to transfer water to partners with poor realizations. Such problems may be more easily resolved between individuals with interlinked social capital such as kinship groups that benefit from relationships of trust that help resolve the inherent time consistency issue in an informal system of exchange. Costs of enforcement within these groups may be lower than with other partners.

Note that, if transaction costs were zero, the probability of observing an exchange between any pair of households in the community, controlling for the benefits of exchange, would be constant across all pairs of households. Hence, observing patterns of exchange that confine transactions to pairs of households with particular social and physical characteristics will reveal the existence of cost advantages related to the social and physical characteristics of the pair. The impact of transaction costs on the configuration of exchanges between potential partners is examined for the Azim 43-L and Fordwah 14-R watercourses of Pakistan.

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<sup>5</sup> A 5-10 minute transfer can be extremely important in terms of its value to the household since these small exchanges help complete watering a field, which would otherwise have to be irrigated from scratch again.

Data collected by the International Water Management Institute during the 1994 *kharif* season record every canal water transaction between households during the season. Further, since the sample is exhaustive within these watercourses, both sides of every transaction can be identified, and measures of social distance (kinship) between exchange partners can be constructed. These data show that among the 595 potential household pairs of Azim 43-L (arising from 35 households), 16% engaged in a transaction during the season while in Fordwah 14-R only 4% of the 4371 potential pairs transacted water.<sup>6</sup>

The transfer matrix in Figure 1 represents the number of exchanges between all potential pairs of households along the Azim 43-L watercourse. Households are numbered in terms of their water turn, with household 1 being closest to the head of the watercourse and 35 at the tailend. Cell  $ij$  of the matrix records the number of times a partial water turn was provided by household  $i$  to household  $j$  during *kharif* 1994. A simple inspection of the transfer matrix reveals preferential exchanges between households who are next to each other in terms of water turn distance (i.e., there is a greater propensity to trade close to the diagonal). Households that are close in terms of water turn distance but far apart in terms of physical distance (i.e., household pairs when one is at the head and the other at the tailend) do not exchange as much. When kinship relationships are examined (illustrated by the different patterns of shading in the Figure), not all kin exchange with each other but there is clearly preferential trading between family members. However, families also tend to be contiguous in terms of water turns, which likely enhances the propensity of kin to exchange. In addition to bilateral exchange, there is also evidence of cluster effects (e.g., households 24 to 30), i.e., with intensive exchanges among members, and few exchanges outside the group, including with neighbors who are close by.<sup>6</sup>

These patterns suggest that transaction costs, as measured by water turn, geographical, and social distance, affect the probability of exchange between households, and that exchange relations are configured in both purely bilateral relationships as well as in clusters of households. We turn to an econometric analysis to test if these relationships are true.

In this endeavor we need to resolve two issues. The first is to characterize the cluster model in a way that clearly distinguishes it from a model of pure bilateral exchanges, and the second is related to the endogeneity of cluster formation. For the first issue, the fundamental feature of the bilateral exchange framework is that whether a link is created or not is largely determined by the characteristics of the pair of partners, while in a cluster model, links are determined by the characteristics of all the members of the cluster, not just the two partners.

In the bilateral exchange model, the probability of exchange between any two farmers  $i$  and  $j$  is a function of some measures of distance  $d_{ij}$  between the two partners:

$$p(e_{ij} = 1) = f(d_{ij}) \quad (\text{bilateral exchange model})$$

where  $e_{ij}$  is a binary variable representing whether the pair (corresponding to each cell in Figure 1) exchanged water at least once during the season. Distance between partners is measured by water-turn distance, geographical distance, and kinship as a measure of social distance. In the cluster model, exchanges take place among members of the same cluster. Exchanges between two partners in a cluster depend on the participation of other members in the cluster, and hence are a function of characteristics of the whole cluster. Hence, a cluster model could be written as:

$$p(e_{ij} = 1) = f(c_{ij}, c_k) \quad (\text{cluster model})$$

where  $c_{ij}$  is a binary variable equal to 1 if  $i$  and  $j$  are members of the same cluster and  $c_k$  are characteristics of the cluster  $k$  to which  $i$  and  $j$  belong.

The econometric estimation can be written as an index function that nests both models:

$$e_{ij}^* = \mathbf{a} \mathbf{1}_{ij} + (\mathbf{b}_{ij} + \mathbf{g}_k) + \mathbf{e}_{ij}, \quad (1)$$

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<sup>6</sup> Note that clusters do not consist solely of water turn neighbors. In the matrix, some households (e.g., 33 and 34) have been placed next to the households they exchange with frequently to uncover a potential cluster configuration.

where  $\mathbf{e}_{ij}$  is an error term. To ensure that  $c_k$  does not capture some unobservable distance measure (and estimation of  $\mathbf{g}_{is}$  therefore, unbiased), we choose for cluster characteristics the average distance (water turn, geographical, or social)  $d$  among all pairs in the cluster, excluding the pair  $ij$ . We will note this variable:

$$c_{k-ij} = \frac{1}{N_k(N_k - 1)} \left( \sum_{mn \in k} d_{mn} - 2 d_{ij} \right),$$

where  $N_k$  is the number of members of cluster  $k$ . The assumption that  $\mathbf{e}_{ij}$  is uncorrelated with  $d_{ij}$  thus ensures that  $\mathbf{e}_{ij}$  is also uncorrelated with  $c_{k-ij}$ . A model of pure bilateral exchange is rejected in favor of a model of cluster formation superimposed on the web of bilateral exchanges if the coefficient  $\mathbf{g}_{is}$  significantly different from zero.

The second issue of endogeneity of clusters is due to the fact that, while we observe that farmers seem to be grouped in clusters within which exchanges are more intense, there is no exogenous definition of these clusters. The clusters are only revealed through the pattern of exchanges. More fundamentally, clusters may not exist beyond consisting of groups of farmers organized for the purpose of exchanging water. This indicates that we cannot test the hypothesis ( $H_0$ ) of a pure bilateral exchange model against ( $H_1$ ) a combination of bilateral exchanges with a given configuration of clusters of farmers  $C_0$ , since there is no external information on the definition of the configuration  $C_0$ .

However, while the choice of a single configuration of clusters  $C_0$  is endogenous, the set  $\{C_s, s \in S\}$  of all potential configurations of clusters is exogenous. We therefore test ( $H_0$ ) against the hypothesis ( $H_2$ ) that there exists a cluster configuration in addition to the bilateral exchange model. Hence, the model of exchanges under ( $H_2$ ) is  $\{($ bilateral exchanges and configuration of cluster  $C_s$ ,  $s \in S\}$ , i.e., (bilateral exchanges and  $C_0$ ) or (bilateral exchanges and  $C_1$ ), etc. The number of potential configurations of clusters is very high, even with a relatively small number of farmers. Yet, since  $H_0$  is embedded in  $H_1$ , itself embedded in  $H_2$ , rejecting  $H_0$  against  $H_1$  is sufficient to reject  $H_0$  against  $H_2$ . In summary, we submit that if we reject the hypothesis that  $\mathbf{g}_{is}$  equal to 0 with at least one cluster configuration  $C_0$ , then we will have shown that there exists at least one cluster configuration that, in combination with bilateral exchanges, better represents the observed exchanges than would a pure bilateral exchange model.

Table 1 reports the analysis for Azim 43-L. By visual inspection of the matrix of exchanges in Figure 1, we chose a cluster configuration  $C_0$ , that consists of 4 clusters of 3, 5, 7, and 7 farmers (22 of the 35 farmers), while the remaining 13 farmers do not belong to any cluster. As discussed earlier, costs of exchange between a pair of households are represented by social distance (kinship) and by physical distance (water turn and geographical distance). To identify the role of these characteristics, we need to control for the other determinants of exchange, particularly the differential benefits of insurance according to where the households are located in the watercourse and the length of their water turns.<sup>7</sup> To identify the impact of cluster characteristics on the probability of exchange between any two partners, we use the percentage of pairs in the cluster (excluding  $ij$ ) that are kin.

Estimates in Table 1 show that the pattern of exchange is affected by water turn and geographical distance, both of which have negative effects on the probability of exchange between households. Kinship is also a powerful explanatory factor.<sup>8</sup> In the bilateral exchange model that focuses on these costs for predicting exchanges, being from the same family offsets the negative effect of water turn distance. Family effects become significantly positive for farmers who are at least two water turns apart. These results confirm the role of distance, and therefore transaction costs, in limiting exchange among community members. When cluster specific effects are introduced, direct family links become insignificant, but the intensity of family links within the clusters significantly increases the probability of exchange between any two households within the cluster. The bilateral exchange model is rejected against the model that includes clusters. Several other cluster configurations also result in rejection of the pure bilateral exchange model. Therefore, our test does not prove nor rely on the fact that the particular configuration that we

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<sup>7</sup> Households located at the tail receive less water, on average, due to seepage losses and are therefore more likely to benefit from insurance. Households with longer water turns have more flexibility in their water management strategies are therefore, less likely to demand insurance.

<sup>8</sup> This is not to imply that *all* kin are involved directly in exchanges. Only 50% of the potential pairs of kin exchanged water at least once during the season.

retain is composed of the actual clusters that farmers have formed. All this analysis shows is that there exists at least one configuration of clusters that outperforms the pure exchange model.

The model of bilateral exchange with clusters explains water exchanges better than the pure bilateral exchange model in the Fordwah 14-R watercourse as well (Table 2). This watercourse covers a much larger area, with 94 households. On the whole, trading is less frequent with only 4% of the 4371 households pairs exchanging full or partial turns at least once during the season, although all households except one had at least one exchange with one partner along the course. Several factors are at play. First, due to administrative preference for the Fordwah distributary, there is less variation in water availability than in Azim, thus providing lesser incentives to trade and insure water realizations. Second, participatory management of water turns is a common practice in Fordwah but not in Azim. Fordwah has a much less inequitable land distribution and a less fragmented social structure than Azim (Strosser and Kuper, 1994) which may explain the differential abilities to engage in joint management. With participatory management in Fordwah, private exchange is a less important mechanism to spread risk. In Fordwah, visual inspection suggests the existence of 7 clusters. Similar to Azim, regression results indicate the role of clusters in explaining exchange. The distance variable retained for characterizing the clusters is the average physical distance among the pairs in the cluster.<sup>9</sup>

These results suggest that clusters play an important role in sustaining mutual insurance and that the community is not the appropriate group for examining the performance of mutual insurance. Drawing from this example, in the following sections we develop a theory of mutual insurance to elucidate the role of transaction costs in determining both the configuration of insurance clusters and the quality of insurance within clusters.

#### 4. Transaction costs of mutual insurance

Two types of transaction costs affect the configuration and quality of mutual insurance. The first are fixed or sunk costs of coordination and information processing that must be incurred for reciprocal exchange between a group of individuals to function. These *association costs* include costs such as those of searching for potential partners, establishing relationships, and coordinating activities. Association costs are thus a function of the number of members in the cluster and independent of the quality of insurance chosen.

The second class of transaction costs, which we shall refer to as *extraction costs*, arise in situations in which exchange is plagued by moral hazard due to imperfect monitoring and enforcement. While most of the literature on cooperation considers these costs as fixed costs, it is likely that enforcement costs vary with the level of transfer requested from the partner. Individuals that need to transfer resources to partners are likely to be more reluctant to comply for larger amounts or for actions which have a higher cost to them. They will expend more effort trying to avoid having to share, hiding their luck, being absent from home when expecting the request, etc. All this suggests that the community has to expend more effort to induce these individuals to comply. Extraction costs are thus assumed to be increasing in the insurance level.

Naturally, association and extraction costs vary across different partner-pairs. Results for water exchange in Pakistan suggest that costs are likely to be lower between households that are kin, and have greater physical proximity. More generally, based on the collective action literature (Olson, 1965; Hirschman, 1970), they should also be smaller in clusters with greater proximity and homogeneity, charismatic leadership, high costs of exit, greater homogeneity and perception of fairness in the distribution of the gains from cooperation, interlinkages among community members, credibility of threats and commitment of sanctions, availability of conflict resolution mechanisms, shared social norms, and high stock of trust capital.

Association costs have important implications for the patterns of exchange within communities. When required to choose among possible partners for exchange, households tend to prefer partners with whom they have

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<sup>9</sup> The regressions for both watercourses were estimated using all three – water turn, geographical, and social -- average distance variables. However, since the variables are highly collinear, the most significant variable was retained for the final specification.

the lowest costs of association.<sup>10</sup> As a result, exchange is often localized to small social spaces. Furthermore, association costs are likely to decrease over time with the accumulation of relation-specific experience or trust. As a result, patterns of exchange tend to display a certain degree of persistence over time. This persistence in reciprocal trade-flows becomes self-sustaining because the mode of exchange itself lowers association costs (Kranton, 1996; Fafchamps, 1999). It, thereby, progressively isolates long-term economic partners from competing sources of demand and supply, and results in localized patterns of exchange. Association costs suggest that informal insurance arrangements will occur in clusters rather than at the community level. Previous studies which have focused on the village as the appropriate insurance group within which one would expect full insurance implicitly assume zero or sufficiently low associations costs among village inhabitants and infinite or extremely high association costs with households outside the village, even though there is reason to believe differently.<sup>11</sup>

The particular structure of association and extraction costs that prevails among members of a community will determine both the configuration of mutual insurance transfers and the quality of insurance achieved. To establish this relation, we develop in the next section a model of mutual insurance where these costs endogenously determine the configuration and performance of informal insurance arrangements.

## 5. A model of costly informal insurance with multiple partners

Consider risk-averse households who face intertemporally variable and independent income streams. All households are similar ex-ante, with identical preferences defined over own income. Each household receives income  $y$  with probability  $\mathbf{p}$  and suffers a loss  $L$  (i.e., receives income  $y - L$ ) with probability  $1 - \mathbf{p}$ . Each household's expected utility in the absence of any kind of informal insurance is

$$Eu = \mathbf{p}u(y) + (1 - \mathbf{p})u(y - L). \quad (2)$$

Since all households are risk averse ( $u' > 0$  and  $u'' < 0$ ) and face uncertain income streams, there are potential gains from state-contingent transfers between them. In order to focus on informal insurance arrangements, alternative consumption smoothing strategies are ignored.

Consider the informal insurance arrangement in which each household which does not suffer a loss agrees to transfer a share  $\alpha$ ,  $0 \leq \alpha \leq 1$ , of the loss to every household which receives income  $y - L$ .<sup>12</sup> No transfers are made if all households receive the same income. Since all households are assumed to be identical, transfer arrangements are symmetric. The overall net transfer given or received by a particular household depends on the income realizations of all households in the insurance cluster, i.e., on the aggregate cluster outcome. If there are  $n$  people in an insurance cluster, expected utility over income for a household is given by:

$$Eu(\alpha, n) = \mathbf{p} \left[ \sum_{x=0}^{n-1} p(x)u(y - (n-1-x)\alpha) \right] + (1 - \mathbf{p}) \left[ \sum_{x=0}^{n-1} p(x)u(y - L + x\alpha) \right]. \quad (3)$$

The first term in square brackets is expected utility when the household suffers no loss. According to the transfer arrangement, the household must transfer  $\alpha L$  to each of its  $n - 1 - x$  partners who received a low income. The probability  $p(x)$  that  $x$  households of the  $n - 1$  partners do not have a loss is distributed as a binomial with

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<sup>10</sup> For example, discussing informal insurance, Posner (1980) notes that households wish to confine their arrangements to “a group to which they belong, whose members know and continually interact with one another and who have broad similar abilities, propensities, character, and prospects.”

<sup>11</sup> Tests of whether insurance deteriorates with ‘distance’ by Rashid (1990, cited in Townsend 1995) in Pakistan and Townsend (1995) in Thailand provide suggestive evidence in support. Both studies, however, test for the village (or region) as the smallest potential insurance group and aggregate up to the national level.

<sup>12</sup> Transfers can also be specified as an absolute amount (as in the sharecropping literature) paid to each household that suffers a loss. What matters is that the total amount transferred or received by a household should be allowed to vary depending on the state of nature.

$p(x) = \binom{n-1}{x} \mathbf{P}^x (1-\mathbf{P})^{n-1-x}$ . Similarly, the second term in square brackets is the expected utility for the household when it suffers a loss. In this case, the household receives a total transfer of  $x\mathbf{aL}$  from the  $x$  households who did not incur a loss. While the share of loss paid out is the same across all states, the actual transfer made (received) by a household in any state depends on the total number of households that incurred a loss (gain).

With both association and extraction costs to insurance, let  $c(\mathbf{a}, n)$  be the total cost associated with informal insurance. As discussed above, the cost need not be in monetary terms, and it can be assumed to be expressed directly in utility terms. We assume that the overall utility obtained from the insurance scheme is separable between income and the cost incurred to enforce the contract.<sup>13</sup> The household expected utility function is thus written:

$$EU(\mathbf{a}, n) = \mathbf{P} \left[ \sum_{x=0}^{n-1} p(x) u(y - (n-1-x)\mathbf{aL}) \right] + (1-\mathbf{P}) \left[ \sum_{x=0}^{n-1} p(x) u(y - L + x\mathbf{aL}) \right] - c(\mathbf{a}, n). \quad (4)$$

Intuitively, the household's motivation in entering a mutual insurance arrangement is to transfer income from good to bad states in order to receive steady income across states of nature, lowering the variance of income without affecting the expected value. This can best be seen by taking a second-order Taylor series approximation of utility  $u$  in each state about expected income. This yields:

$$EU(\mathbf{a}, n) = u(\mathbf{m}) + \frac{1}{2} u''(\mathbf{m}) \mathbf{s}^2 - c(\mathbf{a}, n) \quad (5)$$

where

$$\mathbf{m} = y - (1-\mathbf{P})L$$

and

$$\mathbf{s}^2 = \mathbf{P}(1-\mathbf{P})L^2 \left[ 1 + n(n-1)\mathbf{a}^2 - 2(n-1)\mathbf{a} \right] = \mathbf{s}_0^2 \left[ 1 + n(n-1)\mathbf{a}^2 - 2(n-1)\mathbf{a} \right].$$

In these expressions,  $\mathbf{m}$  and  $\mathbf{s}^2$  are, respectively, the mean and variance of income for a household in an insurance cluster with  $n-1$  partners and a transfer arrangement of  $\mathbf{a}$  while  $\mathbf{s}_0^2$  is the variance of income under autarky (no insurance). The household can use the two instruments -- the number of members in the cluster ( $n$ ) and the transfer arrangement ( $\mathbf{a}$ ) -- to lower variance below autarky levels while leaving mean income unchanged.

The optimal insurance arrangement is derived by maximizing  $EU(\mathbf{a}, n)$  with respect to  $\mathbf{a}$  and  $n$ . Assuming that the cost of increasing cluster size beyond the community is prohibitively large, we include the constraint that cluster size cannot exceed  $N$ , the size of the community.<sup>14</sup> Further, we impose the implementability constraint that utility from insurance has to be at least as great as the autarky utility level:

$$\begin{aligned} & \max_{\mathbf{a}, n} EU(\mathbf{a}, n) \\ \text{s.t. } & EU(\mathbf{a}, n) \geq EU(0, 1) \\ & 0 \leq \mathbf{a} \leq 1 \\ & n \leq N \end{aligned} \quad (6)$$

Using expression (4) for the expected utility of the household, the first-order conditions for an interior solution are:

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<sup>13</sup> Monitoring costs can be thought of as the disutility of lost leisure time. The precise nature of these costs will vary with monitoring technology. For a similar specification, see Newbery and Stiglitz, 1981.

<sup>14</sup> The community may be a village, an ethnic/kinship group, or a watercourse.  $N$  is then, more generally, the largest cluster in which insurance partnerships can be developed, with association costs prohibitively large beyond  $N$ .

$$\begin{aligned}
-u'(\mathbf{m})\mathbf{f}(\mathbf{m})\mathbf{S}_0^2(n-1)(n\alpha-1) &= c_\alpha(\alpha_n) \\
-\frac{1}{2}u'(\mathbf{m})\mathbf{f}(\mathbf{m})\mathbf{S}_0^2\alpha[\alpha(2n-1)-2] &= c_n(\alpha_n)
\end{aligned} \tag{7}$$

where

$$\mathbf{f}(\mathbf{m}) = -\frac{u''(\mathbf{m})}{u'(\mathbf{m})}, \text{ and } \mathbf{S}_0^2 = \mathbf{P}(1-\mathbf{P})L^2.$$

The optimal transfer arrangement and cluster size are such that the marginal benefits of variance reduction are equal to the marginal cost of greater insurance or additional partners. The marginal benefit of a larger cluster or greater insurance increases with the coefficient of absolute risk aversion  $\mathbf{f}$  the level of variance faced by the household in autarky  $\mathbf{S}_0^2$ , and the degree of responsiveness of variance to cluster size. The marginal utility of income,  $u'(\mathbf{m})$ , is a normalization factor.

## 6. Cluster size and optimal insurance

If there were no transaction costs ( $c(\alpha_n) = 0$ ), the first-best transfer arrangement would be to set  $\alpha$  equal to  $1/n$  which is the level of full insurance (losses are divided equally between all cluster members) in a cluster of size  $n$ . From the characterization of variance above, we know that the variance of income faced by each household when there is full insurance within the cluster is  $\mathbf{S}_0^2/n$ ; this residual risk mirrors the aggregate risk faced by the cluster. Therefore, it is in the interest of each household to belong to as large an insurance cluster as possible -- apart from transaction costs, of course.<sup>15</sup>

For an infinitely large cluster, the household receives perfect insurance; i.e., no uncertainty remains and the household receives the mean income in all states. If, however,  $n$  is restricted to be no greater than a particular value  $N$  -- for example, the size of the village -- then the household chooses an insurance cluster of that size and the residual uncertainty is  $\mathbf{S}_0^2/N$ .<sup>16</sup> The literature on mutual insurance generally assumes, implicitly or explicitly, that transaction costs which might prevent the formation of a partnership between all members of a community are zero. This being the case, after the realization of the state of nature, transfers may flow between any two households to bring about risk pooling. If, in an absurd extreme, establishing and maintaining partnerships is indeed costless, there is no reason for a mutual insurance cluster not to be community-wide or world-wide. Real world limits to cluster size must therefore be the result of costs relating to the formation and maintenance of partnerships.

If there are both association and extraction costs to insurance, whether or not there is full or partial insurance and community-level membership or cluster formation depends on the combined costs of increasing cluster size and improving the degree of risk sharing.

To focus on the effects of association costs, consider a simplification of equation (5) that imposes a fixed cost  $A$  to establish a partnership with each household, and restricts extraction costs to zero. That is,  $c(\alpha_n) = (n-1)A$ . Since there are no costs to  $\alpha$  there is full income pooling within the cluster. Knowing this, the household chooses its optimal number of partners to balance the benefits of reduced income variance from

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<sup>15</sup> Determining the optimal solution when there are no transaction costs can be rigorously done with expression (4) for the expected utility. Since there are no costs to insurance, the autarky constraint will not bind and an interior solution is guaranteed for  $\alpha$ . Solving for the first-order condition provides  $\alpha$  equal to  $1/n$ . To determine the optimal cluster size, it is possible to show that the distribution of payoffs faced by a household in an insurance cluster of size  $n$  second-order stochastically dominates the distribution with a cluster of less than  $n$  households. It follows that expected utility increases as cluster size increases (Hadar and Russell, 1969).

<sup>16</sup> Note that we have assumed that there is no covariance between income realizations. A positive covariance of income among members of a finite sized group would confine the effectiveness of informal insurance in reducing individual risk.

increasing  $n$  with the costs of increasing cluster size. Replacing the condition that  $\alpha = 1/n$  in equation (7), optimal cluster size is given by

$$n^2 = \frac{1}{2A} u'(\mathbf{m}) \mathbf{f}(\mathbf{m}) \mathbf{S}_0^2. \quad (8)$$

When costs of forming partnerships are small, there is full insurance at the community level. Alternatively, if costs of establishing partnerships are high, insurance is too expensive to sustain and households are better off in autarky. In between these two extremes, for the interior solution, insurance within the community is characterized by clusters with full insurance within each cluster.

To focus on the case of extraction costs, consider the household's optimization problem when association costs are zero but there are non-zero extraction costs. Since there are no association costs, all  $N$  community members belong to the insurance cluster and only  $\alpha$  is chosen. Although the specifics of the insurance contract and implications for cluster size will vary with monitoring (or enforcement) technology, we can represent this idea with a simple linear technology where the marginal extraction cost is constant and does not depend on the state of nature or on cluster size,  $c(\alpha) = E\alpha$ ,  $E > 0$ . We can rewrite the first-order condition for the choice of  $\alpha$  in equation (7) as follows:

$$\alpha = \frac{1}{N} - \frac{E}{(N-1)N u'(\mathbf{m}) \mathbf{f}(\mathbf{m}) \mathbf{S}_0^2}. \quad (9)$$

In the absence of extraction costs ( $E = 0$ ), there is full community-level insurance. As soon as cost becomes positive,  $\alpha$  decreases below  $1/N$ , leading to partial insurance within the community. Utility falls from its maximum as extraction costs increase. Beyond a certain level of extraction costs, participating in the insurance arrangement leads to utility levels that are lower than those with no insurance at all and the household reverts to autarky.

Comparing Figures 2 and 3 illustrates why both association and extraction costs may yield similar interpretations to traditional empirical tests of full insurance. In Figure 2, as association costs increase, clusters become smaller and the residual risk faced by the household increases. Likewise, in Figure 3, as extraction cost increases, households settle for partial insurance, and therefore face increasing degrees of residual risk. Rejection of full insurance could then be due to association costs, extraction costs, or more likely a combination of the two. The existence of clusters implies that tests of insurance at the village level might also be refuted because the entire village is presumed to be the appropriate group on which to test for efficient risk pooling.

When both costs of association and extraction are positive, different insurance regimes can be summarized graphically in a two-dimensional cost space, with extraction costs on the vertical axis and association costs on the horizontal axis as shown in Figure 4. The five regions -- full community-level insurance, partial community-level insurance, full insurance within clusters, partial insurance within clusters, and autarky -- are optimal responses to the combination of association and extraction costs which satisfy the first order conditions for an interior solution subject to the constraints that the insurance cluster cannot be larger than the community and that utility from the insurance system must be at least as high as that obtained in autarky.

Full insurance at the community level obtains only when the cost of extracting insurance is zero and costs of establishing partnerships are sufficiently low (region 1 on the horizontal axis such that  $A < k/2N^2$  where  $k = u' \mathbf{f} \mathbf{S}_0^2$ ). As association costs increase ( $k/2N^2 < A < k/2$ ), it is too costly to insure at the community level and clusters form (region 2). However, with zero extraction costs, there is full insurance within each cluster. Notwithstanding, in this region, there is less than full insurance at the community level as households aggregate into smaller clusters.

With low costs of organizing partnerships, but positive extraction costs, the optimal insurance arrangement involves a community-wide cluster, but only partial insurance within the community (region 3). If association costs are high and extraction costs sufficiently low, the optimal response is to divide the community into smaller clusters and have partial insurance within each of these (region 4). Finally, if either association costs are very high ( $A > k/2$ ), or extraction costs are substantial ( $E > k(N-1)$ ) or a combination of the two are high, insurance is too expensive and households opt for autarky (region 5).

A discussion of the slopes of the boundaries dividing insurance regimes is instructive since it illuminates the relationship between cluster size and the transfer arrangement at the optimum.<sup>17</sup> The boundary between regions 3 and 4 represents the combination of costs along which the household is indifferent between having partial insurance at the community level and partial insurance in a cluster marginally smaller than the community size ( $n < N$ ). Along this boundary, an increase in association costs creates an incentive to reduce cluster size. A household would be willing to maintain cluster size at  $N$  only if extraction costs were higher (and, therefore, lower insurance provided within the cluster). Notice that utility decreases as costs of association and extraction increase.

The boundary between regions 3 and 5 represents the combination of costs at which the household is indifferent between partial insurance at the community level and autarky. In contrast to the previous case, along this boundary, utility is maintained at autarky levels. An increase in association costs induces a smaller cluster size. Utility is restored to autarky levels only when insurance provision increases within the cluster. This occurs as extraction costs decrease. Likewise, the downward slope of the outer curve between regions 4 and 5 arises from the household's indifference between partial insurance in clusters and autarky.

To summarize, we have identified conditions under which transfers of some form take place (i.e., a risk-sharing institution exists) and under which full income pooling is achieved (i.e., the institution achieves first-best risk sharing). By making the choice of partners endogenous, conditions under which risk-sharing institutions exist in clusters of the community population are identified. Economic theory on the limitations to mutual insurance to date has considered an exogenous group size equivalent to a village or an ethnicity. That assumption restricts insurance regimes only to changes in the degree of insurance at the community level (i.e., to changes along the vertical axis in Figure 4), even though nothing precludes -- and, indeed as we have seen on Pakistan watercourses, there is evidence in support of -- smaller insurance clusters.

## 7. Comparative statics

The benefits of insurance vary with changes in expected income  $m$ , the degree of risk aversion  $\alpha$ , and the income variance the household faces under autarky  $s_0^2$  (see equation 7). As expected income increases (either as  $L$  decreases or  $y$  increases), the marginal utility of income decreases. Assuming constant or decreasing absolute risk aversion,  $k$  decreases, shifting all the boundaries and intersections closer to the origin in Figure 4. This result is in agreement with the frequent observation that informal insurance arrangements are less likely to occur as incomes increase.<sup>18</sup>

Income variance  $s_0^2$  faced by the household under autarky changes with both the magnitude  $L$  and probability of a loss  $1 - p$ . Any factor which increases the variance of income makes the household more tolerant towards costs, shifting all the boundaries and intersections away from the origin. For small losses, then, the household becomes increasingly intolerant to costs, and may prefer autarky to insurance which protects it from this small loss. The effect of reducing the probability of 'success' can be thought of as increasing the likelihood that the household will suffer a loss as well as a reduction in expected income. The optimal response, naturally, is to be willing to tolerate greater costs to establish and maintain exchange agreements. By the same mechanism, households with lower aversion to risk are less willing to tolerate the transaction costs of insurance.

Finally, with the linear specification of transaction costs used above, the size of the cluster  $n$  is a decreasing function of association costs  $A$ , and the quality of insurance  $\alpha$  a decreasing function of the extraction costs  $E$ . The variation of  $n$  with respect to  $E$  is of same sign as the variation of  $\alpha$  with  $A$ , but both can be either positive or negative.

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<sup>17</sup> These boundaries are derived analytically in the Appendix.

<sup>18</sup> For example, Evans-Pritchard (1940, cited in Fafchamps, 1992) suggests that "it is scarcity not self-sufficiency that makes people generous."

To summarize, comparative statics on optimal insurance regimes suggests that the divergence between the optimal informal insurance arrangement and first-best community level risk sharing is smaller when insurance is badly needed: in situations where incomes are lower, losses larger, and the probability of bad realizations higher.

## 8. Empirical analysis of the formation and performance of clusters

Returning to the data, we test some of the predictions of the model regarding the size of clusters and the quality of insurance. For the two watercourses, we use the 11 clusters specified earlier in section 3 (4 in the smaller Azim 43-L and 7 in the larger Fordwah 14-R). Overall, 53% of the farmers belong to an insurance cluster, with greater degree of participation in Azim (63 %) than in Fordwah (49%) (Table 3). About 48% of all potential pairs of farmers pertain to clusters, and about half of the water transfers are done within a cluster. Clusters vary in size from 3 to 11 partners, with an average of 6.2. 22% of the pairs of households in clusters are kin, while overall in these two communities, only 3.8% of the pairs are kin. Within clusters, two-thirds of the pairs exchanged water at least once in the season, and on average they traded approximately two times. Simple correlations show that clusters tend to be larger when risk is higher (further from the head) and members are from the same family. Intensity of exchange is negatively correlated with cluster size. A negative correlation between the percentage of pairs who are kin and average distance between members confirms that clusters with family members can extend further, and this in water turn even more than in physical distance.

The theory developed above predicts that cluster size decreases with association costs and that the quality of insurance decreases with extraction costs. Both the cluster size and the quality of insurance are predicted to increase with exposure to risk. These predictions cannot be tested for the selected clusters (for example, by examining the impact of member characteristics on cluster size) since the cluster itself and the selection of its members are endogenous. Therefore, we approach this choice from an individual's point of view. Each farmer is characterized by indicators of distance to potential partners and of exposure to risk. Physical distance, assumed to be positively correlated to association costs  $A$ , is characterized by the average geographical distance to the closest ten neighbors. Social distance, assumed to be correlated to both association costs  $A$  and extraction costs  $E$ , is characterized by the number of family members that each farmer has along the canal. Risk exposure is captured by location along the canal and length of water turn. Since potential partners are themselves heterogeneous, in reality each farmer will choose not only the size of the cluster but also the specific partners, as a function of his costs of extraction, costs of association, and exposure to risk. The matching that is actually obtained obviously has to be a compromise between the ideal cluster that each one would want for himself and the difficulty of finding partners. Nevertheless, we expect to find that each individual is in a group that corresponds to his desired cluster in terms of size and quality of insurance.

Table 4 reports estimations done at the individual level, i.e., for the 129 farmers that are on either one of the two watercourse command areas. We do not have information on the level of residual variance, nor on the quality of insurance ( $\alpha$  in the model), but only on an intermediate indicator, the intensity of exchange. We therefore explain the size of the cluster to which each farmer belongs (with size 0 for those not belonging to any cluster) and the percent of pairs that exchange at least once in each cluster as a measure of the intensity of exchange:

$$n_i = N_c \text{ if } i \in \text{cluster } c, 0 \text{ if } i \text{ does not belong to any cluster},$$

$$I_i = M_c / [N_c (N_c - 1) / 2] \text{ if } i \in \text{cluster } c, 0 \text{ if } i \text{ does not belong to any cluster},$$

where  $N_c$  is the number of members of cluster  $c$  and  $M_c$  is the number of pairs that exchange at least once in cluster  $c$ . As approximately 43% of the observations do not belong to clusters and hence have zero values, we use a Poisson and a Tobit distribution for these two variables, respectively.

Results in Table 4 confirm that cluster size is an increasing function of the availability of family members (negatively correlated with  $A$  and  $E$ ), decreases with distance (positively correlated with  $A$ ), and decreases in water turn length (negatively correlated with  $\sigma^2$ ). Intensity of transfers increases with the availability of family members (-  $A$  and -  $E$ ), and decreases in water turn length (-  $\sigma^2$ ), but does not seem to respond to distance ( $A$ ).

## 9. Concluding remarks

This paper has drawn attention to the role of transaction costs in shaping both the configuration of transfers and the quality of mutual insurance within communities. Data from water transfers along two watercourses in Pakistan, where water delivery is subject to idiosyncratic random shocks, show that households insure by exchanging water bilaterally with neighbors and family members, and also with members of tightly knit clusters. This pattern of exchange suggests that transaction costs are important in shaping the configuration of insurance transfers.

Given the observed importance of clusters, we have formalized the trade-off between the number of partners in an informal insurance agreement and the degree of risk pooling, as a function of different types of transaction costs: costs of establishing associations that are fixed per partner, and extraction costs to implement income transfers that are a function of the level of insurance. Which of these costs are important in a particular context depends on the specific purpose of the insurance system, the community characteristics, etc. The specificity of these costs influences the size of the cluster as well as the optimal set of state-contingent transfers. In particular, high association costs combined with low extraction costs will lead to clusters with full insurance, while low association costs with high extraction costs will lead to community-level partial insurance. The first case in particular implies that tests of risk pooling at the village level, or other reference group, may be refuted because the village is not necessarily the logical reference group. Analysis of water transfers in Pakistan confirms that the size of the mutual insurance cluster decreases with association costs while the intensity of exchanges decreases with extraction costs.

The results of this paper raise several considerations for policy. First, they suggest that to improve the quality of insurance, it is imperative to focus policy levers on reducing association and extraction costs which lead to lower insurance in the group of interest: lower association costs allow higher quality insurance by broadening insurance clusters to the full community; lower extraction costs allow full instead of partial insurance in the chosen insurance group. Second, evidence of kinship as an organizing principle for self-selected clusters points to the relevance of social capital in promoting informal risk sharing. Risk sharing takes place preferentially in social arenas that facilitate rapid information flows, impose norms of fairness and reciprocity, and apply social sanctions on defaulting parties. These functions of local institutions are non-trivial because they solve inherent problems of coordination, asymmetric information, and contract enforcement that can be prohibitively costly for outsiders to solve. If effective mutual insurance is to transcend clusters of kin, social capital needs to be created. Finally, since local institutions perform important welfare functions, external interventions should enhance instead of crowd out their beneficial roles.

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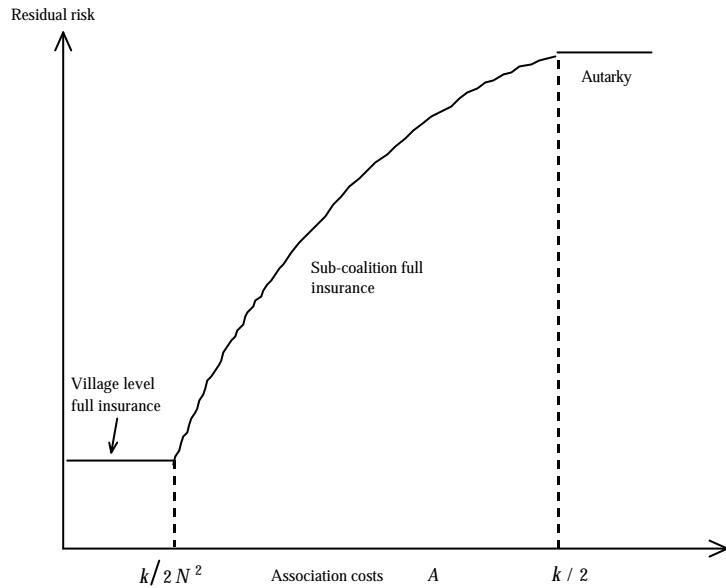
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30																																								
31																																							3	1
32																																							2	3
35																																								1

Farmers' numbers refer to water-turn, numbers in cells are the number of exchanges during the season

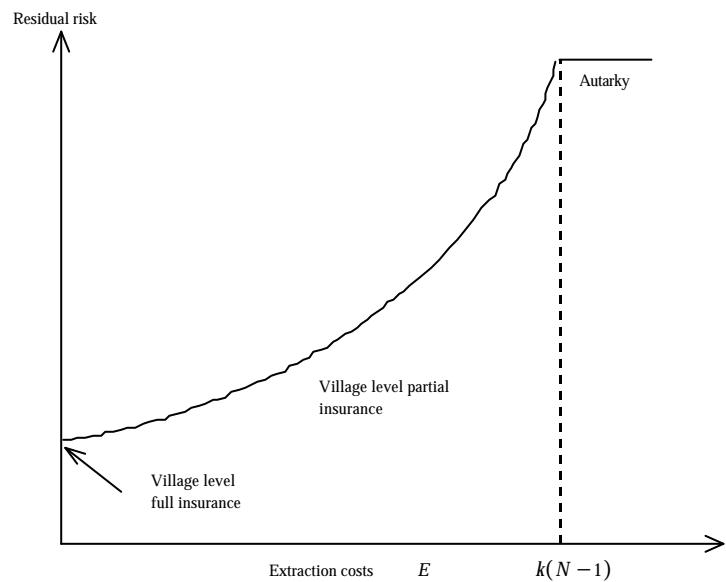
Families identified by color:     

Clusters identified by: 

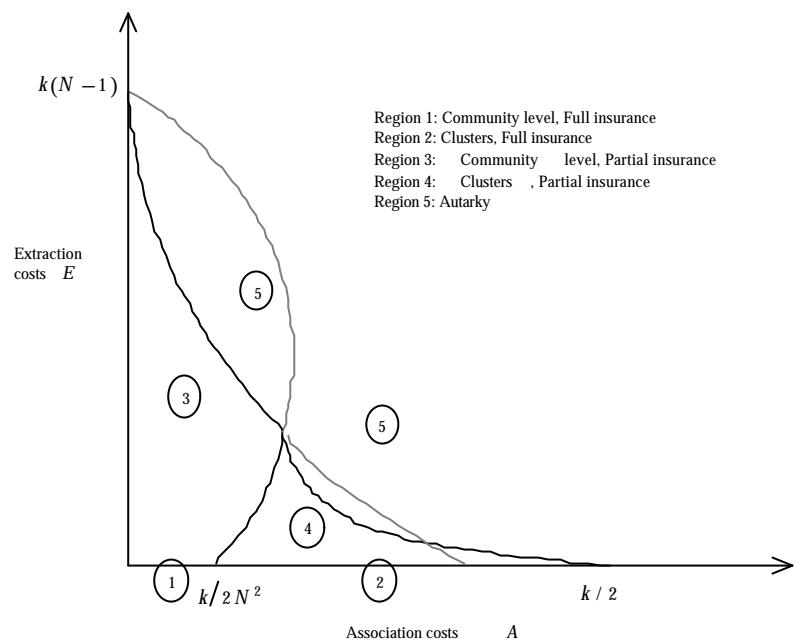
**Figure 1. Matrix of exchanges between farmers of the Azim 43-L watercourse.**



**Figure 2. Residual risk faced by a household as a function of association costs.**



**Figure 3. Residual risk faced by a household as a function of extraction costs.**



**Figure 4. Alternative insurance regimes as a function of association and extraction costs.**

**Table 1. Configuration of water exchanges on Azim 43-L watercourse**

Endogenous variable: probability that the household pair traded at least once

	Mean value of variables	Bilateral exchange model Marginal effect <sup>*</sup>	Bilateral exchange model z <sup>**</sup>	Bilateral and cluster exchange model Marginal effect <sup>*</sup>	Bilateral and cluster exchange model z <sup>**</sup>
<b>Bilateral exchange model</b>					
Physical distance ( $d_{ij}$ )					
Water turn distance	9.0	-0.017	-5.9	-0.011	-4.0
Not geographical neighbors	0.84	-0.178	-5.9	-0.139	-4.4
Social proximity ( $d_{ij}$ )					
Same family	0.05	0.078	1.3	-0.073	-1.3
Same family*water turn distance	0.28	0.025	3.5	0.010	1.3
<b>Cluster model</b>					
Same cluster ( $c_{ij}$ )	0.09	-	-	0.501	8.3
% of other pairs in cluster that are family members ( $c_k$ )		-	-	0.195	2.4
<b>Control variables for the benefits of insurance</b>					
Average length of water turn	4.8	0.003	1.0	0.011	3.6
Location on canal <sup>#</sup>	12.0	0.001	1.2	0.000	-0.2
Number of observations	595	595	595		
Pseudo-R <sup>2</sup>		0.25	0.36		
Log likelihood		-192.5	-165.1		
Test against bilateral and cluster exchange model		chi2(2) = 54.8			
		Prob > chi2 = 0.0000			

<sup>\*</sup>Marginal effects, multiplied by 100, computed at the means of the explanatory variables. Computed as discrete changes for dummy variables (not geographical neighbors, same family, and same cluster).

<sup>#</sup>Location of the household closest to the head of the canal = continuous variable from 1 to 35, from head to tailenders.

<sup>\*\*</sup>z statistic of the corresponding parameter. Standard errors obtained by bootstrapping with 1000 repetitions.

**Table 2. Configuration of water exchanges on F14-R watercourse**

Endogenous variable: probability that the household pair traded at least once

	Mean value of variables	Bilateral exchange model		Bilateral and cluster exchange model	
		Marginal effect <sup>*</sup>	z <sup>**</sup>	Marginal effect <sup>*</sup>	z <sup>**</sup>
<b>Bilateral exchange model</b>					
Physical distance ( $d_{ij}$ )					
Water turn distance	23.8	-0.09	-7.8	-0.05	-5.6
Not geographical neighbors	0.91	-1.18	-7.0	-0.46	-3.7
Social proximity ( $d_{ij}$ )					
Same family	0.04	1.28	3.1	0.44	1.8
Same family*water turn distance	0.61	0.04	2.3	0.03	2.1
<b>Cluster model</b>					
Same cluster ( $c_{ij}$ )	0.09			6.22	15.6
Avg. distance among other pairs in cluster ( $c_k$ )				-0.04	-4.4
<b>Control variables for the benefits of insurance</b>					
Average length of water turn	1.8	-0.001	0.0	0.007	0.2
Location on canal <sup>#</sup>	31.7	0.003	1.8	0.002	1.7
Number of observations	4 371			4 371	
Pseudo-R <sup>2</sup>	0.32			0.43	
Log likelihood	-477.3			-403.0	
Test against bilateral and cluster exchange model		chi2(2) = 148.6		Prob > chi2 = 0.0000	

<sup>\*</sup>Marginal effects, multiplied by 100, computed at the means of the explanatory variables. Computed as discrete changes for dummy variables (not geographical neighbors, same family, and same cluster).

<sup>#</sup>Location of the household closest to the head of the canal = continuous variable from 1 to 94, from head to tailenders.

\*\* z statistic of the corresponding parameter. Standard errors obtained by bootstrapping with 1000 repetitions.

**Table 3. Characteristics of the exchange clusters**

Participation in exchange clusters	Watercourse			
	Azim 43-L	Fordwah 14-R	Overall	
(as a % of all farmers, pairs, or exchanges in the two watercourses)				
Farmers	62.9	48.9	52.7	
Pairs of farmers that exchange at least once	47.3	47.9	47.7	
Exchanges	53.8	50.4	51.4	
<hr/>				
Characteristics of the 11 clusters	Average	St.dev.	Minimum	
Maximum				
Number of members	6.2	2.1	3.0	11.0
% pairs that are kin	22.3	24.1	0.0	71.4
Average water turn distance	4.9	4.7	1.3	16.9
Average geographical distance	1.6	1.4	0.2	5.0
Total number of exchanges	36.9	35.8	14.0	138.0
% pairs that exchange at least once	68.0	20.0	33.3	100.0
Av. number of exchanges per pair	2.2	1.2	0.8	5.3
<hr/>				
Correlation with the number of members				
% pairs that are kin	0.22			
Average location on watercourse	0.51			
% pairs that exchange	-0.52			
Av. number of exchanges per pair	-0.40			
<hr/>				

**Table 4. Determinants of the size and intensity of exchange in clusters**

	Number of members in cluster (Poisson regression)		Percent of pairs that exchange (Tobit)	
	Coefficient	z	Coefficient	z
<b>Transactions costs</b>				
Number of family members along the canal (-A, -E)	0.082	4.3	0.046	2.1
Average distance to the closest ten neighbors (A)	-0.017	-2.7	-0.005	-0.9
<b>Risk exposure</b>				
Location on watercourse	0.002	0.9	-0.002	-0.9
Own water turn length	-0.130	-4.7	-0.042	-1.7
Fordwah 14-R watercourse effect	-0.663	-5.1	-0.528	-3.4
Intercept	1.829	11.2	0.614	3.4
<hr/>				
Number of observations	129		129	
Pseudo R <sup>2</sup>	0.08		0.09	
<hr/>				

## Appendix: Derivation of insurance regime boundaries

The insurance regimes in Figure 4 are derived from the optimization problem:

$$EU(\mathbf{a}, n) = u(\mathbf{m}) + \frac{1}{2} u''(\mathbf{m}) \left\{ \mathbf{p}(1 - \mathbf{p})L^2 \left[ 1 + n(n-1)\mathbf{a}^2 - 2(n-1)\mathbf{a} \right] \right\} - E\mathbf{a} - A(n-1)$$

such that  $0 \leq \mathbf{a} \leq 1$ ,  $1 \leq n \leq N$ , and  $EU(\mathbf{a}, n) \geq EU(0, 1)$ .

The first-order conditions for the choice of  $\mathbf{a}$  and  $n$  for an interior solution are given by

$$k(n-1)(1-n\mathbf{a}) = E \text{ and } \frac{k}{2}\mathbf{a}(\mathbf{a}+2-2n\mathbf{a}) = A \text{ where } k = u' \mathbf{f} \mathbf{s}_0^2 = -u'' \mathbf{p}(1-\mathbf{p})L^2.$$

**1. Boundary between regions 1 and 2** (on the horizontal axis) is the case when the household is indifferent between full insurance at the community level and full insurance in a cluster marginally smaller than the community. It is characterized by the first-order conditions for  $\mathbf{a}$  and  $n$  when extraction costs ( $E$ ) are zero and  $n=N$ . That is,  $\mathbf{a} = \frac{1}{N}$ , and  $\frac{k}{2N} \{ \mathbf{a} + 2 - 2N\mathbf{a} \} = A$ . This implies that  $A = \frac{k}{2N^2}$ .

**2. Boundary between regions 2 and 5** (on the horizontal axis) is the case when the household is indifferent between full insurance in a cluster and autarky. It is characterized by the first-order conditions for  $\mathbf{a}$  and  $n$  when extraction costs are zero and  $n=1$ . That is,  $\mathbf{a} = \frac{1}{n}$ ,  $\frac{k}{2}\mathbf{a}\{\mathbf{a}+2-2n\mathbf{a}\} = A$ , and  $n=1$ . This implies that  $A = \frac{k}{2}$ .

**3. Boundaries between regions 3 and 4, and between regions 3 and 5** are cases when the household is indifferent between partial community-level insurance and partial insurance in clusters marginally smaller than the community (boundary between 3 and 4), and when it is indifferent between partial community-level insurance and autarky (boundary between 3 and 5).

Indifference between partial community-level insurance and partial insurance in clusters marginally smaller than the community is derived from the first order conditions for  $\mathbf{a}$  and  $n$  when  $n=N$ . These are  $k(n-1)(1-n\mathbf{a}) = E$  and  $\frac{k}{2}\mathbf{a}(\mathbf{a}+2-2n\mathbf{a}) = A$ . Together, these conditions imply a curve between  $E$  and  $A$  when a household which faces positive extraction costs and association costs chooses  $n=N$  and an optimal  $\mathbf{a}$ . The curve is characterized by  $A = \frac{k}{2} \frac{k(n-1)-E}{(k(n-1)N)^2} \{ 2EN + k(n-1) - E \}$ . The curve is a parabola, with  $A$  an increasing function of  $E$  in the range  $0 \leq E \leq \frac{k(N-1)^2}{2N-1}$  and a decreasing function beyond; When association costs are zero,  $E = k(N-1)$ , and when extraction costs are zero,  $A = \frac{k}{2N^2}$ , as derived in case 1. In the range where  $A$  is an increasing function of  $E$ , the cluster size and transfer arrangement are complements. The complementarity arises from the structure of variance. In this region, the marginal benefits of risk reduction from changing cluster size is positive with respect to  $\mathbf{a}$ . That is,  $-\frac{d}{d\mathbf{a}} \left( \frac{d\mathbf{s}^2}{dn} \right) > 0$ . Beyond this region, as  $E$  increases to  $k(N-1)$ ,  $A$  decreases as the cross partial of variance with respect to  $\mathbf{a}$  and  $n$  changes sign. Therefore,  $\mathbf{a}$  and  $n$  are substitutes in this part of the boundary.

In addition to this relationship between  $E$  and  $A$ , the optimal solutions are characterized by a participation constraint which requires that utility from insurance exceeds the utility a household retains in autarky. The constraint, when combined with the first-order conditions for  $\mathbf{a}$  and  $n$ , and the condition that  $n=N$  implies that  $\{E - k(N-1)\}^2 - 2Ak(N-1)^2N = 0$ . When association costs are zero,  $E = k(N-1)$  in the positive vertical axis, and when extraction costs are zero,  $A = \frac{k}{2N}$ . In between these points, the curve is downward sloped and convex.

For  $E > k(N - 1)$ , the relationship between  $E$  and  $A$  is upward sloped. However, for these values of  $E$ , the optimal  $\mathbf{a}$  is negative, which violates the constraint that  $\mathbf{a}$  lies between 0 and 1.

Between regions 3 and 5, the household is indifferent between partial community-level insurance and autarky. Therefore, the boundary is defined by the first-order condition for  $\mathbf{a}$  as well as the participation constraint when  $n = N$ . In contrast, between regions 3 and 4, the household is indifferent between partial community-level insurance and partial insurance in a cluster; the boundary is determined only by the two first-order conditions when  $n = N$ .

**4. Boundary between regions 4 and 5** is the case when the household is indifferent between partial insurance in clusters and autarky. This boundary is obtained when the first-order conditions for  $\mathbf{a}$  and  $n$ , and the participation constraint are met. That is,  $k(n-1)(1-n\mathbf{a})=E$ ,  $\frac{k}{2}\mathbf{a}(\mathbf{a}+2-2n\mathbf{a})=A$ , and  $EU(\mathbf{a}n)=EU(0,1)$ . With some algebra, the three conditions can be reduced to  $n\mathbf{a}^2=2A^*$ ;  $n\mathbf{a}=\frac{1}{3}(\mathbf{a}+2)$ ; and  $\frac{2}{3}(1-\mathbf{a})-\mathbf{a}E^*-2(n-1)A^*=0$  where  $E^*=\frac{E}{k}$  and  $A^*=\frac{A}{k}$ . Total differentiation of these 3 conditions with respect to  $\mathbf{a}$ ,  $n$ ,  $E^*$ , and  $A^*$  can be used to show that  $E$  and  $A$  are negatively related along this boundary, suggesting that cluster size and transfer arrangements are substitutes in the region.

As a final point, note that the boundary between regions 4 and 5 intersects the curves for case 3 at the same point. Here, conditions which satisfy all three curves are met:  $n=N$ ,  $EU(\mathbf{a}n)=EU(0,1)$ ,  $\mathbf{a}=\frac{k(n-1)-E}{k(n-1)n}$ . For case 3,

this point is dictated by the participation constraint and the constraint that cluster size not exceed the community size. On the other hand, for case 4, this point is a result of the community-size and the corresponding  $\mathbf{a}$  being chosen as the optimal *interior* solution to the maximization. For this case, as extraction costs increase further, the optimal interior choice of  $n$  is greater than  $N$ , making that range infeasible.

### Additional derivations for the referee

#### Calculation of $\mathbf{m}$ and $s^2$

$$\begin{aligned} \text{With probability } \mathbf{p}(x) & \text{ income is } y^* = y - (n-1)\mathbf{a} + \mathbf{a}L \\ (1-\mathbf{p})p(x) & \text{ income is } y^* = y - L + \mathbf{a}L \end{aligned}$$

$$\begin{aligned} E(y^*) &= \mathbf{p} \sum_x p(x)(y - (n-1-x)\mathbf{a}) + (1-\mathbf{p}) \sum_x p(x)(y - L + x\mathbf{a}) \\ &= \mathbf{p} \sum_x p(x)(y - (n-1)\mathbf{a}) + (1-\mathbf{p}) \sum_x p(x)(y - L) + \sum_x p(x)x\mathbf{a} \\ &= y - \mathbf{p}(n-1)\mathbf{a} - (1-\mathbf{p})L + \mathbf{p}(n-1)\mathbf{a} \\ &= y - (1-\mathbf{p})L \end{aligned}$$

$$\text{var}(y^*) = \mathbf{p} \sum_x p(x)((1-\mathbf{p})L - (n-1-x)\mathbf{a})^2 + (1-\mathbf{p}) \sum_x p(x)((1-\mathbf{p})L - L + x\mathbf{a})^2.$$

Let  $y_0^* = (1-\mathbf{p})L - (n-1)\mathbf{a}$ . Then,

$$\begin{aligned} \text{var}(y^*) &= \mathbf{p} \sum_x p(x)(y_0^* + x\mathbf{a})^2 + (1-\mathbf{p}) \sum_x p(x)(-\mathbf{p}L + x\mathbf{a})^2 \\ &= \mathbf{p} \sum_x p(x)(y_0^{*2} + 2y_0^*x\mathbf{a} + x^2\mathbf{a}^2L^2) + (1-\mathbf{p}) \sum_x p(x)L^2(\mathbf{p}^2 + 2\mathbf{p}\mathbf{a} + x^2\mathbf{a}^2) \\ &= \mathbf{p}y_0^{*2} + (1-\mathbf{p})\mathbf{p}^2L^2 + \sum_x p(x)x[2\mathbf{p}y_0^*\mathbf{a} + 2\mathbf{p}(1-\mathbf{p})\mathbf{a}^2] + \sum_x p(x)x^2[\mathbf{p}\mathbf{a}^2L^2 + (1-\mathbf{p})\mathbf{a}^2L^2] \\ &= \mathbf{p}y_0^{*2} + (1-\mathbf{p})\mathbf{p}^2L^2 + (n-1)\mathbf{p}[2\mathbf{p}y_0^*\mathbf{a}L + 2\mathbf{p}(1-\mathbf{p})\mathbf{a}^2] + (n-1)\mathbf{p}(1+n\mathbf{p}-2\mathbf{p})[\mathbf{p}\mathbf{a}^2L^2 + (1-\mathbf{p})\mathbf{a}^2L^2] \end{aligned}$$

$$\begin{aligned} \frac{1}{\mathbf{p}^2} \text{var}(y^*) &= (1-\mathbf{p})^2 + (n-1)^2\mathbf{a}^2 - 2(1-\mathbf{p})(n-1) + (1-\mathbf{p})\mathbf{p} - 2(n-1)^2\mathbf{a}^2\mathbf{p} + (1-\mathbf{p})(n-1)\mathbf{a}^2 + (n-1)^2\mathbf{a}^2\mathbf{p} \\ &= (1-\mathbf{p})[1 - 2(n-1)\mathbf{a} + n(n-1)\mathbf{a}^2] \end{aligned}$$

#### Derivation of (5)

$$\begin{aligned} u(y - (n-1-x)\mathbf{a}) &= u(\mathbf{m}) + u'(\mathbf{m})(y - \mathbf{m}) + \frac{1}{2}u''(\mathbf{m})(y - \mathbf{m})^2 \\ EU(\mathbf{a}, n) &= \mathbf{p} \sum_{x=0}^{n-1} p(x) \left[ u(\mathbf{m}) + u'(\mathbf{m})(y^* - \mathbf{m}) + \frac{1}{2}u''(\mathbf{m})(y^* - \mathbf{m})^2 \right] \\ &\quad + (1-\mathbf{p}) \sum_{x=0}^{n-1} p(x) \left[ u(\mathbf{m}) + u'(\mathbf{m})(y^* - \mathbf{m}) + \frac{1}{2}u''(\mathbf{m})(y^* - \mathbf{m})^2 \right] - c(\mathbf{a}, n) \\ &= u(\mathbf{m}) + u'(\mathbf{m})E(y^* - \mathbf{m}) + \frac{1}{2}u''(\mathbf{m})\text{var}(y^*) - c(\mathbf{a}, n) \\ &= u(\mathbf{m}) + \frac{1}{2}u''(\mathbf{m})\text{var}(y^*) - c(\mathbf{a}, n) \end{aligned}$$