THE EXPANSION METHOD AS A TOOL OF REGIONAL ANALYSIS

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Standard economic relationships often are estimated econometrically across sets of counties, cities, states, or regions. For example, estimates of demand functions, supply functions, Phillips curves, cost functions, and production functions all have relied, to some extent, on cross-sectional data bases. Although variables are introduced into these relationships to capture spatial dimensions of the behavior in question, the traditional econometric model has no direct way to make the parameters of the underlying behavioral relationship a function of these spatial features.

Casetti (1972, 1982, 1986), writing for geographers, suggests how the traditional econometric model may be expanded to include, as part of its results, a set of equations that would relate the parameters of the regression equation to cross-sectional spatial characteristics. This paper describes Casetti's expansion method and suggests its applicability to a wide range of regional and urban economic situations.

In addition, the paper presents results obtained from the use of the expansion method to estimate the relationship between the odds of being in poverty for female-headed households as a function of AFDC payment levels. Those results, along with others, then are used to demonstrate the power of the expansion technique as a tool of regional economic analysis. Limitations of the technique also are discussed.

The Expansion Model

Consider the set of economic relationships where an underlying relationship between some behavior (y) and some causal factor (x) is tempered by some spatial characteristic (s). Let the relationship between y and x be quadratic (for example, a profit function, a MC or MPP curve, a nonlinear consumption function, and the like) so that:

(1) $y_i = a + bx_i + cx_i^2$

where:

subscript i = the ith cross-sectional observation.

*The author would like to acknowledge the comments of Jarvin Emerson who discussed this paper at the 1990 Mid-Continent Regional Science meetings.

The expansion method posits that the spatial factor s influences this relationship through the parameters a, b, and c because:

(2) $a = f(s_i)$

(3) $b = g(s_i)$

(4) $c = h(s_i)$

In order to estimate the influences of the spatial factors, specific functions for a, b, and c must be chosen. For sake of simplicity, it is assumed that a linear relationship exists between the parameters of the model and s such that:

(2a) $a = a_0 + a_1 s_i$

(3a) $b = b_0 + b_1 s_i$

(4a) $c = c_0 + c_1 s_i$.

Substituting equations (2a) through (4a) into equation (1) yields the terminal equation:

(1a) $y_i = a_0 + b_0 x_i + c_0 x_i^2 + a_1 s_i + b_1 s_i x_i + c_1 s_i x_i^2 + e_i$

$$y_i = (a_0 + a_1 s_i) + (b_0 + b_1 s_i) x_i + (c_0 + c_1 s_i) x_i^2 + e_i$$

where:

e_i = an error term.

This equation easily can be estimated by ordinary least squares (OLS) by introducing into equation (1) s_i and the interaction terms $s_i x_i$ and $s_i x_i^2$. The model tests for spatial effects on the intercept and slope of the relationship by making these functions of s_i. Such spatial interdependence relies entirely on the sign and statistical significance of a_1, b_1, and c_1. Where y_i achieves an optimum with respect to x_i, that optimum, x_i^*, also includes spatial characteristics or:

$x_i^* = (b_0 + b_1 s_i) / 2(c_0 + c_1 s_i)$.

Again, the influence of s_i on x_i^* depends entirely on the signs and statistical significance of b_1 and c_1. Finally, the R^2 change when s_i is introduced into the model provides a simple way of determining the relative contributions of x_i and s_i to the total variation in y_i.

The above illustration can be extended easily in two ways. First, a, b, and c can be made functions of several spatial characteristics (i.e., several s_i's) so that:

(2b) $a = a_0 + \Sigma\, a_k s_{ki}$

(3b) $b = b_0 + \Sigma\, b_k s_{ki}$

(4b) $c = c_0 + \Sigma\, c_k s_{ki}$.

Second, the technique permits many functional forms connecting the parameters of the model to s_i. In theory, the type of functional relationship between the parameters and s_i appears unrestricted. Nonetheless, those functional forms relating a, b, and c to s_i that are linear in the parameters will allow estimation by OLS; those nonlinear in the parameters will not. Thus, $a = a_0 + a_1 s_i + a_2 s_i^2$ or $a = a_0 + a_1(\ln s_i)$ are eligible forms, while $a = a_0 + (\ln a_1)s_i$ or $a = a_0 + a_1^2 s_i$ are not.

Illustrations of the Expansion Method

Casetti notes that the expansion model has been used in estimating fertility and mortality decline, the diffusion of tractors in the U.S., spatial autocorrelation in regression residuals, temporal drifts of model parameters, CBD sales and urban population, and regional variation in food stamp program participation [5, pp. 30-31]. In this section, the expansion method is applied to demand, production, and earnings functions. The illustrations used are not necessarily intended to indicate valuable areas of future research, but to provide a flavor of what the expansion technique tries to do and what its potential may be as a tool of regional economic analysis.

Regional Variations in Demand

The dummy variable method developed by Gujarati [16] represents a special case of the expansion method. Gujarati proposes the introduction of dummy variable regressors to capture the impact of a qualitative variable (e.g., race, sex, war, region, etc.) on the intercept and slope of a function as an alternative to the Chow test of the equivalency of two regressions. Assume that this is a cross-sectional study of demand and the following equation is used to estimate the demand function:

(5) $Q_i = a - bP_i + cY_i + e_i$

where:

P_i = price; and
Y_i = income.

Further, assume that the data are from states or cities in two regions. To test the hypothesis that a, b, and c do not differ by region, a regional dummy variable, R_i, with a value 1 if the ith observation is in region 1 and 0 otherwise that interacts with P_i and Y_i is introduced into equation (5). Such a procedure implicitly defines the linear expansion equations:

(6) $a = a_0 + a_1 R_i$

(7) $b = b_0 + b_1 R_i$

(8) $c = c_0 + c_1 R_i$

so that the terminal model then becomes:

(5a) $Q_i = a_0 - b_0 P_i + c_0 Y_i + a_1 R_i - b_1 R_i P_i + c_1 R_i Y_i + e_i$

$$Q_i = (a_0 + a_1 R_i) - (b_0 + b_1 R_i)P_i + (c_0 + c_1 R_i)Y_i + e_i.$$

The sign and significance of a_1, b_1, and c_1 allow the hypothesis of regional variation in the parameters of the demand function to be tested. Note, however, that the Gujarati procedure restricts the functional relationship between the parameters and the spatial variable to depend only on a qualitative variable having the values of 0 or 1. It thereby limits the type of tests made possible by the more general expansion model.

Spatial Aspects of Production

The more general model appears well suited to the analysis of spatial effects of production relationships. For illustration, consider a Cobb-Douglas production function and assume that it will be estimated at the industry level across cities or states. One may expect that this relationship will be influenced by the extent of localization and urbanization economies present at each location. These easily can be included as factors of production that impact technology, the marginal physical products of capital and labor, economies of scale, and optimum capital-labor ratios. For example, let the production function be given as:

(9) $Q = e^{a}L_i^{b}K_i^{c}$

and define the following expansion equations:

(10) $a = a_0 + a_1A_{Li} + a_2A_{Ui}$

(11) $b = b_0 + b_1A_{Li} + a_2A_{Ui}$

(12) $c = c_0 + c_1A_{Li} + a_2A_{UI}$

where:

A_{Li} = measure of localization; and
A_{Ui} = measure of urbanization.

The linearized natural log form of this model is:

(9a) $\ln Q = a_0 + a_1A_{Li} + a_2A_{Ui} + b_0\ln_{Li} + b_1A_{Li}\ln L_i +$

$$b_2A_{Ui}\ln L_i + c_0\ln K_i + c_1A_{Li}\ln K_i + c_2A_{Ui}\ln K_i + v_i$$

where:

v_i = an error term.

Equation (9a) can be estimated by OLS, and the results can be used in several ways. Returns to scale in this production function equal:

(13) $b + c = b_0 + c_0 + (b_1 + b_2)A_{Li} + (b_2 + c_2)A_{Ui}$.

Thus, the impact of localization and urbanization economies on economies of scale readily can be found. Further, the marginal productivity of labor and capital is given as:

(14) $Q_L = (b_0 + b_1A_{Li} + b_2A_{Ui})e^{a}L^{(b_0 + b_1A_{Li} + b_2A_{Ui} - 1)}K^{(c_0 + c_1A_{Li} + c_2A_{Ui})}$

(15) $Q_K = (c_0 + c_1A_{Li} + c_2A_{Ui})e^{a}L^{(b_0 + b_1A_{Li} + b_2A_{Ui})}K^{(c_0 + c_1A_{Li} + c_2A_{Ui})}$

Consequently, it is straightforward to assess the impacts of these agglomeration economies on the productivity of labor and capital and to obtain specific productivity measures at each location. Finally, the cost-minimizing ratio of capital to labor is given as:

(16) $K/L = (c/b)\,(P_L/P_K)$

$$K/L = ((c_0 + c_1 A_{Li} + c_2 A_{Ui}) / (b_0 + b_1 A_{Li} + b_2 A_{Ui}))\,(P_L/P_K)$$

Hence, even if factor prices equalize across space, each location's capital-labor ratio will differ in equilibrium, with those differences directly related to the expansion parameters b_1, b_2, c_1, c_2 and the measures of localization and urbanization economies A_{Li} and A_{Ui}.

Spatial Differences in Earnings

Earnings profiles demonstrate the relationship between earnings at a particular level of education and experience. Most estimates show that such functions take the form:

(17) $E_i = a + bExp_i + cExp_i^2 + e_i$

where $b > 0$ and $c < 0$. To the extent that location matters, such functions will differ from place to place with the parameters of equation (17) capturing the impacts of specific urban labor market conditions and urban amenities. Using the expansion method, the parameters of earnings profiles thus become functions of local labor market and city amenities so that:

(18) $a = a_0 + \Sigma a_j L_{ji}$

(19) $b = b_0 + \Sigma b_j L_{ji}$

(20) $c = c_0 + \Sigma c_j L_{ji}$

where:

L_{ji} = the jth labor market characteristic or measure of amenity at the ith location.

The terminal equation then becomes:

(21) $E_i = (a_0 + \Sigma a_j L_{ji}) + (b_0 + \Sigma b_j L_{ji})\,Exp_i + (c_0 + \Sigma c_j L_{ji})\,Exp_i^2 + e_i$

The signs and statistical significance of the a_j's, b_j's, and c_j's determine whether the position of the function (i.e., its intercept) or its shape differ by location. The signs and significance of the parameters also determine when earnings peak:

$$(22)\quad Exp_{(peak)} = -(b_0 + \Sigma b_j L_{ji}) / 2(c_0 + \Sigma c_j L_{ji})$$

as well as the growth rates in earnings with respect to additional experience:

$$(23)\quad (dE_i/dExp_i) / E_i = ((b_0 + \Sigma b_j L_{ji}) + 2(c_0 + \Sigma c_j L_{ji}))/E_i$$

An Application: Poverty, AFDC Payments, and Labor Market Conditions

A vigorous, often heated debate has ensued on the ability of the welfare system to reduce poverty rates. Many conservative social scientists and the public in general view the welfare system as paying persons to be poor (i.e., creating poverty rather than eliminating it). Recently, that view has received empirical support from the work of Gallaway and Vedder [11, 12], Murray [28, 29], Anderson [1], Williams [36], and Sowell [31, 32]. The Gallaway-Vedder-Murray (GVM) framework postulates that poverty first declines with increases in welfare and then eventually rises. Using the logit formulation, the function tested takes the quadratic form:

$$(24)\quad LODDS = a + bAFDC + cAFDC^2 + u$$

where:

LODDS = the log-odds of a female-headed household with children being in poverty;
a, b, and c = parameters to be estimated; and
u = an error term.

The alternative position, while admitting the coincident rise in welfare payments and poverty rates, nevertheless attributes the increase in poverty rates to deteriorating labor market conditions. Support for that position is found in the works of Thurow [33], Cain [4], Jones [19], and Gramlich and Laren [15]. The expansion method allows a test of this alternative view by determining the extent to which the parameters a, b, and c of equation (24) are functions of variables that differ across space such as labor market conditions, race, and other factors. For instance, assume that only one variable, Z, which represents some labor market condition to be specified later, affects a, b, and c so that the expansion equations are:

(25) $a = a_0 + a_1 Z$

(26) $b = b_0 + b_1 Z$

(27) $c = c_0 + c_1 Z.$

Substitution yields:

$$\text{(28)}\quad LODDS = a_0 + a_1 Z + b_0 AFDC + b_1 AFDC{*}Z + c_0 AFDC^2 + c_1 AFDC^2{*}Z + u.$$

If female poverty rates were determined according to the GVM model, the coefficients a_0, a_1, b_1, and c_1 should be zero and the terms $b_0 AFDC$ and $c_0 AFDC^2$ should explain most of the variation across observations in LODDS, with b_0 negative and c_0 positive. Alternatively, if the labor market theory were to determine female poverty rates, the parameters b_0, b_1, c_0, and c_1 should be zero and the term $a_1 Z$ should explain most of the variation in LODDS. If the two theories were to overlap in explaining LODDS, then the coefficients b_1 and c_1 should be nonzero. Thus, the expansion method allows a straightforward test of the relative merits of these two competing positions.

The odds of poverty for a female-headed household with children were calculated for each of the 86 central city neighborhoods in 18 relatively large cities as PR/(1-PR), where PR is the poverty rate of such households, and the standard logit analysis then was applied.[1] A description of these neighborhoods is found in Baroni and Green [2]. Those results, given in equation (29), reveal little support for the GVM hypothesis.

$$\text{(29)}\quad LODDS = \underset{(2.43)}{2.794} - \underset{(.999)}{.001185AFDC} + \underset{(.072)}{1.36E\text{-}06AFDC^2}$$

t-values in parentheses
Adj. $R^2 = -0.01127$ $F = 0.5928$

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[1] It is well known that the error in calculating such odds increases as does the number of female-headed households for a given observation, thereby causing heteroscedasticity problems. To overcome such difficulties, weighted least squares regression was applied with weights equal to the square root of 1/(T(PR)(1-PR)), where T represents the number of female-headed households in a particular neighborhood (Kmenta [23, p. 552]).

If there is any support for the GVM position, it is that the coefficients have the anticipated signs.[2]

To test further for spatial variations in the parameters of equation (29), the following variables were introduced and interacted with AFDC and $AFDC^2$: female unemployment rates (URFEMA), female labor force participation rates (PARFEMA), proportion of employees in service occupations (SERV), the proportion of employees in personal service industries (PERS), the percent of employees in blue collar occupations (BLUE), the percent of full-time employees who only worked one to 26 weeks in 1979 (UNDEREM), average hours worked by females (HRS), several measures of migration (MIG), and two racial variables representing black (RACEB) and Hispanic populations (RACEH). The F values for inclusion in the regression model also were calculated. These expansion variables along with AFDC and $AFDC^2$ were placed in the model and a backward stepwise procedure, which was terminated when all remaining variables satisfied the 95 percent confidence level, was used to estimate the equation. The resulting final equation is:

$$
\begin{aligned}
(30)\quad \text{LODDS} = {} & \underset{(0.52)}{.50513} + \underset{(6.51)}{4.74}\text{URFEMA} - \underset{(9.18)}{2.04}\text{PARFEMA} \\
& + \underset{(4.87)}{8.25}\text{MIG} + \underset{(2.00)}{0.24}\text{RACEB} + \underset{(6.70)}{1.50}\text{RACEH} \\
& - \underset{(2.81)}{0.041}\text{AFDC*UNDEREM} + \underset{(2.58)}{.03}\text{AFDC*SERV} \\
& + \underset{(3.05)}{1.67\text{E-}04}\text{AFDC}^2\text{*SERV}
\end{aligned}
$$

Adj. $R^2 = 0.71$ $F = 26.56$

where t-values are in parentheses.

The expansion equations reveal that much of the evidence supports the view that female poverty stems from unfavorable labor

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[2]The particular measure of AFDC payments used in equation (1) is the 1980 per family monthly AFDC payment for the metropolitan statistical area in which a particular female-headed household is located (U.S. Bureau of the Census, *State and Metropolitan Area Date Book 1986* [34]). To eliminate multicollinearity problems between AFDC and $AFDC^2$, deviations from the sample mean for AFDC were used for each observation. The results of equation (1), nevertheless, appear impervious with respect to the measure of AFDC payments used. Similar results were obtained for the nondeviation measure of AFDC per family, AFDC per child, and AFDC per recipient. In no instance was the F value larger than one, and the adjusted R^2s were near zero or negative.

market conditions. Deriving those expansion equations from the parameters of equation (30) yields:

$$(31)\quad a = 0.505 + 4.74\,URFEMA - 2.035PARFEMA + 8.25MIG + 1.50\,RACEH + 0.238\,RACEB$$

$$(32)\quad b = -0.041UNDEREM + 0.03SERV$$

$$(33)\quad c = 1.67E\text{-}04SERV$$

Recall from the equation LODDS = a + b(AFDC) + c($AFDC^2$) that the parameters b and c are the weights on AFDC and $AFDC^2$, respectively, and that the parameter a captures the impact of all non-AFDC factors. Because the equations for b and c do not contain constants (i.e., b_0 and c_0 are not statistically significant), AFDC payments have no independent effects on LODDS.[3] Moreover, the interaction terms obtained by multiplying b and c by AFDC and $AFDC^2$, respectively, only account for 0.05 of the total R^2 value of 0.73. Consequently, the intercept parameter a dominates the model of female-headed household poverty rates. Furthermore, the standardized regression coefficients indicate that URFEMA and PARFEMA are the two most important variables in the equation for a. Thus, labor market conditions play a sizeable role in differences in LODDS. Moreover, the optimal AFDC payments derived from equation (30) also imply that spatial labor market conditions cause such payments to vary from city to city. Taking the derivative of equation (30), setting it equal to zero, and solving for AFDC* yields:[4]

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[3]An alternative specification of equation (2) without any interaction terms but including AFDC and $AFDC^2$ also was estimated as:

$$\underset{}{LODDS} = \underset{(0.07)}{0.07} + \underset{(5.69)}{4.33URFEMA} - \underset{(8.20)}{1.83PARFEMA} + \underset{(4.06)}{6.90\,MIG}$$

$$+ \underset{(6.31)}{1.52RACEH} + \underset{(2.39)}{0.29\,RACEB} - \underset{(1.34)}{904E\text{-}04AFDC}$$

$$+ \underset{(1.87)}{2.30E\text{-}05AFDC^2}$$

Adj. $R^2 = 0.67$ $F = 26.68$

Neither AFDC nor $AFDC^2$ is statistically significant at the normal 5 percent level of error. The optimal AFDC level implied by this equation is \$329, only slightly above that given in equation (2).

[4]The optimal AFDC payment level, AFDC*, is found by taking the derivative of LODDS with respect to AFDC, setting that derivative equal to zero, and then solving for AFDC. Because the measure of AFDC used

$$AFDC^* = \frac{(0.041UNDEREM_i + 0.0744SERV_i)}{0.000334SERV_i}$$

indicating that spatial differences in both underemployment and low paying service jobs affect the optimal AFDC level.

Conclusions: Evaluation of the Expansion Method

The examples suggest that the expansion model can be used:

- To test hypotheses concerning the drift and/or stability of a model's parameters and to obtain functional portraits of this drift;
- To create complex models from simpler ones for research;
- As an organizing scheme within the context of which mathematical models can be classified and related to one another; and
- To interpret complex models in terms of simpler initial model(s) and related expansion equations.

The attractiveness of the expansion method flows from the flexibility it permits in designing models that can be estimated by conventional econometric techniques. One can start from relatively simple functional forms, such as a Cobb-Douglas production function or a quadratic relationship between earnings and experience, and, by introducing expansion equations that make the parameters of the original functions dependent on spatial characteristics, test a wide range of more complex functional forms. The technique allows the model builder to customize the relationship to unique characteristics of each spatial setting. The application of the technique to optimal AFDC

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is the deviation of the AFDC payment for its mean level, the formula for AFDC* is given as:

$$AFDC^* = \frac{(0.041UNDEREM - 0.031SERV + 0.00034AVGAFDC^*SERV)}{(0.000334SERV)}$$

$$AFDC^* = \frac{(0.041UNDEREM + 0.0744SERV)}{(0.000334SERV)}$$

where:

AVGAFDC = the sample average AFDC payment, which equals $310.

payment levels, for example, provided an equation that tailored the level of AFDC payments to various labor market conditions in each city.

The expansion method further generates a relatively large number of testable hypotheses from a simple straightforward structure. Casetti has shown that not only can hypotheses be tested about the stability of parameters in an equation where $y = f(x)$ such as $y_i = a + bx_i + e_i$ with expansion equations $a = a_0 + a_1 z_i$ and $b = b_0 + b_1 z_i$ which yields the terminal equation:

(34) $y_i = (a_0 + a_1 z_i) + (b_0 + b_1 z_i)x_i + e_i,$

but that such an equation has a dual

(35) $y_i = (a_0 + b_0 x_i) + (a_1 + b_1 x_i)z_i + e_i$

so that the stability of the parameters of a different function, $y = h(z)$ can be tested simultaneously. The technique also allows a natural integration of spatial and nonspatial factors and an easy means of assessing the relative contributions of each.

In spite of the value of this technique, and it has many advantages, the technique inherently generates a great deal of multicollinearity.[5] Casetti attempts to bypass the multicollinearity problem by using a backward stepwise regression that eliminates variables from the terminal equation until those that remain have t-values greater than 2. (This technique also is used in the AFDC application given above.) Unfortunately, the results obtained by stepwise methods are a function of the process used to eliminate collinear regressors. The same initial set of variables introduced by a forward stepwise procedure leads, in most instances, to an entirely different final equation.

Other techniques, such as ridge regression and principal components, exist to overcome multicollinearity problems, and these should be evaluated against the stepwise procedures. But even if the multicollinearity problem persists, the expansion method nonetheless provides a powerful modeling tool. Already geographers and demographers have recognized its merit. Preliminary use of the expansion method given in this paper suggests that it also may prove to be an invaluable addition to the regional scientist's tool kit.

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[5]As a demonstration of this problem, the simplest situation of one primary variable, x, and one expansion variable, z, was assumed, and series for x and z were created whereby they were independent of each other. Nonetheless, even where x and z have a zero correlation, the interaction of x and z into the variable xz has a correlation with z of 0.82 and a correlation with x of 0.57. The problem worsens as more z's are included in the expansion equations or when x^2 is introduced with x.

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