UNCERTAINTY, LEARNING, AND THE CLASSICAL ROTATION PROBLEM

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Introduction

When to cut a growing tree is a well known problem in standard capital theory dating back at least to 1849 and the solution by Martin Faustmann. The analogous decision facing a timber owner who is unsure of the future value of his timber provides an excellent example of decision making under uncertainty. If there is the possibility of learning about future timber values, the irreversibility of the cutting decision imparts a quasi-option value to holding the timber another period. Consequently, a timber owner may choose to hold his timber during periods of price uncertainty, even though he would cut his timber given the same current and future expected prices if he were more certain of his expectations.

The first section of this paper briefly reviews the conditions under which quasi-option value exists. The following section summarizes the standard tree-cutting problem and some of the ways other authors have incorporated uncertainty in the standard model. These models have precluded the possibility of learning and hence the quasi-option value of holding timber. In Section 3, an empirical work which does not allow for uncertainty illustrates the erroneous conclusions which may result. A measure of timber price uncertainty is constructed to verify that the time period in question was one of uncertainty about future prices. This measure is compared to another commonly used measure; the two agree on the major periods of stability and instability of prices but not on the relative magnitude of the importance of incorporating learning and uncertainty appropriately in some models. Since any test of the empirical importance of quasi-option value can be quite sensitive to changes in the measure of price uncertainty used, specific assumptions about the stochastic process underlying future timber values appear to be of primary importance.

Quasi-option Value

The key element in the analysis of the rotation problem with uncertainty and in many similar problems is the combination of uncertainty and irreversibility. This section surveys some relatively recent economic literature that has dealt with these issues. The concepts are not at all new but have received curiously little attention. Some recent attention may have been prompted by a desire to refute the conclusions of deterministic models of resource depletion such as suggested by the Club of Rome and others.

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A variety of definitions have been used for irreversibility. Arrow and Fisher (1974) define irreversibility in a technical context. Cummings and Norton (1974) describe an option as irreversible if the costs of reversing it are so large relative to the benefits that reversal will not be undertaken. Flexibility, the obverse of irreversibility, is defined by Jones and Ostroy (1979) in terms of the number of options left open at a given level of costs. For our purposes the most useful definition is that suggested by Henry (1974): "A decision is considered irreversible if it significantly reduces for a long time the variety of choices that would be possible in the future."[1]

As for the effects of irreversibility, Weisbrod (1964) noted that if the costs of reversing a decision are sufficiently high and/or the duration of its effects sufficiently long, the option value of preserving the present state should be taken into account. In his example, when the benefits (demand) for a publicly provided good are uncertain, the option value of retaining the option to consume may be important to individuals as well as the usual consumer surplus. Cicchetti and Freeman (1971) extend this to show there is positive option value (a risk premium) for risk averse individuals if there is uncertainty in either demand or supply. However, Arrow and Lind (1970) demonstrate that with risk averse individuals this risk premium goes to zero, individually and in the aggregate, as the number of individuals sharing the benefits (and the risk) of a single investment increases.

In their study of a proposed hydro-electric project on the Snake River, Fisher, Krutilla and Cicchetti (1972) assumed risk neutrality yet found a bias against irreversible development; that is, even when a cost/benefit analysis indicates an investment is optimal, it will generally be preferable to refrain from investment if disinvestment may be indicated in the near future. The Arrow and Fisher study (1974) clarifies this by showing that even with risk neutrality a "quasi-option value" exists where there is some irreversibility and learning can occur (i.e. information available in one period can affect the next period's expectations). This was actually stated much earlier in the context of investment planning by Hart (1942) who found that "the central problems of uncertainty can be posed and largely solved under the assumptions of 'risk neutrality.' "[2] In the remainder of this paper, I shall use "option value" to refer to the quasi-option value which occurs where there is irreversiblity and the prospect of learning and is not dependent on risk aversion.

The Rotation Problem

The previous section specified when the existence of option value may be anticipated. Given the prevalence of uncertainty and irreversibility, it is logical to ask if it is important to account for option value in either theoretical or empirical models. The remainder of this paper will explore that question in the context of the standard capital theory example of timber cutting, for the decision facing a private timber owner who is unsure of future timber prices

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[1] Henry (1974), p. 1006.

[2] Hart (1942), p. 115.

provides a classic example of decision making under uncertainty where one choice is irreversible.

The tree cutting problem is well known from standard capital theory. Hirshleifer (1970), Samuelson (1976) and Bierman (1968) present excellent surveys of various correct and incorrect versions. It is important to distinquish between those analyses which seek to maximize the present value of only the timber and those which seek to maximize the value of the timber and the land on which it stands. The latter is appropriate here[3]; I assume individual timber owners recognize the opportunity cost of the land and that timber growing continues to be the highest valued use of the land. A third type of model which seeks to maximize timber volume has been shown to be in error although, curiously, it is similar to that still used by the Forest Service.[4]

Assume the timber owner maximizes the present value of his timber holdings with respect to cutting time. If he cuts his timber he will immediately replant his land. In the standard problems, timber prices are constant and growth rates of trees of various ages are known, so the value of a tree t years old is a function f(t), of t only. If r is the market rate of interest and c the cost of planting (and replanting), the present value of the timber owner's returns if trees are cut at age T is:

$$V(T) = \frac{1}{1-e^{-rT}} [f(T)e^{-rT} - c]$$

The optimal rotation period is the solution to:

$$\frac{dV}{dT} = [f(T)e^{-rT} - c] \left[\frac{-re^{-rT}}{(1 - e^{-rT})^2}\right] + \frac{e^{-rT}}{1-e^{-rT}} [f'(T) - rf(T)] = 0$$

or

$$f'(T) = rf(T) + r \left[\frac{f(T)e^{-rT} - c}{1-e^{-rT}} \right].$$

In other words, the optimal T is where the growth in the value of the tree is just equal to the interest on the standing timber plus the interest on the present

[3] The former evidently stems from the time when timber was grown on commonly held ground and has no apparent useful interpretation for tree cutting though still useful for the more general question of when to sell a growing asset.

[4] Hyde (1980), Chapt. 2.

value of future returns from growing timber on the same land.[5]

Note that once timber has been cut, even though the land is replanted, cutting new timber will not be a viable option for a number of years. So holding timber results in a more flexible position for the owner than does cutting timber in that a wider number of options remain open at a given level of costs.

Papers by Kaplan (1972), Norstrom (1975), and Brock, Rothschild and Stiglitz (1982) have included uncertainty in the standard model of asset growth in various ways. These papers formulate the problem as an optimal stopping problem and use dynamic programming results to show the existence of an optimal policy and its method of calculation, and to study the effects of uncertainty.

In Kaplan's model (1972), the asset's value increases over time but at a decreasing rate. The amount of the increase is uncertain; its distribution may be a function of either the current value or age. If it is a function of current value, Kaplan establishes (Theorem 1) that an optimum policy for a multi-period problem consists of a critical value such that whenever the assets' value exceeds the critical value the asset (tree) should be harvested. If the distribution of the increase is instead a function of the asset's age then there is a sequence of similar critical values for each age.

Kaplan's model would not be appropriate for situations where option value exists since limiting the asset's value to an increasing function with diminishing returns either rules out price uncertainty or severely limits the possible variations in price.

Norstrom (1975) assumes the tree's growth is deterministic but its price is determined by a stationary stochastic process. The value of a stand at time t is $P_t f(t)$ where the sequence P_1, P_2, P_3,... is a stationary Markov process which represents the variation in price around its expected value or growth path. Norstrom considers both the case where the value of land is determined exogenously and the case where it is determined endogenously by its future value in growing timber. Comparing his stochastic model to an analogous deterministic one, he concludes (Proposition 2), "The expected present value in a stochastic model is at least as great as the present value in the corresponding deterministic model."[6]

In the analysis of Brock, Rothschild and Stiglitz (1982), the timber owner maximizes only the expected present value of his timber and ignores the opportunity cost of his land. In addition to a discrete time model which is based

[5] The model which ignores the possibility of replanting would similarly conclude that the optimal T is where the growth in the value of the tree is just equal to the interest on the standing timber or the growth rate of the tree equals r.

[6] Norstrom (1975), p. 334.

on a Markov process and similar to Norstrom's, the authors present a continuous time model. This model allows for continuous observation of the value of the tree but it assumes this value follows a stationary diffusion process. The stationarity assumption in this and in Norstrom's model is incompatible with the notion of there being periods of uncertainty and hence is inappropriate for a mode incorporating option value.

In order to include option value a model must be flexible enough to vary the amounts of uncertainty in different periods. Suppose a timber owner has some knowledge of the underlying model currently generating price. If the process is stationary, every period's expected price is the estimated mean, m, and the uncertainty about the price may be characterized by the estimated variance if the distribution of prices is symmetric. If the process is not stationary and the timber owner observes an unusually high price in period t, he may not know whether he has just observed an outlier of the old distribution, or an observation from a new distribution. And if a new distribution, is the mean still m and the variance larger (a mean preserving spread) or has there been a shift in some underlying parameter so that now the expected value of future prices is higher? If the latter, the variance and higher moments may or may not equal those of the original distribution.

Figure 1 illustrates this scenario in the simple case of a few symmetric distributions. The distribution labeled A with mean m_1 and variance s_1^2 is the original one in which the process starts. B is the mean preserving spread with mean $m_2 = m_1$ but variance $s_2^2 > s_1^2$. C is the distribution with higher expected value $m_3 > m_1$, and, in this case, though not necessarily, the original variance $s_3^2 = s_1^2$. Note in particular that, although a Markov transition matrix may describe the transitions between distributions, expected values of price generally are not memoryless; i.e. $E(P_{t+1} / P_t = P_1, P_{t-1} = P_2) \neq E(P_{t+1} / P_t = P_1, P_{t-1} = P_3)$.[7]

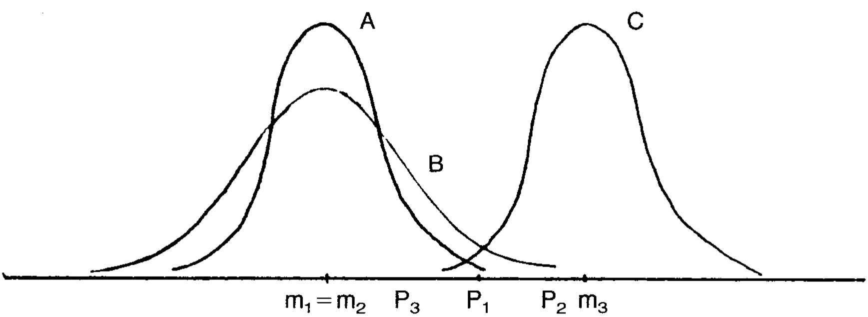

FIGURE 1

Three Possible Distributions of P_t

[7] If transitions between distributions were always equally probable, the expected value of P_t would be constant and thus memoryless.

The statistical properties of non-stationary stochastic processes are not neat and a technical description of this whole process, generalized so that many different distributions are possible, may be unpleasantly complex. However, the previous attempts at incorporating uncertainty in the timber-cutting example eliminate the possibility of option value. In the next section a recent work is used to illustrate the possible consequences when uncertainty is omitted from an empirical analysis.

Price Uncertainty and Empirical Modeling

Peter Berck (1979) studied the Douglas Fir (DF) timber industry to estimate the rate of discount used by private entrepreneurs with rational expectations about future prices. He arrived at the surprising result that they discounted the future at a real rate of 5%. As he does not incorporate any measure of uncertainty, his model cannot explain how this could occur when private corporate returns in the same period were much higher. Samuelson (1976) referring to similarly low rates often used by the forestry literature comments, "The notion that for such gilt-edge rates I would tie up my own capital in a 50-year (much less a 100-year) timber investment, with all the uncertainties and risks that the lumber industry is subject to, at first strikes one as slightly daft."[8]

It is clear from the foregoing discussion of option value that if private timber owners recognize an option value in delaying the harvest of their timber and if the period of Berck's study was one of price uncertainty, then his results are spurious. In particular, timber owners do have higher rates of discount and, without price uncertainty, would have supplied larger amounts of timber during the period studied. Berck's study used data from 1950-1970. In fact, real DF timber prices were fairly stable in the period from 1910 until World War II; the year 1948 marks the beginning of a long period of DF timber price instability. This is illustrated in Figure 2 which shows real prices of DF stumpage from 1910 to 1981.

How might the effects of option value be captured in a model like Berck's? The key element appears to be price uncertainty and a very simple approach might be to measure the error in timber owners' price forecasts. Even with this simplifying assumption we are not too much further ahead as economic theory has not progressed far enough to specify how economic actors form expectations about the future. The remainder of this section deals with a simple and *ad hoc* measure of uncertainty. It is suggested not because it is the "correct" measure but because it overcomes some obvious problems in the usual approach. It is contrasted to the usual kind of measure in order to emphasize the importance of further work in this area.

One way of including price forecasts and uncertainty in a model which seems to tempt some researchers is to design a regression model to simulate the decision maker's forecasting process, to use all the observations to

[8] Samuelson (1976), p. 473.

estimate the regression coefficients and then to let the resulting residuals act as measures of uncertainty. The biggest problem with this approach is that it is analogous to presuming the decision maker acts on information not yet observed; observations from the whole period go into forming the equation

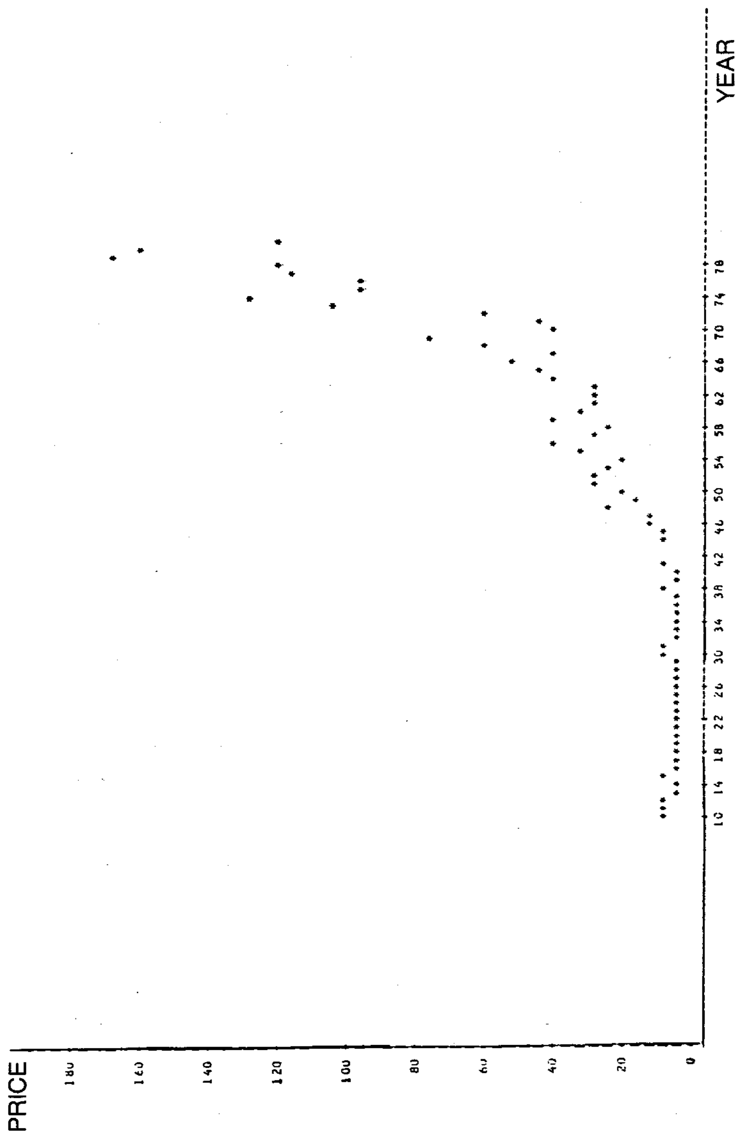

FIGURE 2

Stumpage Prices of Douglas Fir (Prices in constant 1967 dollars per thousand board feet, Scribner scale.)

from which the predictions and residuals for each period are calculated.

Furthermore, a model which allows learning about future timber prices should allow timber owners to revise their price predicting model after each observation. Suppose the timber owner uses the following very simple model to predict future timber prices:

$$\ln P_t = a_0 + a_1 \ln P_{t-1} + a_2 \ln P_{t-2} + a_3 \ln P_{t-3}.$$

If predictions are made at time t about the value, $\ln P_{t+1}$, the prediction should only be based on observations of price available at time t. My experience with the DF timber price series leads me to believe this simple point may be important.

The following method was used to construct an uncertainty variable which allows decision makers to learn and to form predictions based only on observed data. Douglas Fir stumpage prices were used in the regression equation above and for each predicted value, $\ln \hat{P}_{t+1}$, the regression was run using the previous 20 annual observations of price. Then to calculate $\ln \hat{P}_{t+1}$, observed values were plugged in for periods t, t-1, and t-2.

Annual prices were available for the period 1910-81 except for 1942 and 1943. A geometric extrapolation was used for these two years; this allowed a much longer continuous series and seemed somewhat defensible since general price controls were in effect at that time and since the analysis does not focus on individual coefficients.

Predictions were made and the residual, $\ln P_t - \ln \hat{P}_t$, calculated for each year from 1933 to 1981 except for those years, 1942-1946, which depend directly on the extrapolated values. In addition, predictions and residuals were calculated by the more familiar approach of estimating the equation once using all the observations. Table 1 gives the regression results. Table 2 gives both sets of residuals. RES1 is the residual when price predictions are formed from only the last 20 observations of price. RES2 is the residual when price predictions are based on the entire data set.

Table 3 gives averages of the absolute value of the residuals for different periods. Both RES1 and RES2 have relatively high values for the periods 1947-58 and 1964-74. This supports the claim that the period of the Berck study was one of relative uncertainty about future prices. Between 1950 and 1970, the period of the study, only the five years from 1959 through 1963 appear to have had relatively predictable prices, at least by this model. The periods 1933-41 and 1975-81 also appear relatively stable by both measures.

Although they agree on the basic periods of price uncertainty, these two measures differ on the degree of stability in different periods. In particular the average for RES2 in pre-war years is very close to its average in the 1964-74 span, indicating that this measure would not have identified that earlier period as relatively stable. Again, the equation that produced RES2 was estimated from all the observations including the post-war prices which were rising rapidly even in real terms. Similarly prices in the 1959-63 period and the 1975-81 period appear much more predictable when the estimating equation is allowed to vary each year.

TABLE 1

Regression Results

YEAR	a_0	a_1	a_2	a_3	R^2
33	.657	.449	.033	.072	.26
34	.726	.488	-.017	.030	.24
35	.727	.488	-.017	.029	.24
36	.815	.485	.014	-.077	.27
37	.619	.783	-.127	-.064	.55
38	.650	.637	.013	-.083	.44
39	.737	.475	.073	-.019	.30
40	.752	.482	.049	-.010	.27
41	1.148	.464	-.064	-.121	.28
47	.399	.731	.085	-.015	.56
48	.214	.790	.072	.050	.62
49	-.149	.924	.108	.093	.69
50	.156	.691	.192	.072	.67
51	.024	.653	.288	.084	.73
52	-.126	.666	.207	.237	.78
53	-.076	.687	.255	.151	.84
54	.083	.471	.341	.222	.88
55	.285	.426	.269	.244	.86
56	.316	.307	.217	.424	.86
57	.218	.412	.119	.462	.86
58	.411	.390	.042	.489	.86
59	.421	.539	-.095	.454	.83
60	.548	.550	-.255	.569	.84
61	.599	.509	-.208	.543	.82
62	.610	.502	-.198	.538	.80
63	.733	.530	-.221	.484	.75
64	.767	.529	-.217	.470	.71
65	.855	.519	-.246	.487	.67
66	1.059	.520	-.276	.459	.62
67	1.162	.500	-.233	.412	.53
68	1.502	.447	-.272	.403	.45
69	.539	.534	-.159	.496	.56
70	.791	.689	-.321	.435	.62
71	.791	.733	-.537	.597	.56
72	.772	.683	-.519	.641	.55
73	.665	.659	-.461	.633	.60
74	.342	.813	-.485	.603	.57
75	.155	.884	-.454	.563	.63
76	-.023	.941	-.464	.557	.70
77	-.018	.929	-.445	.547	.73
78	.168	.934	-.414	.461	.76
79	.312	.860	-.330	.418	.77
80	-.013	.928	-.282	.384	.79
81	.240	.972	-.397	.394	.82
33-81	.027	.743	-.052	.325	.94

Year refers to the year for which the prediction was made. Thus, regression coefficients for year = 70 were estimated from observations for 1950-69. The final coefficients are for the regression using all the observations.

TABLE 2

Annual Residuals When Last 20 Observations Are Used (RES1) And When All Observations Are Used (RES2)

YEAR	RES1	RES2	YEAR	RES1	RES2
33	-.347	-.528	60	-.141	-.149
34	-.005	-.172	61	.043	-.095
35	.002	-.085	62	-.309	-.275
36	.201	.208	63	-.037	-.037
37	-.341	-.323	64	.260	.225
38	.445	.458	65	.256	.129
39	.045	-.133	66	.353	.166
40	.075	.108	67	-.028	-.209
41	.453	.262	68	.458	.263
47	.281	.157	69	.275	.203
48	.660	.651	70	-.460	.619
49	-.637	-.376	71	.199	-.069
50	.217	.336	72	-.142	.051
51	.320	.180	73	.697	.571
52	.033	.166	74	.450	.362
53	-.401	-.192	75	-.064	-.140
54	-.532	-.349	76	-.048	-.111
55	.298	.366	77	-.080	-.000
56	.375	.236	78	-.044	-.026
57	-.160	-.228	79	.354	.301
58	-.442	-.312	80	-.136	-.049
59	.172	.263	81	-.248	-.308

TABLE 3

Average Size Of Residuals

YEARS	RES1	RES2
1933-41	.213	.253
1947-58	.363	.296
1959-63	.140	.164
1964-74	.326	.261
1975-81	.139	.136

Summary

In theoretical models of decision making, where one decision is irreversible and hence option value may exist, it is important to incorporate uncertainty. The standard capital theory question of when to cut a growing tree has been extended to incorporate uncertainty by several authors, yet each in some way has ruled out the effect of option value. Unfortunately, stochastic processes that are non-stationary and thus allow periods of varying uncertainty do not usually have other appealing properties. Furthermore, it seems that the essence of the learning process may be quite different in different problems and the appropriate method of characterizing learning may vary substantially.

In empirical models like Berck's omission of uncertainty and option value can lead to spurious results. Specifically, it seems likely that the unusually low discount rate estimated by Berck is at least partially the result of ignoring the impact of option value on the tree cutting decision. Empirical incorporation of uncertainty has so far been done on only an *ad hoc* basis as the theory does not dictate any particular measures of uncertainty. Unfortunately, such *ad hoc* methods will often disagree and attention must be paid to improving their properties. This paper has illustrated how two shortcomings of the usual measures may be overcome. It will take much more careful analysis to correctly characterize the nature of uncertainty and learning in such models and to construct meaningful measures of uncertainty.

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