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UNCERTAINTY AND EQUILIBRIUM IN A HOUSING MARKET

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I. Introduction

Traditional models of residential location such as those of Alonso [1], Muth [5], Mills [4] and Beckmann [2] postulate that in a monocentric city land rents must decline with distance from the city center to compensate for increased commuting costs. Derivation of the rent gradient in these models implicitly assumes that commuting costs are known with certainty. While this assumption has often been accurate for extended periods, it fails to describe conditions of the last decade when fuel prices were volatile and subject to considerable uncertainty. Since commuting costs play such a central role in models of residential location, uncertainty about future fuel costs is likely to significantly alter their results. The purpose of this paper is to present a model of residential location which explicitly incorporates this source of uncertainty. It is organized as follows.

In section II we present a simple model of consumer choice and equilibrium in the housing market under conditions of certainty. Section III introduces the problem of uncertainty and examines the way in which uncertain commuting costs alter consumer decisions. In Section IV the effects of uncertainty and the consumer's attitude toward risk are made more precise in a way which allows comparisons to be made among the decisions of similar individuals. Section V considers the structure of equilibrium rents which emerges under uncertainty and points out the way that this equilibrium differs from the standard one. Finally, Section VI contains some concluding remarks.

II. The Standard Model: Certainty and Choice

It will be useful before proceeding to consider a simple model of the monocentric city with certain commuting costs. We shall examine only the consumer side of the market and the adjustment process toward equilibrium; no attempt will be made to incorporate the supply of housing. This omission will slightly weaken some results because we cannot limit the number of consumers who "pile up" at a particular location. However, this difficulty notwithstanding, an examination of consumers' choices and the conditions necessary for equilibrium permit several strong conclusions about homogeneity within consumer classes and the spatial distribution of individuals.

Consumers are assumed to receive utility from housing, a composite consumption good and leisure. Choices are constrained by a time budget and an income budget; income is allocated among rent, consumption and explicit commuting costs, and time among labor, leisure and commuting. Residences are available at any distance from the city center but each individual must

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commute to the central business district. Thus, each possible choice of location incurs for the customer costs of commuting both explicitly and implicitly. The gross price of housing is the sum of rent and commuting cost and if utility is strictly increasing, utility maximization requires the gross price of housing to be minimized for each choice of housing type.

This model of residential choice may be formalized by examining the individual's "bid function." The bid function expresses the rent a consumer could offer for housing of a given type at each possible location while maintaining a fixed level of utility. A family of such functions exists with each member corresponding to a different utility level. If housing of a given type at each location could be obtained at a rental rate equal to the bid given by a particular member of this family, the consumer would be indifferent among all possible locations.

Formally, individuals are assumed to maximize a separable monotone utility function u whose arguments are housing services H , a single composite consumption good x and leisure λ . They are constrained by a time budget and an income budget. Time is allocated among labor L , leisure and commuting time. Income is allocated among consumption purchases, housing rent and explicit commuting costs. Given a residence at distance D from the city center, commuting time is $d(D)$ and explicit commuting costs are $C(D)$. Market rental rates are given by $R(H,D)$, q is the price of consumption, w the wage rate and t the initial endowment of time. The problem of the consumer is then

$$(1) \text{ Maximize } U(h, x, \lambda) \text{ s.t. } \begin{aligned} t &= L + \lambda + d(D) \\ wL &= qx + C(D) + R(H,D) \end{aligned}$$

Let

$$(2) V(q, w, \phi) = \text{Max} \{U(H,x,\lambda) : w(t-\lambda) - qx - \phi(H) = 0\}$$

where $\phi(H) = R(H,D) + C(D) + wd(D)$ represents the gross price of housing. V is the indirect utility function and is strictly decreasing in ϕ . Notice that both time and income budgets have been combined into a single constraint; implicit costs of commuting are evaluated at the consumer's wage rate. Moreover, because the utility function is separate, an increase in implicit commuting costs may be offset either by a reduction in leisure time or reduced labor, and hence consumption, without influencing the marginal rate of substitution between other utility arguments. Maximizing $V(q,w,\phi)$ by choice of D yields a solution to the consumer's problem. If housing type H is chosen, duality assures that maximizing utility is equivalent to minimizing the gross price of housing of that type.

The consumer's family of bid functions is defined implicitly for housing type H by

$$(3) V(q,w,B(H,D,v) + C(D) + wd(D)) = v$$

for v which are solutions to (2). Here, $B(H,D,v)$ is the bid for housing of quality

H at distance D the consumer could offer while maintaining utility v . Since V is monotonic in the gross price of housing, each member of the family of bid functions must be such that the gross price is constant over all distances. That is, for each feasible v , bids must satisfy

$$(4) \quad \bar{B}(H, D, v) = \phi - C(D) - wd(D) \\ \text{where } v = V(q, w, \phi).$$

One possible choice of residential location is the city center where the gross price of housing consists of rent only. If the consumer is willing to bid $\bar{B}(H, O, v)$ for housing at the city center, bids for housing of the same type at other locations must satisfy.

$$(5) \quad \bar{B}(H, D, v) = \bar{B}(H, O, v) - C(D) - wd(D)$$

along the same bid function. Bids must decrease with distance from the city center and the rate of decrease is equal to marginal commuting cost.

To simplify subsequent analysis we shall henceforth assume that both explicit commuting cost and commuting time increase linearly with commuting distance. Thus, each member of the family of bid functions must be a decreasing linear function of distance and we shall refer to marginal commuting costs as the "price of distance" denoted $p(w)$ where w indicates dependence on the consumer's wage rate. Equation (5) may thus be written:

$$(5') \quad \bar{B}(H, D, v) = \bar{B}(H, O, v) - p(w) D$$

and where no confusion is possible we shall, for notational convenience, omit the argument v in subsequent reference to a particular member of the family of bid functions.

If all commuting costs are known with certainty, it is relatively easy to move from individuals' bid functions to market equilibrium prices. Each consumer has a family of bid functions for housing of a given type, each member corresponding to a solution to (5) for some $\bar{B}(H, O)$. Since utility is strictly decreasing in gross price of housing, the objective of each consumer is to obtain housing of a given type for the lowest possible cost. We may formalize the adjustment process by assuming that each consumer submits a series of bids on houses at all possible locations in amounts given by a member of the family in bid functions. A house is obtained by submitting the highest bid for its location. A consumer who does not obtain a house at the initial bid must move to a higher bid function. A consumer who submits the highest bid on at least one location may retract that bid and/or submit lower bids until housing is obtained at the lowest possible cost. Bids must exceed a reservation rent equal to the value of land in alternative uses and assumed constant for any distance.

Equilibrium is attained in the housing market when each consumer obtains a house and no consumer can increase utility either by moving or by offering a

lower bid. To make this precise, if I is the index, set of consumers, $B^i(\bar{H}, \bar{D})$ is the bid function of consumer i , $R(\bar{H}, \bar{D})$ is the equilibrium rental rate, r is the reservation rent and consumer i obtains a house of type \bar{H} at location \bar{D} , then

$$(6) \quad r \leq R(\bar{H}, \bar{D}) = B^i(\bar{H}, \bar{D}) = \sup_{j \in I} B^j(\bar{H}, \bar{D})$$

and $B^i(\bar{H}, \bar{D})$ is the minimum bid for which the right hand equality is satisfied.

From equations (5') and (6) it is clear that if all consumers earned the same wage, equilibrium rents would decrease linearly at rate $p(w)$ to reservation rent at the city's periphery, thus leaving all consumers indifferent among all houses of a given type regardless of location. If wages differ, the form of equilibrium rents may be complex but the foregoing analysis suggests a simple typology of consumers. We will say that two individuals are "similar" if they choose the same housing type; they are "identical" if they also earn the same wage. Notice first that any two identical consumers must, in equilibrium, have the same bid for all houses of the chosen type. Each individual must be indifferent between obtaining his own residence and those obtained by others of his class at equilibrium rents. If a consumer were to "envy" an identical person's choice, i.e., be willing to offer a higher bid, he could obtain housing of this same type at a lower gross price. The existing situation would not be an equilibrium.

Furthermore, our analysis implies that similar classes of consumers will stratify themselves in equilibrium according to their wage rates. No consumer will reside nearer the city center than a similar individual with a higher wage rate. If consumers i and j are similar, have wage rates w^i and w^j and reside at distances D^i and D^j respectively then in equilibrium $B^i(\bar{H}, D) \geq B^j(\bar{H}, D^j)$ where \bar{H} is the housing type chosen. However, if $w^i > w^j$ and $D^i < D^j$ it follows from (5') that because the price of distance is greater for consumer i , $B^i(\bar{H}, D)$ for all $D > D^i$. In particular, this inequality must hold for $D = D^j$ so that the locations chosen cannot be an equilibrium. Consumers with the highest wage rates will thus claim that housing of the given type which is nearest the city center. Consumers with lower wage rates may obtain housing of equal quality, but only by accepting longer commuting distances.

III. Residential Location Choice Under Uncertainty

We now turn to the case where the true cost of commuting is uncertain. If explicit commuting costs are subject to random fluctuations, the price of distance is also a random variable. Each choice of a residential will incur for the consumer an uncertain gross price of housing and, because this is an unavoidable expense, will induce uncertainty as to the actual level of consumption, or possibly of leisure, which will be realized. To model the consumer's choice problem under uncertainty we shall assume that the price of distance, $p(w)$, is a random variable with subjective probability $F(p) = \text{prob}[P(w) \leq p]$ concentrated on the interval $[\alpha(w), \infty)$ where $\alpha(w)$ is the implicit cost of distance for a consumer with wage rate w . Further, we shall assume that the

consumer's utility function is separable, monotonic, strictly concave and of class C^2 .

The problem of the consumer is now one of expected utility maximization and may be stated as

$$(7) \quad \text{Max } E\{U(H,x,l)\} \quad \text{s.t.} \quad \begin{aligned} t &= L + d(D) + \lambda \\ wL &= qx + C(D) + R(H,D) \\ P(w)D &= wd(D) + C(D) \\ \text{prob} &= [P(w) \geq p] = F(p) \end{aligned}$$

where E is the expectation operator. This is equivalent to the problem

$$(8) \quad \text{Max } E\{V(q,w,\phi)\}$$

D

where V is the indirect utility function defined in equation(2) and $\phi = R(H,D) + P(w)D$ is the (uncertain) gross price of housing. The consumer's family of bid functions is now implicitly defined by

$$(9) \quad E\{V(q,w,B(H,D,v) + P(w)D)\} = v$$

for v which are feasible solutions to (7) and by the implicit function theorem, $B(\cdot)$ is a C^1 function of distance. That is, for any member of the family of bid functions, expected utility is constant for all possible locations and hence the bid function must satisfy

$$\frac{d}{dD} E\{V(q,w,\phi)\} = E\{V_{\phi}(q,w,\phi) (B_D(H,D,v) + P(w))\} = 0$$

or, since $P(w)$ is the only source of uncertainty,

$$(10) \quad -B_D(H,D,v) = \frac{E\{(V_{\phi}(q,w,\phi)P(w))\}}{E\{V_{\phi}(q,w,\phi)\}}$$

Equation (10) describes the rate of change of the consumer's bid function as distance from the city center varies. We wish to examine the behavior of a member of the family of bid functions for a fixed housing type. If the consumer were risk neutral we might anticipate a rate of change equal to the expected price of distance, that is $-B_D(\cdot) = E\{P(w)\}$. However, as demonstrated in Appendix (1), strict concavity of the utility function is inherited by the indirect utility function. Applying the result of Appendix (2) to equation (10) yields the conclusion that the rate of decrease of $B(\cdot)$ with distance from the city center must strictly exceed $E\{P(w)\}$. Consumers whose utility functions are strictly concave must exhibit risk aversion in their location decisions. This risk aversion will alter their bid functions and, as we shall show in subsequent sections, change the resulting structure of equilibrium rental values and the spatial distribution of consumers.

IV. Measures of Risk Aversion

Although the standard model of location choice predicts that consumers with equal wage rates must have identical families of bid functions, this is not the case under uncertainty. In this section we shall show that differences in bid functions may arise either from differences in attitudes toward risk or from different subjective probability distributions over future commuting costs. We shall do so by exploiting the fact that explicit commuting costs constitute a relatively small component of consumers' budgets and hence their unpredictability induces a "small risk" in the sense of Pratt [6]. For small risks it is possible to calculate the certainty equivalent of the consumer's bid for any location, the non-stochastic gross housing price which would allow the same (expected) utility as the random gross price represented by the actual bid. The difference between this certainty equivalent and the actual bid is a kind of "risk premium" and for each distance indicates the amount by which the expected gross price of housing must be below that at the city center to make the individual indifferent as to location.

Let π be a risk premium such that the consumer is indifferent between obtaining housing at distance D for the random gross price $[B(H,D) + PD]$ or for the certain price $[B(H,D) + \mu D + \Pi(D)]$ where μ is the mean expected price of distance. The two prices must be such that

$$(11) \quad V(q,w,[B(H,D) + \mu D + \Pi(D)]) = E\{V(q,w,[B(H,D) + PD])\}$$

Expanding by Taylor series about the expected price of distance and ignoring terms of order smaller than σ^2 , the right hand side of (11) may be written

$$(12) \quad E\{V(\cdot)\} = V(\cdot, E\{\phi\}) + \frac{1}{2}V_{\phi\phi}(\cdot, E\{\phi\})D^2\sigma^2$$

where $E\{\phi\} = B(H,D) + \mu D$, $\sigma^2 = E\{(p-\mu)^2\}$ and $V_{\phi\phi}$ is the second derivative of V w.r.t. ϕ . The left hand side of (11) is approximately given by

$$(13) \quad V(\cdot) = V(\cdot, E\{\phi\}) - V_{\phi}(\cdot, E\{\phi\})\Pi(D)$$

Equating (12) and (13) as required by (11) and simplifying yields the fundamental relation

$$(14) \quad \Pi(D) = \frac{1}{2} \left[\frac{V_{\phi\phi}(\cdot)}{V_{\phi}(\cdot)} \right] D^2\sigma^2$$

From the definition of V and the envelope theorem as applied in Appendix (1) it follows that the bracketed part of (14) is equal to $-q \left[\frac{u_{xx}(\cdot)}{u_x(\cdot)} \right]$ at the maximizing values for $\phi = E\{\phi\}$ and is therefore strictly positive. This term is equivalent to Pratt's coefficient of absolute risk aversion (CARA) and serves as an indicator of the consumer's willingness to accept risk. In order to maintain constant expected utility. The consumer's bid function must vary

with distance so as to keep $[B(H,D) + \mu D + \Pi(D)]$ constant for housing of a fixed type. Because $\Pi(D)$ is strictly positive, the rate of decrease of the consumer's bid must exceed the expected price of distance. Furthermore, consumers who are more risk averse in the Pratt sense will include a larger risk premium in their bids for housing at each possible location.

Because one possible choice for the consumer is to reside at the city center where there is no random element to the gross price of housing, the relationships given by (11) and (14) allow us to compute the bid for each possible location given by a particular member of the family of bid functions relative to the bid at distance $D = 0$. From (11), (14) and the definition of Π , it follows that

$$(15) \quad B(H,D) = B(H,0) + \mu D + \frac{1}{2} \left[\frac{V_{\phi\phi}(\cdot)}{V_{\phi}(\cdot)} \right] D^2 \sigma^2$$

for housing of each fixed type; (15) allows us to make the following comparisons among bid functions of different consumers.

Let B^i and B^j be bid functions for consumers i and j such that $B^i(\bar{H}, \bar{D}) = B^j(\bar{H}, \bar{D})$ for a fixed housing type \bar{H} . Then the following relations must hold.

Proposition (1): If consumers i and j have the same wage rates and subjective probability distributions over commuting costs and i is uniformly more risk averse than j for each possible ϕ , then $B^i(\bar{H}, D) > B^j(\bar{H}, D)$ for each $D > \bar{D}$.

Proposition (2): If i and j have the same wage rates and the same CARA for each possible ϕ and their subjective probability distributions over possible commuting costs are respectively F^i and F^j where $\mu_i = \int p dF^i(p) \geq \int p dF^j(p) = \mu_j$ and $\sigma_i^2 = \int (p - \mu_i)^2 dF^i(p) \geq \int (p - \mu_j)^2 dF^j(p) = \sigma_j^2$ with strict inequality at least once, then $B^i(\bar{H}, D) > B^j(\bar{H}, D)$ for $D > \bar{D}$.

The above comparison follows immediately from inspection of (15) but leaves as an unanswered problem the effect of different wage rates on bids of consumers who are otherwise similar. If they have the same CARA for each ϕ , then that consumer with higher wages, and hence a higher implicit price of distance, will have lower bids at all distances. However, Pratt has argued that the coefficient of absolute risk aversion is generally decreasing in its argument (consumption). Thus, the consumer with greater income should tend to include a smaller risk premium in the housing bid and submit a higher bid at each location. The final effect of different wages on consumers' bid functions depends on which of these influences predominates.

V. Uncertainty and Equilibrium

The process of adjustment toward equilibrium and the conditions which characterize equilibrium in the housing market are the same whether commuting costs are certain or uncertain. Uncertainty does, however, alter the bids which consumers submit and hence, the structure of equilibrium rental values. Furthermore, the simple typology of consumers considered in Section II is not sufficient under uncertainty to capture the sources of heterogeneity

among individuals. Consequently conclusions concerning the spatial distribution of individuals and their indifference toward various locations may no longer be valid. We shall present these results as a series of propositions concerning uncertainty and equilibrium in the housing market.

First, keeping in mind the conclusion of section (II) that if all consumers have the same wage rents will fall at a rate equal to the common price of distance, we may demonstrate that uncertainty, even when wages are identical, must alter the shape of the rent gradient.

Proposition (3): If all consumers are risk averse, all earn the same wage and alternative land values are not effected by uncertain commuting costs, then center city rents will be higher under uncertainty than they will in the standard equilibrium model if the actual price of distance is μ .

Proof: Since all consumers attempt to obtain housing at the lowest cost, rental rates at the city's periphery must equal the reservation rent r . Let Γ be the minimum Pratt coefficient of absolute risk aversion (evaluated at the equilibrium expected gross price of housing) over consumers. In the case of certainty, rent at the city center is $R(H,O) = r + \mu\bar{D}$ where \bar{D} is the commuting distance from the periphery. In the case of uncertainty $R(H,O) \geq r + \mu\bar{D} + \bar{D} + \Gamma\bar{D}^2 \sigma^2 > r + \mu\bar{D}$.

Q.E.D.

Proposition (3) indicates that uncertainty and risk aversion must alter the shape of the rent gradient. Without uncertainty and with equal wages, we found in section (II) that every consumer must be indifferent among all housing of a type regardless of location. The next two propositions indicate that even with identical wages, differences in risk aversion or subjective probability estimates render this no longer true.

Suppose consumers i and j choose housing the same type at distances D^i and D^j respectively from the city center. Then the following comparison must hold.

Proposition (4): If consumers i and j have the same wage rate and subjective probability distributions over possible commuting costs and i is uniformly more risk averse than j for each possible ϕ , the $D^i \leq D^j$.

Proof: Suppose, contrary to the proposition that $D^i > D^j$. From the definition of equilibrium $B^i(H,D^i) \geq B^j(H,D^i)$. However, by proposition (1) this implies $B^i(H,D^i) > B^j(H,D^i)$ which cannot be an equilibrium.

Q.E.D.

Proposition (4) indicates that under uncertainty individuals who are identical in the typology of Section II and who have identical expectations will tend to stratify on the basis of attitudes toward risk. A similar conclusion is possible if individuals who are otherwise the same have different expectations.

Proposition (5): If i and j have the same wage rates and the same CARA for each possible ϕ and $\mu_i \geq \mu_j$, and $\sigma_i^2 \geq \sigma_j^2$ with strict inequality at least once, then $D^i \geq D^j$.

Proof: Proposition (5) follows from Proposition (2). The method of proof is identical with that of proposition (4).

Q.E.D.

In Section II we found that location in equilibrium was dependent, for similar individuals, upon their wage rates. Under uncertainty location decisions are influenced by attitudes toward risk and expectations but the effect of different wage rates is ambiguous as noted in IV. Furthermore, the conclusion that consumers who are identical in the typology of section II must be indifferent among residences obtained by members of their class is also rendered invalid by uncertainty as shown by this last proposition.

Proposition (6): Suppose consumers i and j satisfy the hypotheses of proposition (4) or (5). Then there is \bar{D} with $D^i \leq \bar{D} \leq D^j$ such that $B^i(H, D) > B^j(H, D)$ for $D < \bar{D}$ and $B^i(H, D) < B^j(H, D)$ for $D > \bar{D}$.

Proof: From propositions (4) and (5), $D^i \leq D^j$ and by definition of equilibrium $B^i(H, D^i) \geq B^j(H, D^i)$ and $B^j(H, D^j) \geq B^i(H, D^j)$. By continuity of the bid functions there exist \bar{D} , $D^i \leq \bar{D} \leq D^j$ such that $B^i(H, \bar{D}) = B^j(H, \bar{D})$. Thus, by proposition (1) or (2) $B^i(H, D) > B^j(H, D)$ for $D < \bar{D}$ and $B^i(H, D) < B^j(H, D)$ for $D > \bar{D}$.

Q.E.D.

We thus find that several of the central results concerning the shape and interpretation of the city's rent gradient derived in the certainty model of Section II will fail to be true when commuting costs are uncertain. Proposition (3) indicates that even in the simplest case uncertainty must influence the structure of rents. Propositions (4) and (5) show that under uncertainty it is expectations and attitudes toward risk which influence location rather than wages and that in fact the effect of differential wages is ambiguous. Finally, proposition (6) indicates that while it is true that identical consumers should be indifferent in equilibrium among residences chosen by others of their type, uncertainty required that this typology of individuals be expanded to encompass not only housing type and wages but expectations and attitudes toward risk as well.

VI. Conclusion

In this paper we have examined the effect of commuting cost uncertainty on consumers' location decisions and on equilibrium housing rental rates. We have shown that uncertainty must alter consumers' bids for housing at various locations and hence alter the structure of equilibrium rents in a predictable way. In general, we have found that equilibrium rents must decrease more rapidly with commuting distance under uncertainty because of the risk premium inherent in individuals' bids. A consequence of this observation is that center city rents must be higher under uncertainty even if consumers correctly anticipate the actual future price of distance.

Furthermore, in contrast to the standard model of location choice which predicts that cities with heterogeneous populations will tend to stratify on the basis of wages, we find that under uncertainty other influences may determine location outcomes. Those consumers who are most risk averse or who have the most pessimistic estimates of future commuting costs will tend to locate nearest the city center. Under certainty the effect of differential wage rates on location is ambiguous. The fact that higher wages increase the anticipated price of distance is at least partially offset by the tendency of high wages, and hence high income, to reduce risk aversion and so encourage risk taking in the form of longer commuting distances.

The indirect utility function is

$$V(q, w, \phi(H)) = \text{Max}_{H, x, \ell} \{u(h, x, \ell): W(t - \ell) - qx - \phi(H) = 0\}$$

where u is strictly monotone strictly concave and of class C^2 . We wish to examine the behavior of V as ϕ varies for consumers who choose housing of type \bar{H} . First, we show that V is strictly concave. Choose ϕ^i , $i = 1, 2, 3$ where $\phi^3 = \alpha\phi + (1 - \alpha)\phi^2$ for $\alpha \in (0, 1)$. Let $(\bar{H}, \bar{x}^i, \bar{\ell}^i)$ solve the maximum problem for $\phi = \phi^i$ and let $(\hat{x}, \hat{\ell}) = \alpha(x^1, \ell^1) + (1 - \alpha)(x^2, \ell^2)$. Then $w(t - \bar{\ell}^3) - q\hat{x} - \phi^3 = 0$ and so $u(\bar{H}, \bar{x}^3, \bar{\ell}^3) > u(\bar{H}, \hat{x}, \hat{\ell})$. Since u is strictly concave $u(\bar{H}, \hat{x}, \hat{\ell}) > \alpha u(\bar{H}, \bar{x}^1, \bar{\ell}^1) + (1 - \alpha)u(\bar{H}, \bar{x}^2, \bar{\ell}^2)$. Thus, $V(q, w, \phi^3) > \alpha V(q, w, \phi^1) + (1 - \alpha)V(q, w, \phi^2)$ and V is strictly concave. Furthermore, it is clear that V is strictly decreasing in ϕ and, by the implicit function theorem, of class C^2 .

It follows by the envelope theorem that

$$V_\phi(\cdot) = u_x(\cdot) \frac{dx}{d\phi} = -u_x(\cdot) q$$

and

$$V_{\phi\phi}(\cdot) = u_{xx}(\cdot) q^2$$

thus

$$\frac{V_{\phi\phi}(\cdot)}{V_\phi(\cdot)} = -q \frac{u_{xx}(\cdot)}{u_x(\cdot)}$$

We wish to show that if V is a strictly decreasing strictly concave function of P , then

$$E\{V_\phi(\cdot)P\} > E\{V_\phi(\cdot)\}E\{P\}$$

To prove the result, we may apply the following lemma due to Eaton and Rosen [3].

Lemma: Given a random variable X with probability distribution $F(x)$ and support $[a,b)$ and a function $g(x)$ with $x^* \in [a,b]$ such that $g(x) > 0$ for $x > x^*$ and $g(x) < 0$ for $x < x^*$ then $\int_a^b H(x)g(x)dF(x) > 0$ if H is uniformly increasing.

To apply the lemma, note first that $V_\phi(\cdot)$ is negative and strictly decreasing. Define $H(\cdot) = -V_\phi(\cdot)$ and define $g(\cdot) = [p - E\{p\}]$. Then H and g satisfy the hypotheses of the lemma and so

$$\begin{aligned} \int_a^b H(\cdot)g(\cdot)dF(\cdot) &= E\{-V_\phi(\cdot)(P - E\{P\})\} \\ &= -E\{V_\phi(\cdot)P\} + E\{V_\phi(\cdot)\}E\{P\} > 0 \end{aligned}$$

or

$$E\{V_\phi(\cdot)P\} > E\{V_\phi(\cdot)\}E\{P\}$$

Q.E.D.

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