



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

# FINANCING THE ENVIRONMENTAL CAPACITY THROUGH INCOME TAX

*Il-Chung Kim\**

## Introduction

Over the past decade several economists have discussed the issue of optimal public investment in pollution control capital in dynamic optimization models.<sup>1</sup> Their principal focus has been to analyze the optimal mix between private (or directly productive) capital and public capital employed in waste treatment. Unfortunately, their analytical models have ignored the mechanics of financing the public investment. The purpose of this paper is to take up this issue in the context of a growing, decentralized, competitive economy.

Certainly, if the economy is completely centralized and if the government has perfect information on the production possibilities, then it is possible to implement any feasible growth path, even the optimal growth path. If, however, the economy is decentralized and if the government has only a limited set of policy instruments, then it is no longer clear that the optimal path can be implemented. Generally, one might ask two natural questions: (1) Can the publicly optimal investment policy be achieved by the environmental authority with a given set of policy instruments? (2) If the optimal policy is not achievable, what is the nature of the "second-best" policy?

The first question is one of controllability defined, for example, by Arrow and Kurz as follows: "A policy is said to be controllable by a given set of instruments varying over time, in general, which cause the private and government sectors together to realize that policy" [1, p. 120]. When specific policy instruments are introduced and are insufficient to control the publicly optimal policy, a second-best policy is followed, which is the best that can be achieved with the given set of instruments. The characteristics of a second-best policy obviously depend on what kind of policy instruments are used. For the present problem, since environmental quality is a public good by nature, so that its price cannot be suitably determined in the market and its costs cannot be fully recaptured, taxation of current income would seem to be a natural means for financing the public investment.

This paper will explore the above two questions for a prototype competitive economy. In the following section the decentralized model will be fully described. As a point of departure, we will amend Comolli's planning formulation [3] of the problem in order to be able to compare the optimal policy in a

\* Research for this paper was partially supported by the University of Kansas, Grant GST #651419. I wish to give my deep thanks to P. M. Comolli for his helpful comments and advice.

<sup>1</sup> [3], [4], [6], [8], [10], [11]

completely centralized economy with a second-best policy in the decentralized economy. A general analysis of the income-tax financing policy will be undertaken in Section III. A principal result is that such a policy exists in the long run, though it may not correspond to the optimal policy for the completely centralized economy. A full set of conclusions is provided in Section IV.

## Model

We assume that the environmental authority operates with a balanced budget and finances the public investment,  $I_g$ , through an income tax levied at rate  $x$ ,  $0 < x < 1$ . In the Keynesian tradition aggregate consumption,  $C$ , is taken to depend solely on current disposable income, and the ratio of private saving to current disposable income is taken to be a constant number  $s$ ,  $0 < s < 1$ . Denoting aggregate output by  $Y$ , consumption may be written as:

$$(1) \quad C = (1-s)(1-x)Y$$

Because private saving is allocated to private investment,  $I_p$ , we have

$$(2) \quad I_p = s(1-x)Y$$

So that net investment in directly productive capital,  $\dot{K}_p$ , stand as

$$(3) \quad \dot{K}_p = s(1-x)Y - mK_p$$

where  $m > 0$  is a constant rate of depreciation.

All tax revenue is assumed to be allocated to public investment by the environmental authority, thus

$$(4) \quad I_g = xY$$

Following Comolli [3] the stock of public capital,  $K_g$ , is assumed to be measurable, as a capacity for assimilating the economy's effluents,  $E$ . Under this interpretation a constant rate of depreciation seems overly simplistic. Therefore, let us posit that  $K_g$  depreciates exponentially at a rate dependent on its utilization  $E/K_g$ , say  $E/K_g$  to simplify notation. With this assumption, net public investment in the economy is given by

$$(5) \quad \dot{K}_g = xY - mE$$

Finally, it will be assumed throughout that the stock of labor (population) is constant.

The environmental authority, as is common practice, will be taken to use direct control as a means of regulating polluting activities. Assuming that wastes are generated only in production processes, a direct control may mean giving individual firms permits to pollute at some specified levels and

prohibiting them from exceeding these levels. Thus, if a firm violates this prohibition, whether by small or large amounts, it is considered a law-breaker subject to punitive action. It will be supposed that firms can internally reduce their effluents to legally acceptable levels by treatment (e.g., recycling of wastes). Mäler [9] has then shown how the aggregate production function

$$(6) \quad Y = f(K_p, E)$$

may be derived by technical optimization.<sup>2</sup> It will be assumed that (6) has the following properties:

A.1.  $f(K_p, E)$  is concave function that possesses positive and diminishing marginal products and satisfy the Inada conditions.<sup>3</sup>

A.2.  $K_p$  and  $E$  are nonrivalrous, i.e.,  $f_{12} \geq 0$

As Comolli [3] described, the level of environmental quality may be expressed by some numerical index  $Z$ , which seems, like temperature, to be cardinally measurable. It seems reasonable that this index,  $Z$ , has the following functional form

$$(7) \quad Z = Q(K_g, E)$$

for which the following assumptions can be made.

A.3.  $Q$  is concave, and has a positive partial derivative in  $K_g$ , a negative partial derivative in  $E$ , and zero cross partial derivatives.

A.4. Since the choice of zero is arbitrary,  $Q(K_g, E) \geq 0$  as  $K_g \geq E$ .

The concavity assumption of  $Q$  in  $E$  is based on the threshold nature of environmental quality, which implies that the environmental quality would be degraded very sharply beyond some level of effluents. The assumption of zero cross partial derivatives is used for a purely technical purpose. This,

<sup>2</sup> The technical optimization problem is as follows:

$$\begin{aligned} \max Y &= H(K_p^1) \\ \text{subject to } &K_p^1 + K_p^2 \leq K_p \\ &E \geq G(K_p^1, K_p^2) \\ &K_p^i \geq 0, i = 1, 2, 0 \leq E \leq E^* \end{aligned}$$

where  $K_p^1$  and  $K_p^2$  are private capital stocks allocated to production and to waste treatment, respectively, and  $E^*$  is some specified level of wastes at which firms are allowed to discharge. The solution to their maximization problem gives output as a function of total available factor input,  $K_p$  and of the discharge of waste,  $E$ , or  $Y = f(K_p, E)$ . See Mäler [9, pp. 28-30] for more detail.

<sup>3</sup> That is,  $f_1 > 0, f_2 > 0, f_{11} < 0, f_{22} < 0, f_{11}f_{22} - f_{12}^2 \geq 0$   
 $f_1(0, E) = \infty, f_1(\infty, E) = 0, \forall E \geq 0$   
 $f_2(K_p, 0) = \infty, f_2(K_p, \infty) = 0 \forall K_p \geq 0$

however, may be reasonably assumed since there is no reason to say that marginal environmental quality with respect to  $K_g$  (or  $E$ ) is changed due to one unit change in  $E$  (or  $K_g$ ). This environmental quality plays an important role on the social welfare. Thus an aggregate utility function, which is presented to exist for the society, will be  $U(C, Z)$ , where

$$(8) \quad U(C, Z) = u(C) + v(Z)$$

for computational convenience. We introduce two assumptions.

A.5.  $U$  has positive and diminishing marginal utilities.

A.6.  $u'(0) = +\infty = v'(0)$ , and  $v(Z) = -\infty$  for all  $Z, Z < 0$ .

In A.6. the former is reasonable with respect to human health, while the latter serves a purely technical purpose.<sup>4</sup> Social welfare is taken to be measured by the utilitarian criterion

$$(9) \quad J = \int_0^{\infty} U(C, Z) e^{-\rho t} dt$$

with  $\rho > 0$  reflecting the society's constant rate of time preference.

Then the problem of the environmental authority is to maximize (9) subject to (1), (3), (5), (6), and (7) and given initial conditions:  $K_p(0) = K_p^0 > 0$  and  $K_g(0) = K_g^0 > 0$ . The admissible control variables are measurable functions,  $x(t)$ ,  $E(t)$  satisfying  $0 \leq x \leq 1$  and  $0 \leq E \leq K_g$ . The latter is the consequence of our assumptions, A.4. and A.6. that  $v(Z) = -\infty$  for all  $Z, Z < 0$  if and only if  $E > K_g$ . Thus  $E > K_g$  is obviously infeasible.

## An Analysis

**The second-best optimality.** The planning problem of our economy reduces to maximize the current-value Lagrangian with respect to  $x$  and  $E$ . That is,

$$\max L = u[(1-s)(1-x)f(K_p, E)] + v[Q(K_g, E)] + P_p[s(1-x)f(K_p, E) - mK_p] + P_g[xf(K_p, E) - mE] + \mu_1 x + \mu_2(1-x) + \gamma_1 E + \gamma_2(K_g - E)$$

where the costate variables,  $P_p$  and  $P_g$  are the current-value shadow prices of private and public investment, respectively, and  $\mu_1, \mu_2, \gamma_1$  and  $\gamma_2$  are Lagrange multipliers resulting from the inequality constraints on  $x, E$ , respectively.

Necessary conditions for optimality (in this case, second-best optimality) are as follows:

<sup>4</sup> This assumption suggests that the choice of zero for  $Z$  is no longer arbitrary. Nevertheless, there is no loss in generality in our assumption, granted that  $v(Z) = -\infty, Z < \bar{Z}$ , for some  $\bar{Z}$  constant.

$$(10.a) \quad x[(1-s)u' + sP_p - P_g] = 0, (1-s)u' + sP_p - P_g \geq 0; x \geq 0$$

$$(10.b) \quad (1-x)[(1-s)u' + sP_p - P_g] = 0, (1-s)u' + sP_p - P_g \geq 0; x \leq 1$$

$$(11.a) \quad [(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2 \leq 0; \\ E[(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2 = 0; E \geq 0 \\ [(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2 \geq 0$$

$$(11.b) \quad (K_g - E)[(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2 = 0; E \leq K_g$$

$$(12.a) \quad \dot{K}_p = s(1-x)f(K_p, E) - mK_p$$

$$(12.b) \quad \dot{K}_g = xf(K_p, E) - mE$$

$$(13.a) \quad \dot{P}_p = (\rho + m)P_p - f_1[(1-x)\{(1-s)u' + sP_p\} + xP_g]$$

$$(13.b) \quad \dot{P}_g = \rho P_g - v'Q_1 - \gamma_2$$

We note that  $u'$  is the shadow price of consumption and  $(1-s)u' + sP_p$  can be interpreted as the shadow price of disposable income since disposable income is allocated between consumption and private investment in the proportions  $1-s$ ,  $s$ , respectively. Let  $P_1 = (1-s)u' + sP_p$ . In (10.b),  $x=1$  is optimal if  $P_1 \leq P_g$ . But for  $x=1$ ,  $C=0$ , so that  $u'(0) = +\infty$ , by assumption A.6 and the  $P_1 = +\infty$  and  $\partial L/\partial x = -\infty$ , a contradiction. This asserts that income tax rate should be less than 100 percent (i.e.,  $x < 1$ ). Once  $x > 1$ ,  $P_1 = P_g$ . In (10.a),  $x = 0$  is optimal if  $P_1 \geq P_g$ . This cannot be excluded because consumption is positive from (1). If  $x > 0$ , again  $P_1 = P_g$ . In summary,

$$(14) \quad \text{if } x = 0, P_1 \geq P_g \\ \text{and if } 0 < x < 1, P_1 = P_g$$

These results imply that in order to maximize  $L$ , if the shadow price of disposable income is greater than that of public investment, then the income tax should be zero, and in the case of positive income tax, the optimal tax could be achieved when both shadow prices of disposable income and public investment are the same. From (11.a),  $E = 0$  is optimal if  $\partial L/\partial E \leq 0$ . But by Inada condition,  $f_2(K_p, 0) = +\infty$  and hence  $\partial L/\partial E = +\infty$ , a contradiction. This contradiction reminds us of the Ayres and Kneese's [2] material balance view.<sup>5</sup> From (11.b),  $E = K_g$  is optimal if  $\partial L/\partial E = 0$ . However, for  $E = K_g$ ,  $Z = 0$ , so that  $v'(0) = +\infty$  by assumption A.6 and then  $v'Q_2 = -\infty$ , a contradiction. For these reasons,  $E$  should be strictly between zero and  $K_g$  (i.e.,  $0 < E < K_g$ ). Then (11) and (13.b) become, respectively,

<sup>5</sup> This view indicates that tonnage of raw materials extraction utilized by an economy is approximately equal to waste residuals generated by the economy in the long run. This contradiction becomes obvious from this view.

$$(11.c) \quad [(1-x)P_1 + xP_g]f_2 - mP_g = -v'Q_2$$

$$(13.c) \quad \text{and } \dot{P}_g = \rho P_g - v'Q_1$$

From (14), it is known that both cases of zero and positive income taxes are feasible. Hence we now consider each case separately.

Case 1:  $x = 0$

The nature of this case is equivalent to what Keeler, Spence, and Zeckhauser [7] called the "Murky Age" during which no money is spent on pollution control. This case might occur in the economy, either where  $K_g$  is initially large enough to keep acceptable level of environmental quality without making any further public investment, or where, as in many underdeveloped countries, the central planner concentrates on economic development without regard to environmental quality. In any case, the opportunity cost of disposable income is greater than that of public investment. From (10.a) and (11.c),  $(f_2 - m)P_g \leq -v'Q_2$ . This implies that when assimilative investments are not undertaken, firms are also allowed to discharge effluents into the environment even though the value of their net marginal product is less than the value of the deterioration in environmental quality. This might support the authority's neglectful policy on pollution control. However, this might cause serious degradation in environmental quality. In fact, as time goes by, the stock of pollution may increase without any public investment for abating pollution. Since we assume that public capital depreciates at a rate depending upon the level of pollution, the stock of it will approach zero in the long run. This becomes obvious from (12.b),  $K_g = -mE$ , so that, defining  $\mu = E/K_g$  where  $0 < \mu < 1$ ,  $K_g(t) = K_g^0 e^{-m \int_0^t \mu(\tau) d\tau}$  and  $\lim_{t \rightarrow \infty} K_g(t) = 0$ . This process asserts that

the environmental authority should start to invest in environmental capacity after a while. Otherwise, a point of time will come after which environmental capacity is not enough for assimilating pollution to keep a certain level of environmental quality. That is, after some time period,  $E \geq K_g$ , which is not feasible. Thus, the income tax rate will become positive in the long run. Besides, this is not rather interesting case because a steady state doesn't exist. Again, from (12.b),  $\dot{K}_g = 0$  at a steady state, so that  $E = 0$ , a contradiction.

Proposition 1: Under the specification of the model, there does not exist a steady state with zero income tax. Instead, there does exist time  $t$  beyond which income tax becomes positive.

Case 2:  $0 < x < 1$

This is the case of what Keeler, et al. [7] called the "Golden Age," which is characterized by lower private investment, consumption and better environmental quality than the "Murky Age." In this case, (11.c) becomes

$$(11.d) \quad (F_2 - m)P_g = -v'Q_2$$

This asserts that optimal increase in effluents should equate the value of their "net" marginal product with the value of the deterioration in environmental quality. That is, in order to maximize social welfare, the environmental authority should allow the firm to discharge effluents up to the specified level at which the value of effluents' net marginal product equals the value of the deterioration in environmental quality.

Totally differentiating (14) and (11.d),

$$(15) \quad -(1-s)^2 u'' f dx + (1-s)^2 (1-x) f_2 dE = -(1-s)^2 (1-x) u'' f_1 dk_p - s dP_p + dP_g - (f_{22} P_g + v'' Q_2^2 + v' Q_{22}) dE = f_{12} P_g dK_p + (v'' Q_1 Q_2 + v' Q_{12}) dK_g - \frac{v'}{P_g} Q_2 dP_g$$

The matrix of coefficients on the left-hand side of the above system is  $P$  - matrix since all principal minors are positive. Thus, by theorem 4 of Gale and Nikaido [5] there is a unique solution of the functions,  $\hat{x}(K_p, K_g, P_p, P_g)$ ,  $\hat{E}(K_p, K_g, P_p, P_g)$ . Table 1 shows the probable signs of the various partial derivatives obtained from (15) as well as (1) and (6).

**TABLE 1**  
**Summary of Probable Signs**

Derivative of	with respect to			
	$K_p$	$K_g$	$P_p$	$P_g$
X	?	-	-	?
E	+	+	0	+
Y	+	+	0	+
C	+	+	+	?
Z	-	?	0	-

Conclusively, Pontryagin's necessary conditions indicate that the movement along the second-best optimal path is governed by the differential equations for the state variables, (12), and those for the costate variables, (13), with the control variables,  $x$  and  $E$ , being determined by (14) and (11.d).

**Existence of a Steady State.** In the previous section, we discussed that there does not exist a steady state in the "Murky Age." We now show the existence of a steady state in the "Golden Age." In a steady state, the relevant conditions can be summarized as follows:

$$(16) \quad (1-s)u' + sP_p = P_g$$

$$(17) \quad (f_2 - m)P_g = -v'Q_2$$

$$(18) \quad \dot{K}_p = s(1-x)Y - mK_p = 0$$

$$(19) \quad \dot{K}_g = xY - mE = 0$$



$$(20) \quad \dot{P}_p = (\rho + m)P_p - f_1 P_g = 0$$

$$(21) \quad \dot{P}_g = \rho P_g - v' Q_1 = 0$$

$$(22) \quad \text{and } Y = f(K_p, E)$$

$$(23) \quad C = (1-s)(1-x)Y$$

Note that all control and state variables in the above are stationary values. Thus, the existence problem of a steady state becomes that of a solution in the above equation system. Under assumption (A.3), we may reasonably assume that  $Q(K_g, E) = \beta(K_g - E)$ ,  $K_g > E$ . This helps to make the proof of existence easier without loss of generality. Then (17) and (21) become

$$(17') \quad (f_2 - m)P_g = \beta v'$$

$$(21') \quad \text{and } \rho P_g = \beta v'$$

It follows then from (17') and (21') that

$$(24) \quad f_2(K_p, E) = \rho + m$$

By Inada condition, there exists a solution  $\tilde{E}(K_p)$ , with

$$(25) \quad E'(K_p) = -f_{12}/f_{22} \geq 0, \text{ for any } K_p > 0$$

Eliminating  $x$  between (18) and (19) and substituting for  $Y$  in (22) yields

$$(26) \quad sf[K_p, \tilde{E}(K_p)] = m[K_p + s\tilde{E}(K_p)]$$

If there exists a solution  $K_p$  for equation (26), the existence of a steady state will be realized. This is so because  $\tilde{Y}$  is determined by (22) and then  $\tilde{x}$  is determined by either (18) or (19), the other being automatically satisfied by (26). Then  $\tilde{C}$  is determined by (23) and thereby  $u'$ , so that  $\tilde{P}_p$  and  $\tilde{P}_g$  are defined by (16) and (20). The  $\tilde{K}_g$  is also determined by either (17') or (21'), in view of assumptions (A.3) and (A.5), the other being automatically satisfied by (24). Thus, the proof of the existence of a steady state is reduced to that of the existence of a solution for (26). For this, we shall make the following assumptions.

A.7  $f(K_p, E)/(K_p + E)$  can be made arbitrarily small for  $K_p + E$  large.

A.8 If  $K_p$  converges to zero and  $(K_p, E)$  vary along  $f_2 = \rho + m$ , then  $K_p/E$  converges to zero.<sup>6</sup>

<sup>6</sup> Two more alternative assumptions can be made:

$f_2(0, E) = \rho + m$ , some  $E > 0$

or  $f_1(0, 0) > m/s$

The former is similar to A.8 and the latter is tenable in view of the Inada condition. The proof of the existence under each assumption can be done very similarly to Arrow and Kurz [1, p. 136].

A.7 implies that aggregate output per total inputs used converges to zero (for fixed labor forces) when  $K_p$  approaches to infinity.

Now, let us define  $F(K_p) = sf[K_p, \tilde{E}(K_p)] - m[K_p + s\tilde{E}(K_p)] = 0$  from (26). To guarantee that  $F$  has a solution in  $K_p$ , we require

$$(27) \quad \lim_{K_p \rightarrow \infty} F(K_p) < 0$$

$$(28) \quad \text{and } F(K_p) > 0, \text{ for some } K_p < +\infty$$

By assumption A.7, as  $K_p$  tends to infinity,

$$sF[K_p, \tilde{E}(K_p)] < ms[K_p + \tilde{E}(K_p)] < m[K_p + s\tilde{E}(K_p)],$$

so that (27) holds. Since  $f$  is concave, we have the well-known inequality

$$f_1[K_p, \tilde{E}(K_p)](K_p - 0) + f_2[K_p, \tilde{E}(K_p)](\tilde{E}(K_p) - 0) \leq f(K_p, \tilde{E}(K_p)) - f(0, 0)$$

Since  $f(0, 0) \geq 0$  and  $f_2[K_p, \tilde{E}(K_p)] = \rho + m$ ,  $f[K_p, \tilde{E}(K_p)] \geq f_1[K_p, \tilde{E}(K_p)] + (\rho + m)\tilde{E}(K_p)$ . Then it is sufficient for (28) to hold that

$$sf_1[K_p, \tilde{E}(K_p)]K_p + s(\rho + m)\tilde{E}(K_p) - m[K_p + s\tilde{E}(K_p)] \geq 0.$$

$$(29) \quad \text{That is } sp \geq (m - sf_1) [K_p/\tilde{E}(K_p)]$$

Since  $s > 0$  and  $\rho > 0$ , the LHS is positive, so that a sufficient condition for (29) is that

$$(30) \quad \lim_{K_p \rightarrow 0} p(m - sf_1)[K_p/\tilde{E}(K_p)] \leq 0$$

Here, since  $f_1 > 0$ ,  $m - sf_1 < m$ . Hence, by A.8

$$\lim_{K_p \rightarrow 0} \sup (m - sf_1)[K_p/\tilde{E}(K_p)] < \lim_{K_p \rightarrow 0}$$

$\sup m[K_p/\tilde{E}(K_p)] = 0$  so that (30) certainly holds.

**Uniqueness of the Steady State.** We have just shown that the solution to the equations (24) and (26), say  $K_p$  and  $E$  does exist under the assumptions A.1, A.7 and A.8 when  $Q$  is linear function. We also know that once  $K_p$  and  $E$  are determined, all other variables — such as  $x$ ,  $C$ ,  $Y$ ,  $P_p$ ,  $P_g$  and  $K_g$  — are uniquely determined. Thus, the uniqueness question of the steady state boils down to that of a solution for equations (24) and (26). That is, it is whether these curves intersect only once in  $K_p - E$  plane.

Now let us define

$$(31) \quad \phi(K_p, E) = (\rho + m) - f_2(K_p, E)$$

$$(32) \text{ and } \psi(K_p, E) \equiv sf(K_p, E) - m(K_p + sE)$$

In fact, the function  $\phi(K_p, E) = 0$  represents  $\dot{P}_g/P_g = 0$  in view of (21), (17') and (21'), while the function  $\psi(K_p, E) = 0$  represents  $\dot{K}^p + s\dot{K}_g = 0$  from (18), (19) and (22). Both of these functions may be regarded as the long-run functions. The long-run function  $\phi(K_p, E) = 0$ , with derivatives

$$\partial\phi/\partial K_p = -f_{21} \leq 0$$

$$\text{and } \partial\phi/\partial E = -f_{22} > 0$$

may be represented by a curve in  $K_p - E$  phase plane with slope

$$dE/dK_p|_{\phi(K_p, E) = 0} = -(\partial\phi/\partial K_p)/(\partial\phi/\partial E) = -f_{12}/f_{22} \geq 0$$

where this curve starts from some positive  $E$  because  $\phi(K_p, E) = 0$  would, otherwise, not be satisfied in view of Inada condition. Similarly, the curve in the plane for  $\psi(K_p, E) = 0$ , with derivatives,

$$\partial\psi/\partial K_p = sf_1 - m \text{ and } \partial\psi/\partial E = sp > 0$$

has the slope

$$dE/dK_p|_{\psi(K_p, E) = 0} = -(\partial\psi/\partial K_p)/(\partial\psi/\partial E) = (m - sf_1)/sp$$

whose sign depends on the sign of  $m - sf_1$ . Now for our purpose, we need to check

$$dE/dK_p|_{\phi=0} \geq dE/dK_p|_{\psi=0}$$

which is equivalent to

$$(33) \quad -f_{12}/f_{22} \geq (m - sf_1)/sp$$

Naturally, if  $f_1(K_p, E) > m/s$  then the steady state is unique without further analysis; that is,  $\phi(K_p, E) = 0$  is nondecreasing in  $K_p - E$  phase plane, while  $\psi(K_p, E) = 0$  is strictly decreasing. In view of (30), however, this is the case only when  $K_p$  approaches zero. Alternatively, the sufficient condition for uniqueness is that  $f_{21}K_p + f_{22}E < 0$  from (29) and (33)<sup>7</sup>; both curves are nondecreasing in the phase plane, but the slope of  $\psi(K_p, E) = 0$  is greater than that of  $\phi(K_p, E) = 0$ . This condition implies that effluents' marginal product is decreasing as  $K_p$  and  $E$  increase in any fixed proportion. This condition will not hold if there are increasing returns. Hence, it may be more reasonable to say

<sup>7</sup> This can be shown in the following way:

From (29),  $\dot{E}(K_p)/K_p \geq (m - sf_1)/sp$

Combining (33) with this,  $\dot{E}(K_p)/K_p \geq (m - sf_1)/sp > -f_{12}/f_{22}$

Hence  $f_{21}K_p + f_{22}E < 0$ .

that  $f_1(K_p, E) > m/s$  for very small  $K_p$  and  $f_1(K_p, E) < m/s$  for larger  $K_p$ . That is,  $\psi(K_p, E) = 0$  curve falls and then rises in the phase plane. Thus, without additional restrictions, unfortunately, it is impossible to be specific about the direction of the inequality in (33). Hence, the  $\phi = 0$  curve may intersect the  $\psi = 0$  curve at numerous points.

Before concluding, it may be instructive to characterize some aspects of a steady state. Condition (17) has the same form as in Comolli's [3] publicly optimal policy. Moreover, equation (24), which implies that effluent's "net" marginal product equals the "social rate of interest" at any stationary point, is exactly the same as the one in Comolli's. This result asserts that the level of effluent could be controlled optimally by the environmental authority even in the decentralized economy just as by the one with full power over the economy. On the other hand, private capital's "net" marginal product necessarily differs from the "social rate of interest." To see this suppose  $f_1 - m = \rho$ . From (20),  $P_p = P_g$ . Let  $P$  be the common value of these shadow prices. Then (16) simplifies to  $u' = P$ . This notes that all the shadow prices of consumption, private and public capital are the same. If we let  $K = K_p + K_g$ , it follows from (18), (19), and (23) that  $Y = C + m(E + K - K_g)$ . This implies  $\dot{K} = 0$ , which defines a stationary locus in Comolli's publicly optimal model. The inequality among the shadow prices of consumption, private investment and public investment is an indicator of the failure to achieve the optimum possible in perfectly planned economy. Moreover, it may be noted from (20) and (24) that when  $Q$  is linear, the ratio between the shadow prices of private and public capital is equal to the marginal rate of technical substitution of private capital for the joint input of effluents at a stationary point.

The foregoing results are summarized succinctly in the following.

**Proposition 2:** Under the specification of the model, there exists a steady state with positive income tax if  $Q$  is linear, even though the uniqueness of it is uncertain. At any stationary point, effluent's net marginal product equals the "social rate of interest." Private capital's net marginal product, however, is not equal to the "social rate of interest" unless the stationary solution for the second-best policy happens to be that of the publicly optimal policy.

## Concluding Remarks

We have seen that the optimal policy is not controllable when environmental authorities with balanced budgets impose only income tax and use direct control on a firm's polluting activities, and private savings are a fixed proportion of disposable income. This is caused by the differences among the shadow prices of consumption, private capital and public capital. We also saw that without making any public investment in environmental capacity, there does not exist a steady state, ending up with serious deterioration in environmental quality. These results warn any country against expanding its economy without taking serious consideration of its environment. It is suggested that in order to maximize the social welfare, environmental authorities should finance public investment with an optimal tax rate at which the shadow price of disposable income equals that of public capital, and allows firms to

increase effluents up to the specified level at which the value of effluents' net marginal product equals the value of the deterioration in environmental quality.

In fact, this paper is an attempt to reduce a gap between literatures on public investment in environmental capacity and fiscal aspect of taxation, but we must admit that our results are restricted to a special case.

## REFERENCES

1. Arrow, K. J. and Kurz, M. *Public Investment, the Rate of Return, and Optimal Fiscal Policy*: John Hopkins Press for RFF, Baltimore, 1977.
2. Ayres, R. V. and Kneese, A. V. "Production, Consumption and Externalities," *American Economic Review* 59 (3), June, 1969, 282-297.
3. Comolli, P. M. "Optimal Public Investment in the Environmental Capacity," *Journal of Economics* 7, (Feb., 1981), 5-8.
4. d'Arge, R. C. and K. C. Kogiku. "Economic Growth and the Environment," *Review of Economic Studies* 40, (Jan., 1973), 61-77.
5. Gale, D. and H. Nikaido. "The Jacobian Matrix and Global Univalence of Mappings," *Mathematische Annalen* 159, (1965), 81-93.
6. Gruver, G. W. "Optimal Investment in Pollution Control Capital in a Neoclassical Growth Context," *Journal of Environmental Economics and Management* 3, (Oct., 1976), 165-177.
7. Keeler, E., Spence, M. and R. Zeckhauser. "The Optimal Control of Pollution," *Journal of Economic Theory* 4, (1972), 19-34.
8. Mäler, K. G. *Environmental Economics: A Theoretical Inquiry*, John Hopkins Press for RFF, Baltimore, 1974.
9. \_\_\_\_\_. "Macroeconomic Aspects of Environmental Policy," in E. S. Mills, ed. *Economic Analysis of Environmental Problems*, Columbia University Press for NBER, New York, (1975), 27-56.
10. Plourde, C. G.: "A Model of Waste Accumulation and Disposal," *Canadian Journal of Economics* 5, (1972), 119-125.
11. Smith, V. L.: "Dynamics of Waste Accumulation," *Quarterly Journal of Economics* 86, (1972), 600-616.