LEVELS OF SPATIAL VARIATION WITHIN A METROPOLITAN AREA

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The great majority of studies of the residential structure of urban areas have focused on the variations across census tracts in average characteristics of the population and housing. This raises two issues: On the one hand, census tracts have either been assumed to be sufficiently uniform so as to justify ignoring any variations in those characteristics within the tracts, or such variations were considered to be unimportant. On the other hand, very little consideration has been given to the possibility that the variations observed across census tracts might actually be the product of variation in those qualities occurring across larger regions. The geographical scale at which urban residential variation occurs is the subject of empirical investigation in this paper.

Occasional studies of urban areas have been conducted using more than one level of aggregation, so the issue is not a new one. For example, Duncan and Duncan [5] carried out the analyses in their famous study of occupational segregation using both census tracts and larger zone-sector segments. Rees [11] conducted factorial ecological studies of Chicago using both tracts and larger community areas with suburban municipalities. In both instances, similar results were obtained. However Duncan, Cuzzort, and Duncan, in their methodological study [4], given examples of the ways in which the level of analysis can dramatically influence the conclusions one might draw and discuss some of the issues involved. Their conclusion concerning the importance of scale is reemphasized by Clawson [3] in his study of land development in the northeastern United States.

One approach to these issues relating to the level of aggregation has been through discussions of the so-called "ecological fallacy," first raised by Goodman [6]. An important generalization and extension is the more recent work of Alker [1] in developing a range of spatial and temporal "fallacies" relating to aggregation and disaggregation. Haggett, Cliff, and Frey [7] have posed the question as being one of establishing the geographical scale at which variation occurs and relationships obtain. They discuss a number of analytical techniques, including polynomial trend surface analysis, fourier and spectral analysis, and analysis of variance, that might be employed in addressing such questions. It is a version of the latter approach that will be employed in this paper. In addition to this direct analysis to determine levels of variation, several conventional analyses of urban phenomena will be carried out at different levels in order to further consider the importance of scale.

Partitioning Of Variation

Determination of the amounts of variation in population and housing characteristics occurring at various geographical scales within an urban area

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can be accomplished using procedures associated with the analysis of variance. Kish [8], Stokes [13], and Moellering and Tobler [9] have proposed approaches for the partitioning of variation across a hierarchically-nested set of areal divisions. The methodology and its application to geographical problems have been reviewed by Haggett, Cliff, and Frey [7]. The very straightforward approach taken here is similar to all of these, but bears perhaps the closest resemblance to the nested analysis of variance model with fixed effects described by Scheffé [12]. The brief presentation here follows closely that of Moellering and Tobler [9], and the reader is directed to that paper for further details on the general approach being used.

The analysis begins with a set of N observations on the characteristics of individual units which are aggregated into successively-larger areal units which constitute a nested set of subdivisions. In this study, individuals are aggregated into census tracts, which in turn are aggregated into a smaller number of larger analysis regions. The n regions constitute a subdivision of the entire metropolitan area, while n_i tracts produce a subdivision of each region i. Each tract j in region i contains n_{ij} individuals. It follows that

$$(1) \quad \sum_{i}^{n} \sum_{j}^{n_i} n_{ij} = N.$$

Take x_{ijk} as an observation of the value of some characteristic of the k^{th} individual in the j^{th} tract in the i^{th} region. Then this observation can be seen as a product of effects operating at the different scales and can be taken as the result of the effects of the grand mean, μ; the i^{th} region, α_i; the i^{th} tract in the i^{th} region β_{ij}; and the specific effect associated with the k^{th} individual within that tract, γ_{ijk}:

$$(2) \quad x_{ijk} = \mu + \alpha_i + \beta_{ij} + \gamma_{ijk}$$

At issue, then, are the relative magnitudes of the effects associated with each of the levels, α_i, β_{ij}, and γ_{ijk}. The separation of the effects becomes clearer when they are expressed in terms of deviations from the appropriate means:

$$(3) \quad (x_{ijk} - X) = (X_i - X) + (X_{ij} - X_i) + (x_{ijk} - X_{ij})$$

where X is the grand mean across all individuals, X_i is the mean across all individuals in the i^{th} region, and X_{ij} is the mean across all individuals in the i^{th} tract of the i^{th} region. Thus, deviations of individual values from the grand mean are sums of the deviations of region means from the grand mean, tract means from region means, and individual observations from tract means. The total deviation has been partitioned into the deviations associated with the effects at the region, tract, and individual levels. Squaring both sides of this equation gives the following (as generally occurs in the analysis of variance, the cross-product terms on the right-hand side drop out):

$$(4) \quad (x_{ijk} - X)^2 = (X_i - X)^2 + (X_{ij} - X_i)^2 + (x_{ijk} - X_{ij})^2$$

Then summing over all individuals, tracts, and regions gives the following partitioning of the total variation:

$$\sum_{i}^{n}\sum_{j}^{n_i}\sum_{k}^{n_{ij}}(x_{ijk} - X)^2 = \sum_{i}^{n}\sum_{j}^{n_i}\sum_{k}^{n_{ij}}(X_i - X)^2 + \sum_{i}^{n}\sum_{j}^{n_i}\sum_{k}^{n_{ij}}(X_{ij} - X_i)^2 + \sum_{i}^{n}\sum_{j}^{n_i}\sum_{k}^{n_{ij}}(x_{ijk} - X_{ij})^2 \quad (5)$$

The total variation of individual values around the grand mean is the sum of the variation of the region means around the grand mean (effectively weighted by the numbers of individuals in each region, given the summations over individuals, k, and tracts, j), the tract means around the region means (again weighted by the numbers of individuals in each tract), and the variation of individual values around tract means. Since the variation is the sum of the squared deviations, this partitioning might be more familiarly represented as:

$$SS_{total} = SS_{regions} + SS_{tracts} = SS_{individuals} \quad (6)$$

Each of the terms on the right-hand side of the equation represents the amounts of the total variation that can be attributed to that level. For example, SS_{tracts} is the quantity of the total variation occurring at the tract level, variation across tracts within regions. This partitioning of the total variation serves as the basis for analyzing the levels of variation of different characteristics within an urban area.

The analysis being undertaken here is a purely descriptive one, given the assumption of a complete census of the observational units as a basis for the partitioning. Thus, there are no issues of estimation, of the inference of population parameters from sample statistics, and no reasons for tests of statistical significance. Degrees of freedom and distributional characteristics are likewise irrelevant.[1]

The City, the Areal Divisions, and the Data

The Indianapolis, Indiana Standard Metropolitan Statistical Area was chosen as the setting for the analysis. In addition to being a middle-sized SMSA (population just over one million in 1970) in the middle of the country, Indianapolis possesses several other advantages that make it an appropriate choice. The Indianapolis SMSA has a single central city, and its growth has not been strongly influenced by other large urban areas in the vicinity. The city is located on a flat plain and is relatively symmetrical. A generous area, extending fairly uniform distances in all directions, is included within the SMSA boundaries (and hence is tracted). Finally, the additional uniformities imposed by the rectilinear land survey provide a regular pattern of boundaries ideal for aggregation into larger territories.

Census tract data are used in the analysis, and tracts will therefore constitute the lowest level of geographical aggregation above the individual observational units which can be considered. The individual units and the 253 tracts in

[1]This represents a significant departure from the Moellering and Tobler analysis [9] and more closely parallels the approach taken by Kish [8].

the Indianapolis SMSA are thus the two lowest scales at which variation will be examined, but the question was also posed regarding variation at a scale larger than tracts. To examine this, a system of 16 regions was created through the aggregation of census tracts. Within the more densely-populated Marion County at the heart of the SMSA, township boundaries were used to establish nine square, basically equal-sized regions. The seven surrounding counties were each taken as a region, making the total of sixteen. The division of the SMSA into these regions is shown in Figure 1, with the numbers of census tracts shown for each region. The regions vary widely in both area, as can be seen from the map, and in population, from 10,293 to 273,598. No claim is being made for this particular aggregation; it represents a compromise between a uniform areal division (the eight counties would come close) and uniformity in terms of population. This is a compromise reflecting established, recognized political boundaries in the region.[2]

For some purposes, an even coarser division of the SMSA into only three areas will be employed.[3] This involves further aggregation of the 16 regions.

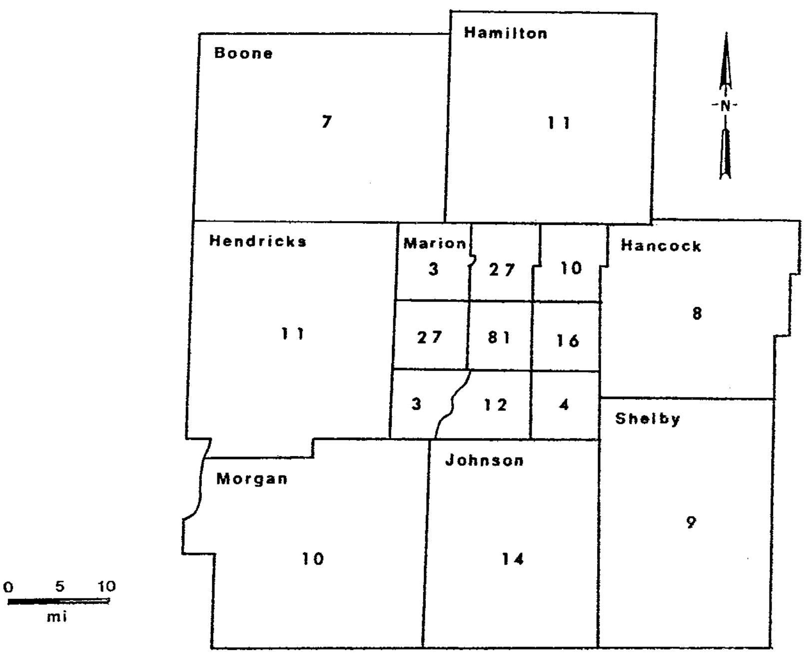

Figure 1 — Sixteen analysis regions in the Indianapolis SMSA (Numbers of census tracts indicated in each region)

[2]A question not being explored in this research is the effect of different aggregation strategies.

[3]The central city-suburban division, which would be appropriate for most SMSAs, was rendered less interesting for Indianapolis by the consolidation of the governments of the City of Indianapolis and Marion County in the late 1960s. In this consolidation, the older central city area was combined with many newer suburban areas. The present central city now includes nearly all of Marion County (except for a few small municipalities remaining independent). Therefore, the central city-suburb division for Indianapolis fails to reflect the differences that it does in most metropolitan areas. For this reason, the central city-suburb distinction is ignored in favor of the more useful division of the SMSA into three major areas.

Center Township within Marion County is the older inner-city portion of the SMSA. The remainder of Marion County is the intermediate area, while the surrounding seven counties outside of Marion County form an outer ring. This division is shown in Figure 2.

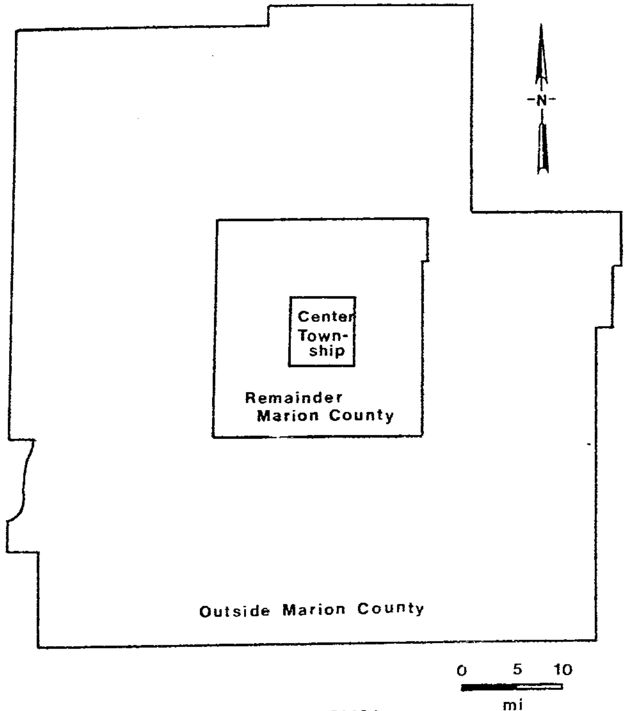

Figure 2 — Three major areas in the Indianapolis SMSA

The partitioning of variation analysis will be applied to four common characteristics of the population and housing in 1970 for which interval scale information is available at the level of the individual observational units — education, income, value, and rent [14]. For education, the distribution is given for persons aged 25 years and over of the numbers of years of school completed. Families are categorized by their total annual incomes during the preceding year. Value and rent refer to owner- and renter-occupied housing units, respectively. In all cases, given the grouped data provided in the census tract reports, all individual units were assumed to have the value of the midpoint of the interval in which they were reported.[4] These frequency distributions for each tract provided the raw data for the computations of the means and the sums of squared deviations required in the analysis.

[4]For observations in the highest, open-ended intervals, those values were selected that would have been midpoints had those intervals been the sizes of the intervals immediately below them. These gave mean values for the SMSA population fairly close to those reported by the census using their more detailed sets of intervals for which the data were originally collected.

Levels Of Variation

Following the procedures outlined above, the total variation across the individual observational units was partitioned to the levels of individuals, tracts, and regions for education, income, value, and rent. The sums of squared deviations attributable to variation at each of these levels were computed. The results are presented in Table 1, including the percentage of the total variation attributable to each level.

TABLE 1. PARTITIONING OF TOTAL VARIATION

Level	Sum of Squares	Percent
Education		
16 Regions	567,000	10.2
253 Tracts	368,000	6.6
Individuals	4,630,000	83.2
Total	5,570,000	100.0
Income		
16 Regions	1.84×10^{12}	8.9
253 Tracts	1.64×10^{12}	7.9
Families	1.72×10^{13}	83.2
Total	2.07×10^{13}	100.0
Value		
16 Regions	4.50×10^{12}	21.6
253 Tracts	4.90×10^{12}	23.6
Housing Units	1.14×10^{13}	54.8
Total	2.08×10^{13}	100.0
Rent		
16 Regions	7.27×10^{7}	29.6
253 Tracts	4.22×10^{7}	17.2
Housing Units	1.30×10^{8}	53.2
Total	2.45×10^{8}	100.0

In every instance, the majority of the overall variation occurs at the individual level. That is, variation between individuals within each tract is greater than variation between tracts (or at any higher level). Thus, any analysis of tract-level variation of these four characteristics necessarily ignores much of the variation within the metropolitan area.

While there were differences, the variation occurring above the individual level was divided between the tract and region levels with greater variation at the level of the 16 regions in three of the four cases. That is, more of the variation in those instances occurred across the 16 regions than across the census tracts within each of those regions.

The most striking difference comes between the two population characteristics, education and income, and the two housing characteristics, value and rent. For both education and income, fully five-sixths of the total variation occurred at the individual level, within tracts, while just over half of the variation in value and rent was within tracts. Put another way, the population is more heterogeneous within census tracts compared with its distribution across tracts, while housing is relatively more homogeneous. This might, of course, be attributed to the more permanent nature of housing, and this is compatible with an earlier observation that, looked at over an extended period of time, housing characteristics seem to possess greater regularity in their patterning within a metropolitan area than do population characteristics [10].

From the practical standpoint of considering the implications of using different systems of areal divisions for analysis, the variations across the more aggregated units can be compared with the *total* variations of the characteristics across tracts within the metropolitan area. Since there is certain to be some variation across tracts within these more aggregated units, such as the 16 regions, the variation at these higher levels will necessarily be less; something will be lost through the aggregation. The variation at the higher level expressed as a percentage of the total variation across tracts gives an indication of how much remains with the shift to the more aggregated units. Table 2 gives these percentages of the total tract variation remaining at the levels of the 16 regions, and the three large areas. At the regional level, about one-half of the tract variation is retained. Thus, in the shift from the traditional fine division of the metropolitan area into 253 census tracts to the system of only 16 regions, only about one-half of the variation is lost. Even more striking is the percentage of the tract-level variation remaining with aggregation into only three large areas. From just over one-fourth of the tract variation, in the case of value, to fully one-half of the variation, in the case of rent, is retained. With only the three areas, a notable proportion of the total variation remains, suggesting that some of the processes operating to create differentiation in the metropolitan area operate at a very broad scale.

TABLE 2. PERCENT OF TOTAL TRACT VARIATION

Variable	16 Regions	3 Areas
Education	60.7	39.2
Income	52.9	34.0
Value	47.9	26.8
Rent	63.3	52.2

One further question involves the possibility that the distribution of the total variation across levels might differ in different parts of the metropolitan area. To examine this, the partitionings of the total variation across individual units and tracts (with regions dropped) are determined in each of the three large areas, with the results presented in Table 3. For each of the four variables, a similar pattern occurs: Center Township and the area outside Marion County, the inner and outer areas, have a much higher percentage of the total variation at the individual level. The intermediate ring, the remainder of Marion County outside of Center Township, has relatively less individual-level variation, with more of the total variation at the tract level. Evidently the oldest inner area and the outer area combining new suburban development with older rural areas have greater heterogeneity, while the relatively new but more completely-developed intermediate ring is more homogeneous.

TABLE 3. PARTITIONING OF VARIATION IN THREE AREAS

Area	Percent		
	Tracts	Individual Units	Total
Education			
Center Township	6.8	93.2	100.0
Remainder Marion Co.	16.0	84.0	100.0
Outside Marion Co.	6.1	93.9	100.0
Total SMSA	16.8	83.2	100.0
Income			
Center Township	6.6	93.4	100.0
Remainder Marion Co.	15.2	84.8	100.0
Outside Marion Co.	5.5	94.5	100.0
Total SMSA	16.8	83.2	100.0
Value			
Center Township	24.3	75.7	100.0
Remainder Marion Co.	45.0	55.0	100.0
Outside Marion Co.	21.9	78.1	100.0
Total SMSA	45.2	54.8	100.0
Rent			
Center Township	14.3	85.7	100.0
Remainder Marion Co.	36.6	63.4	100.0
Outside Marion Co.	27.4	72.6	100.0
Total SMSA	46.8	53.2	100.0

Analyses At Different Levels

A significant proportion of the total variation across tracts in four basic population and housing characteristics remains at the higher level of aggregation to 16 regions. This suggests that relationships frequently observed between variables at the tract level might also obtain at this higher level of aggregation. Two sets of analyses, one a regression analysis involving the determinants of population density, and one involving a principal components factorial ecology analysis of a larger set of variables are undertaken to empirically address the question of the effect of the level of aggregation on the analysis.

The negative-exponential decline of population density with distance from the center of the city must certainly be one of the most commonly-noted phenomena associated with the spatial differentiation of urban areas. With the application of a logarithmic transformation, the natural log of population density should have a negative linear association with distance. The analysis of urban population densities is extended here in two ways. First, the percentage of housing units that are single-family dwellings is an alternative indicator of density; though in some ways not as sensitive, this measure overcomes the distorting effects of nonresidential land use on gross population density. Second, the period of construction of the housing has also been suggested as an additional determinant of density.

Linear regression, using just distance and both distance and housing age as predictors of either of the density measures were conducted using both the 253 census tracts and the 16 regions as units of analysis. The density variables are gross population density (population divided by the total area of a unit) and the percentage of all housing units single-family. The independent variables are the distance from the center of the city (Monument Circle) to the center of each area, in miles, and, as an indicator of housing age, the percentage of housing units built before 1940.

The results of the analyses are shown in Table 4. In most of the cases, the similarity between the results for the regressions using the 16 regions versus the 253 tracts is very strong. The estimates of the regression coefficients show the least variation between the two sets of analyses; there are no cases in which a substantially different interpretation might be made. The coefficients of determination, R^2, are uniformly higher for the regressions using the smaller number of areas. This is a phenomenon observed by others in situations involving aggregation, and has been explained by Blalock [2] as resulting from the fact that the aggregation in effect served to "control" for some of the disturbing influences on the dependent variable. The determination of the relative importance of the two independent variables through the comparison of the standardized regression or beta coefficients, is the one aspect most affected by the level of analysis. The multiple regressions with the percent single-family as the dependent variable exhibit this clearly. With the 16 regions as the units of analysis, the beta coefficient for distance is about twice the size of the housing-age beta, suggesting a considerably greater importance for the former as a factor affecting the percent single-family. Shifting the analysis to tracts produces a much greater decline in the distance beta than the housing-age beta, leading one to conclude that these variables have rather more similar effects on the dependent variable. The changes in the relative variances of the independent variables account for the differences and illustrate that the question of relative importance is more dependent upon the context in which it is asked [2].

Factorial ecology utilizes multivariate techniques such as principal components analysis or factor analysis to attempt to determine a small number of dimensions of variation across urban subareas associated with a larger number of variables. This is therefore an appropriate technique to determine the extent to which the relationships among a large number of variables remain constant at different levels of aggregation. In this research, principal components analyses are conducted on a set of 15 common census population and housing characteristics using both the 16 regions and the 253 tracts as units of analyses. The factors with eigenvalues greater than one are retained and rotated to the normal varimax position.

The variables included are new variables, not those used in the analyses partitioning the variation (though some are related). Brief labels are included in Table 5. The population variables are defined as follows: percentage of the population black, percentage foreign stock, percentages aged under 18 and 65 and over, percentages of employed persons in white-collar (professional, managerial, sales, and clerical) and managerial and professional occupations, percentage persons aged 25 and over who have graduated from high school,

TABLE 4. REGRESSION ANALYSIS: DETERMINANTS OF DENSITY

	16 Regions as Units	253 Tracts as Units
Log Density as Dependent Variable		
Simple Regression		
R^2	0.823**	0.552**
Distance coefficient	-0.168	-0.159
Constant	8.31	8.94
Multiple Regression		
R^2	0.856**	0.569**
Distance coefficient	-0.187	-0.155
Housing-age coefficient	0.0173	0.00775
Constant	7.99	8.55
Distance beta	-1.01	-0.720
Housing-age beta	0.208	0.132
Percent Single-family as Dependent Variable		
Simple Regression		
R^2	0.465*	0.279**
Distance coefficient	0.909	1.39
Constant	70.7	62.4
Multiple Regression		
R^2	0.631*	0.394**
Distance coefficient	1.20	1.24
Housing-age coefficient	-0.277	-0.248
Constant	75.8	74.8
Distance beta	0.902	0.469
Housing-age beta	-0.463	-0.344

*$p < 0.01$ **$p < 0.001$

and the percentage of families with incomes below the poverty level. Housing characteristics included in the analyses are: percentage of units vacant, single-family, lacking some or all plumbing facilities, built before 1940 and after 1960, owner-occupied, and with more than one person per room

The results of the principal components analyses are shown in Table 5. To show the basic structure with greater clarity, only factor loadings greater than 0.50 are given. Using both regions and tracts, three factors emerged: Factor one is primarily associated with socioeconomic status, with occupation, education, poverty, plumbing facilities, housing age, and crowding all having factor loadings in appropriate directions. Interestingly, the proportion foreign stock is positively related to socioeconomic status. Factor two might be labeled race and resources, with a high percentage black being associated with poverty and with vacant, multi-family, and renter-occupied housing. Factor three is unambiguously associated with age — both of the population and housing. The primary question here relates not to the particular factors emerging from the analyses of this particular set of variables for Indianapolis, but to the comparison of the results for the two different levels of aggregation. The similarities are overwhelming — the particular loadings greater than 0.50 shown for each factor are the same for the analyses using the 16 regions and the 253 tracts as units of analysis. The general magnitudes of the loadings are similar, with 14 of the 18 loadings shown varying by no more than 0.15. The structure of interrelationships within this set of 15 variables, as shown by the principal components analysis, is completely captured at the rather aggregate levels of the 16 regions. There is no essential difference from the results obtained with the presumably more detailed analysis involving the 253 census tracts.

TABLE 5. PRINCIPLE COMPONENTS ANALYSES: FACTOR LOADINGS*

Variable	Regions as Units			Tracts as Units		
	Factor One	Factor Two	Factor Three	Factor One	Factor Two	Factor Three
Black	—	0.88	—	—	0.62	—
Foreign	0.90	—	—	0.71	—	—
Under 18	—	—	0.69	—	—	0.80
65 and Over	—	—	-0.97	—	—	-0.90
White Collar	0.95	—	—	0.95	—	—
Manager & Prof.	0.96	—	—	0.90	—	—
High School	0.90	—	—	0.86	—	—
Poverty	-0.66	0.52	—	-.58	0.66	—
Vacant	—	0.87	—	—	0.83	—
Single-family	—	-0.97	—	—	-0.86	—
No Plumbing	-0.84	—	—	-0.56	—	—
Old Housing	-0.56	—	-0.78	-0.61	—	-0.61
New Housing	0.55	—	0.77	0.58	—	0.62
Owner-Occupied	—	-0.91	—	—	-0.87	—
Crowded	-0.90	—	—	-0.76	—	—

*Only loadings greater than 0.50 are shown.

Conclusion

The results of this analysis clearly point away from census tracts as being a level especially strongly associated with the variation of key population and housing characteristics in a metropolitan area. On the one hand, the majority of the total variation of education, income, value, and rent across the individual observational units came at the lowest level, the level of those units. That is, more of the variation was within the tracts, rather than between them. Moving in the other direction, nearly one-half or more of the total variation across tracts was captured by a set of only 16 large regions which were aggregations of tracts. Even a very coarse subdivision of the metropolitan area into three large areas retained a surprising proportion of the total tract-level variation in these characteristics.

Within the context of these general results, a number of interesting details arose. The proportion of the total variation that came within tracts was far greater for the population than the housing characteristics. This suggests a fundamental difference in the manner in which population and housing are distributed and differentiated across the metropolitan area. In addition, different parts of the metropolitan area exhibited clear differences in the distribution of the variation across the different levels. Both the center and the periphery showed much higher variation at the level of the individual, while more of the variation in the intermediate ring came at the higher levels. These findings point to interesting possibilities for further research.

The comparison of the analyses of the determinants of density and the factorial ecologies at the levels of both the census tracts and the 16 larger regions showed very few differences. This suggests that the basic processes operating to differentiate and structure the metropolitan area may be operating at a very large scale, far above the level of the individual census tract. In particular, the ecological differentiation implied by the analyses associated with factorial ecology may not be a neighborhood-level phenomenon at all.

The aim of this paper has been to pose questions concerning the levels of variation of fundamental characteristics in a metropolitan area. The research certainly raises more issues than it answers, but it does suggest that there is far greater complexity in the structuring of urban areas, at a variety of geographical scales, than has thus far been examined.

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