THE USE OF ALTERNATIVE MEASURES OF ACCESSIBILITY IN ESTIMATING SPATIAL RELATIONSHIPS*

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Introduction

Examination of the literature discussing the effects of accessibility on the spatial distribution of economic phenomenon (in particular property values and rents, and population densities) reveals that there is no clear consensus among investigators as to what observed variable adequately or appropriately measures the concept of accessibility or spatial proximity. The accessibility measure suggested by theoretical constructs, whether a point or aggregate notion, is often neglected and replaced in empirical analysis by a surrogate measure that is easily derived from available data.

For example in a simple von Thunen type residential land use model, relationships such as rent and density gradients have distance from the core (i.e., the central business district or CBD) as the primary explanatory variable. Transportation costs act as centripetal forces in spatial decisions since they are positively related to distance while decreasing land values or rents, which are inversely related to distance, act as centrifugal forces. It is generally acknowledged, however, that distance is, when more realistic models are developed, simply a surrogate measure for a more general accessibility concept. Nonetheless, such general acknowledgments in principal are not always adhered to in practice as is evidenced by the somewhat cavalier manner in which operational measures of accessibility are applied or interchanged in applied spatial analysis. The implicit justification for the interchange among accessibility variables appears to be the existence of a high correlation between distance and these alternative measures. That is, given the strong correlation among many accessibility measures it is presumed that it makes little if any difference which of the measures is employed in the statistical verification of theoretically derived hypotheses.

This paper summarizes the results from some simple comparisons when alternative measures of accessibility are used to statistically describe characteristics of house value, rent, and density gradients. It is argued that high correlations among alternative operational measures of accessibility are not sufficient to justify their interchange in commonly applied estimation

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procedures. Gradient estimates are provided for the Los Angeles metropolitan area to illustrate the types of problems that can arise and the different qualitative conclusions that can be obtained when alternative measures are employed. It is hoped that this exercise offers some insight into what factors are important in choosing among operational measures of accessibility. Admittedly, rather adventuresome comparisons are made in later sections. And obviously, the illustrations and conclusions apply only to the particular sample chosen. As such, it may be most appropriate if this discussion is perceived as an extended caveate about the use of accessibility metrics.

A brief outline on the derivation of density and rent gradients from simple land use models is presented in the next section. The general concept of accessibility and the conditions which allow for the statistical substitution of alternative measures are then discussed. Statistical estimates of rent, value and density gradients follow illustrating the empirical differences that can arise when alternative measures of accessibility are used.

The data for the statistical analysis are drawn from a regional travel time study conducted in the greater Los Angeles metropolitan area. The sample data consist of observations on distance and travel time for trips to and from the core communities. The observation periods include the A.M. peak period, the P.M. peak period, and the nonpeak period. Measures of point and aggregate accessibility are considered.

A Simple Land Use Model

A suitable theoretical base for this discussion on the use of alternative accessibility measures is provided by the simple location model developed by Muth [18]. Though other theoretical foundations for discussing the empirical implications of alternative accessibility measures are available, notably the literature on the derivation and calibration of spatial distribution models such as gravity models and intervening opportunities models, Wilson [28], Kirby [13, 14] and that on transportation choice behavior, Burns and Golob [5], the perspective from using a simple land use model seems appropriate given the continued interest in density gradient characteristics, White [29] and the increasing use of information available in rent and house value gradients to calculate benefits from public programs, Freeman [9, 10]. The choice among alternative measures of accessibility is especially critical under the latter because of the need to completely specify a rental or value function that distinguishes between pure accessibility effects and certain attractor variables. This discussion should not be interpreted as a criticism of Muth but rather to suggest where elements of his model are applicable to an investigation of our problem. Nor is the discussion meant to imply that Muth's model is alone among simple land use models. Others, notably those of Alonso [1] and Mills [16] exist. Muth's model is chosen from personal preference and used only as representative of many others.

Muth initially assumes that the city is located on a featureless plain with all economic activity occurring in the central business district (CBD). The model is monocentric. It is assumed that $T_k > 0$ and that $Ty > 0$. Transportation costs (T) increase with distance to the CBD (k) and income (Y). Each individual is described by a utility index function $u = u(x, q)$ where x is a composite good with constant price π, and q is housing with a variable rent r(k). It is also assumed that households undertake a fixed number of trips to the CBD.

Each household is confronted with the problem of maximizing u subject to Y and all prices:

(1) $L = u(x, q) + \lambda[Y - \pi X - r(k)q - T(k, y)]$

From this problem are derived the (traditional) first order conditions

(2) a. $\mu_x - \lambda\pi = 0$

b. $\mu_q - \lambda r(k) = 0$

c. $-\lambda[r'(k)q + T_k(k, y) = 0$

d. $Y - \pi X - r(k)q = 0$

which yield equilibrium values X*, q*, and k* in addition to a value for λ. Most attention has focused on the spatial equilibrium condition summarized by Equation (2-c). Recognizing from (2-a) that λ is greater than zero suggests that the bracketed term in (2-c) must equal zero. This implies that r'(k) is negative if (as does not seem unreasonable) transportation costs increase with distance. Rents decline with distance measured from the CBD. Similarly, the condition on the bracketed term implies that when a household is in spatial equilibrium the savings in housing expenditures from a small increase in distance (a move outward) just equals the additional transportation costs incurred because of the increased travel.

(3) $-r'(k)q = T_k(k, y)$

If you assume, as Muth does, that the equilibrium described by Equation (3) is stable, that the income compensated demand for housing has unitary price elasticity and that marginal transportation costs are constant with respect to distance, it can be shown, Muth [18, pp. 70-92] that the rent function is described by the expression

(4) $r(k) = r_o e^{-r\ k}$

And if you are willing to further assume that the production function for housing is linear homogeneous it can be shown that density at any distance k, D(k) is given by:

(5) $D(k) = D_o e^{-D\ k}$

Thus, the Muth model under a few simplifying assumptions is able to generate both a negative exponential rent gradient and a negative exponential density gradient, two relationships that emphasize the role of accessibility in the household location decision and that receive considerable empirical attention.

Though the Muth model is naive in its assumption of a single center it has heuristic appeal since the rent and density functions are expressed in terms of distance from the core area. This provides an intuitive interpretation to increases in accessibility and the concomitant changes in rents and densities. Variations in k can be associated with a particular movement within the geographic area. Increases in k are associated with movement towards the periphery and decreases in k are associated with movement towards the core.

The approach suggested by Muth, however, can be generalized to a multinucleated situation without a respecification of the model by a redefinition of k. Define k not as distance to the core, but as a one dimensional measure (conceptual to this point) of accessibility to all relevant economic centers. Such a definition incorporates the notion or the possibility that there exist multiple points of economic opportunity which affect choice. There are costs, however, attached to this redefinition. Along with a possible increase in the correspondence between theory and reality is a loss in our ability to attach an intuitive a priori interpretation to the accessibility variable. Though variations in k are representative of spatial movement and thus changes in accessibility, they can no longer be associated with a specific directional geographic movement. That is, increases in k do not necessarily imply movements towards the periphery or hinterland.

Nonetheless, the redefinition of the accessibility variable does not violate the concepts of a transportation cost function, rent function or density function. The latter two are defined with reference to that point associated with the highest degree of accessibility instead of the CBD, while the transportation cost function reflects the generalized cost of travel. Accordingly, any relationship between these concepts and radial distance may be random or spurious; direct or indirect.

Of greater consequence, the redefinition of k introduces a kind of identification problem. In general form (i.e., without explicit functional relationships) the location models for a monocentric metropolitan area and a multinucleated metropolitan area are indistinguishable, differing only in the definition of the accessibility variable. Regardless of spatial structure, households move until they balance at the margin increases in transportation costs against savings in housing expenditures. Thus, the conceptual framework provides no guidance in application for choosing among possible accessibility measures. This raises the empirical problem of how one specifies the accessibility variable. That is, there exists a major problem in defining an operational measure of accessibility to be used in the estimation of rent and density gradients, and auxiliary relationships tied to these such as benefit functions. Obviously no a priori answer is available. The query by nature is empirical and the answer is probably unique to the geographic area studied.

The Concept of Accessibility[1]

Accessibility measures the spatial proximity between two or more points and reflects the ease with which economic agents are able to spatially interact. Accessibility is related to overcoming the cost (both time and money) of movement between different locations, Burns and Golob [5]. For the purpose of the subsequent analysis accessibility is viewed as an intrinsic characteristic of a location with respect to overcoming or minimizing the costs of what has been commonly referred to as "spatial friction" or the inertia introduced by space and time. It is an exogenously determined attribute associated with points on a geographic plane.

Within the literature discussing operational measures it is possible to discern two concepts of accessibility: (1) point accessibility, and (2) aggregate accessibility, Ingram [12], Dalvi and Martin [8]. Point accessibility alludes to the spatial proximity to a specified (dominant) center of economic activity. This concept is typically associated with the monocentric models of site choice and land use, and traditional house value and density studies. Aggregate accessibility refers to the spatial proximity of a location relative to more than one location of economic activity implying that a variety of economic opportunities exist, and no one opportunity or location dominates the site choice problem. Some opportunities, however, may be relatively more important than others in the spatial calculus conducted by individuals. Aggregate accessibility may be associated with multinucleated models of land use and many transportation related questions such as trip distributions and and mode choice. Obviously, point accessibility is a special case of aggregate accessibility.

In Table 1 are displayed specifications for various measures of point and aggregate accessibility that have been discussed or suggested as appropriate in the literature. Given our focus the list is not comprehensive but rather emphasizes those measures used in conjunction with simple land use models. d_i is the distance between the ith location and the dominant core or CBD, d_{ij} is the distance between the ith and jth location, N is the total number of locations, V is an unknown constant which is sometimes set equal to the value of the radius of the smallest circle circumscribing all locations, E_j is the employment in the jth location and E_T is total employment, derived by summing over E_j for all j.

Perusal of the table reveals that the measures are defined in terms of distance. This is somewhat misleading since travel time or travel cost per unit distance may be substituted for distance to provide additional operational measures of accessibility. Also, where the general expression $a + b(d_{ij})^k$ is adopted, for most empirical analysis it is assumed that a = 0, b = 1, and k = 2 simplifying the expression to d_{ij}^2. Lastly, the expressions in the bottom

[1]For a theoretical discussion of accessibility measures using an axiomatic approach see Weibull [27]. Operational measures are discussed in Ingram [12]. Different metrics are discussed in Perreur and Thisse [20].

TABLE 1: Alternative Accessibility Measures

Point Accessibility Measures: a_i

1. Distance to CBD: $a_i = d_i$

2. Travel Cost to CBD: $a_i = 100 \cdot d_i^{-k}$ (reciprocal)

   $a_i = 100e^{-di}$ (exponent)

3. Logarithm of Distance: $a_i = Ln(d_i)$

Aggregate Accessibility Measures: A_i $(A_i = \Sigma a_i)$

1. Distance to Multiple Centers: $A_i = \sum_{j=1}^{k} d_{ij}$

2. Mean Opportunity Distance: $A_i = (\Sigma dij/N)/V$

3. Aggregate Accessibility: $A_i = \sum_{j=1}^{k} \{100 \exp[-d_{ij}^{k} V^{-1}]\}$

Employment Potential Measures

1. Multiple Center Index: $\bar{A}_i = \sum_j (E_j/a+bD_{ij}^{k})$

2. Relative Multiple Center Index: $\bar{A}_i = \sum_j [E_j/E_T)/a+bD_{ij}^{k}]$

3. Relative Aggregate Measure: $\bar{A}_i = \sum_j 100 \exp E_j/E_T - d_{ij}^{k} V^{-1}$

group are aggregate measures of accessibility that attempt to incorporate the premise that some spatial opportunities are more important than others in location decisions. Thus distances to the various opportunities are weighted by the importance of each location as measured by absolute or relative employment. Similar to the use of distance as a measure of spatial friction, other measures of economic importance may be substituted for employment such as retail sales or population. The measure of importance depends on the nature of the underlying hypotheses and model motivating the statistical analysis. For example, if the location decision is based on the availability of final commodities, retail sales may be used rather than employment, the use of which is premised on labor market considerations. It should be evident from this brief discussion that in specifying an operational measure of accessibility questions arise with respect to both the form (e.g., mono-nucleated or multinucleated) and the base (e.g., distance, travel time, or travel cost per unit distance).

To provide additional insight into where the emphasis has been in the empirical research the reader is referred to Table 2. In this table are displayed measures of accessibility that have been employed in a sample of empirical studies investigating rent, house value, and population density. The studies are either intracity or intercity. Though differences exist in the measures employed, it is evident that measures based on distance dominate and that of these simple relations of distance are in the majority. Emphasis will be placed on these types of measures in subsequent analysis contrasting estimated rent, value and density gradients.[2]

The Econometric Problem

Admitting that there exist a variety of measures for accessibility, and that there are different concepts of accessibility, introduces the potential that an incorrect measure will be utilized in the statistical estimation of spatial phenomenon. This raises the question of whether this type of error leads to statistical or interpretative problems that make conclusions based on the errors in variable tenuous or invalid. Accordingly it seems appropriate to briefly discuss the errors in variable problem in the general context of the standard linear regression model and to view an error in variable as an error in specification.

[2] One reviewer has suggested that there is little point in defining a pure, distance-oriented, measure of accessibility. It can be argued, however, that the inclusion of attractor variables in measuring accessibility may hinder the interpretation of statistical results when studying rent or density gradients because observed differences in such a measure would reflect spatial variation in both components. See Ingram [12] and Dalvi and Martin [8]. In gravity models there is also some ambiguity concerning the estimation and interpretation of normalizing factors, Rose [25].

TABLE 2: Selected Studies

Notation:

d_{ij}: Distance between ith and jth location

E_j : Employment in jth location

t_{ij}: travel time between ith and jth location

TC_{ij}: Travel Cost between ith and jth location

	Accessibility Variable: Type of Study	
	Intra-city Study	Inter-city Study
Anderson & Crocker [2]	LnD_{ij}	
Brigham [3]		D_{ij} $\Sigma(E_j/a+bd^k_{ij})$
Brodsky [4]	d_{ij} $L_n(d_{ij})$	
Burns & Mittelbach [6]	D_{ij}	
Clark [7]	$\Sigma E_j/T_{ij}$	
Harris, et. al [11]	TC_i	
McDougall [15]		$\Sigma 100 \exp [-d^k_{ij}V^{-1}]$
Mohring [17]	t_i D_{ij}	
Muth [18]		$Ln(d_{ij})$
Oates [19]		$Ln(d_{ij})$
Pollakowski [21]		$\Sigma E_j/a+bd^k_{ij}$
Ranich [22]	D_{ij}	
Ridker & Henning [24]		t_i
Seyfried [26]	D_{ij}; LnD_{ij}	
Yeates [30]	$Ln(d_{ij})$	

Assume that the true regression model is given by Equation (6)

(6) $Y = X\beta + U$

where

$$Y = \begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix} \quad X = \begin{bmatrix} X_{11} \cdot & \cdot & \cdot & \cdot X_{1j} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ X_{n1} \cdot & \cdot & \cdot & \cdot X_{nj} \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_j \end{bmatrix} \quad U = \begin{bmatrix} U_1 \\ \cdot \\ \cdot \\ \cdot \\ U_n \end{bmatrix}$$

Instead of having X as the observation matrix however, suppose that we use X* which differs from X in that the last (jth) explanatory variable is measured with error. Thus the jth column in X* differs from that in X, while all other columns are identical. In the context of this problem the jth variable in X* corresponds to one of the discussed measures of accessibility, but it is an incorrect measure.

With the error in the observation matrix the regression model is given by:

(7) $Y = X^*\beta + U$

and the least squares estimator of the unknown coefficient vector, β^* is

(8) $\beta^* = (X^{*\prime}X^*)\ X^{*\prime}Y$

Extracting the ith element from β^* and taking the expected value we have

$$E(\beta^*_i) = \beta_i + r_{ik}\beta_k$$

(9) $i \neq k$

$$E(\beta^*_k) = r_{kk}\beta_k$$

$i = k$

where β_i, $i = 1, k$, are the coefficients in the true model and r_{ik}, $i = 1, k$, is the (i, k)th element from the matrix $R = (X^{*\prime}X^*)^{-1}X^{*\prime}X$.

Consider briefly the matrix R, comparing its definition with that of the least squares estimator given by Equation (8). It is obvious that R is a matrix of coefficients from the auxiliary regressions of X on X*, and that R is equivalent to the unit matrix except for the last (kth) column. Thus, r_{ik} in Equation (9) is the regression coefficient from the auxiliary regression:

(10) $X_k = r_{1k}X_1 + r_{2k}X_2 + \cdot\cdot\cdot\ r_{kk}X^*_k$

Using this we may infer from Equation (9) that β_i^*, $i \neq k$ will be an unbiased estimate of β_i if the true explanatory variables are independent of each other (orthogonal); that is, if r_{ik}, is equal to zero. The coefficient for the kth explanatory variable (the variable measured with error) is always biased except when r_{kk} is equal to one. This implies that for a linear specification of the underlying relationship (Equation 6) the surrogate measure of accessibility (X_i^*) differs from the "true" measure of accessibility (X_i) by only a constant, while if a logarithmic or semi-logarithmic form is estimated it implies the surrogate measure is proportional to the true measure. Thus, the condition of a strong correlation between the true measure and alternative measures is not sufficient to justify their interchange in a regression framework.

Statistical Comparisons

A number of stochastic models were estimated using ordinary least squares to compare the effect of using alternative accessibility measures to empirically describe density gradients, rent gradients, and house value gradients. Representative results are summarized in Tables 1 through 5.[3] Two operational models provide the basis for the estimated equations: (1) a mononucleated model presuming that the city of Los Angeles is the dominant core; and (2) a multinucleated model assuming Los Angeles, Long Beach, Riverside, and San Bernardino are each centers of economic activity. Both distance and travel times are used as base indicators of spatial friction. In the multinucleated situations distance refers to the sum of the driving distances to each of the centers of activity, while travel time reflects the sum of the travel times to each of the cities. The last equation in each group uses the average total driving times for both peak periods and the off-peak period. Both linear and semi-logarithmic specifications are estimated. Comparisons are made on the basis of the coefficient of determination and the precision of the slope coefficient estimates. Since each of the specifications possess the same number of explanatory variables adjustments to R-squares are unnecessary for comparative purposes. Also, because the purpose of this discussion is to compare the outcomes when alternative operational measures of accessibility are employed, and because the specifications are admittedly naive, little attempt is made to interpret specific results. Rather emphasis is on cross-model explanatory power and qualitative outcomes to determine the appropriate form and base.

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[3]The distance and travel time data were drawn from a preliminary report on regional travel times for the Los Angeles metropolitan area conducted by the Southern California Automobile Association. Density, rent and value data are drawn from published census information. The results displayed are representative of a larger set of outcomes, some of which used employment to weight observations. The results were insensitive to this weighting scheme.

Density Gradient

Consider first the estimated parameters describing the density gradient within the Los Angeles metropolitan area summarized in Table 3. Examination of the coefficient estimates indicates that for all cases the estimated density gradient is negatively sloped, supporting the general hypothesis derived from the simple location model. A comparison of the coefficients of determination for the single center model with those for the multiple center model suggests that the monocentric model explains relatively more of the observed variation in population density or its logarithm than does the multinuclear model. This would seem to indicate that regardless of the apparent variety of opportunities within the Los Angeles metropolitan area and the extensive personal transportation system, the Los Angeles region empirically corresponds to an area dominated by a single center vis-a-vis the distribution of its population.

Examination of the results from the mononucleated model further suggests that in terms of explanatory power accessibility measured by the morning peak period driving time is marginally most powerful. A possible explanation (but one offered only as suggestive of an area of further study) is that for the commuting trip which is primarily job oriented the effective constraint is determined by when the job begins each day. Thus individuals are not so concerned with the cost of an off-peak period trip or the cost of the journey home because there is more leeway or discretion in undertaking home oriented or consumption oriented activities.

Given the relatively high R-squared values for the single center model and the relatively high degree of precision displayed by the coefficient estimates it seems appropriate to conclude that even though the Muth location model is naive in the number of factors considered as important in the location decision, it displays a high degree of explanatory power. Similarly, because the coefficient estimates for the semi-logarithmic and linear forms do not differ substantially within each specification class but do differ substantially between classes, it appears that the choice of the form of the accessibility measure is more critical than the choice of its base (time, distance, etc.).

Rent Gradient

In relation to our results from the estimation of the density gradient there are conflicting results vis-a-vis estimates of the rent gradient summarized in Table 4. A comparison of the coefficients of determination indicates that contrary to the density gradient analysis the specifications based on a multinucleated concept of accessibility have greater explanatory power than those based on the notion of a single center urbanized area. For the linear monocentric model the R-squared values range from .13 to .42 while for the linear multinuclear models the values range from .54 to .63. Analogous comparisons exist for the semi-logarithmic specifications. Also in conflict with our estimated density gradients is the complete absence of a negatively sloped rent gradient. Referring to the multiple center specifications it is seen that all of the slope coefficient estimates are

TABLE 3: Density Gradient (Standard Errors)

Mononucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
7777.2	-86.80 (22.71)					.715
8514.9		-85.39 (19.91)				.754
8123.5			-58.178 (19.29)			.628
8162.2				-87.96 (24.3)		.695
8354.3					-84.59 (21.64)	.706

Mononucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
9.049	-.022 (.006)					.698
9.232		-.021 (.005)				.734
9.123			-.014 (.005)			.603
9.1086				-.021 (.007)		.648
9.123					-.020 (.006)	.676

Multinucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
76.054	-.145 (.134)					.279
63.996		-.071 (.098)				.190
63.476			-.063 (.087)			.190
64.614				-.079 (.122)		.170
63.721					-.073 (.111)	.173

Multinucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
4.8375	-.006 (.003)					.437
4.4122		-.003 (.003)				.329
4.4201			-.003 (.002)			.345
4.4062				-.003 (.003)		.262
4.4317					-.003 (.003)	.314

TABLE 4: Rent Gradient (Standard Errors)

Mononucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
134.37	0.171 (9.262)					.125
126.01		0.331 (0.329)				.260
115.96			-.434 (.253)			.416
129.43				0.286 (0.372)		.201
124.32					.378 (0.328)	.294

Mononucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
4.899	-.0007 (.003)					.076
4.848		-.002 (.002)				.021
4.7705			.003 (.002)			.382
4.866				.002 (.003)		.155
4.8301					.002 (.002)	.245

Multinucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
71.400	1.02 (.338)					.628
126.84		.631 (.262)				.541
125.37			.585 (.227)			.561
90.404				.852 (.313)		.588
103.77					.769 (.285)	.585

Multinucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average Time	R-Squared
4.8476	.004 (.001)					.646
5.0192		.002 (.009)				.587
4.8951			.003 (.001)			.644
4.9000				.003 (.001)		.620
4.9414					.003 (.001)	.625

positive and significant. This seemingly perverse outcome may be a result of not having standardized for differences in housing services since implicit in the estimated stochastic models is the assumption that housing services are independent of location. This may not be the case if larger and newer homes are constructed on the less accessible sites; that is, if development takes place in peripheral (in terms of accessibility and not geographic location) areas. Alternatively it is possible, as suggested by others, that a positive rent gradient may be observed if there exists a positive externality component in total rents.[4] The effect of positive externalities may outweigh the negative effects on site choice of reduced accessibility resulting in a positively sloped rent gradient.

The R-squared values for the various stochastic specifications utilizing a multinucleated concept of accessibility also indicate, contrary to our findings vis-a-vis density gradients, that the specifications using distance as an indicator of spatial friction display marginally greater explanatory power. As with our previous results, however, the coefficient estimates and summary statistics suggest that the crucial decision is related to the form of the accessibility measure rather than the base.

The House Value Gradient

Examination of the coefficient estimates for the house value gradient summarized in Table 5 reveals generally poor results in terms of the precision of the coefficient estimates and the overall explanatory power of the estimated relations. As with the specification of the rent gradient, however, there is the assumption in the house value relation that the level of housing services is independent of location. Thus, the relatively poor statistical results may not be indicative of the inappropriateness of the theoretical framework but rather that the stochastic models do not standardize for differences in housing services. Thus, the positive coefficients in the rent gradient and the insignificant coefficient estimates for the house value relation may reflect the presence of compensating changes in the level of housing services.

In an attempt to correct for this possibility the rent and house value gradients were reestimated using rent per room and value per room as the dependent variable. The premise behind this transformation is that housing rents or values depend on the level of housing services provided by a housing unit and that housing services are directly related to the size of the structure. The size of the structure is measured approximately by the number of rooms. The results from the reestimation are presented in Tables 6 and 7.

Rent/Room and Value/Room Gradients

Examination of the summary statistics in Table 6 indicates that standard-

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[4]Richardson [23] discusses the conditions within a traditional location model incorporating externalities which lead to positive rent gradient when there is a declining density profile.

TABLE 5: House Value Gradient (Standard Errors)

Mononucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
27859	-37.69 (101.1)					.099
26524		1.941 (94.75)				.006
24438			39.02 (76.81)			.135
26709				-2.756 (105.8)		.007
26194					9.602 (95.49)	.027

Mononucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
10.255	-.003 (.004)					.236
10.203		-.001 (.003)				.099
10.135			-.0003 (.003)			.029
10.223				-.002 (.004)		.139
10.200					-.001 (.003)	.092

Multinucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
6596.2	-6.49 (14.91)					.116
4637.3		3.26 (10.73)				.081
6229.1			-3.63 (9.48)			.102
4822.0				2.60 (13.36)		.052
5242.4					.543 (12.14)	.012

Multinucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
7.3049	.004 (.012)					.079
6.8765		.005 (.009)				.156
7.6995			.001 (.008)			.042
6.9546				.005 (.011)		.131
7.0875					.044 (.010)	.117

TABLE 6: Rent/Room Gradient (Standard Errors)

Mononucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Time	Average Time	R-Squared
38.680	-.094 (.064)					.365
38.805		-.077 (.061)				.319
36.918			-.025 (.052)			.125
38.289				-.074 (.069)		.274
27.005					.020 (.010)	.453

Mononucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Time	Average Time	R-Squared
3.6585	-.003 (.002)					.407
3.6596		-.002 (.002)				.346
3.6043			-.0008 (.002)			.144
3.6480				-.002 (.002)		.312
3.2969					.003 (.001)	.463

Multinucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Time	Average Time	R-Squared
27.27	.043 (.027)					.393
28.666		.031 (.019)				.394
28.194			.031 (.017)			.436
26.206				.045 (.023)		.460
26.358					.040 (.023)	.457

Multicentered Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Time	Average Time	R-Squared
3.3211	.001 (.0008)					.402
3.3609		.0009 (.0006)				.403
3.3050			.0009 (.0005)			.447
3.2930				.001 (.0007)		.466
3.3147					.001 (.0006)	.461

TABLE 7: House Value/Room Gradient (Standard Errors)

Mononucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
4918.7	-9.238 (15.92)					.153
4771.0		-3.758 (15.0)				.067
4918.7			-6.185 (14.19)			.116
4734.1				-3.296 (16.76)		.053
2328.7					5.25 (2.31)	.519

Mononucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
8.5175	-.003 (.003)					.284
8.4827		-.002 (.003)				.160
8.4197			-.003 (.002)			.280
8.4871				-.003 (.003)		.177
7.9183					.001 (.0004)	.564

Multinucleated Model: Linear

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
1953.6	13.93 (5.80)					.540
2678.9		8.76 (4.35)				.474
2673.8			8.06 (3.81)			.492
2087.1				12.24 (5.20)		.533
2328.7					10.82 (4.76)	.519

Multinucleated Model: Semi-logarithmic

Constant	Distance	A.M. Peak (Into)	P.M. Peak (Out of)	Off Peak	Average	R-Squared
7.8599	.003 (.001)					.565
7.9854		.002 (.008)				.524
7.9913			.002 (.0007)			.536
7.8743				.003 (.0009)		.571
7.9183					.002 (.001)	.564

izing for size of structure has increased the precision of the coefficient estimates and that relatively more of the observed variation in rents per room is explained by the regression models using an accessibility measure based on the notion of multiple centers than those models based on the notion of a single center urban area. Single center models, however, using distance, A.M. peak travel time and average travel time do compare favorably in terms of the proportion of the variation explained. Perhaps more revealing is that for those accessibility models with comparable explanatory power there are conflicting results as to whether the rent gradient is positive or negative or constant. In the single center models using distance and A.M. peak travel time the rent gradient estimates suggest a negatively sloped relationship while for the same specification in the multiple center context the rent gradient estimates suggest a positive slope.

The statistical results summarized above are illustrative of the types of problems that can arise when there is a question (conceptually or empirically) about the appropriate concept of accessibility. In the case assuming a single centered urban area an investigator is led to the conclusion that the rent gradient is non-positive. Under a multiple center specification of the urban area the conclusions are opposite; the estimated rent gradient is non-negative. And, a comparison of these results with those for value per room summarized in Table 7 reveals analogous ambiguities. In the single center model the value gradients are non-positive, whereas in the multiple center model the value gradients are estimated to be non-negative.

Conclusion

It has been argued that in the empirical and theoretical literature discussing spatial economic phenomenon there is some confusion or ambiguity concerning the appropriate concept and measure of accessibility. As such, the conclusions derived from an investigation of spatial phenomenon may differ according to the concept and measure of accessibility employed in the analysis. This follows from an errors-in-variables problem. To demonstrate the nature and possible consequences of this problem a variety of density, rent and value gradients were estimated using different measures of spatial friction (i.e., distance of travel time) and different assumptions about urban structure (i.e., mononucleated urban area vs. a multinucleated urban area). The empirical results for this particular sample suggest:

a. The density gradient is unambiguously negative.

b. The rent gradient is either positive or negatively sloped depending on the measure of accessibility.

c. The house value gradient is either positive or negatively sloped depending on the measure of accessibility.

d. Density follows a distribution consistent with a monocentric urban area.

e. House rents and values follow distributions consistent with a multicentric urban area.

And, in general, we may conclude that:

f. The most appropriate functional specification appears to be the semi-logarithmic specification.

g. The critical question appears to be concerned with the form of the accessibility measure (i.e., a single center measure or a multiple center measure) rather than the base of the measure (i.e., distance or travel time).

These conclusions suggest further that for urban policy questions involving accessibility considerations such as new residential and working sites, service facility locations and transportation links, it is of primary importance to determine which is the most appropriate concept of accessibility. That is, does the urban area empirically approximate an area dominated by a single center or does it resemble an urban area characterized by multiple centers of activity. As our results from the Los Angeles metropolitan area indicate, the answer is not obvious. For here is an area that many have characterized as a multinucleated urban area but, as evidenced by the density gradient estimates, may be reflective of a dominant core structure.

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