MARKET VERSUS GOVERNMENTAL INCENTIVES FOR INSULATION USE IN STRUCTURES

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In the residential and commercial sectors, approximately 65-70 percent of total BTU consumption is for space heating. In the industrial sector, space heating accounts for 35 percent of the energy consumed [13]. Thus, if energy conservation is to occur, one major factor deserving the attention of individuals who buy energy and governmental officials who propose policies for energy conservation is the reduction in heat loss through the envelopes of structures. (The envelope consists of walls, doors, windows, floor and ceiling.) The heat loss through the envelope depends upon the difference in exterior and interior temperatures and the thermal conductivity of the envelope. The outside temperature is fixed by nature. The inside temperature is fixed by the occupants, and the thermal conductivity is established by design. This paper considers one aspect of the design of structures: the economic feasiblity of wall and ceiling insulation use and the merits of governmental incentives for increasing insulation use.

The Economics of Insulation Use in New Structures

Wall and ceiling insulation reduces the thermal conductivity of walls and thus will reduce cold air infiltration and the heat loss through walls and ceilings. Accordingly, the use of insulation decreases the amount and cost of energy required for space heating. However, the use of insulation entails two types of costs: the cost of the insulation and the cost of installation. Thus, from the viewpoint of an owner, it is desired that the total costs of insulation use are minimized where

(1) total cost = cost of insulation + annual cost of heating + cost of installation

In Equation 1, the costs of heating will be a given amount if there is no insulation. With the purchase of some insulation, the annual costs of heating are reduced, yet the costs of insulation rise. This paper considers the thickness of insulation where the total costs on an annual basis are minimized.

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The costs of the insulation and installation are converted to annual costs by multiplying the present value of costs by a capital recovery factor (C_R):

$$(2) \quad C_R = (i^*(1 + i^*)^n)/((1 + i^*)^n - 1)$$

where

i^* = the opportunity cost of funds

n = the life of the asset

Besides the capital recovery factor described above there are several other environmental and economic factors which must be considered in the determination of the total costs to the owner. The environmental factors are the thermal conductivity of the envelope and insulation and the amount of time in a year when heating is required. The economic factors are the cost of the insulation, the cost of and inflation rate on energy, the cost of installation, taxes, depreciation and alternative financing arrangements. These factors lead to the following annual cost equation:

$$(3) \quad TC = a\chi k_B k_D C_R + 24DC_B k_R(1-t)/(\chi R_i + R_w)\ LC_R$$

where

TC = total annual costs

a = the cost of one square foot of insulation one inch thick

χ = the thickness of the insulation

k_B = factor to accommodate financing through borrowed funds (see Appendix)

k_D = the depreciation factor (see Appendix)

C_R = the capital recovery factor

D = the number of degree days (A degree day is defined as a 24-hour period in which the average temperature is one degree below 65°F)

C_B = the cost of one BTU in cents (varies by type of energy source)

k_R = the inflation factor on the energy source (see Appendix)

t = the tax rate

R_i = the thermal resistance of the insulation used in the envelope

R_w = the thermal resistance of the wall without insulation

L = fixed installation cost

Equation 3 shows the annual costs of the insulation plus the annual costs of the energy for space heating plus the fixed, installation costs on an annual basis. By differentiating Equation 3 with respect to χ, setting the differential equal to zero and solving for χ, the amount of insulation which minimizes TC can be determined:

$$(4) \quad \chi = (1/R_i) \quad (\sqrt{24DC_Bk_RR_i(1 - t)/(aC_Rk_DR_B)} - R_w)$$

We shall now discuss how the variables in Equation 4 affect the optimum amount of insulation to be used.

Environmental Effects. The environmental factors are R_i, R_w and D. Consider R_i first. R_i is the thermal resistance of an inch of insulation. As R_i increases (depending upon the type of insulation), the optimum thickness of the insulation is reduced as expected. Similarly, as the thermal resistance of the wall envelope (R_w) increases, the optimum thickness of insulation is reduced.

Another environmental variable is D, the number of degree days. As D increases, the optimum thickness of insulation increases due to the saving in heat from extra insulation. The D for various locations is available in published manuals and ranges up to 10,000 for northern parts of the United States mainland, except Alaska [10]. In moderate climates of the midwest, the degree days range from 6,000 to 8,000. For eastern mid-atlantic areas the degree days range from 3,000 to 6,000. Also, since D is based upon one hour per day, it is multiplied by 24 to give the total temperature difference for a typical season. In the simulations below, $R_i = 5.4$, $R_w = 4$ and D = 10,000 which is appropriate for colder climates in the northern parts of the United States. In a later section on governmental incentives, the variations in insulation thickness by degree days (and thus regions) are analyzed.

Cost and Inflation Factors. The cost of insulation, a, is a constant which is assumed to be 25¢ in the study. This value is for rigid fiberglass and urethane and is based upon cost data published by R. S. Means Co. [4].[1] Generally, the range of possible values for "a" combined with the R_i do not cause significant variations in χ, the optimum thickness of insulation. Rather, the variations in χ are most sensitive to the range of values for k_R, the inflation factor, for C_B, the cost of fuel and for i*, the opportunity cost of funds. (The k_R factor which converts present costs to current costs with inflation is presented in the Appendix.)

Although the inflation rate on fuel oil and coal was six percent in 1976, it was 50 percent in 1974 and averaged 27 percent for the 1973-75 period [14]. Other energy sources such as gas and electricity have shown more moderate changes for the 1973-76 period. Given these considerations, inflation rates of four percent to 22 percent are assumed on annual energy costs. Inflation has a positive relation to χ, the optimum amount of insulation. That is, as the expected rate of inflation on energy sources increases, the optimum amount of insulation increases.[2]

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[1]Blanket insulation is less costly but has a lower R value.

[2]It should be noted that the Energy Research and Development Administration assumed inflation rates of 10 percent for fuel costs in a recent study [11].

While expected inflation rates can vary, so can the cost per BTU, C_B, of alternative fuel sources. Equation 3 shows that as C_B increases, the optimum thickness of insulation increases as well. Table 1 presents the costs per BTU of stoker coal, #2 fuel oil, propane gas, natural gas and electricity. The range in BTU costs is from 0.0002 cents to 0.0012 cents. Except where noted, the analysis below takes a conservative approach and uses a cost per BTU, C_B, of 0.0004 cents.

i^*, the opportunity costs of funds, is used in the k_R, C_R and k_B factors. While it is not evident in most cases, the i^* variable has the following effect on χ: As i^* increases, the optimum amount of insulation to be used decreases. This is especially evident when i^* is relatively high at rates of 20-25 percent. The reason for the sensitivity of χ to i^* stems from the use of funds in their most profitable use. At relatively high values of i^*, the investor is encouraged to place funds in the high yielding use and away from the less profitable use-insulation.

Optimum Thickness of Insulation

Given an appropriate set of values for the coefficients in Equation 4, one can determine the optimum thickness of insulation. Some optimum values are presented in Tables 2 through 5. Several warnings are in order. The optimum values calculated in the tables are for walls in new structures. The values should not be applied to retrofitting the walls of older structures with insulation. Since windows and doors are not included, the optimum thickness would be somewhat higher because doors and windows have very low thermal conductivity coefficients. (Windows are especially wasteful of energy). Finally, in areas where snowfall is heavy, the ceiling insulation values will be somewhat smaller because of the insulating factor of snow.

In each of the tables which follow, the inflation rate is allowed to vary from four percent to 22 percent. The optimum thickness of the insulation is then determined as other economic variables are assigned values. For example, in Table 2, the optimum amount of insulation, χ in inches, is given as the inflation rate, the life of the insulation and the term of the loan to finance the insulation vary. Also, note that the opportunity cost of funds is 10 percent and the borrowing rate is nine percent. Other constants are listed below Table 2.

A review of the tables gives the following generalizations pertaining to economic variables influencing insulation use:

1. As the expected inflation on energy increases, the optimum thickness of the wall insulation increases. The reason for this result is that as the cost of fuel increases, the savings in fuel costs from more insulation overrides the costs of insulation.

2. As the life of the insulation (or structure), n, and the term of the loan, nl, to finance the purchase of insulation decreases, the optimum amount of wall insulation decreases. This relation-

TABLE 1: Cost Comparisons of Conventional Energy Sources for Residential Space Heating

Energy Source	Unit	Cost Per Unit*	1000 BTU Per Unit+	Source Space Heating Efficiency‡	Cost Per Million BTU	Cost Per BTU
Coal	Ton	$25.00	24,000.0	50%	$ 2.08	0.000208¢
#2 Fuel Oil	Gallon	$ 0.45	138.0	60%	5.43	0.000543¢
Propane	Gallon	$ 0.40	95.0	70%	6.01	0.000601¢
Electricity	kwh	$ 0.04	3.4	98%	12.00	0.001200¢
Natural Gas	Thousand Cubic Feet	$ 2.27	1,000.0	70%	3.24	0.000324¢

* Source: Federal Energy Administration, [12].

+ Source: Arola, [3].

‡ Source: ASHRAE, [2].

TABLE 2. Values of x with i* = 0.05, il = the letter i and the number 1, nl = the letter n and the number 1, r varies and n and nl vary.

r	n=30 nl=20	n=20 nl=15	n=15 nl=10	n=10 nl=5
0.04	3.22	2.62	2.23	1.73
0.10	5.43	3.72	2.91	2.08
0.16	9.63	5.37	3.81	2.48
0.22	17.40	7.79	5.00	2.97

i* = the opportunity cost of funds

il = the borrowing rate

n = life of asset

nl = term of loan

r = the inflation rate

a = the cost of one square foot of insulation 1 inch thick= 25¢

C_B = the cost per BTU = 0.0004¢

ship exists because as the length of the life of the insulation is shortened, the savings in fuel is reduced. Yet the costs of insulation remain at previous levels. However, the reduction in the optimum amount of insulation as the life of the insulation is reduced does not include a salvage value. If the insulation has some market value at the end of its life, then the optimum insulation will be higher.

In line with the above discussion, a recent study shows that households change residences in California after five years [5]. Thus, the lower values for n and nl may be appropriate.

3. A very important variable in the use of wall insulation is the desired profit rate, i*. Note that as i* increases from five percent in Table 2 to 25 percent in Table 4, the optimum thickness of wall insulation decreases. For example, in Table 2 with an inflation rate of 10 percent and n = 30 and nl = 20, the optimum thickness of insulation is 5.43 inches. For comparable values in Table 4 with i* = 25 percent, the optimum thickness is 1.10 inches. The explanation for this relation is that as the profit rate increases, it pays the investor to put funds in the most profitable use and avoid tying up funds in insulation which do not yield as high a profit. However, the five percent and 25 percent values for desired profit rates are relatively low and high values. A moderate profit rate is to be found in Table 3 where i* = 0.10. Depending upon n and nl values and the inflation rate, the optimum amount of insulation ranges from 1.36 inches to 9.23 inches.

In conjunction with this important variable, it should be noted that a discount rate of 10 percent was used by the Rand Corporation in determining the cost effectiveness of various energy conservation measures [7].

4. As the cost of the fuel per BTU increases, the amount of insulation used increases. However, the cost per BTU, it appears, does not have as strong an effect as the expected inflation rate (Table 5).

5. While the tables have been prepared under the assumption that wall insulation thicknesses are continuous, there are limits on the thickness of insulation due to construction methods. In practice, the wall thickness is 3 1/2 inches. However, recently, 2 x 6 framing studs have been used to increase the insulation thickness. Thus, under current construction methods, the limit of insulation thickness is six inches. However, with relatively high inflation rates on energy, insulation of nearly double the amount is optimal. On the other hand, in most cases ceilings can accommodate the thicker insulation.

TABLE 3. Values of x with i* = 0.10, i1 = 0.09, r varies and n and n1 vary.

r	n=30 n1=20	n=20 n1=15	n=15 n1=10	n=10 n1=5
0.04	2.01	1.81	1.63	1.36
0.10	3.16	2.50	2.10	1.63
0.16	5.30	3.52	2.73	1.95
0.22	9.23	5.03	3.55	2.32

TABLE 4. Values of x with i* = 0.25, i1 = 0.09, r varies and n and n1 vary.

r	n=30 n1=20	n=20 n1=15	n=15 n1=10	n=10 n1=5
0.04	0.83	0.82	0.80	0.75
0.10	1.10	1.06	1.00	0.89
0.16	1.53	1.39	1.26	1.06
0.22	2.26	1.86	1.59	1.25

6. The set of tables and the ensuing generalizations applied to insulation used in a business. This means that depreciation was used in computing the optimum insulation. While the owner-occupant of a non-income producing residential structure cannot take depreciation, the generalizations listed apply to the owner-occupant as well.

Governmental Incentives to Alter Insulation Use

In this section of the study a number of governmental incentives are proposed and examined for their effectiveness in changing insulation use and for the costs to the government if enacted. No analysis is made of the incidence of the incentives for different income classes or different industries. Rather, the focus is on the ability of incentives to increase insulation use (and, in turn, energy conservation) and the costs to the state government.

No Incentives. Recently, many state governments as well as the federal government have proposed that incentives be used to increase the amount of insulation in structures where space heating is required. Yet, as the inflation rate on energy increases, so does the optimum thickness of insulation in new structures. With reference to energy conservation, a household or business conserves on energy without any special incentives when prices increase at relatively high rates such as 16 to 22 percent. (Except when $i^* = 0.25$, the lowest insulation thickness is 1.95 inches, Table 3.) In short, no incentives are required to promote insulation use when inflation rates on energy are relatively high. Rather, the appropriate public policy would be to educate the public as to the merits and costs of alternative types of insulation.

There are two exceptions to the above generalization. The first pertains to profitable firms. If i^*, the opportunity cost of funds, is relatively high, then the optimum amount of insulation is relatively low and defeats the goal of energy conservation. For example, in Table 4 with $i^* = 25$ percent, the thickest insulation is 2.26 inches for an assumed 30-year life of the structure. Thus, if effective incentives are to be provided, the one group which should receive them are the successful business firms. This type of policy would raise serious income redistribution questions.

Another exception to the generalization will occur when the occupant and the owner of a structure are not the same. According to 1970 Census of Housing figures, approximately 25 percent of residential units are rented while 75 percent are owner-occupied [9]. Without more information, a similar proportion can be assumed for commercial and industrial property. Clearly, a significant proportion of structures fit the category of rental property. Unfortunately, if renters pay the costs of energy, then owners have no incentive to use insulation. Additionally, unless renters have long term leases (which is frequently the case for industrial and commercial firms), the renters have no incentive to retrofit structures with insulation. Thus, for a major proportion of structures, any incentives would be inapplicable unless the insulation costs were 100 percent covered.

TABLE 5. Values of x with $i*$ = 0.15, il = 0.09, n=30, n1=20, r varies and C_B varies.

C_B / r	C_B=0.0002	C_B=0.0004	C_B=0.0006	C_B=0.0008
0.04	0.78	1.41	1.90	2.30
0.10	1.25	2.04	2.70	3.23
0.16	2.08	3.25	4.14	4.90
0.22	3.58	5.37	6.75	7.91

Before analyzing a solution to the above exceptions (ASHRAE Standards 90-75), an analysis of governmental incentives for insulation use is presented.

Governmental Incentives. Although a number of incentives have been considered by governments in different regions of the United States, the effect of these incentives on the optimum thickness of wall and ceiling insulation is not too great unless the total cost to the government becomes relatively large.[3] To analyze the strength of incentives on insulation use in different regions, refer to Tables 6 and 7. In Table 6, the optimum thickness of insulation is presented where the inflation, r, varies, the degree days applicable for regions of the United States varies and "a", the cost of a square foot of insulation equals 25¢. Clearly, as the degree days decrease from 10,000 (for a city such as Duluth, Minnesota) to 4,000 (for a city such as Amarillo, Texas), the optimum thickness of insulation decreases. However, it should be noted that the analysis excludes any of the savings in cooling costs from insulation use. If the cooling cost savings were included in Equation 4, the optimum insulation thickness would be higher than indicated in Table 6, especially for the regions with 4,000 to 6,000 degree days.

In Table 7, the optimum thickness of insulation is given with r and the degree days varying and with "a" equal to 15¢. The difference in the "a" value between Tables 6 and 7 can be considered as a 40 percent subsidy to homeowners. While the subsidy does increase the optimum insulation thickness for each combination of r and degree day from Table 6 to Table 7, the difference in the thickness runs only from approximately one to two inches. Furthermore, in those regions with relatively low degree days, an insulation incentive of a sizable magnitude may force cuts in more effective alternative energy saving programs such as mass transit. If, however, the inflation rate on energy is held in the lower range for r (due to price controls, for example), then to promote energy conservation through insulation use subsidies for all regions will be required.

ASHRAE Standards 90-75. As an alternative to the use of incentives to promote insulation use in new structures, many governments, including the federal government, have considered the adoption of building codes. In conjunction with the goal of energy conservation, the American Society of Heating, Refrigerating and Air Conditioning Engineers, Inc. has proposed a set of standards which some state governments have adopted [1]. While the set of standards deals with conservation in all aspects of energy use (lighting, heat transfer systems, etc.), there is one section pertaining to the thermal conductivity of walls. That section shows the inverse relation of windows and insulation thickness to meet the minimum thermal conductivity of wall space. That is, as window space in the wall increases, the required insulation thickness of the wall increases. This section now considers

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[3] In an earlier study, Gale and Hennessy reviewed personal income tax credits, profit tax credits, property tax exemptions, governmental loan and interest guarantees and an excise tax on energy sources [6].

TABLE 6. Values of x with i*=0.10, i1=0.09, n=30, n1=20, a=25¢ and r and Degree Days Vary

r \ D	10,000	8,000	6,000	4,000
0.04	2.01	1.72	1.39	1.00
0.10	3.16	2.75	2.29	1.73
0.16	5.30	4.66	3.94	3.08
0.22	9.23	8.18	6.98	5.56

TABLE 7. Values of x with i*=0.10, i1=0.09, n=30, n1=20, a=15¢ and r and Degree Days Vary

r \ D	10,000	8,000	6,000	4,000
0.04	2.81	2.44	2.01	1.51
0.10	4.29	3.77	3.16	2.45
0.16	7.06	6.24	5.30	4.19
0.22	12.13	10.77	9.23	7.40

the costs of implementing the standards.

There would be two costs associated with the adoption of ASHRAE Standards. The first cost is the administrative cost for the state government. The second cost is the implementation cost to the owners of new structures.

With regard to the governmental costs, the administration of the policy could easily follow the procedure now in use by some governments. Under current building codes in some localities, a permit to build is given if the building meets the relevant codes. In many cases, the permit is certified if a registered engineer has approved and specified that the existing standards are met. State and local governments generally accept permit applications completed by a professional engineer as proof that codes will be followed.

Alternatively and traditionally, governments employ building inspectors to ensure that codes and standards are met. Thus, for most local governments the machinery is already present for the enforcement of the ASHRAE Standards. As a result, the costs of enforcement of the program would be slight. A training period and temporary staff to provide training to inspectors would be required. Considering the costs of training plus the extra labor time for monitoring the ASHRAE Standards, the administrative costs are negligible.

On the other hand, the costs of compliance to the owners of structures is not so slight. As a percent of total costs, compliance with current codes and regulations is approximately one to two percent of total costs.[4] In the case of ASHRAE Standards, there would be the added design costs and structure costs. It is assumed that these costs are approximately 0.5 percent to two percent of the total costs of a structure. In the initial phases of implementation of the ASHRAE Standards, the design costs will be relatively high because of the learning time required by the architects and engineers. Accordingly, the costs of learning by these groups can be expected to be one percent of total costs. However, the material costs will rise if current window space in residential structures is to be maintained. These costs are assumed to be one to two percent of total costs. Thus, the total increase in construction costs if the ASHRAE Standards are applied is assumed to be 2.5 percent to five percent of the total costs of the structure. For a house with normal window space (25 to 30 percent of floor space) and a total cost of $45,000, the extra costs of construction will range from $1,125 to $2,250.

It should be noted that if the window space in residential structures is reduced to meet the ASHRAE Standards, the total structure cost may actually decline since windows are a relatively costly portion of the exterior envelope. Additionally, as designers and engineers become accustomed to the ASHRAE Standards, the learning costs will be zero.

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[4]The costs assumed in this section are based upon information provided by design engineers for several contracts and R. S. Means Co. [4].

Finally, the adoption of mandatory standards entails a philosophical question of freedom of choice. Some authors have rejected mandatory standards because they reduce freedom of choice for the owner and the designer. If designer freedom is limited, innovations in housing design may actually reduce energy saving technological developments. Thus, some authors have proposed that the ASHRAE Standards be provided only as a reference [5]. On the other hand, some authors feel that the overriding social concern calls for active use of ASHRAE to conserve on energy [7].

Summary

This study has analyzed the incentives which influence the amount of insulation used in structures. The first general conclusion of the study is that inflation rates on energy of 10 percent or greater dictate an insulation level of at least three inches in residential structures. Except where profit rates are abnormally high, the same result holds true in the commercial and industrial sectors. Thus, if the inflation rate on energy remains relatively high, the appropriate governmental policy is to provide information and education pertaining to insulation use and energy conservation in general.

Some traditional incentives such as exemptions from property taxes and credits on personal income or profits taxes would have a slight effect on the amount of insulation to be used. It is clear from the data in the report's tables that these measures would provide marginal changes if the losses in state government revenues are to be relatively small.

Another disadvantage of the above incentives (including energy inflation) is that in many cases the owners and occupants of a structure (residential, commercial or industrial) are not the same persons or firms. In the case of residential structures, 25 percent are rented while 75 percent are owner-occupied. Without more information, a similar proportion is assumed for commercial and industrial sectors. Unfortunately, if renters pay the cost of utilities, then owners have no incentive to use insulation. Unless renters have long term leases (which is frequently the case for commercial and industrial firms), the renters have no incentive to retrofit structures with insulation. Thus, for a major proportion of new structures, the incentives would be inapplicable.

A possible solution to the above rental problem is to adopt some regulations such as ASHRAE Standards 90-75. The use of the ASHRAE Standards would insure that the rental units would meet specifications for energy conservation. Furthermore, the ASHRAE Standards could be adopted with a relatively low cost to the state. However, the cost to owners will be two to five percent of the total structural cost in the first few years of adoption.

In short, this study has found that energy inflation and profit rates are crucial variables in determining the extent to which owners of structures use insulation. Various types of tax incentives would have relatively small effects on insulation use. The ASHRAE Standards, at least, can be expected to insure insulation in rental units where incentives would be ineffective otherwise. However, as mentioned in the discussion on the ASHRAE Standards some economists and politicians view standards as a restraint in freedom of choice even though energy conservation is a social goal.

APPENDIX

Given the initial cost of fuel, the inflation of fuel prices can be incorporated by multiplying the fuel cost in the initial year by (1+r) for the fuel cost in year 1 and so forth for later years. (r = the inflation rate.) A method has been developed by Smith to convert these increasing costs to annual costs by using a present value concept [8]. The critical factor is the relation between the inflation rate, r, and the opportunity cost of funds, i*. Depending upon the relation of r and i*, k_R is one of the following:

$$r > i^*: \quad k_R = (1/(1+i^*))(F/A)^w_n \, C_R$$

$$r < i^*: \quad k_R = (1/(1+r))(P/A)^w_n \, C_R$$

$$r = i^*: \quad k_R = (n/(1+r))C_R$$

where

$$w = ((1+r)/(1+i^*)) - 1 \qquad \text{if } r > i^*$$

$$w = ((1+i^*)/(1+r)) - 1 \qquad \text{if } r < i^*$$

n = the life of the asset

$$(F/A)^w_n = ((1+w) - 1)/n$$

$$(P/A)^w_n = ((1+w)^n - 1)/(w(1+w)^n).$$

To calculate the k_D factor required consideration of the life of the asset, the tax bracket, t, and the capital recovery factor, C_R. First, define the depreciation rate, E, as the following:

$$E = (1/n) + (0.15)\ (i^*). \quad \text{(Straight line depreciation)}$$

Then, to convert the depreciation factor to current costs:

$$k_D = 1 - ((E)(t)/C_R).$$

k_B is the factor which shows the effects from financing the insulation, in part, through borrowed funds. Several variables must be taken into account to determine the annual costs of borrowing. They are: the interest rate on borrowed funds, the term of the loan, the tax bracket of the borrower, the down payment of the borrower and the opportunity costs of funds. Giving due consideration to these factors yields the borrowing factor, k_B.

$$k_B = p + ((1 - p)/nl)(P/A)^{i^*}_{nl} + (1 - p)(il)(1 - t)(P/A)^{i^*}_{nl}$$
$$- ((1 - p)/nl)(il)(1 - t)(P/G)^{i^*}_{nl}$$

where

p = down payment percentage of purchase price

nl = term of loan

il = interest rate on borrowed funds

$$(P/A)^{i*}_{nl} = ((1 + i*)nl - 1)/(i*(1 + i*)nl)$$

$$(P/G)^{i*}_{nl} = (1/i*)(((1 + i*)nl - 1)/(i*(1 + i*)nl)) - (nl/)(1 + i*)nl)).$$

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