The Delineation of a Hierarchical Economic System Using Heuristic Programming*

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Empirical studies of hierarchical central place systems require a methodology for grouping central places. The purpose of this paper is the presentation of a methodology for performing this task.[1]

The first section of the paper contains a review of the literature on techniques that have been used to solve the hierarchical grouping problem. A proposed methodology for ranking and grouping central places is included in the second section. The methodology includes (a) the development of an index which can be used to rank communities in a central place system and (b) description of a heuristic programming algorithm which can be used to group the communities by hierarchy. In the third section the methodology is applied to parts of the Rocky Mountain region. This includes Wyoming and parts of six adjacent states--Colorado, Utah, Montana, South Dakota and Nebraska. The final section includes summary comments on the methodology.

The primary conclusion of this paper is that heuristic programming presents encouraging possibilities for delineating central place systems when the data are taxonomic.

Relevant Literature

The theoretical justification for a hierarchical grouping of central places has been provided by A. Losch [18] and W. Christaller [9] and in a generalized form by B. Berry and W. Garrison [5]. Essentially, central place theory is founded on the ordering of economic activities according to the size of market areas. For example, a commercial bank is more likely to have a larger market area than a grocery store, and a new car dealer will have an even larger market area. Given this hierarchical nature of goods and services, central places can

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*The findings reported herein have been supported by a contract from the Bureau of Health Manpower Education, National Institute of Health (Contract No. NIH 72-4083). This paper reflects the opinions and findings of the authors and does not reflect in any way the official opinions and policies of the funding agency.

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[1]It should be noted that this paper is not a development or extension of central place theory, rather this paper is an application of central place theory.

be grouped according to the types of goods and services sold in them. Central places which sell goods of a higher order (goods with larger market areas) are grouped at a higher level in the central place hierarchy.

The literature on central place theory reveals that the usual empirical procedure is the identification of groups of central places in the hierarchy for some study area, using information on retail establishments by central place function [4, 6, 7, 8, 11, 12, 13, 19, 25]. The final results of central place analysis usually include a specification of the number of hierarchies and the types of businesses and numbers of establishments characteristic of each group. Sometimes efforts are also made to define threshold business functions--those which are pivotal in causing marginal central places to be placed in one group rather than another. The notion of threshold has also been extended to defining population thresholds for central place groups in the hierarchy and to defining population thresholds for businesses [2, 6, 11].

The purpose of this section of the paper is to review the variety of grouping methodologies which has been applied to data on business functions in analyzing central places. Many of the studies have relied upon somewhat arbitrary and subjective grouping procedures, placing a great deal of importance on the familiarity of the analyst with his study area.

Studies particularly guilty of arbitrary grouping procedures generally occurred early in the chronology of empirical work on central place systems [1, 8, 13, 25]. J. Brush's study of southwestern Wisconsin was typical of these early studies [8]. For his rural study area, Brush identified a three-fold classification of central places and the functions which were typical of each. Hamlets were defined as containing at least five residential structures and between one and ten retail and service units. Villages contained a minimum of ten retail and service units, at least four additional retail businesses, and three additional services. Towns were required to contain fifty retail units, a weekly newspaper, a high school, and four professions (physician, dentist, veterinarian, and lawyer). Despite these detailed descriptions of the grouping requirements, Brush did not present a rationale for choosing these particular delimitations between central place groupings.

Similar arbitrariness is evident in studies using multiple forms of data in addition to functional characteristics [2, 6, 22]. An example is the hierarchical ranking of trading centers by Rand McNally in their 1972 Commercial Atlas and Marketing Guide [22]. Sales of general merchandise, newspaper circulation, and to a small degree, number of banks, retail sales, and location of the city relative to other centers were considered in the formation of groups of cities in the United States. Merchandise sales and newspaper circulation data were each assigned weights of approximately 40 percent in selecting and ranking centers, while the other data received the remaining 20 percent of total consideration. The subjectivity of the methodology is particularly critical for remote, sparsely populated areas, where isolated cities are assigned trade center designation due to their isolation from other centers.[2]

J. Borchert and R. Adams also used multiple criteria to identify six hierarchical classes of retail centers in the upper Midwestern United States [6]. Employing information from the Dunn and Bradstreet Reference Book [14], they first combined retail businesses and select services into 52 groups of functions. The number of these functions and population was then tabulated for each place in a sample of 311 central places. A majority of functions was found to occur in places above a certain population size which could then be used to define classes of retail centers. These population thresholds defined four of the classes, but it was necessary to employ wholesale trade in defining the two classifications with a population of 9,500 and over. While their methodology seems superior to that used by Rand McNally, some subjective judgments were required by Borchert and Adams: (a) only 38 of the 52 functions were useful in identifying sharp population thresholds, but the precise technique for deciding delimitations was not discussed; and (b) larger places were classified by retail and/or wholesale sales data rather than by functional activity, thus introducing an alternative criterion to the grouping methodology. Moreover, it is not clear how these alternative criteria were analyzed to form additional hierarchical groups.

During the past decade some new methodologies have been applied to the problem of hierarchical grouping. One method of addressing the grouping problem has recently been suggested by R. Preston [21]. The technique incorporated the use of a moving average similar to that employed in time series analysis. Estimates of surplus retail sales were calculated for central places in the study area and used as measures of centrality. These centrality values were converted into a moving average of first differences of successively higher ordered centers. A graph of the moving average allowed the delineation of groups where the largest changes in slope occurred. This procedure seems subjective both from the point of view of choosing an optimal number of groups and of optimally placing central places into a predetermined number of groups--the basic deficiency being the reliance upon visual inspection.

The most promising methods of standardizing grouping procedures are based upon linkage analysis and distance-to-nearest-neighbor grouping. In the late 1950's Berry and Garrison adopted the distance-to-nearest-neighbor method in forming hierarchical groups of central places in Snohomish County, Washington [4]. Borrowing from the work of P. Clark and F. Evans in plant ecology [10], Berry and Garrison defined a group as a set of centers such that each center within the group was closer in number of economic functions to some other member of the group than to a center outside the group. John Marshall has criticized this procedure by arguing that it is not strict enough in its grouping require-

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[2]Figures and information were obtained from Richard L. Forstall, editor of the 1972 Commercial Atlas and Marketing Guide [22], in a telephone conversation.

ment--that for ideal grouping each member of the group should be closer to all other members than to any point outside that group [19, p. 55]. Upon applying this stricter grouping requirement, Marshall found it necessary to alter the empirical values of centrality for some central places in order that this criterion might be satisified. From the point of view of those who desire value-free and standardized procedures, Marshall's correction appears to be inadequate.

The concept of nearest neighbor has recently been extended to the grouping of centers with multidimensional characteristics using cluster analysis, linkage trees, and various other grouping algorithms [3, 16, 17, 20]. The usual procedure is an iterative process of initially grouping the pair of central places which are "nearest" each other in centrality value. This pairing process continues until the linking of places results in the desired number of separate groups containing linked places or until the number of groups formed is such that the value of an objective function has reached a predetermined critical level. For example, as the linking process adds points to a group, an average of intragroup distances could be calculated. As the grouping process continues, any natural clusters will show up as jumps in the average intragroup distance.

The remainder of this paper is devoted to the explanation and application of a methodology which belongs to this more recent set of grouping techniques.

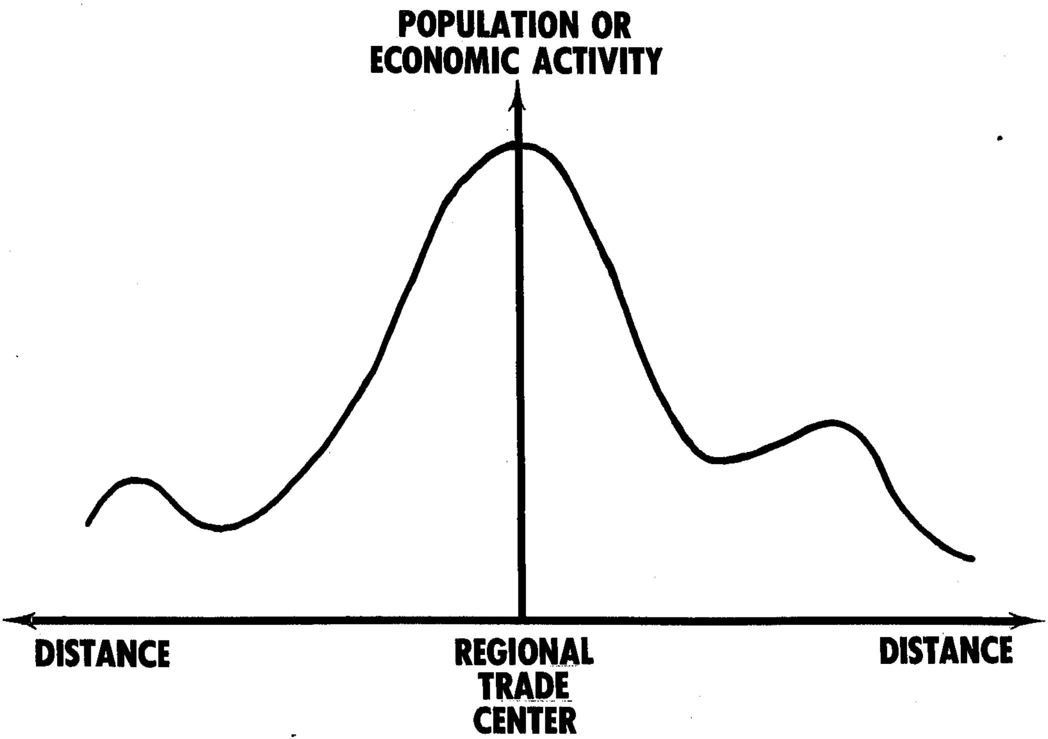
Methodology

Virtually all of the methodologies used to delineate hierarchies of central places are based, in greater or lesser degree, on the assumption that regional trade centers exist, that population and economic activity decline as movement occurs away from these trade centers, that intervening sub-regional trade centers interrupt the decline of population and economic activity (see Figure 1), and that population dispersion is a regular and continuous function except for the intervening trade centers. In the intermountain region, it is convenient to describe the area as having islands of population and economic activity with little or nothing in between (see Figure 2). In spite of the island's description, hierarchical levels of economic activity exist--some islands have a greater range of economic activities than others. The question that emerges is: How can the region be defined in a meaningful hierarchical system?

The methodology which initially appeared to have the most potential for optimally combining communities into hierarchical groups was combinatorial programming [23]. Combinatorial programming provides unique solutions to the hierarchical grouping problem. An example is shown in Figure 3. The figure is based on the assumption that four communities (A, B, C, and D) are to be optimally combined to satisfy some objective, given a single measure of centrality for each community. The combinatorial programming methodology initially calculates all possible groupings (note the linkages in the diagram) and then selects those groups which meet the objective specified by the analyst. Unfortunately, the large amount of data and the large number of communities included in the study of the intermountain region precluded the use of exact combinatorial methods.

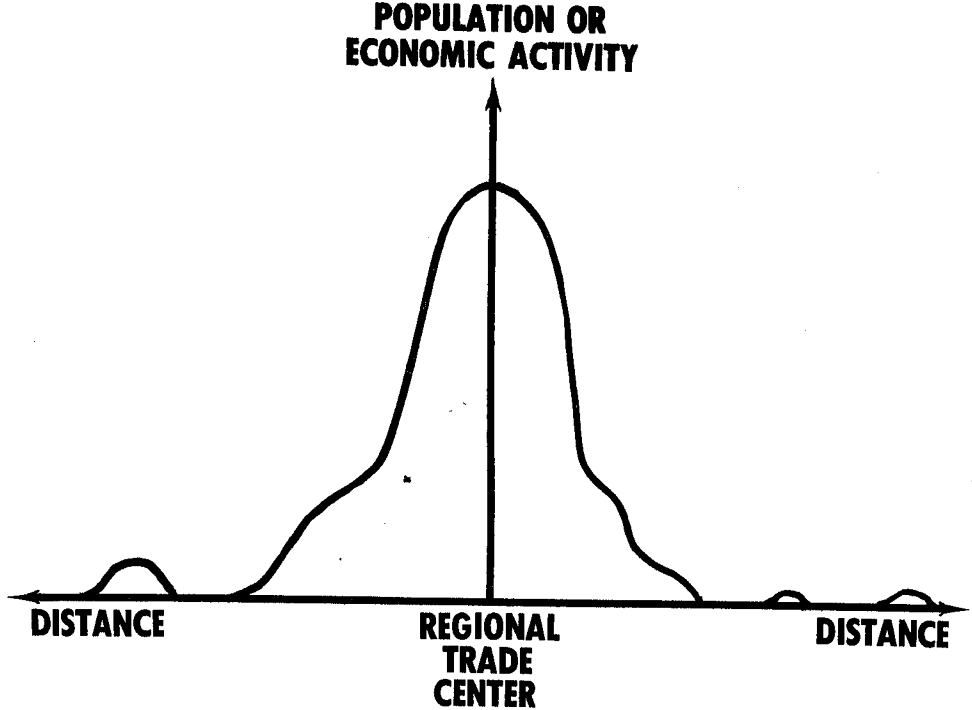
EXPECTED POPULATION DISTRIBUTION AS A LINEAR MOVEMENT OCCURS AWAY FROM A REGIONAL TRADE CENTER.

FIGURE 1

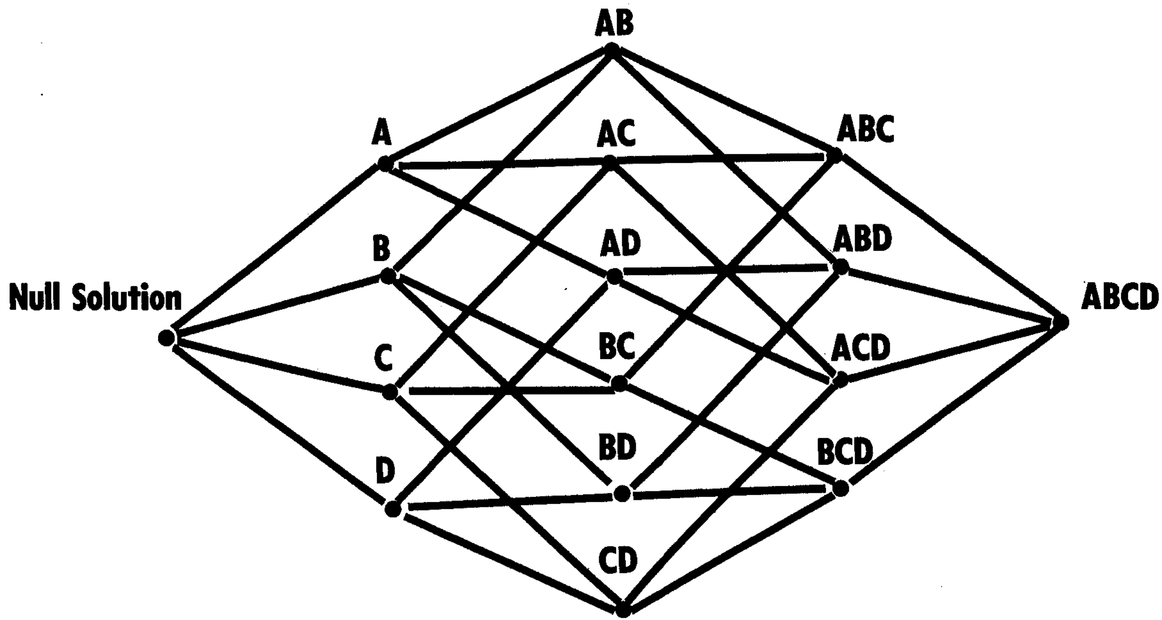


EXPECTED POPULATION DISTRIBUTION AS A LINEAR MOVEMENT OCCURS AWAY FROM A REGIONAL TRADE CENTER IN THE INTERMOUNTAIN REGION.

FIGURE 2



NUMBER OF COMBINATIONS FOR ONE ECONOMIC ACTIVITY AND FOUR COMMUNITIES.
FIGURE 3
($2^4=16$, including the null solution)



The number of central places in this study area was 545, and the data were collected on 114 variables. To make the data manageable for the illustration included in this paper, both the numbers of variables and communities were reduced to a matrix smaller than the initial dimensions of 545 x 114. This reduction process was accomplished, first, by using factor analysis to reduce the number of variables from 114 to 39, and, second, by assuming that communities with a population of less than 1,000 comprised the bottom hierarchy.[3, 4] These two procedures reduced the data from a matrix with dimensions of 545 x 114 to one with dimensions of 123 x 39.

The reduction of the data still did not allow the employment of combinatorial programming. Even for a matrix with dimensions of 123 x 1, this method would require enormous storage capacity on a high speed computer. It would involve the computation of all possible combinations of 123 communities, or a total number of combinations equal to 2^{123} (including the null solution). This process would total approximately 64,000,000 followed by thirty zeroes. While certain algorithms will optimally solve combinatorial problems of this size, A. Scott has noted that their ability to handle more than 90-100 communities and one economic activity is doubtful [23, p. 37]. Moreover, the computer time needed would be very lengthy.

In view of the data processing limitations imposed by both time and storage capacity, it was necessary to select a sub-optimal method of areal delineation (i.e., one that gropes for the optimum, but one that does not necessarily achieve the optimum). The specific programming algorithm chosen was that developed by Ward [28]. This heuristic algorithm might provide the unique optimal solution. Realistically, it was expected that this algorithm would provide a sub-optimal solution that converged toward the optimum.[5]

The Ward algorithm requires the minimization of the error sum of squares, ESS, for some variable X_i. The objective function can be written as follows:

$$(1) \qquad \text{Minimize ESS} = \sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}$$

[3]Data sources included [14, 22, 26 and 27].

[4]Briefly, a 114 x 114 matrix was formed containing partial correlation coefficients of the variables from which the influence of population had been removed. A principal axis factor analysis was then performed, and four factors rotated to yield the factor matrix. Variables chosen were those for which at least 50 percent of the variance was explained by any one of the first three factors. For the fourth factor, the maximum variance explained was 45 percent. For this factor, variables chosen were those for which approximately 25 percent or more of the variance was explained by the fourth factor.

[5]Theoretically, it is possible that the Ward algorithm would provide a different solution for successive runs on the computer. Practically, the data were organized in rank-order form prior to the commencement of the analysis with the result that this modification provided the same hierarchical result for repeated runs on the computer.

The computation begins with n groups in the hierarchy. Subsequent computations result in the combination of the n groups into a smaller group, for example, k (k is the parameter defining the number of groups chosen for the system of cities studied). For the data being reported herein, the number of groups was assumed to be seven ($k = 7$). Furthermore, for the purposes of data processing, the parameter used for the analysis was set equal to six. Therefore, the generalized form of the objective function is as follows:

$$(2) \qquad \text{Minimize ESS}_{(k \text{ groups})} = \text{ESS}_{(\text{Group 1})} + \text{ESS}_{(\text{Group 2})} + \ldots \text{ESS}_{(\text{Group } k)}$$

The Ward algorithm begins with $k = 123$ ($\text{ESS}_{k = 123} = 0.0$) and combines communities until $k = 6$.[6] Communities are combined such that no other combination could produce a smaller increase in the value of the objective function. Since all combinations are not included when this algorithm is used, it is probable that the final solution will have a higher objective function value than if all combinations had been used [23, Ch. 9].

Since the algorithm permits the computation of a hierarchy with one variable at a time, it would appear necessary to compute 39 different hierarchies--one for each of the economic variables. Rather than compute 39 different hierarchies and then find an average hierarchy, the 39 variables were collapsed into a single and proportional index of centrality, C_i.

If X_{ij} is the number of establishments for Standard Industrial Classification (SIC) code j ($j = 1, \ldots, n$) in community i ($i = 1, \ldots, m$), then a proportional weight for each type of business, P_{ij}, can be computed for each community as follows:

$$(3) \qquad P_{ij} = \frac{X_{ij}}{\sum_{i=1}^{m} X_{ij}}$$

, where P_{ij} is the proportional weight contributed to community i by business establishments of type j.

The proportions can then be summed over each j to compute C_i,

$$(4) \qquad C_i = \sum_{j=1}^{n} P_{ij}$$

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[6]The parameter $k = 6$ was used, since Group 7 was assumed to include all communities with a population less than 1,000.

The variable C_i is a centrality index for each community and is similar to those computed by W. Davies [11] and J. Marshall [19]. The single index of centrality can be used in conjunction with the Ward algorithm to compute a hierarchical system among the n communities.

The single proportional variable, as defined above, requires a strong assumption, namely, that each economic activity defined by an SIC code is, in the aggregate, of equal importance to any other economic activity in the economic system. For example, in the analysis reported herein, the assumption would mean that 86 bicycle stores would have the same weight as 599 new and used car dealers.

This assumption seems reasonable. The effect of the centrality index is not inconsistent with the conclusions of central place theory since it is expected that scarce activities will increase the influence of a community as a central place. The centrality index incorporates this concept by giving more weight to an establishment performing a rare activity than to an establishment performing a more ubiquitous activity. It also incorporates the intuitively reasonable idea that communities with a greater number of establishments of any one business type should also have a greater influence as a central place. It is interesting to further note that the subset of variables provided by factor analysis excludes some of the more numerous activities. For example, the factor loadings on grocery stores, drug stores, and hotels and motels were not sufficiently high to merit their inclusion. Thus, it appeared that less ubiquitous variables such as bicycle shops and book stores, although less numerous, had more of their variance explained by the individual factors determined through factor analysis.

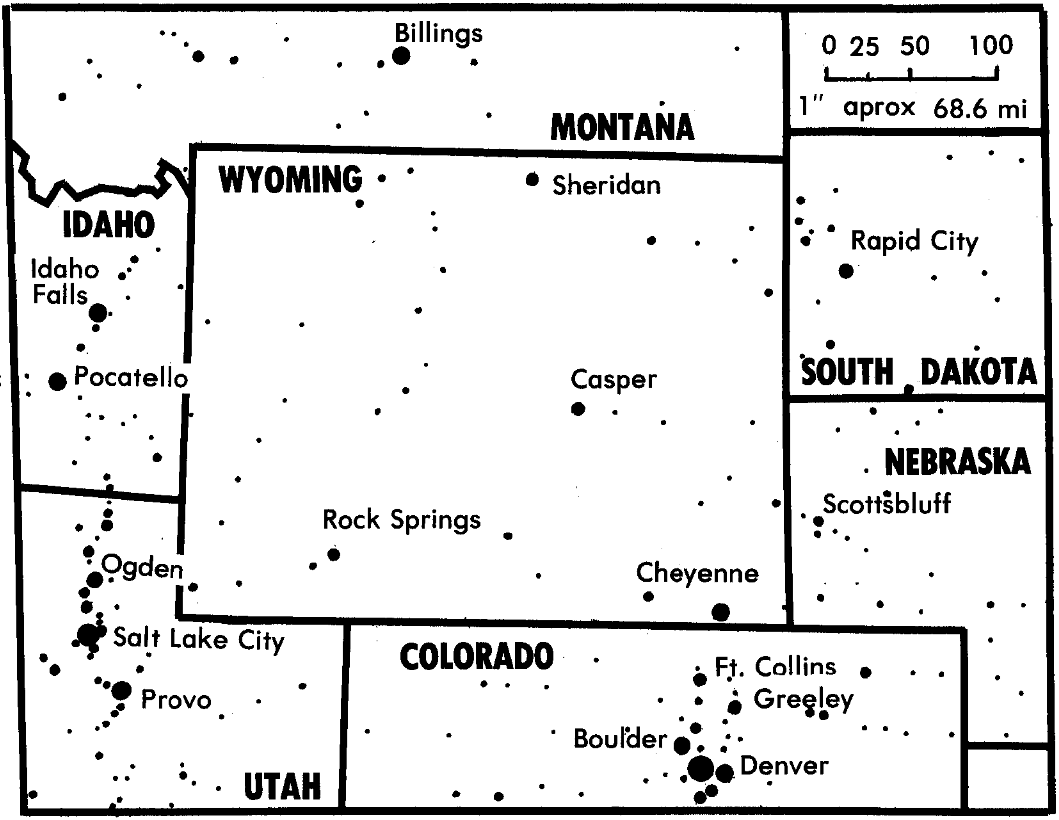
A final methodological comment seems appropriate on the determination of the number of hierarchical groups. It was mentioned in the previous section that some studies employ methodologies that search for the number of groups. In this study, the objective was the optimization of a given number of groups. The number of groups utilized was seven. However, the study has not been completed, and it is possible that the number of groups in the final report will be different. In any case, the number seven was not chosen arbitrarily as it conforms with the conclusions of the hierarchical studies of Saskatchewan [15] and Minnesota [6]--two regions which appear similar to the intermountain study area.

The above explanation describes the methodology used for estimating the hierarchical structure for 123 communities in the intermountain region with populations of 1,000 or more. The empirical definition of the hierarchical system is reported below.

An Application

The region to be studied was defined to include Wyoming and parts of six adjacent states--Colorado, Utah, Idaho, Montana, South Dakota, and Nebraska (see Figure 4). This intermountain region was selected because of its low population density and its predominantly rural nature. The specific reasons

MAP OF INTERMOUNTAIN STUDY REGION.
FIGURE 4



for including the portions of the several states can be summarized as follows: The portions of states adjacent to Wyoming were included if services in the adjacent states were expected to affect the shopping and other patterns of Wyoming residents. Table 1 (A and B) provides a comparative description of the intermountain region.

Both Denver, Colorado and Salt Lake City, Utah were included, since it was expected that these two cities would provide both wholesale and retail services to Wyoming, and that these two cities would be regional trade centers. (It was not expected that these two cities would be of equal importance in the hierarchical system.) Other major cities in adjacent states included in the study were: Idaho Falls, Idaho; Billings, Montana; Rapid City, South Dakota; Scottsbluff, Nebraska; Provo, Utah; and Ogden, Utah. Each of these cities was expected to be part of the intermountain trade service region, but each of these cities was not expected to fit into the same hierarchical group.

The number of groups selected for the heuristic programming algorithm was six, and the hierarchical system was defined to include seven groups. The seventh group was assumed to include all communities with a population under 1,000. A few words of explanation on this assumption are necessary.

The heuristic programming algorithm optimized the objective function noted in equation (1) above. For each unit increase in the number of groups, the algorithm tended to divide upper level groups (i.e., the ESS was greater for the higher hierarchical levels than for the lower hierarchical levels). Table 2 illustrates the hierarchical differences that resulted when the parameter k was increased from five to six and then to seven. An increase in k from five to six resulted in a division of the third group into two groups. A similar result occurred when the number of groups was increased from six to seven. Experience with the program indicated that a large increase in the parameter k was required to discriminate between communities with smaller numbers of businesses and SIC codes (i.e., those communities in the seventh hierarchical level).

The number of groups is described in Table 3. The range of activities for each of the first six groups is included in Table 4. Thirty-nine SIC codes were used for this paper, and each of these activities was derived for the first two groups, G_1 and G_2. These two levels were described, respectively, as (a) a regional trade center (Denver, Colorado) and (b) a sub-regional trade center (Salt Lake City, Utah). The principal reason that these communities emerged in different groups is the relatively larger number of businesses in Denver (i.e., 6,947) as compared to the number in Salt Lake City (i.e., 3,855).

The next hierarchical level, G_3, primary wholesale/retail shopping centers, included four cities. In this group, the first decrease in the total number of SIC codes appeared. One city in this level had wholesale footwear (SIC code 5039); three cities had wholesale hardware (SIC code 5072) and wholesale produce (SIC code 5048); and each of the four cities had wholesale men's clothing and furnishings (SIC code 5036) and wholesale furniture and home furnishings (SIC code 5097). Each of the four cities had all 34 of the remaining activities.

TABLE 1: Comparison of the Intermountain Region to the United States

A. General Characteristics

	Total Population	Total Land Area (square miles)	Total Central Places
Intermountain Region	2,797,700	230,451	545
United States	230,211,900	3,536,855	20,768
Region/U.S. (%)	1.2	6.3	2.6

B. Central Place Characteristics

	Intermountain Region		United States	
Central Places (population)	Number of Central Places	Percentage of Total	Number of Central Places	Percentage of Total
100,000 or more	4	.7	396	1.9
25,000 - 99,999	9	1.7	1,905	9.2
1,000 - 24,999	104	19.1	8,952[a]	43.1
999 or less	428	78.5	9,515	45.8
Totals	545	100.0	20,768	100.0

Source: Statistical Abstract, 1972.

[a] It was assumed that the 627 urban places in the United States with a population below 2,500 had populations between 1,000 and 2,499.

TABLE 2: Descriptive Impact of Increasing the Number of Groups in the Hierarchy from Five to Seven

	Number of Places for Each Parameter (k)		
	k = 5 groups	k = 6 groups	k = 7 groups
Group 1	1	1	1
Group 2	1	1	1
Group 3	11	4	4
Group 4	29	7	7
Group 5	81	29	8
Group 6		81	21
Group 7			81

TABLE 3: Description of the Seven Hierarchical Groups

Group	Hierarchical Designation	Description of Group
1	G_1	Regional Trade Center
2	G_2	Sub-regional Trade Center
3	G_3	Primary Wholesale/Retail Trade Centers
4	G_4	Primary Shopping Centers
5	G_5	Secondary Shopping Centers
6	G_6	Convenience Centers
7	G_7	Minimum Convenience Centers and Hamlets

TABLE 4: Frequency Table: Number of Times Each SIC Code Did Not Exist for Each of the First Six Groups

SIC Code	Code Description	G_6	G_5	G_4	G_3	G_2	G_1
5039	Wholesale Footwear	81	29	7	3		
5072	Wholesale Hardware	80	25	2	1		
5048	Wholesale Produce	76	23	2	1		
5036	Wh. Men's Cl. & Furn.	81	28	6			
5097	Wh. Furn. & Home Furn.	79	26	3			
5499	Health Food Stores	78	18	1			
5953	Bicycles	80	19				
5942	Book Stores	77	16				
3993	Sign Painting	74	8				
5221	Plumbing & Heating	74	8				
6120	Savings & Loan Co.	68	5				
5231	Paint & Glass Stores	66	4				
5661	Shoe Stores	64	2				
4832	Radio Stations	58	5				
5933	Second-hand Stores	56	3				
5714	Drapery & Upholstery	54	4				
5451	Dairy Products	47	3				
5462	Bakeries	46	4				
5971	Jewelry Stores	42	5				
8931	Accountants	43	2				
5252	Farm Machinery	30	3				
5722	Appliances	31	1				
5962	Feed Stores	28	3				
722	Veterinarians	26	4				
5592	Trlr. & Camper Dealers	67					
5730	Music & TV Stores	46					
5621	Women's Clothing	35					
5712	Furniture Stores	27					
5311	Department Stores	23					
5531	Auto Equipment Stores	23					
5511	New & Used Car Dealers	19					
7900	Amusement & Recreation	17					
8111	Lawyers	15					
5211	Bldg. Materials	10					
7538	General Auto Repair	8					
6020	Banks	6					
5812	Restaurants	5					
7200	Personal Services	2					
5541	Service Stations	1					
n		81	29	7	4	1	1
$\Sigma_j X_{ij}$		1733	248	21	5	0	0
Max $\Sigma_j X_{ij}$		3159	1131	273	156	39	39
$\Sigma_j X_{ij}$ as % of Max.		54.9	21.9	7.7	3.2	0.0	0.0
No. of Businesses		3918	5099	3432	3949	3851	6947

The fourth group of seven cities was described as one having primary shopping centers. In this group, all cities had the full range of retail shopping facilities and services. The seven cities had approximately 43 percent of the possible wholesale activities (i.e., for the seven cities, a maximum of 35 wholesale activities were possible and 15 wholesale activities were present). One of the cities lacked one retail activity, a health food store (SIC code 5499).

The fifth group of 29 cities was described as having secondary shopping centers. At this level a decline in the number of retail shopping facilities and services was evident. The secondary shopping center group included cities with relatively few of the maximum possible wholesale activities (i.e., 131 of a possible 145 wholesale activities were absent--approximately 90 percent).

The sixth group, convenience centers, included 81 communities and less than two percent of the maximum possible wholesale activities. Further, at least one of each retail shopping and service activities was lacking for each of the communities in this group. Most of the communities possessed some combination of service stations, personal services, restaurants, banks, automobile repair centers, building material centers, legal services, amusement and recreation services, and new and used car dealers. With somewhat less frequency, communities in this group had automobile equipment dealers, department stores, and furniture stores.[7]

The seventh and final group, minimum convenience centers and hamlets, was arbitrarily defined to include all communities with a population under 1,000. Table 5 provides an aggregate description of this group vis-a-vis the other six groups. It may be observed that the 422 communities in G_7 had 2,360 businesses (approximately one-third of the number in Denver) and that each of these communities had an average of 5.6 businesses. Table 5 also indicates that the percentage of the maximum number of SIC codes declined from 100 percent for G_1 and G_2 to 11.4 percent for G_7. Line 5 includes the total number of businesses in each hierarchy, and line 6 provides the average number of businesses in each hierarchy for G_3, G_4, G_5, G_6, and G_7.

Summary Comments

The methodology presented in this paper represents an effort toward standardizing the techniques for hierarchical grouping of communities. It has several advantages over past methods as well as some deficiencies.

First, though requiring the aid of a computer when a large number of places and variables are involved, the procedures and concepts employed are relatively easy to comprehend. The concept of a centrality index based upon functional complexity and numbers of establishments seems reasonable in light of theo-

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[7]Cities in this group also had grocery stores, a varible not included as a result of the factor analysis.

TABLE 5: Description of the Seven Groups

Characteristics of Each Group	G_7	G_6	G_5	G_4	G_3	G_2	G_1
(1) Number of communities (i.e., *n*)	422	81	29	7	4	1	1
(2) Maximum number of SIC codes (if each community in each group had all 39 SIC codes)	12238	3159	1131	283	156	39	39
(3) Actual number of SIC codes	1397	1426	883	252	150	39	39
(4) Actual number/maximum number (%)	11.4	45.1	78.1	89.0	96.2	100.0	100.0
(5) Number of businesses	2360	3918	5099	3432	3949	3851	6947
(6) Average number of businesses in each community	5.6	48.4	175.8	490.7	987.5	3855	6947

retical conclusions from central place theory (i.e., communities are ranked in terms of the complexity of their structures as central places). In addition, the Ward algorithm uses the concept of variance in the objective function, thus incorporating a statistically sound measure of intragroup dispersion.

Second, the data sources used for this paper are relatively accessible and eliminate the necessity of expensive surveys and field work. The data are, of course, subject to the deficiencies in Dunn and Bradstreet's Reference Book and the Yellow Pages of the telephone books.

Third, the method does not rely upon populations of communities. This approach improves the quality of the results from the viewpoint of central place theory since population may reflect the internal influence of economic activity other than central place functions. For example, the dominance of manufacturing in a community may cause population to rise even though the community's position as a central place remains constant.

The major weakness of the proposed methodology appears to be the difficulty in delineating threshold functions (i.e., the key functions which determine the group placement of marginal communities). The primary reason is that the centrality index used is based on both types of functions and numbers of establishments. For the results of this paper, no attempt was made to determine rules for making a specific identification of threshold functions and numbers of establishments.

Finally, the authors believe that the heuristic programming method represents an improvement in the traditional taxonomic techniques employed in central place analysis.

APPENDIX

Listing of 39 SIC Codes for Four Factors[a]

Factor 1

SIC Code Number	Description	Factor Loading
5541	Service Stations	0.92
5712	Furniture Stores	0.91
5722	Appliance Stores	0.90
5531	Automotive Equipment	0.88
5661	Shoe Stores	0.88
5511	New and Used Cars	0.88
3993	Sign Painting	0.84
7900	Amusement and Recreation	0.82
5221	Plumbing and Heating	0.81
5592	Household Trailer Dealers	0.80
5971	Jewelry Stores	0.79
5311	Department Stores	0.76
5933	Second-hand Stores	0.76
7200	Personal Services	0.76
5621	Women's Clothing	0.76
7538	General Auto Repair	0.76
5714	Draperies and Upholstery	0.75
5252	Farm Machinery	0.75
6020	Banks	0.74
5812	Restaurants	0.73
5730	Music and TV Stores	0.73
5211	Building Materials	0.73
5962	Feed Stores	0.71
4832	Radio Stations	0.71

[a]The description of the SIC codes has been modified in those instances for which an SIC code and description did not exist for some of the data. For example, the SIC code designation 5499 (Miscellaneous Food Stores) has been redesignated as Health Food Stores, the only type of business in this designation.

Listing of 39 SIC Codes for Four Factors (continued)

Factor 2

SIC Code Number	Description	Factor Loading
8111	Lawyers	0.95
5036	Wholesale Men's Clothing & Furn.	0.89
5499	Health Food Stores	0.89
5451	Dairy Products	0.85
5994	News Dealers & Newsstands	0.77
722	Veterinarians	0.73
5462	Bakeries	0.71
	Factor 3	
5072	Wholesale Hardware	0.74
5097	Wholesale Furniture & Home Furn.	0.72
8931	Accountants	0.72
	Factor 4	
5953	Bicycle Stores	0.67
5048	Wholesale Produce	0.58
5942	Book Stores	0.56
5039	Wholesale Footwear	0.49

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