TESTING FOR SPATIALLY AUTOCORRELATED DISTURBANCES WITH APPLICATION TO RELATIONSHIPS ESTIMATED USING MISSOURI COUNTY DATA*

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Regional scientists deal with a wide variety of problems. Considerable use is made of census and other cross section data for examining the relationships between variables of interest and for testing hypotheses. The popularity of cross section data is in large part due to our natural interest in the interrelationships among variables over space. But even when we are more concerned about the performance of selected variables over time, cross section data are often used simply because appropriate time series data are not always available.

Some of us are more familiar with time series analysis. We are thus sensitive to potential statistical problems in that domain, but tend to ignore analogous problems in cross section analysis. Yet the nature of available cross section data suggests that they may pose estimation and inference problems at least as severe as those posed by time series data.

This paper deals with the problem of spatial autocorrelation of cross section disturbances. This problem has recently been called to our attention in articles by Berry [1] and Fisher [5]. This paper has two main sections. In the first section, the general nature of the problem is discussed. The second section reports the results obtained from the application of tests for autocorrelated disturbances. It also reports the estimates obtained by modifying the estimation procedure to correct for autocorrelated disturbances.

Parts of the paper exploit the analogy between this problem and the autocorrelation problem sometimes encountered in time series analysis. While these problems are analogous, the cross section version of the problem is usually harder to deal with because of the lack of one way causation, greater mathematical complexity, and greater irregularity of the observational units used. Thus the time series analogue serves, at best, only as a point of departure for dealing with the cross section version of the problem.

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General Nature of the Problems Posed by Interdependent Disturbances

The general nature of estimation and inference problems posed by the lack of independence among disturbance terms is well known. Least squares procedures are commonly used to estimate the parameters of relationships of interest to us. Dependence among the error or disturbance terms associated with these relationships does not by itself bias the least squares estimators, but it does mean that the appropriate (but often unknown) generalized least squares estimator is more efficient. Dependence may also bias the estimators of the variances of parameter estimators. In most instances the variance estimators are biased downward. Thus least squares estimators are inefficient but tend to give results which "look good" because of the downward bias in their variance estimators.

Sources of Interdependence Among Disturbances

The problem of interdependent disturbances, if it exists, usually has its origins in economic, sociological or other relationships similar to those being estimated. Certain of our theories suggest that the relationships which we attempt to estimate require large geographic areas to work themselves out completely. Other relationships are stochastic and thus can safely be treated as deterministic only for moderately large geographic units. Using small geographic areas as observational units thus effectively increases the number of variables needed to adequately specify these relationships. It also increases the probability that some relevant variables will be omitted. The effects of these variables are thus relegated to the disturbance terms. These effects are often widely diffused or the values taken by the omitted variables themselves may be similar for many observations.

Testing for Autocorrelated Disturbances

The considerations discussed above suggest the potential for autocorrelated errors. They also suggest the sort of observational units which might lead to more or less serious autocorrelation. Unfortunately, they usually do not provide a very sound basis for assessing the strength of the interdependence in any given estimation situation. For that purpose statistical tests are useful.

Various tests have been proposed for detecting spatially correlated errors. Of these, Moran's test [6] is perhaps most easily adapted to testing for correlation among regression residuals. This required adaptation has been provided by Cliff and Ord [3]. The test statistic used is

$$(1) \qquad r = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} W_{ij}\, \hat{v}_i\, \hat{v}_j / T \sum_{i=1}^{n} \hat{v}_i^{\,2}$$

where

$$T = \sum_{\substack{i=j \\ i \neq j}}^{n} \sum_{j=1}^{n} W_{ij}/n, \tag{2}$$

$\hat{v}_i$, $\hat{v}_j$ = the residuals associated with the i^{th} and j^{th} observation respectively,

W_{ij} = a measure of the ties between or contiguity of observations i and j, and

n = the number of observations.

The form of this statistic is similar to that of the well-known Durbin-Watson d statistic. The distributions of both depend upon the regressors used to generate the residuals and thus vary from problem to problem. The distribution of r has not been tabulated but can be approximated for each problem using the same principles that have been used to approximate the distribution of the Durbin-Watson statistic. In practice, however, calculating the moments required to use the usual Beta distribution approximation is somewhat more tedious for r than for d. Thus a normal approximation is more commonly used. The work of Cliff and Ord [4] provides some support for this approach. The r statistic which they examined was based on deviations from the sample mean but hopefully their finding of approximate normality is also valid for r as defined in (1).

Although the calculations needed to implement this test are troublesome, the largest barrier to using this test will usually be specification of the W matrix. This matrix must specify which disturbances are believed to be correlated as well as the relative weights to be attached to each pair. It seems possible to take a cue from the Durbin-Watson test and set W_{ij} equal to zero except for those pairs suspected of being most highly correlated. The problems of identifying those pairs still remains.

For many variables the "closest" observations may not be physically adjacent but may be those linked by the fastest means of transportation or those which are similar. Furthermore, unless the phenomena generating the interdependence is a "one-way" process, the number of common "links" to other observation units may influence the strength of the resulting correlation.

Estimation in the Presence of Autocorrelated Disturbances

How should the estimation method be altered if the null hypothesis of spatial independence is rejected? For time series problems this issue has been fairly well resolved. In that domain it is known that the Durbin-Watson test is particularly effective for detecting autocorrelation of the sort generated by a first order Markov scheme. Thus if the null hypothesis is rejected, it is advisable to use (as at least a first approximation) procedures consistent with a first order autoregressive scheme.

If the first order autocorrelation coefficient is known some variant of generalized least squares is commonly and appropriately used. The structure of the correlation matrix of the disturbance terms is known so generalized least squares can be applied directly, but typically the data are transformed as implied by the first order process and least squares procedures are applied to the transformed data. The resulting estimates usually differ from the generalized least squares estimates due to the neglect of the initial (or "border") observation but the loss of efficiency is often small. This loss of efficiency can be avoided by a simple modification of the customary transformation which makes the transformation used completely consistent with the covariance matrix. Even if the first order autocorrelation coefficient is unknown the estimation procedures are not unduly complicated.

As one might expect, the situation is not quite as simple in spatial estimation problems. The alternative hypotheses for which the Moran test is the most powerful are not well known. This ignorance is due in part to the fact that the test is not a single test but a whole class of tests.

Generalized least squares still provides an appropriate estimation framework, but to be made operational this framework must be combined with some assumption about the nature of the interdependence among the disturbances. There are at least three ways of characterizing this interdependence. One can, as Berry [1] suggests, describe the interdependence in terms of the diffusion process believed to generate it. Whittle in his classic work [8] used autoregressive schemes to describe the interdependence among observations corresponding to the points of a rectangular plane lattice. Fisher [5] used the covariance matrix to characterize the interdependence.

Theoretically, these ways of characterizing the interdependence are just three ways of describing exactly the same situation. However, in practice this is not likely to be the case. Once one of these characterizations is chosen, the principle of Occam's razor suggests that a simple, first order, form of that characterization should be used. But a simple form of one characterization is often consistent only with complicated versions of the other two characterizations. This point may be illustrated by considering the simple autoregressive scheme:

$$v_i = C(v_{iN} + v_{iS} + v_{iE} + v_{iW}) + e_i \tag{3}$$

where

v_i, v_{iN}, v_{iS}, v_{iE}, v_{iW} are the disturbance terms associated with the i^{th} observation, and the observations directly north, south, east, and west, respectively, of the i^{th} observation. The subscript notation suggests that the observations correspond to points of a rectangular plane lattice.

e_i is a "transformed" disturbance term associated with the i^{th} observation. e_i is assumed to have a zero expected value and unit variance for each value of i. It is assumed that the covariance of e_i and e_j is zero unless i equals j.

C is a parameter of the autoregressive scheme which is restricted to values in the open interval from -.25 to .25.

This autoregressive scheme is probably the simplest non-degenerate two dimensional scheme. It is possible to transform this scheme into its moving average representation. This moving average representation could under some conditions be equivalent to the diffusion process consistent with the autoregressive scheme. In either case v_i would be expressed as a linear function of an infinite number of the e's. Van der Pol and Bremmer [7, p. 367] have provided an integral form which (with minor corrections and conversion from an improper to a proper integral) can be used to generate the coefficients of this moving average scheme. The leading (largest) coefficients are presented in Table 1 for C equal to .1 and .2. The values of these coefficients are not intuitively obvious nor are these coefficients related in any obvious way to C. Similar statements can be made about the covariances of the v_i's. Selected elements of this covariance matrix are presented in Table 2.

Given the propensity to start with a simple assumption, these three characterizations lead to three different approaches having different observational implications. Ideally the choice among these approaches could be made in part on empirical bases, but the difficulties of deriving the observational implications and the irregularities (of size, location and structure) of the observational units are likely to frustrate any attempt to distinguish among these approaches empirically. Thus it appears that the choice must usually be made on the basis of intuitive plausibility and operational considerations.

Diffusion processes appear to have the edge as far as plausibility is concerned, since they provide simple, mechanistic explanations of the problem. They also have implications for spatial analysis which extend beyond explaining autocorrelated errors. If one believes that the interdependence of the disturbance terms is the result of a diffusion process which has distributed the effects of omitted variables spatially, he is likely to suspect that effects of the "independent" variables included in the relationship to be estimated may also be diffused or distributed spatially.

It is less clear as to which approach has the operational advantage. Perhaps it is the covariance approach. It does not require spatial stationarity since it allows the covariance between one pair of observations to be different from the covariance of a second pair when elements of these pairs are separated by the same distance. Yet this very flexibility has its disadvantages. It may make it more difficult to arrive at reasonable variances and covariances which lead to a positive definite matrix. Furthermore, the potential of using sample residuals to help estimate the matrix or to test the accuracy of the matrix elements is limited in the absence of

TABLE 1: Selected Coefficients of the Moving Average Representation of a Simple Autoregressive Scheme

m	p	Moving Average Coefficients[a] C = .1	C = .2
0	0	1.04406	1.27025
1	0	.11014	.33781
1	1	.02275	.16006
2	0	.01186	.09868
2	1	.00359	.06234
2	2	.00074	.03009
3	0	.00130	.03092
3	1	.00051	.02288
3	2	.00013	.01289
3	3	.00003	.00628
4	0	.00015	.01017
4	1	.00007	.00824
4	2	.00002	.00519
4	3	.00000	.00281
4	4	.	.00137
5	0	.00002	.00346
5	1	.00001	.00296
5	2	.00000	.00202
5	3	.	.00119
5	4	.	.00063
5	5	.	.00031
6	0	.	.00120
6	1	.	.00106
6	2	.	.00077
6	3	.	.00049
6	4	.	.00028
6	5	.	.00014
6	6	.	.00007
7	0	.	.00042
7	1	.	.00038
7	2	.	.00029
7	3	.	.00019
7	4	.	.00012
7	5	.	.00006
7	6	.	.00003
7	7	.	.00002

[a]The mp^{th} coefficient is associated with the "transformed" disturbance term located m units east and p units north of v_i. The moving average representation is symmetric.

TABLE 2: Covariances of v_i and Disturbance Terms Located m Units East and p Units North of v_i

m	p	Covariances	
		C = .1	C = .2
0	0	1.14132	2.25708
1	0	.24317	1.23354
1	1	.07452	.82677
2	0	.03992	.56802
2	1	.01572	.43323
2	2	.00406	.25631
3	0	.00594	.24670
3	1	.00281	.20334
3	2	.00005	.13230
3	3	.00021	.07445
4	0	.00084	.10418
4	1	.00046	.09005
4	2	.00016	.06299
4	3	.00004	.03814
4	4	.00001	.02089
5	0	.00012	.04326
5	1	.00007	.03854
5	2	.00003	.02848
5	3	.00001	.01833
5	4	.00000	.01065
5	5	.	.00574
6	0	.00002	.01776
6	1	.00001	.01614
6	2	.00000	.01244
6	3	.	.00842
6	4	.	.00515
6	5	.	.00291
6	6	.	.00155
7	0	.	.00722
7	1	.	.00666
7	2	.	.00531
7	3	.	.00374
7	4	.	.00239
7	5	.	.00142
7	6	.	.00079
7	7	.	.00042

any assumptions (except positive definiteness) about the structure of the matrix. The covariance approach also avoids the "border" problem since all available observations can be used for estimation purposes. For large problems, however, it means that a fairly large covariance matrix must be inverted or a transformation consistent with the covariance matrix must be found.

On the other hand the autoregressive and diffusion process approaches lead directly to simple transformations valid for all but the "border" observations. Unfortunately, the number of "border" observations may be moderately large. These observations can be salvaged but the problem of finding the transformations appropriate for the "border" observations can be troublesome.

Autocorrelation of the Disturbances of an Economic Base Model

Many relationships have been studied using cross section data. Braschler [2] has used census data to examine and predict changes in employment in Missouri counties. The models used have been extensively tested using 1950 and 1960 data and will soon be updated using 1970 data. Autocorrelated disturbances seemed possible but no test had been made. Since this model will be used in the future, it seems appropriate to test for autocorrelated disturbances. It is likely that this problem is somewhat persistent. Thus if the disturbances were interdependent in 1950 and 1960 they are likely to be interdependent in 1970 as well.

The basic model used is of the economic base type and assumes that total employment in a region is linearly related to employment in the basic exogenous sectors in that region.

Several versions of the basic model were used in this study. These versions differed only with respect to the specification of the variables used. All of them have the general form:

(4) $$E_i = aX_i + b_1 AG_i + b_2 MAN_i + v_i$$

For version 1:

E_i = total employment in the i^{th} county in 1950,

X_i = 1 for all counties ($i=1, \ldots, 114$),

AG_i = agricultural employment in the i^{th} county in 1950,

MAN_i = manufacturing employment in the i^{th} county in 1950,

v_i is the disturbance term associated with this relationship for the i^{th} county in 1950, a = is the intercept coefficient,

b_1 and b_2 are employment multipliers for the agricultural and manufacturing sectors, respectively.

Version II is the same as version I except that 1960 census data was used. The primary reason for using these two versions was to obtain at least a crude measure of the persistence over time of the dependence existing among the error terms.

For version III the variables are defined as the change in employment between 1950 and 1960. Thus version III represents the difference between version I and version II, assuming that the multipliers (but not the intercept coefficient) were the same in both 1950 and 1960. The potential advantages of version III are two-fold. First, the observational units used vary somewhat in size and economic structure. The formulation of versions I and II assumes that the intercept coefficient and the multipliers are the same for all counties. Version III allows each county to have a unique intercept in 1950 and 1960 and merely requires that the change in the intercept from 1950 to 1960 be the same for all counties. If the differences in the counties can be reflected in the intercept terms and if these differences are persistent over time then version III would be an improvement. Second, while the relationship between the basic sectors and total employment at a given time are of considerable interest, the relationship between changes in basic employment and changes in total employment are often of greater interest.

In version IV, X_i is set equal to 1950 employment. In most applications version III has been fitted to fairly homogeneous groups of counties. When the observational units are all of approximately the same size, as measured perhaps by employment or population, the intercept term may be meaningful. Its interpretation is less clear when heterogeneous observational units are used. Part of this meaning may be restored by redefining X_i as outlined above.

Testing for Autocorrelated Disturbances

The residuals of these formulations were used to test for interdependence among the disturbance terms. The W matrix was constructed by assuming that the disturbances of adjacent counties would be most highly correlated. Other measures of proximity such as travel time, similarity of the counties, etc. are viable alternatives for selection of most correlated disturbances but were not used here. Missouri's counties form a fairly regular pattern but deviate considerably from a hypothetical lattice or checkerboard pattern. The number of counties considered adjacent to any one county was limited to four by selecting one county in each of the four cardinal directions. In the case of a county having two counties to its north (or south or east or west) the one having the largest common boundary was selected. Border counties thus had at most three adjacent counties. W_{ij} was arbitrarily set equal to 1 if county j is adjacent to county i and set equal to 0 otherwise. The W matrix was thus almost symmetric.

The results obtained are presented in Table 3. The least squares estimates of a, b_1, and b_2 are presented in the first three columns of the table. The standard errors of these estimates are presented in parentheses directly below the estimates. These standard errors were of course calculated using standard formulas and thus are probably too small. The fourth and fifth columns present the numerator of the test statistic, r, and the residual sums of squares, respectively. The sixth column presents values calculated for r. The estimates of the mean and standard deviation of r under the null hypothesis are presented in the seventh and eighth columns. The last column presents the z statistic obtained by dividing the difference between r and $\bar{r}$ by the estimated standard deviation.

These results support the various hypotheses to varying degrees. Positive autocorrelation seems more likely, but negative spatial correlation of the disturbances is possible. Thus a two-tailed test is appropriate. By consulting a table of percentage points for the normal distribution it can be seen that all but one of the "z" statistics are significantly different from zero at the five per cent level of significance.

The results also suggest that the spatial correlation of disturbances is persistent over time. The numerator of r is about the same for version I and II but is considerably smaller for versions III and IV. These facts are consistent with the persistence hypothesis but could be partly attributable to other factors as well.

Correcting for the Interdependence Among the Disturbances

As outlined above there are several ways of dealing with the interdependence among the disturbance terms. For this paper an autoregressive approach was adopted. Specifically, it was assumed that the dependence among the error terms is consistent with the symmetric autoregressive scheme presented above.

C is an unknown parameter which is permitted to have values in the range -.25 to .25. If one aims to more efficiently estimate the parameters a, b_1, and b_2 by using the autoregressive transformation an estimate of C must be obtained. In this study Whittle's method [8] was used. As applied to the problem at hand his method involves the selection of alternative values for C, transforming the variables E_i, X_i, AG_i, and MAN_i (and thus implicitly v_i) as suggested by the autoregressive transformation, and then using the resulting residual sums of squares, U(C), to determine the "best" value of C. Whittle has shown the maximum likelihood estimates of C are those which minimize K(C) times U(C). K(C) is a complex function (attributable to the fact that the Jacobian of the transformation from the e_i's to the v_i's is not equal to one) which has a value of 1 for C = 0, increases as C becomes different from zero, and tends to infinity as C tends to $\pm$.25. Fortunately, K(C) is easy to approximate except for values of C near $\pm$.25.

C was assigned values of 0, .025, .05, . . . , .225, and the resulting values of K times U were calculated. The value of C which gave the lowest value for KU was selected. A finer (intervals of .005) grid symmetric around

TABLE 3: Parameter Estimates, Standard Errors and Test Statistics for Various Versions of the Economic Base Model

Version	Estimates of:			Test Statistics					
	a	b_1	b_2	$\hat{v}'W\hat{v}$	$\hat{v}'\hat{v}$	r	$\bar{r}$	$S(r)$	z
				(1,000,000's)	(1,000,000's)				
I	-111.3492 (1024.6590)	1.4174 (.4081)	3.8117 (.0615)	1252.1511	2068.8872	.1703	-.0139	.0691	2.6654
II	-170.2653 (1169.2235)	2.0369 (.8224)	3.3610 (.0535)	1377.7681	3325.7332	.1166	-.0145	.0691	1.8975
III	382.9192 (225.2562)	1.4237 (.1990)	2.7836 (.0256)	51.2792	112.6492	.1281	-.0149	.0692	2.0566
IV	-.0125 (.0049)	1.0508 (.0878)	2.8436 (.0343)	79.7377	109.0966	.2057	-.0087	.0695	3.0870

this value of C was then constructed except for versions I and II. The value of C which minimized KU was then selected as the estimate of C.

The results are presented in Table 4. Two sets of results are presented for each version; results are presented both for C = 0 and for C equal to the estimated value. Use of the transformation procedure reduced the number of available observations by about forty per cent; thus for values of C tending to zero the parameter estimates need not converge to the results presented in Table 3. The standard errors shown in parentheses immediately beneath the estimates were calculated in the usual way; thus all of them tend to be too small. Those associated with C = 0 are probably small because the interdependence among the disturbance terms was neglected in their calculation. Those associated with estimated values of C were calculated as if the estimated value of C was known to be the true value. For large samples the estimator of C is independent of the estimators of the other parameters. Ignoring the interdependence which exists in small samples tends to underestimate the standard errors.

The values of KU corresponding to C = 0 and C equal to its estimated value, respectively, provide information which can be used in an alternate test of the hypothesis that C = 0. The chi-square statistic (one degree of freedom) presented in the last column of Table 4 was calculated using the formula

$$(5) \qquad (n-4)(\ln K(0)U(0) - \ln K(\hat{c})U(\hat{c}))$$

where

$n-4$ is the number (71) of transformed observations minus the number (4) of parameters estimated,

$\ln K(0)U(0)$ and $\ln K(\hat{c})U(\hat{c})$ are the natural logarithms of KU evaluated at C equal 0 and C equal to its estimated value, respectively.

The calculated chi-square statistics for versions III and IV are greater than tabulated values corresponding to levels of significance of ten per cent and one per cent, respectively.

Conclusions and Implications

The results presented above suggest the existence of spatial autocorrelation among the disturbances of the economic base models applied to 1950 and 1960 Missouri county data. They also suggest that this pattern of interdependence may be stable over time. Thus the potential for interdependent disturbances must be recognized when updating this model or when estimating the parameters of related models. The results also suggest that at least some of the interdependence which exists between the disturbance terms is consistent with a first order autoregressive scheme.

TABLE 4: Parameter Estimates, Standard Errors and Test Statistics Obtained Using a Simple Autoregressive Transformation

Version	Estimates of: a	b_1	b_2	C	Test Statistics: KU	Chi-square
					(1,000,000's)	
I	-58.7518 (761.0707)	1.3703 (.3231)	4.1803 (.3080)	0	413.4585	
	142.3078 (833.2928)	1.2137 (.3472)	4.4355 (.3225)	.05	403.8289	1.576
II	416.1054 (683.7889)	1.1261 (.5052)	3.9202 (.1936)	0	436.6117	
	833.2570 (807.0985)	.7057 (.5710)	4.0702 (.2068)	.075	419.9106	2.615
III	600.3192 (314.0458)	1.7104 (.2996)	3.0878 (.1849)	0	78.8579	
	724.4145 (355.7464)	1.8154 (.3258)	3.1069 (.1926)	.075	75.1809	3.1987
IV	.0304 (.0414)	1.3206 (.2149)	2.9229 (.3005)	0	82.4439	
	-.0616 (.0183)	1.3423 (.2499)	3.3069 (.2059)	.1625	73.2333	7.9391

The implications for estimation procedures are more limited. The "problem" of interdependent disturbances can be "solved" in many ways. Not all of these methods have been discussed above. Other remedies include reduction of the number of observations by aggregation of counties, for example, into larger multi-county regions. It appears that this remedy would indeed solve part of the "problem". Versions III and IV were fitted to data from thirty-five regions formed by a very arbitrary aggregation of counties. The resulting "z" statistics were negative, small in absolute value, and very insignificant. A second alternative remedy involves the identification, measurement, and inclusion (in the model) of those omitted variables causing the interdependence. This remedy is often harder to apply but may reduce the interdependence and may also reduce the probability of specification bias.

The "problem" is not really the elimination of the interdependence, but rather is that of exploiting this interdependence to obtain more efficient estimators. When viewed in that framework many of the available remedies appear to be less effective. Aggregation, improved specification, and the autoregressive transformation approach all give up part of the potential efficiency gains by reducing the number of observations actually and effectively available to estimate the parameters of the model.

Part of this loss of efficiency due to the reduction of the effective number of observations is more apparent than real; another part of this loss is avoidable. In the case of the model used in this study this loss could have been avoided or at least offset by using data for those counties of other states which border Missouri counties. The data from these observations could then be used to transform the observations corresponding to the "border" counties. An alternative way of salvaging these "border" observations is available using information contained in the inverse of the covariance matrix for the disturbances. Examination of this matrix for selected values of C suggests that it should not be too hard to salvage the border observations even without collecting additional data. The inverses examined were approximations and were based on the rectangular lattice assumption so it seems appropriate to ignore all but the first or second order modifications which they suggest. Thus the transformation for most "interior" counties would remain the same, minor adjustments might be in order for counties not on the "border" but adjacent to "border" counties, and a transformation not very different from the intuitively reasonable transformation (i.e., the transformation obtained by ignoring the observations on one or two sides) would be in order for the border counties.

This issue is further complicated by the fact that interdependence of the disturbances is not the only troublesome factor to be reckoned with in cross sectional estimation. In the case of the model used in this paper the number employed in each sector is reported on the basis of residence rather than place of employment. In general one would expect "errors of observation" of this and other types partly because the measurements chosen for the variables do not correspond exactly to their theoretical counterparts and partly because of errors due to the procedure used to estimate the variables themselves. Heterogeneity of disturbance variances and cross-correlation of the independent variables and the disturbance terms are also likely to be common in models designed for cross sectional analysis. The necessity to be concerned about problems or potential problems of this sort is likely to reduce the attention that can be paid to interdependent disturbances and may also influence the methods chosen to deal with this interdependence.

Finally, it must be remembered that the empirical results presented above are not independent. The four versions employed are variants of the same model and their estimates and test statistics were all calculated from the same basic data. The results have various implications for related models estimated under similar circumstances, but these implications may not be valid for other models since the diffusion processes and other forces leading to interdependent disturbances may be quite different for other relationships.

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