URBAN TRANSPORTATION: CONGESTION, WELFARE, AND OPTIMAL PRICING*

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Vickery has stated that "In no major area are pricing practices so irrational, so out of date, and so conductive to waste in urban transportation. Two aspects are particularly deficient: the absence of adequate peak-off differential and the gross underpricing of some modes relative to others" [4, p. 452].

Our task here is first to review briefly the pricing policy of public facilities such as transportation, and then to examine the optimal pricing strategy given the present pricing practices in transportation.

We believe that the pricing policy of public facilities should be not only to achieve efficient allocation of scarce resources, but also to promote social welfare. Pricing in transportation should then be used as a possible means to direct a smooth flow of traffic and to redistribute income.

Therefore, in order to give a meaningful discussion of the pricing policy of public facilities such as transportation we should put together all three strands of economic disciplines: welfare economics, public finance, and regulatory institution.

As it is well known, congestion results from not using a proper price mechanism: a too low price is charged so that an excessive number of road users is on the roads. This would imply inefficient allocation of road users, for congestion results, from too many users during peak-hours and fewer users during off-peak hours.

If we can persuade the peak-hour users to switch to the off-peak-hour users, we may be able to solve the problem of congestion. This is where the price mechanism comes in. (Of course, here we are ignoring the fact that people have to use roads during peak periods to get to work.) A relatively

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high price charged during the peak periods and a relatively lower price during the off-peak periods would accomplish the transfer of some of the peak-period users to the off-peak period users.

The proper use of the price mechanism will thus solve the problem of congestion, but it will create another formidable problem -- the problem of inequity. A higher price charged to the peak-period users reduces congestion, but at the same time prevents the poor from using the roads. We can conceive a situation where the peak period road use is limited only to the rich. The poor will be forced to give up the use of their automobiles and to use mass transit.

We face, therefore, a problem -- a choice of various combinations of efficiency and equity. What we should aim at, in this regard, is to set price so as to maximize welfare and to achieve allocative efficiency.

We will start out, therefore, with a more general social welfare function, i.e., a linear function of individual utility functions with weights. With this formulation we explicitly take into consideration the fact that society does not value equally a dollar received by its members.

Our aim is then to accomplish Pareto-Optimality by setting up proper price levels and by redistributing income. With the above formulation of the social welfare function, we can argue that the movement toward a more efficient allocation of resources would be necessarily a good thing only if accompanied by a suitable redistribution of income.

For achieving Pareto-Optimality, marginal cost pricing has been proposed. Pricing practices used in today's traffic, however, do not follow marginal cost pricing.

Peaking of demand and limited capacity of highways make it impossible to adjust price to continuously varying marginal cost, more specifically, marginal social cost. Also revenues are normally required to cover cost for operating, maintaining, and expanding traffic facilities. Moreover some transportation mode is not paying its marginal social cost as discussed below.

We are therefore faced with a problem involving maximization of social welfare in the presence of many added constraints. That is, we are dealing with a problem in the area of the second best. In such a case prices which deviate from marginal cost in a systematic manner will be required for Pareto-Optimality. In what follows various second best cases are discussed, indicating what level price should be and how income should be redistributed.

We present argument in terms of choice between automobile trip and bus trip in the following, but this argument can well be presented in terms of choice between peak-period trip and off-peak period trip.

Theoretical Framework

In this economy there are n consumers ($i = 1, 2, \ldots n$), and each derives satisfaction from travelling t_1^i, auto travel, t_2^i, bus travel, and from consuming units of composite commodity x^i. Each consumer possesses a utility function of the form

$$(1) \qquad u^i = u^i \; (t_1^i, t_2^i, x^i)$$

Utility functions are quasi-concave, continuous, and twice differentiable.

This economy has two modes of transportation, whose facilities are provided by public or semi-public agencies.

Average input quantity required to provide a unit of t_j is represented by composite variables (including fuel, tires, vehicles, etc.)[1] g, i.e.,

$$(2) \qquad g_1 = g_1 \; (t_1, t_2) \quad \text{and} \quad g_2 = g_2 \; (t_1, t_2)$$

g_j is a convex twice continuously differentiable input function of total traffic t. In the above formulation we have explicitly taken into consideration the interdependent congestion: automobile trip contributing to the congestion of bus trip and vice versa.

The total amount of resources used for travelling is G

$$(3) \qquad G = g_1 \; t_1 + g_2 \; t_2$$

G and X are related according to the following transformation function:

$$(4) \qquad f(G, X) = 0$$

The price of g in terms of X is denoted by π. With the price P_j per unit of t_j, X as numeraire, and y^i as the i^{th} consumer's income, the budget constraint of the i^{th} individual is

$$(5) \qquad P_1 \; t_1^i + P_2 \; t_2^i + x^i = y^i$$

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[1]The input commodity, g, may include consumers' time as well. Then in order to preserve a single price we assume equal valuation of time by all consumers.

Now our task is to maximize social welfare -- a linear function of individual utility functions with weights β^i. That is:

$$(6) \qquad \text{Max } Z = \Sigma\, \beta^i\, u^i\, (t_1^i,\, t_2^i,\, x^i)$$

subject to various constraints in order to find price levels and income redistribution.

<u>Case I. Two Travel Modes Model Without Special Constraints</u>. Each consumer has a choice of purchasing various combinations of t_1, t_2, and x. Our problem here is to find optimal levels of P_1 and P_2 in terms of the numeraire X and the optimal condition for income redistribution that will give the maximum social welfare. We can formulate our problem as follows:

$$(7) \qquad \text{Max } Z = \Sigma\, \beta^i\, u^i\, (t_1^i,\, t_2^i,\, x^i)$$

subject to

$$(8) \qquad \Sigma\, t_1^i = t_1 \qquad\qquad \mu_\beta$$

$$(9) \qquad \Sigma\, t_2^i = t_2 \qquad\qquad \mu_\lambda$$

$$(10) \qquad g_1\, t_1 + g_2\, t_2 = G \qquad\qquad \mu_\gamma$$

$$(11) \qquad \Sigma\, x^i = X \qquad\qquad \mu_x$$

$$(12) \qquad f(G,\, X) = 0 \qquad\qquad \mu_\phi$$

where the Greek letters at the right denote the Lagrange multipliers associated with the constraints. The first order conditions for a maximum are:

$$(13) \qquad \Sigma\, \beta^i\, \lambda^i\, t_1^i - \mu_\beta\, \Sigma\, \frac{\partial t_1^i}{\partial P_1} - \mu_\lambda\, \Sigma\, \frac{\partial t_2^i}{\partial P_1} - \mu_x\, \Sigma\, \frac{\partial x^i}{\partial P_1} = 0$$

$$(14) \qquad \Sigma\, \beta^i\, \lambda^i\, t_2^i - \mu_\beta\, \Sigma\, \frac{\partial t_1^i}{\partial P_2} - \mu_\lambda\, \Sigma\, \frac{\partial t_2^i}{\partial P_2} - \mu_x\, \Sigma\, \frac{\partial x^i}{\partial P_2} = 0$$

$$(15) \qquad \mu_\beta - \mu_\gamma \{ g_1 + t_1 \frac{\partial g_1}{\partial t_1} + t_2 \frac{\partial g_2}{\partial t_1} \} = 0$$

$$(16) \qquad \mu_\lambda - \mu_\gamma \{ g_2 + t_2 \frac{\partial g_2}{\partial t_2} + t_1 \frac{\partial g_1}{\partial t_2} \} = 0$$

$$(17) \qquad \mu_\gamma - \mu_\phi \frac{\partial f}{\partial G} = 0$$

$$(18) \qquad \mu_x - \mu_\phi \frac{\partial f}{\partial X} = 0$$

$$(19) \qquad \beta^i \lambda^i - \mu_\beta \frac{\partial t_1^i}{\partial y_i} - \mu_\lambda \frac{\partial t_2^i}{\partial y_i} - \mu_x \frac{\partial \chi^i}{\partial y_i} = 0$$

We use these first order conditions and the following relations (20), (21), (22), and (23)

$$(20) \qquad S_{kj} = \sum_{i=1}^{n} \left(\frac{\partial t_k^i}{\partial P_j} - t_k^i \frac{\partial t_k^i}{\partial y^i}\right)$$

for $k = 1, 2, 3, \quad j = 1, 2$

where S_{kj}'s are the Hicks-Slutsky pure substitution effect of the price change in P_j on t_k. Of course, when $k = 3$, (20) is

$$(21) \qquad S_{3j} = \sum_{i=1}^{n} \left(\frac{\partial x^i}{\partial P_j} - \chi^i \frac{\partial x^i}{\partial y^i}\right) \quad j = 1, 2$$

$$(22) \qquad \frac{\mu_r}{\mu_x} = - \frac{dX}{dG} = \pi$$

where π is the price of g in terms of X. And also

$$(23) \qquad P_1 S_{11} + P_2 S_{21} + S_{31} = P_1 S_{12} + P_2 S_{22} + S_{32} = 0$$ [2]

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[2] See [2, p. 105].

From these equations (identities) and the first order conditions we obtain finally

$$(24) \qquad \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix} \times \begin{bmatrix} MC_1 - P_1 \\ MC_2 - P_2 \end{bmatrix} = 0$$

where

$$(25) \qquad MC_1 = \pi \; \{g_1 + t_1 \frac{\partial g_1}{\partial t_1} + t_2 \frac{\partial g_2}{\partial t_1}\}$$

$$(26) \qquad MC_2 = \pi \; \{g_2 + t_2 \frac{\partial g_2}{\partial t_2} + t_1 \frac{\partial g_1}{\partial t_2}\}$$

They represent the marginal social cost of the trips. The first matrix on the left is positive.[3] We can conclude then that

$$(27) \qquad MC_1 = P_1 \quad \text{and} \quad MC_2 = P_2$$

And we can show that the condition for optimal redistribution of income is

$$(28) \qquad \beta^i \lambda^i = \beta^j \lambda^j$$

where λ's and β's, respectively, represent the marginal utilities of income and their social weights. We obtain (28) only when $P_i = MC_i$. The relation (28) implies that an additional dollar of income spent by any consumer incurs a social cost exactly equal to unity, and that each consumer gets the same level of satisfaction from the additional dollar of income spent.

All of these results are expected -- maximized social welfare, optimal distribution of resources, and optimal redistribution of income -- in an ideal economy where price reflects marginal social cost and perfect competition prevails. In the following cases we look into more realistic situations where price does not reflect marginal social cost and/or price is required to deviate from marginal social cost in order to render a higher social welfare.

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[3]See [2, p. 106]. This is the second order condition for maximization.

Case II. The Optimal Toll in Two Travel Modes Model. Assuming that because of congestion they create the marginal social cost of the automobile users is higher than that of the bus users, our problem here is to find the optimal toll, ρ, for the automobile users.

We can formulate this problem as follows:

(29) $\quad \text{Max } Z = \Sigma\, \beta^i\, u^i\, (t_1^i,\, t_2^i,\, x^i)$

subject to

(30) $\quad P_1 = P_2 + \rho \qquad\qquad \mu_x$

(31) $\quad \Sigma\, t_1^i = t_1 \qquad\qquad \mu_\beta$

(32) $\quad \Sigma\, t_2^i = t_2 \qquad\qquad \mu_\lambda$

(33) $\quad g_1\, t_1 + g_2\, t_2 = G \qquad\qquad \mu_\gamma$

(34) $\quad \Sigma\, x^i = X \qquad\qquad \mu_x$

(35) $\quad f(G,\, X) = 0 \qquad\qquad \mu_\phi$

where P_1 is the price of automobile travel and P_2 is the price of bus travel.

Since the derivation of the above with respect to ρ gives $\mu_{\lambda} = 0$, the first order conditions are the same as Case I. That is

(36) $\quad MC_1 = P_1$

(37) $\quad MC_2 = P_2$

(36) - (37) will give

(38) $\quad \rho = MC_1 - MC_2 > 0$

The price of automobile use is higher than that of bus use exactly by the amount of the discrepancy between the two marginal social costs. That is, automobile users should pay a toll of ρ.

The condition for optimal income redistribution is the same as Case I.

Case III. The Deficit Constraints in Two Travel Modes Model. Here we face the situation where government (or semi-government) agencies providing facilities for bus and automobile transportation set a deficit limit. For the prices the transportation users pay do not cover the cost of operating and maintaining the facilities.[4] Since this case has been discussed intensively by Baumol and Bradford [1] among others, we present only conclusions:

$$MC_1 < P_1 \quad \text{and} \quad MC_2 < P_2$$

That is the prices of the travels should be set higher than the marginal social cost.

The condition for the optimal income redistribution in this case is:

$$W^i \lambda^i = W^j \lambda^j$$

where

$$W^i = \frac{\beta^i}{\frac{\partial t_k^i}{\partial y^i} (MC_1 - P) + 1}$$

where k = 1 and 2; 1 for auto users and 2 for bus users. The implication of this optimal condition is that income redistribution would favor those who would concentrate their marginal spending on commodities whose marginal cost is low, i.e., bus travel.

Case IV. The Optimal Subsidy in Two Travel Modes Model. In this situation the automobile users are paying only their private cost, i.e., average cost rather than marginal social cost, ignoring the cost they are imposing on their fellow travelers and society. In this case, as the following analysis will show, the bus users are rewarded subsidy for their participation in mass transit to avoid congestion. Our problem is to find an optimal subsidy to the bus users explicitly taking into consideration congestion interdependence between the two travel modes, automobile and bus.

We formulate this problem as follows:

(39) $$\text{Max } Z = \Sigma \beta^i u^i (t_1^i, t_2^i, x^i)$$

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[4] The case of the cost of expanding facilities is discussed later.

subject to

(40) $P_1 = \pi\, g_1$ $\quad\quad \mu_\alpha$

(41) $P_2 = \pi\, g_2 + \rho$ $\quad\quad \mu_\tau$

(42) $\Sigma\, t_1^i = t_1$ $\quad\quad \mu_\beta$

(43) $\Sigma\, t_2^j = t_2$ $\quad\quad \mu_\lambda$

(44) $g_1\, t_1 + g_2\, t_2 = G$ $\quad\quad \mu_\gamma$

(45) $\Sigma\, x^i = X$ $\quad\quad \mu_x$

(46) $f(G, X) = 0$ $\quad\quad \mu_\phi$

From the first order conditions we derive:

$$(47) \quad \begin{bmatrix} MC_1 - P_1 \\ \\ MC_2 - P_2 \end{bmatrix} = \frac{\mu_\alpha}{\mu_x} \; \frac{\pi \dfrac{dg_1}{dt_1} - \dfrac{S_{22}}{S}}{\pi \dfrac{dg_1}{dt_2} + \dfrac{S_{12}}{S}}$$

And for the optimal ρ, we derive

$$(48) \quad \rho = MC_2 - \pi\, g_2 - \frac{\left\{\pi \dfrac{dg_1}{dt_2} + \dfrac{S_{21}}{S}\right\}}{\left\{\pi \dfrac{dg_1}{dt_1} - \dfrac{S_{22}}{S}\right\}} (MC_1 - \pi\, g_1)$$

For $\rho < 0$, i.e., an optimal study is obtained under the following inequality holds:

$$(49) \quad \frac{\dfrac{t_1}{g_1}\zeta_{12} + \dfrac{t_2}{g_1}\zeta_{22}}{\dfrac{t_1}{g_2}\zeta_{11} + \dfrac{t_2}{g_1}\zeta_{21}} < \frac{\eta_{12}}{\eta_{22}}$$

where ζ_{ij} is cost elasticity defined as

$$(50) \qquad \zeta_{ij} = \frac{\partial g_i}{\partial t_j} \frac{t_j}{g_i}$$

and η_{ij} compensated demand elasticity,

$$(51) \qquad \eta_{ij} = s_{ij} \frac{p_j}{t_i}$$

Interpretation of (49) for an optimal subsidy is not easy to make, but following R. Sherman's assumption [3, p. 567], i.e.

$$\frac{\dfrac{\partial g_1}{\partial t_2}}{\dfrac{\partial g_1}{\partial t_1}} = \frac{\dfrac{\partial g_2}{\partial t_2}}{\dfrac{\partial g_2}{\partial t_1}}$$

Namely assuming the same relative marginal contribution from each travel model in the congestion of each other, we can write (49) as follows:

$$\frac{\zeta_{12}}{\zeta_{11}} = \frac{\zeta_{22}}{\zeta_{21}} < \frac{\eta_{12}}{\eta_{22}}$$

That is, subsidy should be paid to the bus users when a compensated increase in bus fare P_2 causes a greater relative increase in congestion in both automobile travel and bus travel through an induced increase in automobile travel than a decrease due to a reduced use in bus travel.

The condition for optimal income redistribution is the same as Case III.

<u>Case V. The Capacity Constraint in Two Travel Modes Model</u>. Here we introduce the capacity constraint. Highway space is limited thus causing untolerable delay and congestion. We assume that too many automobiles on highways are causing the congestion. Our problem here is to find the optimal pricing under such highway capacity constraint.

We formulate this problem as follows:

$$(52) \qquad \text{Max } Z = \Sigma\, \beta^i u^i \, (t_1^i, t_2^i, x^i)$$

subject to

(53) $\Sigma \; t_1^i = t_1$ $\qquad \mu_\beta$

(54) $\Sigma \; t_2^i = t_2$ $\qquad \mu_\lambda$

(55) $g_1 \; t_1 + g_2 \; t_2 = G$ $\qquad \mu_\gamma$

(56) $t_1 = K$ $\qquad \mu_\psi$

(57) $\Sigma \; x^i = X$ $\qquad \mu_x$

(58) $E = E \; (K)$ $\qquad \mu_\varepsilon$

(59) $f(G, X, E) = 0$ $\qquad \mu_\phi$

where K is a constant representing the capacity of highways. E represents the amount of resources used for construction and expansion of highways.

$$g_1 = g_1 \; (t_1, t_2, K) \quad \text{and} \quad g_2 = g_2 \; (t_1, t_2, K)$$

Under these conditions we derive from the first order conditions and other identities discussed before:

$$\text{(60)} \quad \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} \dfrac{\mu_\beta}{\mu_x} & - P_1 \\ \dfrac{\mu_\lambda}{\mu_x} & - P_2 \end{bmatrix} = 0$$

Since the first matrix is nonsingular, we conclude that

(61) $P_1 = \dfrac{\mu_\beta}{\mu_x}$ and (62) $P_2 = \dfrac{\mu_\lambda}{\mu_x}$

But

$$(63) \quad \frac{\mu_\beta}{\mu_x} = \pi \{g_1 + t_1 \frac{\partial g_1}{\partial t_1} + t_2 \frac{\partial g_2}{\partial t_1}\} + \pi \{t_1 \frac{\partial g_1}{\partial K} + t_2 \frac{\partial g_2}{\partial K}\} + \frac{\partial X}{\partial K}$$

$$(64) \quad \frac{\mu_\lambda}{\mu_x} = \pi \{g_2 + t_1 \frac{\partial g_1}{\partial t_2} + t_2 \frac{\partial g_2}{\partial t_2} = MC_2$$

Therefore from (62) and (64) we can say that $P_2 = MC_2$, i.e., P_2 bus fare being equal to its marginal social cost, but from (61) and (63) P_1 is higher than MC_1 which is the first term of the right-hand side of (63). The second term is the cost-saving derived from capacity expansion ($\frac{\partial g_1}{\partial K}$ and $\frac{\partial g_2}{\partial K}$ are negative). The third term is the real cost of expansion. We can write therefore that

(65) P_1 = the marginal cost of t_1 + the Cost of Expansion

That is, the automobile users should be paying not only their marginal social cost but also the expansion cost. The expansion cost is, however, lower than the real cost of expansion by the amount of reduced cost of congestion.

Concluding Remarks

In the above discussion we have shown various second best solutions in the current transportation service. It was shown that given today's pricing policy of transportation service, a strict adherence to the marginal cost pricing principle would not yield Pareto-Optimality. Rather a systematic deviation from marginal cost is required for the attainment of Pareto-Optimality.

As shown, when the users of one transportation mode fail to pay its real social cost, but also impose as a form of congestion greater external dis-economies of a substitute mode, then the optimal (second best) price for that substitute mode will lie below its marginal cost and sometimes even below average cost, and may require a subsidy.

What is not discussed in this paper is the interdependence between peak periods and off-peak periods. That is, the optimal solution for travel modes in peak periods may not be the optimal solution for off-peak periods, for the optimal solution is obtained for peak-periods on the assumption of the existing interdependent congestion, which does not exist in off-peak periods. Therefore, the optimal solution for off-peak periods is necessarily different from that of peak periods. We may describe this situation as saying that the first best solution in peak periods is a second best solution for off-peak periods, or vice versa.

A more preferable approach, which was not taken up in this paper, would be to obtain economic welfare maximizing prices in all periods.

Also left out in this analysis is the case of increasing returns to scale, which will have a crucial bearing on the investment policy of transportation service.

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