

DYNAMICS OF REGIONAL ECONOMIC DEVELOPMENT

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Dynamic concepts of economic development can differ from the static short run equilibrium concepts of economic activity in a significant manner. This is particularly noticeable in the interpretation of the roles of autonomous investment, exports and imports of economic activity in each case. Apart from such considerations of basic interpretations, an economic model should be subject to an extension by exogenous introductions of such dimensions as space, distance, and location.

Regional Accounts: What are Their Purposes?

Traditionally, national and regional income account schemes have emphasized the exchange market aspect of economic activity. Such an emphasis may or may not be relevant in relation to the strength and growth of a particular society or region, or its ability to survive. If the survival and growth of a region is of importance, then the respective regional accounts should be assumed to be complete in the sense that all production, consumption, investment and export-import activities are properly imputed and accounted for in a total sense. For example, all economic activities of self-sufficient economic units outside of exchange markets, all flows of people, know-how, brain power, etc. should be imputed for the estimations of total consumption, investment, export and import activities for a particular region. In the subsequent treatment it is assumed that such an imputation and accounting scheme is possible.

Some Notes on Multiplier Models

Multiplier models are typically static short-run equilibrium models embodying within themselves no specific concepts of a dynamic growth of economic activity. The development over time of economic activity can be described only by such means as comparative statics. In particular, the interpretation of the roles of autonomous investment, exports and imports can become quite different if one introduces a growth mechanism [1]. This important aspect for interpretation will be illustrated subsequently. In particular, a high level of exports does not assure, necessarily, that a region is assuring its future for a long run growth of economic activity.

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A Model of the Dynamics of Regional Economic Development

Consider a system of n regions R_k, $k = 1,2,3,\ldots\ldots, n$. Let $Y_k(t)$ be the total gross regional product of the region R_k in the sense that all consumption, investment, export, import and production activities have been imputed and accounted for all self sufficient and market exchange economic activities. Let $P_k(t)$ be the pool of productive means (labor, capital, land, technological know how, managerial talent, etc. in an effective productive combination in a specific institutional, technological, cultural, demographic, etc. setting) for the particular region R_k. The fullest possible employment of resources for each and all regions is assumed. Let r_k be the regional rate of return from the pool of productive means, respectively. Then the following linear production relationship is assumed:

$$(1) \qquad Y_k(t) = r_k \, P_k(t); \; k = 1,2,3, \ldots\ldots, n$$

By definition, the total private and public investment, $I_k(t)$, is the rate of formation of new productive means contributing to the current realized production capacity:

$$(2) \qquad I_k(t) = dP_k(t)/dt = (1/r_k) \; [dY_k(t)/dt]$$

The product saved from consumption, $O_k(t)$, is not immediately converted to investment, but represents an outlay for the formation of future new productive means. Let $\theta_k(t)$ be the lag time distribution associated with the process of converting these outlays, $O_k(t)$, into investment $I_k(t)$. By the principle of convolution,

$$(3) \qquad I_k(t) = \int_0^t \theta_k(t-z) \; O_k(z) \; dz$$

The total private and public consumption, including the consumption of all productive means (due to wear, tear, obsolescence, etc.) is convoluted over the past and present gross regional product $Y_k(t)$ with a respective lag time distribution, $\phi_k(t)$ as follows:

$$(4) \qquad C_k(t) = b_k \int_0^t \phi_k(t-z) \; Y_k(z) \; dz + (1-b_k) \; Y_{sk}$$

b_k is the marginal propensity to consume for the region R_k, and, respectively, $1-b_k$ is the propensity to save product from consumption. Y_{sk} is the subsistence level of regional gross product for R_k. The reason why it is introduced to the consumption equation in the above manner becomes clear in the subsequent treatment.

Let $E_k(t)$ be the total exports from the region R_k into the other regions, and let $M_k(t)$ be the imports into R_k from these other regions. Then, in a more explicit form,

$$(5) \qquad E_k(t) - M_k(t) = \sum_{\substack{i=1 \\ i \neq k}}^{n} [E_{ki}(t) - M_{ik}(t)]$$

where $E_{ki}(t)$ is the export into the ith region from R_k and $M_{ik}(t)$ is the import from R_i into R_k. As will be pointed out later, these imports and exports may be lagged results of such activities as economic aid. By definition, if $E_k(t) - M_k(t) = 0$, one says there is a "balance of payments" for the region R_k. Such a balance rarely occurs in practice, especially in such situations as the interstate trade in USA.

In a general setting, $E_k(t)$ and $M_k(t)$ can be functions of all the regional gross products $Y_1(t)$, $Y_2(t)$, ..., $Y_n(t)$ and/or their derivatives. By the definition, the gross regional product for the region R_k is as follows:

$$(6) \qquad Y_k(t) = C_k(t) + O_k(t) + E_k(t) - M_k(t)$$

It is assumed that the resources are fully utilized within the technological, institutional and demographic constraints. This assumption does not, necessarily, exclude the possibility that there could exist some slacks of factors of production or "hidden unemployments" of productive means, or a subsidy of an idle portion of a society.

The Laplace transforms of the equations (2), (3), (4), (5), and (6) are, respectively, as follows:

$$(7) \qquad \mathcal{I}_k(s) = (1/r_k)\ [s\mathcal{Y}_k(s) - Y_{ok}]\ ; \ Y_{ok} = Y_k\ (0)$$

$$(8) \qquad \mathcal{I}_k(s) = \theta_k\ (s)\ \mathcal{O}_k\ (s)$$

$$(9) \qquad \mathcal{C}_k(s) = b_k\ \phi_k\ (s)\ \mathcal{Y}_k\ (s) + (1/s)\ [(1-b_k)\ Y_{sk}]$$

$$(10) \qquad \mathcal{E}_k(s) - \mathcal{M}_k\ (s) = \sum_{\substack{i=1 \\ i \neq k}}^{n} [\ \mathcal{E}_{ki}\ (s) - \mathcal{M}_{ik}\ (s)]$$

$$(11) \qquad \mathcal{Y}_k(s) = \mathcal{C}_k\ (s) + \mathcal{O}_k\ (s) + \mathcal{E}_k\ (s) - \mathcal{M}_k\ (s)$$

Equations (7) and (8) can be solved for $\mathcal{O}_k(s)$. This result and the appropriate expressions in equations (9) and (10) can be substituted into equation (11). This expression can then be solved <u>implicitly</u> for $\mathcal{Y}_k(s)$:

$$(12) \qquad \mathcal{Y}_k(s)\ [s - r_k\ \theta_k\ (s)\ (1 - b_k\ \phi_k\ (s))] + r_k\ \theta_k\ (s)\ [\mathcal{E}_k\ (s) - \mathcal{M}_k\ (s)]$$
$$= Y_{ok} - r_k\ (1-b_k)\ Y_{sk}\ [\theta_k\ (s)/s]$$

$Y_{ok} = Y_k\ (0)$; Y_{sk} is the subsistence gross regional product for the region R_k.

In general, $E_k(s)$ and $M_k(s)$ are functions of s and $y_1(s)$, $y_2(s)$, $y_3(s)$, ..., $y_n(s)$. Therefore, depending on the exact nature of these functions, equation (12) must be solved explicitly for $y_k(s)$, whereby one obtains a system of equations for $y_1(s)$, $y_2(s)$, ..., $y_n(s)$ corresponding, respectively, to the expressions for the regions R_1, R_2, ..., R_n.

"Balance of Payments" and The Closed Behavior for Economic Development

By definition, region R_k experiences a general balance of payments if the following condition is satisfied:

$$(13) \qquad E_k(s) = M_k(s) \text{ or, equivalently, } \sum_{\substack{i=1 \\ i\neq k}}^{n} E_k(s) = \sum_{\substack{i=1 \\ i\neq k}}^{n} M_k(s)$$

A much stronger condition, by definition, is the region by region balance of payments for the region R_k:

$$(14) \qquad E_{ki}(s) = M_{ik}(s) \text{ for all } i,\ k = 1,2,3,....,n$$

This latter condition is sufficient but not necessary for the former condition for the balance of payments for R_k. In either case, for the region R_k, the effects of exports will balance and cancel the effects of imports whereby the economic behavior of the region is characterized by intra-regional determinants of consumption and investment. In such a case equation (12) reduces to the form:

$$(15) \qquad y_k(s)[s - r_k\ \theta_k\ (s)\ (1-b_k\ \phi_k(s))] = Y_{ok} - r_k\ (1-b_k)\ Y_{sk}\ [\theta_k\ (s)/s]$$

This is an explicit solution for $y_k(s)$.

One should note that the balance of payments assumption does not represent a "typical" case, especially, if one imputes all types of imports and exports (e.g. brain drain, immigration of labor and talent, technological know-how, etc.).

Special Cases of a Closed Economic Development of Region R_k

Equation (15) can be utilized to illustrate a number of special cases of closed regional economic development.

1. If there are no investment and consumption lags, i.e. $\theta_k(s) = \phi_k(s) = 1$, and $Y_{sk} = 0$, then

   $Y_k\ (t) = Y_{ok} \exp A_k\ t$; $A_k = r_k\ (1-b_k)$

II. If there is no consumption lag, i.e. $\phi_k(s) = 1$, and if the investment lag is an exponential one with a mean lag time 1/a (i.e. $\theta_k(s) = a/(s+a)$), then

$$Y_k\ (t) = (Y_{ok} - Y_{sk})\ [\exp\ (-at/2)]\ [\cosh\ (a/2)\sqrt{1 + (4A_k/a)}\ t +$$

$$(1/\sqrt{1 + (4A_k/a)})\sinh\ (a/2)\sqrt{1 + (4A_k/a)}\ t\ ] + Y_{sk}$$

As the mean investment lag 1/a goes to infinity, the growth of the economic development goes to zero. As 1/a goes to zero, then

$$Y_k\ (t) = (Y_{ok} - Y_{sk})\ \exp A_k t + Y_{sk};\ Y_{ok} \geq Y_{sk}$$

One should note why the subsistence gross regional product was introduced as it was in equation (4). The subsistence economy barely survives without growth when $Y_{ok} = Y_{sk}$.

III. If there is no investment lag, and the consumption lag is an exponential one with a mean lag time 1/c, then

$$Y_k\ (t) = (Y_{ok} - Y_{sk})\ \exp(-(g_k - 1)\ (r_k/2)\ t)\ \cdot$$

$$[\cosh\ r_k\sqrt{(1/4)\ (g_k - 1)^2 + 4g_k\ (1-b_k)}\ t +$$

$$((g_k + 1)/\sqrt{(g_k - 1)^2 + 16g_k\ (1-b_k)}\ )\ \cdot$$

$$\sinh\ r_k\sqrt{(1/4)\ (g_k - 1)^2 + 4g_k\ (1-b_k)}\ t +$$

$$Y_{sk}\ ;\ g_k = (1/r_k)\ c$$

If the mean consumption lag 1/c becomes infinite, then

$$Y_k\ (t) = (Y_{ok} - Y_{sk})\ \exp\ (r_k t) + Y_{sk}$$

If 1/c becomes zero, then

$$Y_k\ (t) = (Y_{ok} - Y_{sk})\ \exp A_k t + Y_{sk}$$

The intra-regional economic development becomes the more sensitive to business cycles the higher the order of exponential consumption and/or investment lags become.

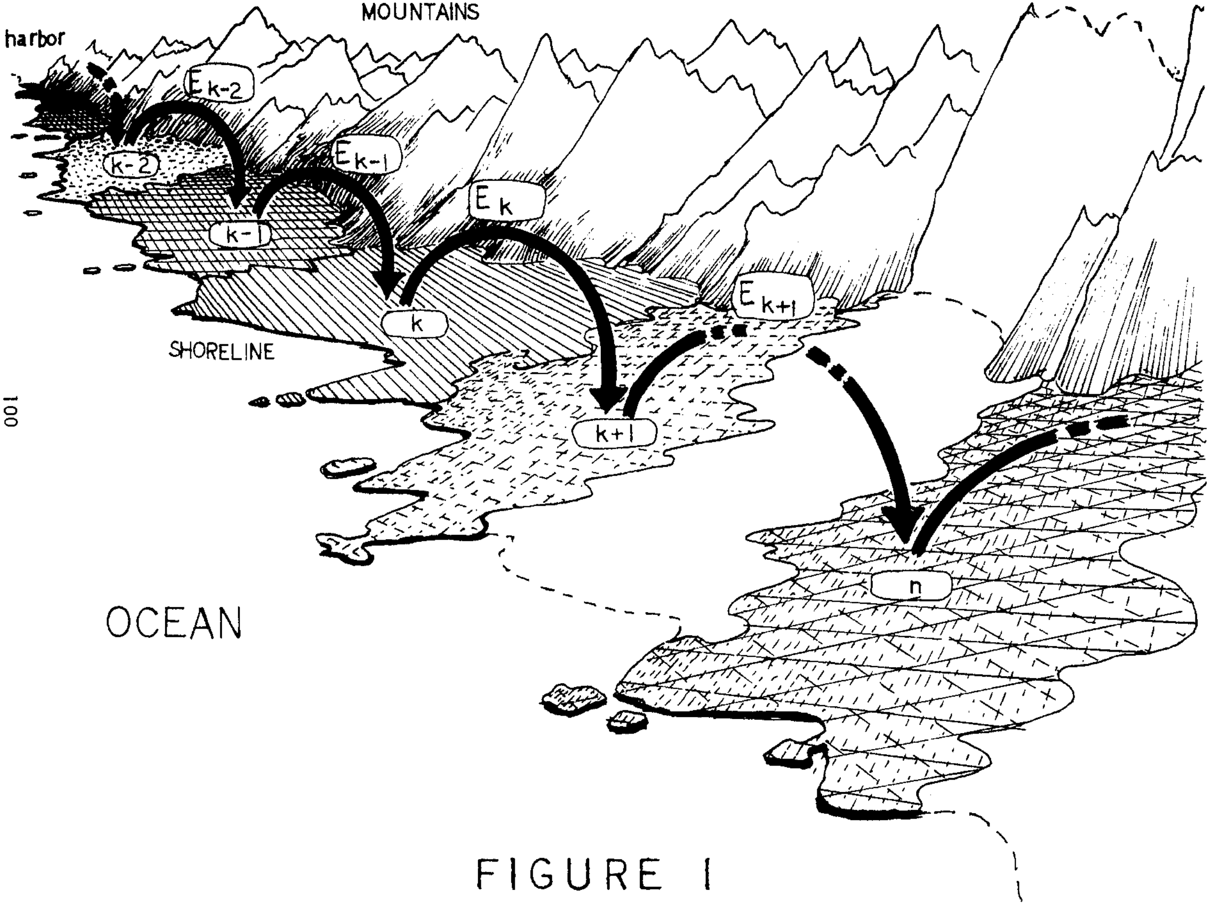


FIGURE I

A Special Case of Open Economic Development: Externally Forced Regional Business Cycles

Assuming no internal investment or consumption lags, if one assumes in equation (12) that $Y_{ok} = Y_{sk} = 0$ and that

$$M_k\ (t) - E_k\ (t) = (Q_{ok}\ r_k)\ [1-\cos wt] \text{ or } M_k\ (s) - E_k\ (s)$$

$$= [Q_{ok}\ r_k\ w^2]\ /\ [s\ (s^2 + w^2)]$$

then

$$Y_k\ (t) = (Q_{ok}\ r_k\ /\ (1-b_k))[(w^2/\ (A_k{}^2 + w^2))\ \exp\ (A_k t) - 1] +$$

$$[(Q_{ok}\ r_k{}^2)\ /\ (A_k{}^2 + w^2)][A_k\ \cos(wt) - w\ \sin\ (wt)\ ]$$

$Q_{ok}\ r_k$ is the amplitude of the net imported force of economic development, i.e. $M_k(t) - E_k(t)$. It should be noted that the "unbalanced payments" concept of imports minus exports is an important one here, and is a reverse one to the one obtained from a simple multiplier model, as will be pointed out subsequently.

A Note on Causes of Business Cycles

In the analysis of regional economic stability one should distinguish between the following possibilities of business cycles for any region R_k:

a. Intraregional causes of business cycles

b. Interregional causes of business cycles

c. Noneconomic causes of business cycles

d. Combination causes by above sources of business cycles

In regional economics, there are two distinct dimensions for business cycles: temporal ones and spatial ones. The concept of spatial business cycles is a very important one. In such a case, the basic model introduced here must be augmented by additional exogenous assumptions which bring into the picture the concepts of space, distance and location. The net result should be a synthesis of a spatio-temporal structure.

A Simple Illustration: The Role of Imports minus Exports in Multiplier and Economic Development Models

If one assumes no investment and consumption lags, $Y_{ok} = Y_{sk} = 0$, and $E_k(s) - M_k(s) = [E_k - M_k]/s$ in equation (12), then the solution for $Y_k(t)$ would turn out to be as follows:

$$Y_k\ (t) = [M_k - E_k] \exp [r_k\ (1-b_k)\ t] + [1/(1-b_k)]\ [E_k - M_k]$$

$E_k-M_k = -(M_k-E_k)$ is an autonomously determined difference between exports and imports for the region R_k. The second term in the above expression is the usual multiplier effect on exports minus imports. In a pure multiplier model the regional gross product is the higher the larger this exports minus imports term is. However, if one introduces a growth process, the interpretation of this difference between exports and imports has a new meaning: The imports minus exports becomes now the amplitude of growth of regional gross product for R_k. For an increased growth it is desirable to have more imports than exports. For example,

a. if more people and investment goods are imported into the region R_k, the more is the amplitude of its growth due to such (properly imputed) inflows of productive means; or

b. if more consumer goods are imported, the more domestic product can be saved from consumption for investment, providing thereby an increased amplitude of growth.

For a long run growth of a region it is desirable to have more imports than exports, this being a favorable "unbalance of payments," provided that productive means are continuously employed for growth. This is, indeed, an interpretation contrary to one provided by a pure multiplier model [1].

Effects of Lags in Economic Aid and Import Flows

Let $S_k(t)$ represent a flow of productive means into region R_k. Such a flow could be a result of an economic aid or relocation of industries, or migration of talent or skilled labor, etc. There can be various transportation and implementation lags before such source flows become an effective force for economic development, that is, usable productive means. If one means by the imports $M_k(t)$ a readily usable flow of productive means, then one can introduce a lag time distribution $X_k(t)$ relating $M_k(t)$ to $S_k(t)$ by the following convolution integral:

$$M_k(t) = \int_0^t X_k(t-z)\ S_k(z)\ dz \tag{16}$$

If one takes the Laplace transform across this equation, one obtains, respectively, the following relationship of the Laplace transforms:

$$\mathcal{M}_k(s) = \mathcal{X}_k\ (s)\ \mathcal{S}_k\ (s) \tag{17}$$

Example: If one assumes no investment and consumption lags, $E_k(t) = 0$, $Y_{ok} = Y_{sk} = 0$ and that $\mathcal{M}_k(s) = \mathcal{X}_k(s)\ \mathcal{S}_k(s)$ in equation (12),

then

$$\mathcal{Y}_k\ (s)\ [s - r_k\ (1-b_k)] = r_k\ \mathcal{X}_k\ (s)\ \mathcal{S}_k\ (s)$$

Let $S_k(t) = M_o u(t)$ dollars per annum economic aid into region R_k, $u(t)$ is a unit step function. Then $\mathcal{S}_k(s) = M_o/s$. If there is no lag of this economic aid, then

a. $Y_k(t) = [M_o/\ (1-b_k)][\ \exp A_k t - 1];\ A_k = r_k\ (1-b_k)$

If there is a simple exponential lag with a mean lag time $1/m$, then $\mathcal{X}_k(s) = m/(s + m)$. In such a case

b. $Y_k(t) = [M_o/(1-b_k)][1/(m + A_k)][m \exp A_k t + A_k \exp(-mt)] - [M_o/(1-b_k)]$

The GAP between equations a and b is the difference between the two:

$$\text{GAP} = [M_o/\ (1-b_k)][A_k\ /\ (m + A_k)][\ \exp A_k t - \exp(-mt)]$$

Such a GAP is a measure of inefficiency of economic aid due to transportation, implementation, and administrative lags.

Interregional Dynamic Interactions

In a setting of dynamic interregional interactions, the regional imports and exports are, in general, functions of the gross regional products of all involved regions, as pointed out in the connection of equation (12). Once such dependencies are defined, equation (12) can be solved explicitly for $\mathcal{Y}_k(s)$, yielding a system of equations for the n regions R_k, $k = 1,2,3,\ldots,n$. For a particular system of interacting interregional economic development, one can introduce a variety of specific interpretations. Such interpretations can be arrived at by introducing exogenous interrelationships between the n regions, such as those regarding the distance, location, and spatial relationships between these regions. It seems appropriate to provide examples, excluding or including such exogenous additions, for the dynamics of interregional economic development.

In order to simplify the subsequent examples, it will be assumed for the n regions, R_k, $k = 1,2,\ldots n$, that $b_k = b$, $r_k = r$, and that there are no regional consumption nor investment lags. Further, $Y_{sk} = 0$. Therefore, equation (12) takes the following special form:

(18) $\quad y_k(s)\,[s-r\,(1-b)] = Y_{ok} + r[M_k(s) - E_k(s)]$

It should be noted that the imports minus exports really is the driving function for the respective differential equation for regional economic development.

Example 1: Push or Spill-Over by Region R_1

Assume a case where a center region R_1 pushes or spills over its gross regional product equally to the remaining regions R_i, $i = 2,3,\ldots,n$. Let $Y_{01} > 0$, but let $Y_{02} = Y_{03} = \ldots = Y_{0n} = 0$.

Let $aY_1(t)$ be the exports from the region R_1 to each of the remaining regions R_i, $i = 2,3,\ldots,n$; $0 \le (n-1)\,a + b \le 1$. The respective system of equations is as follows:

$$\begin{bmatrix} [s-r(1-(n-1)a-b)] & 0 & \ldots & 0 \\ -ra & [s-r(1-b)] & \ldots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ -ra & 0 & \ldots & [s-r(1-b)] \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \\ \cdot \\ \cdot \\ y_n(s) \end{bmatrix} = \begin{bmatrix} Y_{01} \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

After solving for $y_i(s)$, $i = 1,2,3,\ldots,n$, and taking the respective inverse Laplace transforms, one obtains the following solutions:

(19) $\quad Y_1(t) = Y_{01} \exp[r(1-(n-1)a-b)t]$

$$Y_2(t) = Y_3(t) = \ldots = Y_n(t)$$

$$= [Y_{01}/(n-1)][\exp\,[r(1-b)t] - \exp\,[r(1-(n-1)a-b)t]]$$

It is seen that regions $R_2, R_3,\ldots,R_n$ will overtake region R_1 in economic growth. This kind of a model may caricature in part the process of suburbanization at the expense of the center city. One should note that so far no specific spatial structure has been implied. Such spatial structures must be introduced by exogenous additions to the above interregional model for economic development.

Example 2: Pull or Suction by Region R_n

Dynamically quite a distinct process from the push case is the pull or suction of economic activity by the region R_n from the remaining regions R_{n-1}, $R_{n-2},\ldots,R_1$. This kind of a case caricatures such processes as the rural to urban migration. The exports from the region R_i, $i = 1,2,\ldots,n-1$, to region R_n is of the form $E_i(t) = aY_n(t)$. Except for the region R_n imports for all other regions are zero. Utilizing equation (18) under these assumptions and with the assumption that $Y_{01} = Y_{02} = .. = Y_{0n} = Y_0$ (i.e. one has an initially uniform level of economic activity for all the regions), the respective system of equations is as follows:

$$\begin{bmatrix} [s-r(1-b-a)] & 0 & \cdots & 0 & ra \\ 0 & [s-r(1-b-a)] & \cdots & 0 & ra \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & \cdots & & [s-r(1-b+(n-1)a)] \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = \begin{bmatrix} Y_0 \\ Y_0 \\ \vdots \\ Y_0 \end{bmatrix}$$

One solves first for $y_n(s)$, then for $y_{n-1}(s)$, etc. By taking the respective inverse Laplace transforms, one finds the following results:

(20) $$Y_n(t) = Y_0 \exp [r(1-b + (n-1)a)t]$$

$$Y_{n-1}(t) = Y_{n-2}(t) = \ldots = Y_1(t)$$

$$= Y_0 \; [[(n+2)/(n+1)] \exp [r(1-b-a)t] -$$

$$[1/(n+1)] \exp [r(1-b+(n-1)a)t]]$$

These expressions are valid until $t = t_0 = [(n+1)ar]^{-1} \ln(n+2)$. Thereafter all other regional gross products remain zero except $Y_n(t)$ which becomes for $t \geq t_0$ as follows:

(21) $$Y_n(t) = Y_o \; [\exp \; (1-b + (n-1)a)rt_o] \exp [r(1-b)(t-t_o)]$$

Remarks on Exogenous Introduction of a Spatial Structure, Distance and Location

The regions $R_1, R_2, \ldots, R_n$ could have multitude of different spatial interrelationships. Such spatial relationships must be specified outside of the dynamic economic interaction model discussed here. Figure 1 illustrates a specific spatial setting where a distant harbor stimulates a push type chain reaction in an essentially one dimensional array of regions. The situation could be equally well a reverse one where the harbor stimulates a pull or suction type chain reaction. With such explicit introduction of a spatial pattern (and, thus, possibly, distance and location for each region and from region to region) the push and pull chain reaction will now be presented in terms of simple examples. It is again assumed that there are no consumption nor investment lags and that $rk = r$, $b_k = b$ and $Y_{sk} = 0$ for $k = 1,2,3,\ldots; n = \infty$.

Example 3: Push Chain Reaction Along a One Dimensional Array of Regions

In a one dimensional chain reaction of the push type, the exports from the region R_k to R_{k+1} are assumed to be of the form $aY_k(t)$. Thus, the imports to R_k come from R_{k-1} and there are no imports to R_k from R_{k+1}. With these assumptions equation (18) leads to the following system of equations ($Y_{02} = Y_{03} = \dots = Y_{0k} = 0$ for $k = 2,3,\dots.$):

$$\begin{bmatrix} [s-r(1-b-a)] & 0 & 0 & \dots \\ -ar & [s-r(1-b-a)] & & \dots \\ 0 & -ar & [s-r(1-b-a)] & \dots \\ 0 & 0 & -ar & \dots \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \\ y_3(s) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} Y_{01} \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

Let $A = r(1-b-a)$. Then one finds

(22) $$y_k(s) = (ra)^{k-1} \, Y_{01} \, (s-A)^{-k} \quad \text{and}$$

$$Y_k(t) = (ra)^{k-1} \, [Y_{01}/(k-1)!] \, t^{k-1} \exp At$$

This set of solutions represent a growing boom wave propagating away from the source R_1 in the direction of increasing k. This model, again, could represent the process of propagating suburbanization at the expense of the center city areas.

Example 4: Pull or Suction Chain Reaction Along a One Dimensional Array of Regions

In the pull or suction case one assumes $Y_{01} = Y_{02} = \dots = Y_{ok} = \dots\dots = Y_0$ whereby one starts with an initially uniform level of economic activity for all regions, $k = 1,2,3,\dots.$ In the one dimensional pull or suction model the exports from the region R_{k+1} to the region R_k are assumed to be of the form $aY_k(t)$. The net export flows are reversed in relation to the previous case. Therefore, the resulting chain reaction is distinctly different in its dynamic nature from the previous example. It will be shown that this latter process leads into a spatial business cycle situation in contrast to the propagating boom wave generated by the process in example 3. In a way, then, rural to urban migration may be spatially a more unstable process than the suburbanization process around center cities.

The system of equations associated with the pull chain reaction case assumed here is as follows:

$$\begin{bmatrix} [s-r(1-b+a)] & 0 & 0 & \cdots & 0 \\ ar & [s-r(1-b+a)] & 0 & \cdots & 0 \\ 0 & ar & [s-r(1-b+a)] & \cdots & 0 \\ 0 & 0 & ar & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \\ y_3(s) \\ \vdots \end{bmatrix} = \begin{bmatrix} Y_0 \\ Y_0 \\ Y_0 \\ \vdots \end{bmatrix}$$

Solving first for $y_1(s)$, then for $y_2(s)$, etc. yields the following general solution for the region R_k:

(23) $$y_k(s) = Y_0 \sum_{i=1}^{k} (-ra)^{i-1} [s-(1-b+a)r]^{-i}$$

For the regions R_1, R_2, R_3 and R_4 the inverse Laplace transforms yield the following results:

(24) $$Y_1(t) = Y_0 \exp A't \qquad ; A' = r\,(1-b+a)$$

$$Y_2(t) = [1-rat]\, Y_0 \exp A't$$

$$Y_3(t) = [1-rat + (1/2)\,(ra)^2\, t^2]\, Y_0 \exp A't$$

$$Y_4(t) = [1-rat + (1/2)\,(ra)^2\, t^2 - (1/6)\,(ra)^3\, t^3]\, Y_0 \exp A't$$

Equations (23) and (24) are valid only for $0 \leq t \leq 1/ra$. For $t \geq 1/ra$, $Y_2(t) = 0$, and the region R_1 becomes isolated from the other regions:

(25) $$Y_1(t) = [Y_0 \exp (A'/ra)] \exp r\,(1-b)\,(t-(1/ra)) \text{ and}$$

$$Y_2(t) = 0 \text{ for all } t \geq 1/ra$$

For $(1/ra) \leq t \leq (5/3)\,(1/ra)$,

(26) $$Y_3(t) = [(1/2)\, Y_0 \exp (A'/ra)] \exp r\,(1-b)\,(t-(1/ra))$$

$$Y_4(t) = [Y_0 \exp (A'/ra)][(1/3) - (1/2)\, ra\,(t-(1/ra))] \cdot \exp r\,(1-b)\,(t-(1/ra))$$

For $t \geq (5/3)\ (1/ra)$.

(27) $$Y_3(t) = (1/2)\ Y_0\ [\exp\ (5/3)\ (A'/ra)]\ \exp\ r(1-b)\ (t-\ 5/3ar))$$

$$Y_4(t) = 0\ ;\ \text{etc.}$$

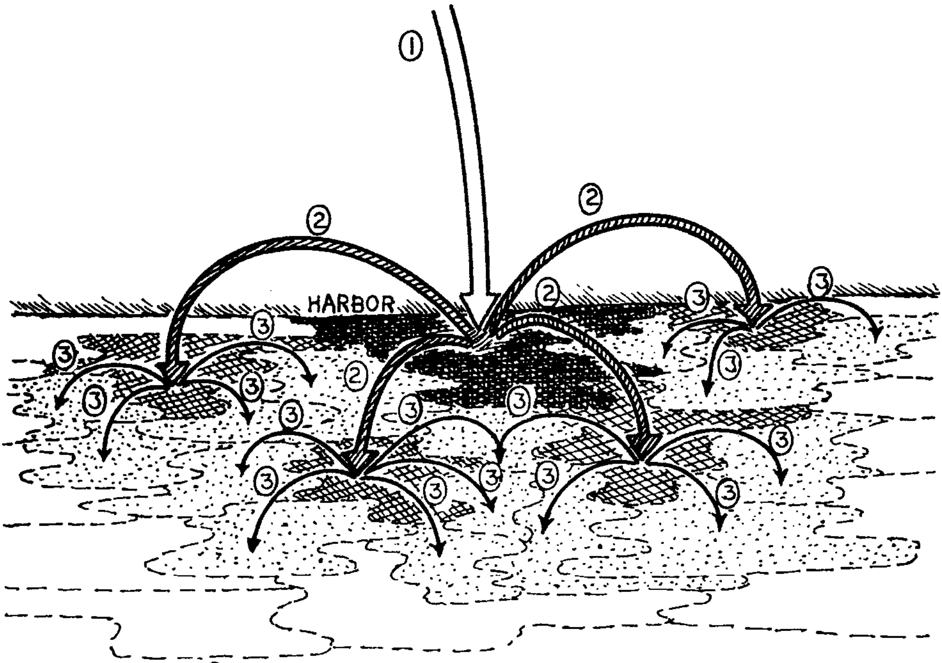
The end result is the fractionation of regional economic development where the regions R_4 and R_2 become completely deserted, and the regions R_1 and R_3 become isolated pockets of economic development. Thus, an initially homogeneous system of regions breaks into a system of fractionated pattern of alternating regions of booms and depressions. This illustrates a case of spatial business cycle, a spatial instability of regional economic development.

Remarks on Hierarchical Interregional Interactions

Examples 1 and 2 could be interpreted as interactions between a particular region and the rest of the regions, at a distance. Examples 3 and 4 could be interpreted as neighborhood or neighbor-to-neighbor interactions stimulated by a leading region. One could, also, introduce a variety of hierarchical interactions. Figure 2 illustrates a particular case of such hierarchical interactions. Net imports are treated as flows of sources for economic development. A primary flow arrives at the harbor region from the "rest of the world." The harbor region, in turn, generates secondary flows into its surrounding regions. These latter regions generate ternary flows into their surrounding regions, etc. In general, such flows can be of a combined push-pull type, and can depend on levels, rates of changes, and so on, of all the gross regional products of all regions in the system of regions. Noting equation (12), all the possible effects of lags can be incorporated into such interactions. Further (equation (16)), import-export lag effects can be included.

Concluding Remarks

The linear model for the dynamics of interregional economic development presented here can be extended in a variety of directions. First, one can introduce the role of regional as well as interregional fiscal policies into the model. For such fiscal policy models one can, also, introduce lags for taxation and transfer payments. Such lags can be identified with administrative delays in fiscal policy making and execution. Second, one can introduce through the production function and import-export flows specific roles of particular factors of production such as labor and labor migration, movements and diffusion of technological know-how and innovations, effects of brain drain, and relocation of industries. Third, it is possible to modify the model to include such nonlinear effects as economies or diseconomies of scale of productive institutions. Fourth, one can combine with the model specific regional diffusion models [1], which bring the roles of space, distance, and location more explicitly into the framework of a total dynamic model for interregional economic development.



(1) PRIMARY SOURCE FLOW FROM OUTSIDE

(2) SECONDARY SOURCE FLOWS WITHIN REGION

(3) TERNARY SOURCE FLOWS WITHIN REGION

FIGURE 2

REFERENCES

1. Jutila, S. T. "A Linear Model for Agglomeration, Diffusion, and Growth of Regional Economic Activity," Regional Science Perspectives, 1 (1971), 83-108.