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A MODEL OF REGIONAL INCOME DIFFERENCES AND ECONOMIC GROWTH*

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This paper is inspired by recent advances accomplished in the theory of international trade concerning the resource allocational and welfare implications of the inter-industry factor-price differentials by writers such as Bhagwati [7], Bhagwati and Ramaswami [8], Bhagwati and Srinivasan [9], Batra [1], Batra and Pattanaik [2, 3, 4], Batra and Scully [5], Magee [14], and Jones [12] among others. The two-commodity, two-factor, factor-price differential model developed by these authors can be easily adapted with slight modifications to analyze the implications of the observed phenomenon of inter-regional income differences for resource allocation, welfare and economic growth. One purpose of this paper is to construct a closed economy model where each of the two regions produces one commodity with the help of two factors of production, capital (K) and labor (L), but pays different prices for the use of the two factors. We then analyze the effects of economic growth stemming from the expansion of factor supplies on national income in the presence of a stable inter-regional factor-price differential which may arise from different degrees of unionization in the two regions, differences in regional economic structures characterized by the nature and size of industrial enterprises, differences in production functions, differential tax rates on the factor use, etc. It is worth pointing out here that although such differentials are known to exist in several countries, their effect on the growth rate of an economy has received relatively little attention.

The inter-regional factor-price differential itself may be altered in the course of expansion in factor supplies, as has been empirically demonstrated in the case, for example, of Canada and the United States, where it has been observed that such differentials have converged over time.[1] The second purpose of this paper is then to explore the consequences of a change in the inter-regional factor-price differential for the national rate of economic growth. Some interesting policy devices suggest themselves from this discussion. For if the society's objective is to raise the rate of growth in national income, and if there does exist a certain relationship between this growth rate and the factor-price differential, then the Federal Government should introduce policies that alter the differential in the desired manner by pursuing a suitable tax-cum-subsidy policy on the use of the two factors in different

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regions.[2]

Assumptions and the Model

Unless otherwise specified, the following assumptions will be maintained throughout the paper.

1. The economy is divided into two regions, region 1 and region 2, each producing a different commodity, X_1 and X_2, respectively. It is assumed that the technology used in producing the two goods differs between the two regions. Each region may exchange its good with the other, but as far as the nation is concerned, there is no international trade with other nations. This implies that the demand for the two goods in the two regions exactly equals their supply in equilibrium.

2. Each commodity requires labor (L) and capital (K) for its production. Production functions exhibit constant returns to scale but diminishing returns to factor proportions. For simplicity, and without loss of generality, we assume that X_1 is always capital-intensive relative to X_2 in terms of the physical units of the two factors.

3. Factor supplies, inelastic at any moment of time, grow at exogenously given rates. Both factors remain fully employed at all times, full employment being maintained by the presence of perfect factor-price flexibility and perfect inter-regional mobility of factors.

4. There is perfect competition in product as well as capital markets, but the labor market is distorted by a wage-differential. In other words, while the return on capital is always equal as between the two regions, the wage rates are dissimilar. This differential is assumed to be exogenously determined in the sense that its existence is not attributable to any of the variables in the model itself.

5. It is assumed that expressions for the demand for the two commodities can be obtained from an aggregate utility function and the constraint provided by the available resources. This utility function is assumed to be homothetic, so that the proportion in which the two commodities are consumed is solely determined by the commodity-price ratio.

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[1] This view has been expressed by several regional economists in recent years. For evidence on the convergence of the wage-differential in Canada, see Peitchinis [15], and for that in U.S.A. see Scully [16] and Batra and Scully [6].

[2] We do not pursue this point further in the main body of the text and leave it to the interested reader to draw his own conclusions. In this connection, the article by Bhagwati and Ramaswami [8] which examines the optimum policy in the presence of the inter-industry wage-differential in a static model can offer valuable suggestions.

The model implicit in these assumptions can be summarized by the following set of equations:

(1) $X_1 = F_1\ (L_1, K_1) = L_1\ f_1\ (k_1)$

(2) $X_2 = F_2\ (L_2, K_2) = L_2\ f_2\ (k_2)$

(3) $w_1 = (f_1 - k_1\ f_1') = \alpha\ p\ (f_2 - k_2\ f_2') = \alpha\ w_2$

(4) $r\ = f_1' = pf_2'$

(5) $L_1 + L_2 = L$

(6) $K_1 + K_2 = L_1\ k_1 + L_2\ k_2 = K$

(7) $U = U\ (X_1, X_2)$

with

(7*) $\dfrac{X_1}{X_2} = f\ (p),$

where X_i = output in the ith region, $k_i = (K_i/L_i)$ is the capital/labor ratio, w_i = the wage rate, r_i = the rate of return on capital, α = the extent of the inter-regional wage differential, p = the price of X_2 in terms of X_1, U = social utility, and L_i and K_i respectively stand for the amount of labor and capital used in the ith region (i = 1, 2).

The first two equations describe the production functions, the next two the determination of factor rewards (here f_i' = marginal product of capital and $f_i - k_i\ f_i'$ = marginal product of labor), equations (5) and (6) specify the full employment condition and finally equation (7) describes the aggregate utility function, with (7*) showing that this function is homothetic.

Some Properties of This Model

Before proceeding directly to the objectives of this paper, some relations, which will be used intensively in the analysis of subsequent sections, must be obtained. To begin with, we derive a relationship between the commodity-price ratio and the slope of the inter-regional marginal rate of transformation, by which we mean the rate at which the output in one region is changed as a result of a unit change in the output in the other region.[3] Differentiating (1) and

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[3] The inter-regional transformation curve is the direct counterpart of the usual transformation curve between commodities. This transformation curve may be interpreted as the locus of all efficient outputs produced in the two regions constrained by the full employment of factors.

(2) totally and dividing through by dX_2, we have

$$\frac{dX_1}{dX_2} = \frac{f_1' \, dK_1 + (f_1 - k_1 \, f_1') \, dL_1}{f_2' \, dK_2 + (f_2 - k_2 \, f_2') \, dL_2}$$

where as before f_i' is the marginal product of capital and $(f_i - k_i \, f_i')$ is the marginal product of labor. Using (3) and (4), and from (5) and (6) the fact that

$$dL_1 + dL_2 = 0$$

$$dK_1 + dK_2 = 0,$$

we have

(8) $$\frac{dX_1}{dX_2} = - p \left[\frac{f_2' \, dK_2 + \alpha \, (f_2 - k_2 \, f_2') \, dL_2}{f_2' \, dK_2 + (f_2 - k_2 \, f_2') \, dL_2}\right] = -\beta p$$

where $$\beta = \frac{f_2' \, dK_2 + \alpha \, (f_2 - k_2 \, f_2') \, dL_2}{f_2' \, dK_2 + (f_2 - k_2 \, f_2') \, dL_2} \gtreqless 1 \text{ if } \alpha \gtreqless 1.$$

If $\alpha = 1$, i.e., if $\beta = 1$, so that there is no inter-regional wage differential, the marginal rate of transformation between the outputs produced in two regions equals the commodity-price ratio, as in Figure 1, where TT' is the concave to the origin, inter-regional transformation curve, and the production point is given by P where the price line AB, indicating the commodity-price ratio, is tangential to TT'. If the wage-differential is introduced, two effects follow: First the transformation curve shrinks towards the origin[4] to TP'T', and second, the price line is no longer tangential to the transformation curve as shown by the fact that A'B' intersects TP'T' at the production point P'.[5]

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[4]See Hagen [10].

[5]The transformation curve in Figure 1 has been drawn concave to the origin in spite of the presence of the inter-regional wage-differential. In doing so we are following the tradition set by Hagen [10], Bhagwati and Ramaswami [8], and Batra and Pattanaik [2]. Recently, it has been shown by Bhagwati and Srinivasan [9], Jones [12] and Kemp and Herberg [13] that the shape of the transformation curve, unlike the case in the absence of the wage-differential, has no bearing on the response of outputs to changes in their prices. In view of this result, the transformation curve has been drawn concave to the origin throughout the paper irrespective of, as we show later, whether or not the supply curves have their usual positive slope. The credit for initiating this controversy goes to Johnson [11].

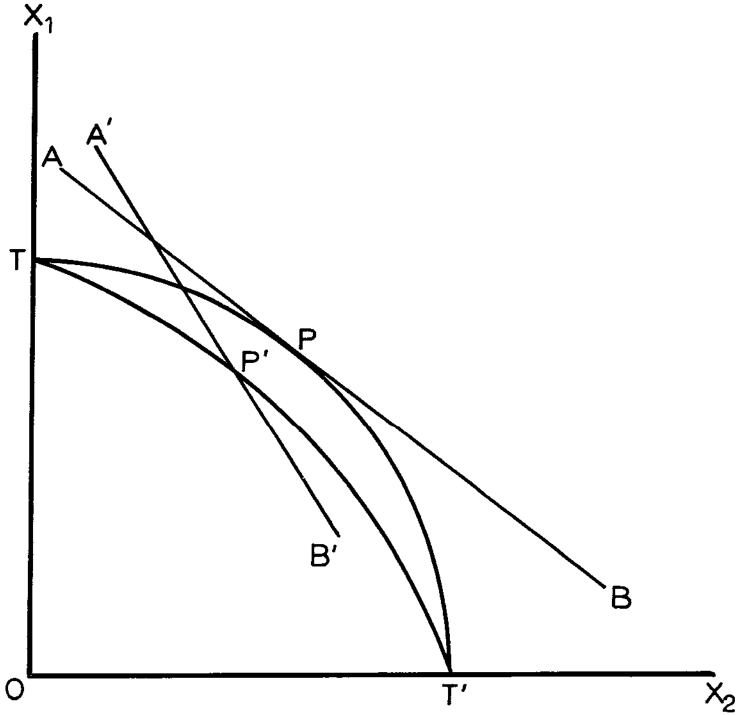

Figure 1

Since A'B' is drawn steeper than the marginal rate of transformation at P' (given by the slope of TP'T' at P'), the wage-differential is paid by region 2, which from (4) means that $\alpha < 1$, so that $w_1 < w_2$. This may also be confirmed from (8), where, with $\alpha < 1$ and hence $\beta < 1$,

$$\left|\frac{dX_1}{dX_2}\right| < p.$$

Next, we examine the effects of a change in the commodity-price ratio on the outputs in the two regions in the presence of the wage-differential. But first expressions for the elasticity of the factor substitution (σ_i) in each region must be obtained. Let ω_i be the wage/rental ratio in the ith region. Then from (3) and (4)

$$(9) \qquad \omega_i = \frac{f_i}{f_i'} - k_i .$$

Differentiating (9) with respect to ω_i, we obtain

$$\frac{dk_i}{d\omega_i} = - \frac{f_i'^2}{f_i f_i''} ,$$

whence

$$(10) \qquad \sigma_i = \frac{\omega_i}{k_i} \cdot \frac{dk_i}{d\omega_i} = - \frac{f_i' \; (f_i - k_i \; f_i')}{f_i \; k_i \; f_i''} ,$$

which in view of diminishing returns to factor-proportions that imply $f_i'' < 0$, is positive. Differentiating (1)-(6) with respect to p, keeping α, K and L constant and using (10), we obtain

$$(11) \qquad \frac{\partial X_1}{\partial p} = \frac{A}{(k_1 - k_2) \; (k_1 - \alpha \; k_2)}$$

$$(12) \qquad \frac{\partial X_2}{\partial p} = \frac{-Z}{(k_1 - k_2) \; (k_1 - \alpha \; k_2)}$$

where $$A = -\left[\frac{\alpha L_1 f_2 \{f_1 - f_1' (k_1 - k_2)\} f_1 k_1 \sigma_1}{f_1' (f_1 - k_1 f_1')} + \frac{L_2 f_1^2 f_2 k_2 \sigma_2}{p^2 f_2' (f_2 - k_2 f_2')}\right] < 0,$$

$$Z = -\left[\frac{\alpha L_1 f_2^2 f_1 k_1 \sigma_1}{f_1' (f_1 - k_1 f_1')} + \frac{L_2 f_1 \{f_2 + f_2' (k_1 - k_2)\} f_2 k_2 \sigma_2}{p^2 f_2' (f_2 - k_2 f_2')}\right] < 0,$$

because $[f_1 - f_1' (k_1 - k_2)] = (f_1 - k_1 f_1') + f_1' k_2 > 0$, and $[f_2 + f_2' (k_1 - k_2)] = (f_2 - k_2 f_2') + k_1 f_2' > 0$. With $A < 0$ and $Z < 0$, it is clear that the sign of $\partial X_i/\partial p$ depends on the sign of the denominator. If there is no wage-differential, that is, if $\alpha = 1$, there is no problem in interpretation. Here $\partial X_1/\partial p < 0$ and $\partial X_2/\partial p > 0$, which means that the supply curve of each output has the normal positive slope. However, if $\alpha \neq 1$, then it is possible that the supply curves may have the "perverse" negative slope, provided, of course, that $(k_1 - k_2)(k_1 - \alpha k_2) < 0$.

It will now be fruitful to introduce a distinction between, what is by now widely known, the factor-intensities in the "physical" sense and those in the "value" sense.[6] The sign of $(k_1 - k_2)$ furnishes the relationship between factor-intensities in the physical sense, because here only the physical units of the factors employed in the two regions are being compared. Since we have assumed X_1 to be capital-intensive relative to X_2, $(k_1 - k_2) > 0$. The sign of $(k_1 - \alpha k_2)$ on the other hand describes the inter-regional factor-intensity relationship in the value sense, because here we compare the ratios between the value of factors employed in the two regions. This is because $(k_1 - \alpha k_2)$ can be written as:

$$(k_1 - \alpha k_2) = k_1 - \frac{w_1}{w_2} k_2 = w_1\left[\frac{K_1}{w_1 L_1} - \frac{K_2}{w_2 L_2}\right]$$
$$= \omega_1 \left[\frac{rK_1}{w_1 L_1} - \frac{rK_2}{w_2 L_2}\right],$$

which is nothing but the comparison of the ratios between factor shares in the two regions. Now since $w_1 \neq w_2$, it is possible that the sign of factor-intensities in the value sense may be opposite to the sign of factor-intensities in physical terms. Since $(k_1 - k_2) > 0$, a necessary condition for this reversal to occur is that $\alpha > 1$ or $w_1 > w_2$. In other words, if $\alpha < 1$, that is, if the differential is paid by the second region, which is relatively labor-intensive in the physical sense, the physical and value factor-intensities possess the same sign. Stated differently, if a region pays the differential on its intensive factor, the physical and the value factor-intensities cannot be reversed. Evidently then, the "perverse," negative slope of the supply curves require that $\alpha > 1$, or the differential is paid by a region on its non-intensive factor.

[6]For recent articles that make use of the terminology, see Batra and Pattanaik [3], Jones [12] and Magee [14].

Factor Growth

Let us now examine the effects of growth in factor supplies on the change in real national income in the presence of the stable wage-differential. Differentiating the social utility function given by (7) totally, we get

$$dU = U_1 \, dX_1 + U_2 \, dX_2,$$

where $U_i = \partial U/\partial X_i$ is the marginal utility of the output produced in the ith region. Remembering that X_1 is the numeraire, let the change in real income (dy) be measured by the change in aggregate welfare. Then

$$dy = \frac{dU}{U_1} = dX_1 + \frac{U_2}{U_1} dX_2 = dX_1 + p dX_2. \tag{13}$$

On the other hand, from (7*), we obtain

$$\frac{dX_1}{X_1} - \frac{dX_2}{X_2} = \sigma_D \frac{dp}{p}, \tag{14}$$

where σ_D is the elasticity of substitution between the two commodities. Using (13) and (14) and differentiating (1)-(6), we obtain the following generalized system of equations:

$$\begin{bmatrix} 1 & -1 & -p & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -L_1 f_1' & 0 & -f_1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -L_2 f_2' & 0 & -f_2 & 0 \\ 0 & 0 & 0 & -k_1 f_1'' & \alpha p k_2 f_2'' & 0 & 0 & -\alpha(f_2 - k_2 f_2') \\ 0 & 0 & 0 & f_1'' & -p f_2'' & 0 & 0 & -f_2' \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & L_1 & L_2 & k_1 & k_2 & 0 \\ 0 & \frac{f_1'}{f_1} & -\frac{f_2'}{f_2} & 0 & 0 & \frac{1}{L_1} & -\frac{1}{L_2} & -\frac{\sigma_D}{p} \end{bmatrix} \begin{bmatrix} dY \\ dX_1 \\ dX_2 \\ dk_1 \\ dk_2 \\ dL_1 \\ dL_2 \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ p(f_2 - k_2 f_2') d\alpha \\ 0 \\ dL \\ dK \\ 0 \end{bmatrix} \tag{15}$$

The system given by (15) can be solved to obtain the effects of a change in any parameter. To begin with, let us suppose that $dL = 0$, $d\alpha = 0$, and $dK > 0$, so that the supply of labor is unchanged while that of capital increases, with a constant inter-regional wage-differential. From (15) and (10) we obtain:

$$(16) \qquad \frac{1}{y}\frac{dy}{dK} = \frac{\dfrac{-(k_1 - \alpha k_2)(k_1 - k_2)}{y}\left[\dfrac{f_1 - pf_2}{k_1 - k_2}\right] + B}{(k_1 - \alpha k_2)(k_1 - k_2) + C},$$

where

$$B = \frac{\alpha k_2 f_1 \sigma_2}{L_1 (f_1 - k_1 f_1') \sigma_D} + \frac{k_1 f_2 \sigma_1}{L_2 (f_2 - k_2 f_2') \sigma_D} > 0,$$

$$C = \frac{\alpha p k_2 f_1 \left[f_2 + \rho_1 f_2' (k_1 - k_2)\right] \sigma_2}{\rho_1 f_1' (f_1 - k_1 f_1') \sigma_D} + \frac{k_1 f_2 \left[f_1 - \rho_2 f_1' (k_1 - k_2)\right] \sigma_1}{p \rho_2 f_2' (f_2 - k_2 f_2') \sigma_D} > 0,$$

and $\rho_i = L_i/L$. $C > 0$, because

$$f_2 + \rho_1 f_2' (k_1 - k_2) = \frac{1}{\alpha\rho}\left[f_1 - f_1' (k_1 - \alpha k)\right] > 0, \text{ and}$$

$$f_1 - \rho_2 f_1' (k_1 - k_2) = \alpha\rho \left[f_2 - f_2' \left(k_2 - \frac{k}{\alpha}\right)\right] > 0 .$$

Using (3) and (14), (16) can be written as

$$(17) \qquad \frac{1}{y}\frac{dy}{dK} = \frac{Er\left[1 + \dfrac{w_2 (\alpha - 1)}{k_1 - k_2}\right] + B}{Ey + C}$$

where

$$E = \frac{(k_1 - \alpha k_2)(k_1 - k_2)}{y} .$$

As shown before, E may be positive or negative in the presence of the wage-differential, but in the absence of the wage-differential, $E > 0$. The following results may now be derived.

1. If there is no wage-differential, so that $\alpha = 1$, then from (17) it is clear that $(1/y)(dy/dK) > 0$. In other words, an increase in the supply of capital necessarily raises real income in the absence of the wage differential.

2. Assume now that a wage-differential exists. If $\alpha < 1$, so that $w_1 < w_2$, then with $k_1 > k_2$, we know that $E > 0$. The sign of $(1/y)\ (dy/dK)$ then depends on the sign of the numerator of (17). If $w_2\ (\alpha - 1)/\ (k_1 - k_2) > -1$, both terms in the numerator are positive and $(1/y)\ (dy/dK) > 0$. However, if $w_2\ (\alpha - 1)/\ (k_1 - k_2) < -1$, $(1/y)\ (dy/dK)$ will be positive (negative) if the first term exceeds in value (falls short of) the second term.

3. If $\alpha > 1$, so that $w_1 > w_2$, then E may be positive or negative. If $E > 0$, then $(1/y)\ (dy/dK) > 0$. On the other hand, if $E < 0$, both the numerator and the denominator may possess either sign. Hence, it follows that with $\alpha > 1$, $(1/y)\ (dy/dK) \gtrless 0$ provided $E < 0$.

To sum up, <u>even if the price-output responses are normal (which requires $E > 0$: see section II), the rate of growth of income may be negative, whether $\alpha > 1$ or $\alpha < 1$. On the contrary, even if price-output responses are "perverse" (which requires $E < 0$ and $\alpha > 1$), the rate of growth of income may be positive.</u>

For a visual examination of these results, let us analyze Figures 2 and 3, where the original transformation curve is omitted for the sake of simplicity. The pre-capital accumulation equilibrium point in both diagrams is given by P which lies on the price line given by (1). As capital stock increases, the transformation curve shifts out in a biased way to GG', showing that this outward shift is biased towards the capital-intensive commodity X_1. If the commodity prices are kept constant, then from the Rybczynski theorem we know that the output of the labor-intensive commodity X_2 must decline. This situation is depicted by the new production point P', which lies at the intersection of GG' and line (2) which is drawn parallel to line (1). However, in the new equilibrium, commodity prices must shift in favor of the second commodity whose output has declined at constant commodity prices. The final equilibrium point is then given by P'', which lies at the intersection of GG' and line (3) reflecting a higher relative price for the second commodity than before. The price-output response in both diagrams is assumed to be normal. In Figure 2, drawn on the assumption that $\alpha < 1$, social welfare declines from U to U' as a result of capital accumulation, whereas in Figure 3, drawn on the assumption that $\alpha > 1$, social welfare rises from U to U''. This is then a geometrical illustration of the results derived above.

Similar results can be derived if the increase in factor supplies takes the form of an increase in the size of the labor force alone. By setting dK and $d\alpha$ equal to zero in (15), we obtain

$$(18) \qquad \frac{1}{y}\frac{dy}{dL} = \frac{E\left[\dfrac{pk_1\ f_2 - k_2\ f_1}{k_1 - k_2}\right] + F}{Ey + C}$$

$$\text{where} \qquad F = \frac{k_2\ f_1\ \sigma_2}{L_1\ f_1'\ \sigma_D} + \frac{\alpha\ k_1\ f_2\ \sigma_1}{L_2\ f_2'\ \sigma_D} > 0\ .$$

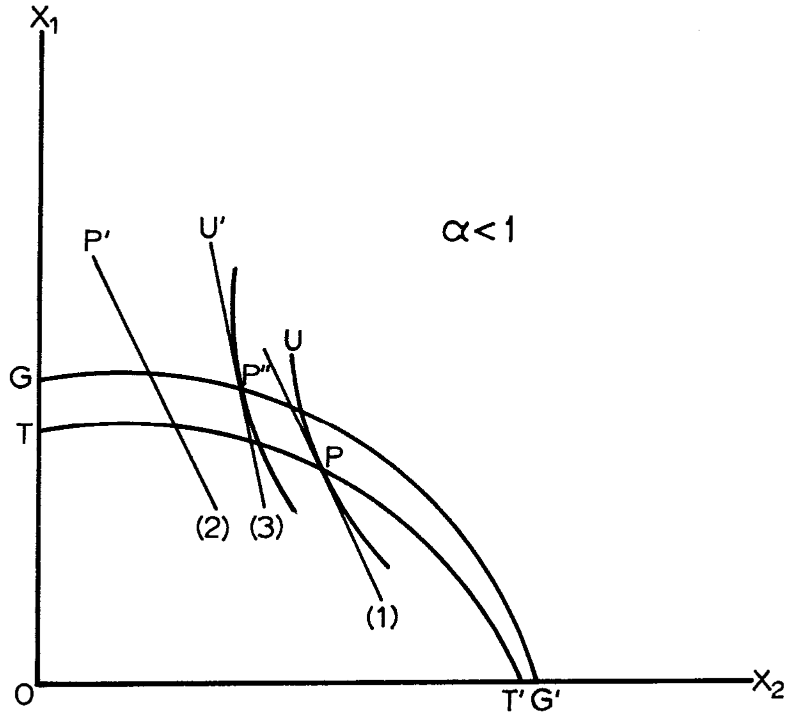

Figure 2

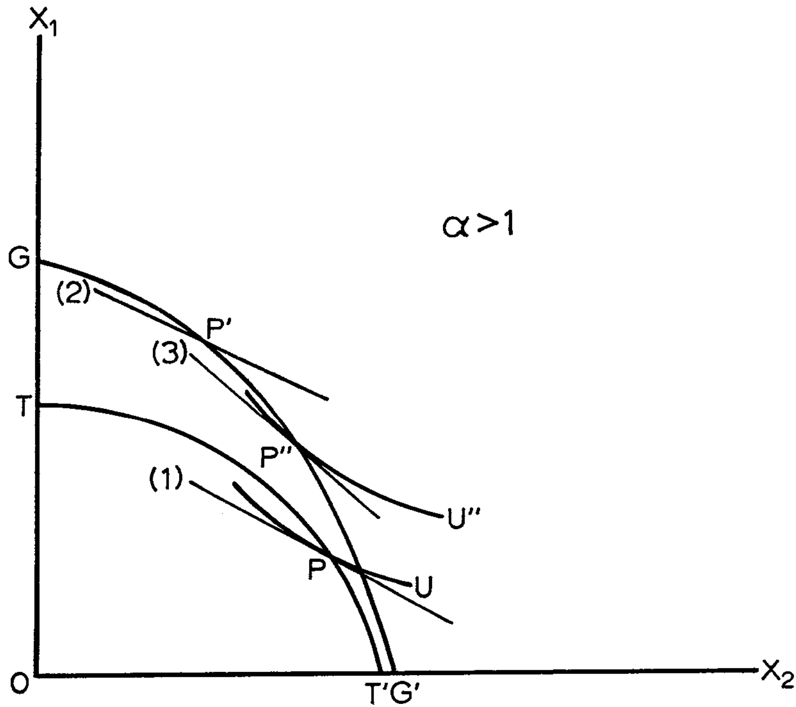

Figure 3

Using (3) and (4), (18) can be written as

$$(19) \qquad \frac{1}{y} \cdot \frac{dy}{dL} = \frac{E\left[\frac{w_2 \, (k_1 - \alpha \, k_2)}{(k_1 - k_2)}\right] + F}{Ey + C}$$

The interpretation of (18) proceeds in much the same way as that of (17). The interesting result derived from (17) that the rate of growth of real income may be negative in the presence of expansion in the supply of capital can also be derived from (19), where it is the supply of labor that has risen. However, there is still a basic difference between the case of pure labor growth and that of pure capital growth. In the case of the latter, factor growth can lead to a loss of real income irrespective of the sign of E; but with labor growth alone, the necessary condition for the decline in real income is that E is negative, for the sign of E is the same as the sign of $(k_1 - \alpha \, k_2)/(k_1 - k_2)$, and with $F > 0$, the numerator of (19) is necessarily positive. Hence for $(1/y)\ (dy/dL)$ to be negative, the denominator must be negative; since both C and y are positive, E must be negative to achieve the possibility of the loss in real income as a result of the labor growth alone. On closer examination, the reason for this difference in the two cases of factor growth can be seen to lie in the fact that the capital market is perfect and only the labor market is distorted.

The two cases discussed above describe the extremes where either the supply of capital or that of labor has been permitted to grow. The more realistic situation is one where both capital and labor are growing. Is it then possible to encounter a negative growth of income in the face of growing factor supplies, and, if yes, under what conditions? Suppose that $dK = \lambda k dL$, where $k = K/L$ is the overall capital/labor ratio and λ is a constant. If $\lambda = 1$, $dK/dL = k$, so that both capital and labor are growing at the same rate. This situation may be called the case of steady state. Otherwise, $dK/dL \gtrless k$ if $\lambda \gtrless 1$. When both factors are growing,

$$\frac{1}{y}\frac{dy}{dL} = \frac{1}{y}\frac{\partial y}{\partial L} + \frac{1}{y}\frac{\partial y}{\partial K}\frac{dK}{dL} = \frac{1}{y}\frac{\partial y}{\partial L} + \frac{\lambda k}{y}\frac{\partial y}{\partial K}.$$

This from (16) and (18) can be written as

$$(20) \qquad \frac{1}{y} \cdot \frac{dy}{dL}\bigg|_{\frac{dK}{dL} = \lambda k} = \frac{E\left[\frac{(pk_1 \, f_2 - k_2 \, f_1)\ (1 - \lambda)}{(k_1 - k_2)} + \frac{\lambda y}{L}\right] + G + H}{Ey + C},$$

where $$G = \frac{\alpha\, pk_2\, f_1\, \sigma_2 \left[f_2 + f_2' \{\lambda \rho_1\, (k_1 - k_2) - k_2\, (1 - \lambda)\}\right]}{L\, \rho_1\, f_1'\, (f_1 - k_1\, f_1')\, \sigma_D}$$

and $$H = \frac{k_1\, f_2\, \sigma_1}{L\, p\rho_2\, f_2'\, (f_2 - k_2\, f_2')\, \sigma_D} \left[f_1 - f_1' \{\lambda\, \rho_2\, (k_1 - k_2) + k_1\, (1 - \lambda)\}\right].$$

Both G and H are positive, because

$$f_2 + f_2' \left[\lambda\, \rho_1\, (k_1 - k_2) - k_2\, (1 - \lambda)\right] = (f_2 - k_2\, f_2') + \lambda\, kf_2' > 0,$$

and

$$f_1 - f_1' \left[\lambda\, \rho_2\, (k_1 - k_2) + k_1\, (1 - \lambda)\right] = (f_1 - k_1\, f_1') + \lambda\, kf_1' > 0,$$

Using (3) and (4), (20) can be written as

(21) $$\frac{1}{y}\frac{dy}{dL}\bigg|_{\frac{dK}{dL} = \lambda k} = \frac{E\left[\frac{w_2\, (k_1 - \alpha\, k_2)\, (1 - \lambda)}{(k_1 - k_2)} + \frac{\lambda y}{L}\right] + G + H}{Ey + C}$$

The following results can now be derived from (21).

1. First, consider the case where the economy is in steady state growth, that is, both capital and labor are growing at the same rate, so that $\lambda = 1$. With $\lambda = 1$, (21) reduces to

$$\frac{1}{y} \cdot \frac{dy}{dL}\bigg|_{\frac{dK}{dL} = k} = \frac{(E\, y/L) + G + H}{Ey + C}$$

From the definitions of G, H and C, one can see that,

$$G + H = C/L,$$

so that

$$\frac{1}{y} \cdot \frac{dy}{dL}\bigg|_{\frac{dK}{dL} = k} = \frac{1}{L}, \text{ or } \frac{dy}{dL} = \frac{dL}{L} = \frac{dK}{K}.$$

In other words, if both capital and labor are growing at the same rate, national income also grows at this rate. The presence of the wage-differential makes no difference to this result. Thus we conclude that the decline in real income is impossible in the steady state growth path.

2. Let $\lambda > 1$, so that $dK/dL > k$, which means that capital is growing faster than labor. If there is no wage-differential $(1/y)\ (dy/dL)$ can be easily seen to be positive. However in the presence of the regional wage-differential, real income may decline even if $E > 0$.

3. If $\lambda < 1$, so that $dK/dL < k$, which means that labor is growing faster than capital, the necessary condition for the real income to decline is that $E < 0$. In other words, if the value and physical factor-intensities possess the same sign, so that $E > 0$, the real income must rise if labor is growing faster than capital.

Change in the Wage-Differential

In this section we examine the implications of a change in the inter-regional wage-differential for the outputs in the two regions and national income under the assumption of constant factor supplies. With $dK = dL = 0$, (15) can be solved to obtain

$$(22) \qquad \frac{dX_1}{d\alpha} = - \frac{Q + (\alpha - 1)\ L_1\ R/\sigma_D\ \alpha}{Ey + C},$$

$$(23) \qquad \frac{dX_2}{d\alpha} = \frac{S - (\alpha - 1)\ L_2\ R/\sigma_d\ \alpha}{Ey + C}$$

and

$$(24) \qquad \frac{1}{y} \cdot \frac{dy}{d\alpha} = - \frac{(\alpha - 1)\left[T + R/f_1\ \sigma_D\ \alpha\right]}{Ey + C},$$

where
$$Q = \frac{f_1}{\alpha\ pf_2'}\left[K_1\ \sigma_1\ \{f_1 - f_1'\ (k_1 - k_2)\} + K_2\ f_2\ \sigma_2\right] > 0,$$

$$S = \frac{f_2}{\alpha\ pf_2'}\left[K_1\ f_1\ \sigma_1 + \alpha\ pK_2\ \{f_2 + f_2'\ (k_1 - k_2)\ \sigma_2\}\right] > 0,$$

$$R = k_1\ f_1\ k_2\ \sigma_1\ \sigma_2 > 0, \text{ and}$$

$$T = \frac{(f_2 - k_2\ f_2')}{\alpha\ f_2'\ y}\left[K_1\ f_1\ \sigma_1 + \alpha\ pK_2\ f_2\ \sigma_2\right] > 0\ .$$

The effects of a change in α on the outputs in the two regions can be derived from (22) and (23). Consider first the case where the physical and the value factor-intensities have the same sign, so that $E > 0$. With $E > 0$, the denominator of (22) and (23) is positive. If $\alpha > 1$, then it is clear that $dX_1/d\alpha < 0$, but $dX_2/d\alpha$ may have any sign. However, if σ_D is very large and R relatively small, $dX_2/d\alpha$ will be positive. In the extreme case where commodity prices are kept constant (which, for example, would occur if we introduced international trade and assumed that the home country was small, or if σ_D, the elasticity of substitution, is equal to infinity), the second term in the numerator of (23) will disappear, so that $dX_2/d\alpha > 0$. If on the other hand α is less than unity, $dX_2/d\alpha > 0$, but the sign of $dX_1/d\alpha$ is uncertain. Here again if σ_D is large or R small, $dX_1/d\alpha < 0$. Thus we may conclude that under the 'normal' price-output response and large values of elasticity of commodity substitution, $dX_1/d\alpha < 0$ and $dX_2/d\alpha > 0$. Now $\alpha > 1$ implies that the differential is paid by region 1 which produces X_1, whereas $\alpha < 1$ implies that the differential is paid by region 2 producing X_2. Furthermore when $\alpha > 1$, a rise in α signifies a rise in the inter-regional wage-differential, whereas when $\alpha < 1$, the rise in the wage-differential requires a decline in α. We have already established that $dX_1/d\alpha < 0$ if $\alpha > 1$ and $dX_2/d\alpha > 0$ if $\alpha < 1$. All this discussion suggests that a rise in the wage-differential paid by any region causes a decline in its output. The effect on the output of the other region is uncertain, but most likely, its output will rise.

If $E < 0$, that is, if the price-output response is "perverse" the results derived above may be reversed, because now the denominator may be negative.

By contrast, the implications of a change in α for real income are less complicated. It can be easily observed that if $E > 0$, $dy/d\alpha \lesseqgtr 0$ if $\alpha \gtreqless 1$. If $\alpha > 1$, then, as already noted, a rise in α implies a rise in the wage-differential, but a decline in national income, because now $dy/d\alpha < 0$. Similarly, if $\alpha < 1$, a decline in α implies a rise in the wage-differential, and hence a decline in national income, because here $dy/d\alpha > 0$. Thus we may conclude that in the presence of the 'normal' price-output response, a rise in the inter-regional wage-differential results in a decline in real income and vice versa.

This result is reversed if E is negative and is large enough to render the sign of the denominator of (24) negative.

The last result is surprising and paradoxical, for it implies that a rise in the wage-differential between the two regions could actually result in a rise in the real income and vice versa. Therefore, in order to convince the reader and ourselves, we consider Figures 4 and 5 which are drawn for the case of $\alpha < 1$. Figure 4 depicts the normal case where a rise in α, which with $\alpha < 1$ implies a decline in the wage-differential, shifts the transformation curve TPT' to the dotted curve TP'T'. The output of X_2 rises from that given by the production point P to that indicated by point P'. Furthermore, the relative price of X_2 declines as shown by the fact that line (2) is less steep than line (1). This occurs because when $\alpha < 1$, the wage-differential was initially paid by X_2 producers, and with the decline in the differential, the cost disadvantage to X_2 also declines. (See Batra and Pattanaik [2] and Jones [12]). It may be observed that welfare rises from U_1 to U_2 as a result of a decline in the inter-regional wage-differential.

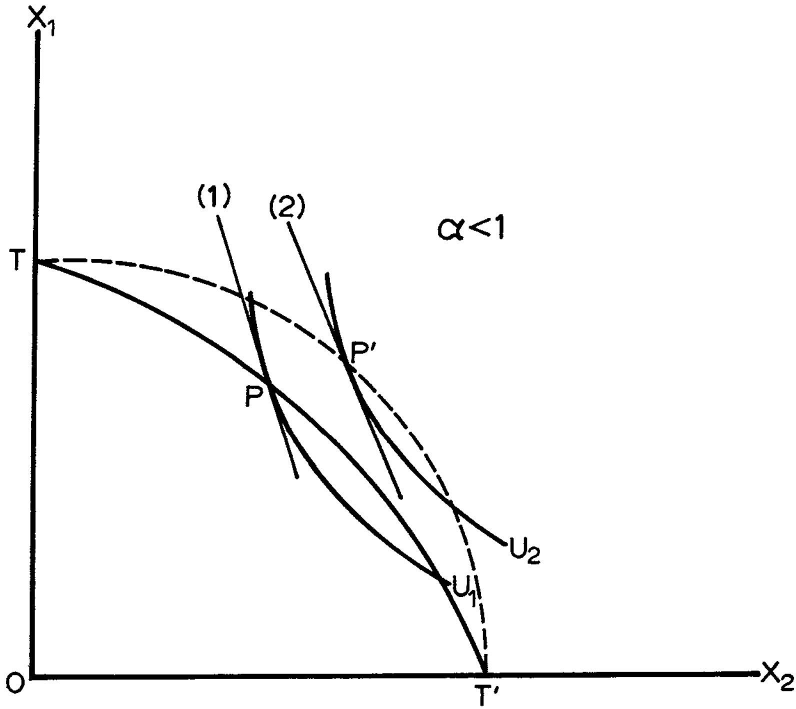

Figure 4

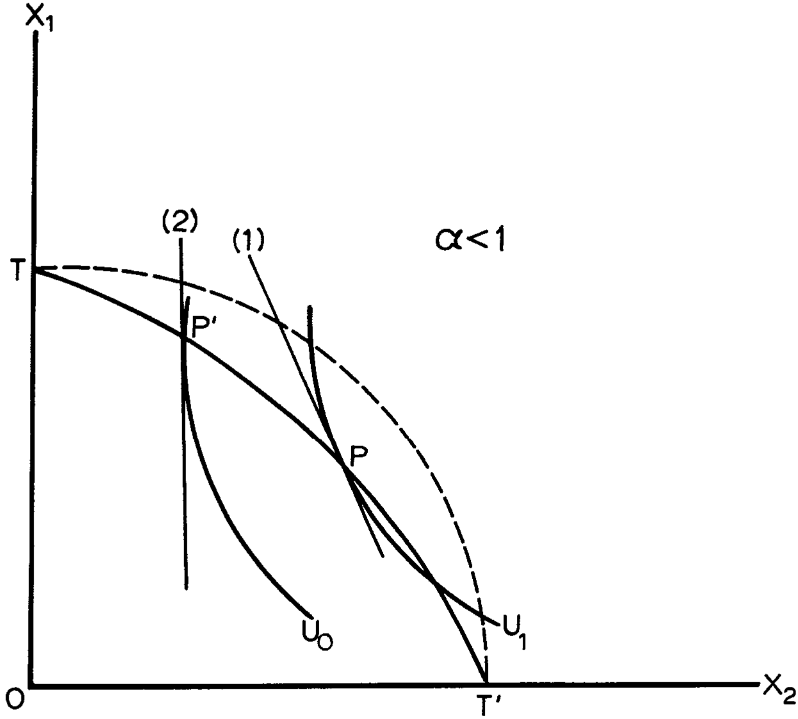

Figure 5

Figure 5 on the other hand depicts the perverse case where the output of X_2 declines as a result of a decline in the wage-differential, and so does welfare from U_1 to U_0.

Factor Growth and the Change in the Wage-Differential

The stage has now been set for combining the analyses of the last two sections and exploring the consequences of the change in the wage-differential on the rate of growth of real income. We shall examine the case where both capital and labor are growing and see how the rate of growth of income is influenced by the change in the wage-differential which may occur in the growth process.

In order to facilitate the exposition of this section, the following results derived in the last two sections, may be recapitulated.

1. If $E > 0$, the real income necessarily rises at a given inter-regional wage-differential, provided the rate of growth of labor is equal to or greater than the rate of growth of capital. However, if capital is growing faster than labor, the real income may decline at the constant wage-differential.

2. If $E < 0$, the real income may decline when capital and labor are growing at unequal rates.

3. At a given capital/labor ratio in the economy, a decline (rise) in the differential necessarily leads to a rise (decline) in the real income, if $E > 0$. If $E < 0$, then the real income and the wage-differential may be positively related.

This last result suggests that a convergence of the inter-regional differential will result in a rise in the rate of growth of real income consequent upon the expansion of factor supplies, provided $E > 0$. However, if $E < 0$, the convergence of the differential may actually cause a decline in the rate of growth.

Of some interest is the case where the real income is unambiguously rising at the unchanged wage-differential, but the change in the differential may actually lead to its decline. On closer examination, this may occur in the following cases:

1. The economy is in the steady state path, but i) the wage-differential is rising and $E > 0$, or ii) the differential is declining but $E < 0$.

2. Labor is growing faster than capital, $E > 0$, and the wage-differential is rising.

Concluding Remarks

In the foregoing analysis we have examined the implications of factor supply expansion for real national income in the presence of an inter-regional wage differential. We have shown that an increase in factor supplies may actually lead to a decline in real income in the presence of a constant wage-differential. Furthermore a decline in the differential may cause a decline in the rate of growth of real income. Although these results are very interesting, simply because they are paradoxical, in our view they are mere theoretical possibilities not likely to apply to realistic situations. A closer scrutiny of our equations in sections three and four will show that only for extremely large values of the inter-regional wage-differential, (α), is it possible to discover these paradoxes in reality. For reasonable values of α and the factor-intensities in the two regions, the paradoxes may disappear. Nevertheless, our analysis does suggest the possibility of a correlation between the rate of growth of income and the inter-regional wage differential. From the weight of our analysis, we suggest that the observed convergence of the regional wage differential in Canada and the United States had beneficial effect on the growth rates in the two countries. Whether this beneficient effect was statistically significant or large enough to have perceptible influence on per capita incomes is an empirical question. We do hope that our results will induce an empirical economist to provide answers to this query.

On the other hand, the conclusions of this paper are subject to the simplifying assumptions underlying the analysis. A closer approximation to the problems of a regionalized economy would thus necessitate a closer investigation of these assumptions. This is particularly true of those assumptions which have been made with regard to factors of production. On the one hand, there arises the question of whether the results will hold in the presence of three or more inputs. This element can be incorporated into the analysis without much difficulty and it may be shown that the basic qualitative results remain unaltered. Of greater importance perhaps is the assumption relating to factor mobility. Indeed it can be argued that a major cause for the very existence of inter-regional income differences is the fact that productive factors are not perfectly mobile whether inter-regionally or inter-industrially, and that the extent of such income differences is largely determined by the degree of factor movements called forth by various economic changes (factor growth, changing tastes, technological progress, etc.). A better understanding of the functioning of the regional economy thus requires a closer scrutiny of the determinants of inter-regional income differences by generalizing the model so as to incorporate these determinants. This is an important and fruitful field of theoretical as well as empirical investigation, but is obviously outside the scope of the present study.

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