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Time Preference, Abatement Costs, And International Climate Policy: An Appraisal Of IPCC 1995

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Abstract

This paper is an appraisal of current economic methodology in the analysis of the social rate of time preference and discounting, abatement cost methodologies, and value of life estimates as they relate to climate change. The paper makes a case for using a zero rate of time preference when assessing climate policies. Furthermore, it argues that the currently estimated disparity in the cost of greenhouse gas abatement between developed countries and developing countries may be highly inaccurate. Finally, the paper integrates discount rates, abatement costs, and value of life estimates to highlight important and contrasting implications of international climate policy for developing and high income countries. The context of the paper is the forthcoming report Second Assessment Report of the Intergovernmental Panel for Climate Change.

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Introduction

Working Group III (WG III) of the Intergovernmental Panel on Climate Change (IPCC) was convened to assess the economic implications of climate change. The resulting draft report, which is a part of the Second Assessment Report of the IPCC, summarizes and examines the relevant economic literature, focussing on issues central to policy making in this context. It is intended to be an authoritative and widely supported commentary on the economics of climate change that will be the foundation of future climate change negotiations and policies.

The draft 1995 WG III report, identifies four key questions relating to climate change policies (chap 5, p 1):

- 1) by how much should greenhouse gas (GHG) emissions be reduced?
- 2) when should emissions be reduced?
- 3) how should emissions be reduced?
- 4) who should reduce the emissions?

The first two questions are inextricably linked with the choice of the discount rate which influences policies affecting both the timing and the extent of GHG reductions. The third question may be viewed from a purely technocratic perspective; however, the credibility of the international mechanism chosen to bring about the requisite technology and policy changes plays an important role in their practical implementation. The fourth question is linked to the third, in the sense that the location of reductions in emissions is influenced by economic wealth, technology, and equity.

Our analysis appraises the draft 1995 WG III report in this context. It examines the above questions within the broad setting of IPCC 1995, highlighting the more controversial aspects of the debate.

Discounting Discounting

Discounting brings together two aspects of modern economics that have traditionally been separate -- positive and normative microeconomics. Choosing the appropriate discount rate for global warming involves taking an ethical stand on how this generation values future generations, as distinct from the former's evaluation of its own future. At the same time, it invokes the positive concern of the degree to which current investments in GHG abatement displace current consumption and/or other investments in the economy (IPCC 1995, chap 4, pp 3-5).

Consider a private investment in a project whose benefits accrued to the agent during her lifetime. Suppose also, that this investment displaced another similar private investment. In this situation, the appropriate rate of discount is the private discount rate, that is the rate at which the individual agent values future cash flows in terms of present dollars. Typically, this is measured as the opportunity cost of capital. In the case of public policies whose impacts are spread over multiple generations, such as investing in GHG abatement, the issue of intergenerational equity becomes central to the decision making process. In this case, the appropriate discount rate is the social rate of time preference (SRTP) that reflects the value that *society* places on future levels of consumption (Feldstein 1964, p 364).

The basic idea here is that individuals make two different types of decisions -- "private" decisions reflecting personal interests, and "public" decisions which take account of social responsibilities for other individuals both in the present and future generations. Market or private discount rates reflect the former context, whereas the social discount rate (the SRTP) reflects the latter (Markandaya and Pearce 1988, p 48). This distinction is important because the SRTP need not have any systematic relationship with the private discount rate. Nor does it represent any inconsistency in human behavior: people appear to exhibit different preferences in the private and public domain since the economic choices they make in the two situations are fundamentally different (Lind 1982b, p 57).

The SRTP is represented by the following equation (IPCC 1995, chap 4, p 7):

$$\text{SRTP} = \rho + \theta g$$

where ρ is the pure rate of time preference

θ is the elasticity of marginal utility with respect to consumption

g is the rate of growth of consumption in the future.

The pure rate of time preference is the rate at which time is discounted: a unit of utility today is preferred to a unit of future utility. This arises out of the myopic nature of individual decision making. θg , on the other hand, is the discount rate that is used to compare different consumption levels, accounting for the declining marginal utility of consumption.

The above equation for SRTP was first derived rigorously by Otto Eckstein (1957, pp 74-76), though it was implicit in Ramsey (1928), which included a verbal explanation attributed to Keynes. For that reason, we refer to it as the *Ramsey-Keynes-Eckstein* equation. In 1965, Tjalling Koopmans developed an independent derivation for this equation as the first

order equilibrium condition for maximizing the discounted sum of utility over an infinite horizon for a perfectly competitive economy (see appendix for derivation).

The second component of the SRTP is based on the underlying growth model used in the analysis at hand. Cline (1992, p 248), for instance, places this at approximately 1% on the basis of the projected growth rate of per capita GDP for the period 2050 to 2100 and the Golden Rule growth model.¹ This part is, therefore, relatively non-controversial. It is the first part, the pure rate of time preference, that is surrounded by controversy. This is because in an intergenerational context, a positive value for ρ involves an intertemporal, interpersonal comparison: it implies that the welfare of future generations is worth less than that of the present generation.

A key feature of the climate change problem is the extremely long time scales involved. Whereas public policy typically addresses problems over a 5 or 10 year period, climate change requires a much longer perspective which spans several decades. Therein lies the philosophical problem with discounting -- it eliminates the value of future benefits and costs that may accrue from investments made today. Consider, for instance, \$100 of GHG investment undertaken today which yield \$150 of benefits in the form of avoided climate change 70 years in the future. The present value of the benefits, using an SRTP of 3%, is approximately \$18.95.² In fact, any discount rate in excess of 0.5809% would make this present value less than \$100, thereby making the investment economically inefficient in a traditional benefit-cost sense.

¹ The Golden Rule growth model maximizes steady state per capita consumption (Cline 1992, p 248).

² Nordhaus (1994) uses a pure rate of time preference of 3% per annum to discount future utilities.

The discount rate is also a major determinant of the optimal timing of projects (Lind 1982a, p 2). In a re-examination of the Nordhaus DICE model, Chapman, et. al. (1995, p 7) argue that a lower rate of time preference implies not only a greater level of action, but also a greater urgency (see figure 1). Similarly, Conrad (1995, p 20) concludes that in an optimal stopping rule model for global warming, small changes in ρ lead to large changes in the time to begin investment in GHG reductions (see table I below).

Table I: Optimal Timing of GHG Reduction Investments	
ρ	Time to begin GHG reductions (years)
0.02	0
0.03	0
0.04	92
0.05	122
0.06	140
<i>Source: Based on Conrad (1995).</i>	

For an individual, a positive pure rate of time preference may be justified by the risk of death: the individual may not be alive in the future to enjoy the benefits of increased consumption. However, in the case of societies, this argument breaks down when one takes into account the immortality of society as opposed to the mortality of the individual (Markandaya and Pearce, 1988, p 33). In a similar vein, Arrow et. al. (IPCC 1995, chap 4, p

8) argue that to make ρ positive on this basis amounts to the implicit premise of a breakdown of public policy.

In the context of global warming, where we are concerned with the impacts of public policy over long time horizons that span several generations, there is no ethical basis for a positive pure rate of time preference. The current generation is no more important than those in the future, unless the former were faced with a high probability of death. Rawls (1971, pp 293-298) has argued for a zero rate of time preference on the basis of intergenerational justice. According to him, time preference has no intrinsic ethical appeal since it allows the current generation to assess the situation of future generations on the basis of the former's own view of its present circumstances. In other words, ".....the living take advantage of their position in time to favor their own interests." Likewise, Sen believes that "in the case of the environment, a fundamental right of future generations may be violated by environmental degradation caused by the current generation, which causes an 'oppression' of future generations, even if they are richer and have a lower marginal utility of consumption." (Sen, 1982, as quoted in IPCC 1995, chap 4, p 5). Furthermore, discounting implies that the preferences of current generation are superimposed on future generations, and are used to make decisions that will affect the latter. According to Mishan (1975, pp 208-09), this introduces an arbitrary generational preference in the analysis. He believes that in this case, a zero rate of time preference is appropriate.

In a survey instrument, Case (1986) found that responses were largely consistent with a zero discount rate in the intergenerational context, or for events in the far future. This is

especially true when current activities are characterized by irreversibilities, as in the case of global warming.

That intergenerational equity should be addressed specifically and separately from issues of allocative efficiency has been well illustrated by Howarth and Norgaard (1995). Using an overlapping generations framework with intergenerational asset transfers, competitive production and exchange of goods, GHG emissions, and energy taxes, they argue that efficiency considerations alone, as reflected in a simple cost-benefit analysis, may lead to significantly lower welfare levels for future generations. For climate policy to be "optimal" in the true sense of the word, equity issues must be made to specifically bear upon the analysis. In the Howarth-Norgaard model, intergenerational equity is achieved via income transfers between the present and future generations. Given these, cost-benefit analysis leads to an efficient intertemporal resource allocation.

There may be some situations where ρ is set at a positive value for mathematical reasons. For example, in the case of an infinite horizon problem, convergence of the integral requires $\rho > 0$. Any positive ρ would satisfy the requisite mathematical condition. In keeping with the recommendation of Arrow et. al. (IPCC 1995, chap 4, pp 9-10), in this case ρ should be set at as small a value as possible so as to minimize the distortion on account of this factor. Furthermore, this should be stated explicitly and should not be defended on the grounds of economic reasoning. In addition, for a finite time horizon, ρ can be zero without causing computational problems.

It is interesting that in an earlier draft of the IPCC report, Arrow et. al. (1994, p 16) had suggested that the central value of the pure rate of time preference should be 0.1%.

However, this suggestion was removed from the present draft. This only serves to emphasize the extremely controversial nature of the issue.

IPCC 1995 sets out the various aspects of the debate succinctly. However, it does not suggest a range of appropriate values that policy makers might use. To this extent, it fails to bring the controversy surrounding discount rates any closer to resolution.

Polluter Pays

A review of the IPCC 1995 damage estimates from global warming impacts leads to the conclusion that developing countries might have to bear a disproportionate share of the damages. This would be due to their greater vulnerability on account of geographic, climatic and socio-economic factors. Thus, while for developed countries the value of economic damages from 2XCO₂ is estimated to be in the range of 1-2% of GDP, the corresponding figure for developing countries is 2-9% (IPCC 1995, chap 6, p 48).^{3,4}

In addition, despite a wide variation in model results, IPCC 1995 concludes that there is a significant potential for negative or low cost CO₂ reduction options in developing countries relating to end-use and conventional energy supply technologies in the short and medium run. The UNEP 1994 study shows that for the period 2005/2010 to 2020/2030, the average emissions reduction cost is less than \$37 per tonne of CO₂ (as quoted in IPCC 1995, chap 9, p 33). A similar study for the USA, EMF 1993, concluded that a 20% reduction in

³ 2XCO₂ refers to the situation where the concentration of GHGs has increased so as to raise global mean surface temperature by an amount equivalent to that produced by a doubling of the base year concentration of CO₂.

⁴ The damage estimates represent the range of the results from existing studies. They are not central values.

CO₂ emissions would require a carbon tax which is an order of magnitude higher in 2010 (as quoted in IPCC 1995, chap 9, table 9.1.1.1.3.1, p 107).

Economic efficiency, as distinct from equity, mandates that pollution damages be paid by the agent for whom it is the cheapest, regardless of whether they are the polluter or the victim. According to this principle, then, GHG abatement should be concentrated in developing countries until all the "cost-effective" options are exhausted. Three issues should be kept in mind here. First, not all studies reveal such a sharp dichotomy in abatement costs between developed and developing countries. According to the OECD Model Comparison Project, there are likely to be fairly large losses in GDP for China and other developing nations from reductions in energy use. A 2% reduction in the rate of growth of CO₂ emissions, for instance, would cause a percentage loss in GDP in China that is only slightly lower than the average for the global economy. Stabilization of emissions at 1990 levels, on the other hand, might be more harmful to China (as quoted in IPCC 1995, chap 9, table 9.1.4.1.1.1, p 120). Conversely, Rubin et. al. (1992) show that fairly large GHG emissions reductions can be achieved in the USA at negative or low cost per tonne of CO₂ equivalent reduced. Likewise, Krause et. al. (1995, p ii) argue that OECD countries could decrease carbon emissions by at least 50% of 1990 levels over the next 30-50 years at a negative net economic cost.

A second and more fundamental question is why nations have not taken advantage of the "free lunches" available to them. Perhaps, this is because the "free-lunches" arise out of market and institutional imperfections that prevent cost effective emission reduction measures from being implemented. According to Rubin et. al. (1992), there is evidence to show that

households and industry are unwilling to undertake large scale investment in energy efficiency improvements if the payback periods are too long. In effect, this implies that private discount rates are up to an order of magnitude higher than those typically used in studies that assess the potential for GHG reductions. The same holds true for minimum acceptable rates of return in industry. This creates a gap between the technically achievable levels of GHG reductions and the level that can be realistically reached within the present economic and policy framework (Rubin et. al. 1992, pp 263-264). The key issue, therefore, is the identification of the institutional mechanisms that will bring private and social discount rates into correspondence (Howarth and Norgaard 1995, p 113), and whether the underlying imperfections can be removed cost effectively by policy measures (IPCC 1995, chap 9, p 55). This issue has not been adequately addressed in the literature so far.

A third issue arises from the fact that, at present, there are few mitigation cost studies for developing countries. While the results for developed countries have been improved over multiple generations of models, estimates for the former are only beginning to come in. Furthermore, developing country studies tend to mimic the studies for developed countries, without assessing their applicability to their particular situations. IPCC 1995 argues that these studies are often based on assumptions that may not be valid for the developing world: these include perfectly competitive market behavior, existence of futures markets and perfect information. Also, they typically do not model informal, non-market sectors. An implication of this is that they are unable to take account of the increasing integration over time of traditional sectors into the formal economy (IPCC 1995, chap 9, pp 25-26). All of the above factors point to the fact that the results for developing countries should be viewed with

caution. At the same time, this does not absolve them from their responsibilities. Developing countries are likely to account for a significant share of future GHG emissions, and must therefore, supplement developed country efforts towards emission reductions.

The above findings of IPCC 1995, viz., the possibility of much higher economic damages and the potential for large reductions in CO₂ emissions at relatively low cost in developing countries, are the outcome of its efficiency-based focus on reducing emissions. A fourth, broader issue is the feasibility, both economic and political, of such approaches in mitigating global warming. This issue has been examined in some detail by Chapman and Drennen (1990). They consider seven possible scenarios focussing on the use of fossil fuels and forestation to address the feasibility question. Each scenario is assessed in light of its effectiveness, as reflected in the length of time for which a doubling of CO₂ concentrations is deferred; and its implication on international equity (as captured by the per capita emissions differential between developed and developing countries) which determines its political feasibility. The authors conclude that scenarios based on energy growth rate policies alone fail to satisfy the equity and effectiveness criteria (p 25). A multi-pronged strategy focussing on increased energy taxation, reduced population growth, and forestry would be the preferred option. This would not only delay CO₂ doubling till more than three centuries into the future (if the doubling occurs at all), it would also allow for closing the gap between the per capita emissions of developing and developed countries. The latter is, we believe, a minimum feasibility requirement of any practical international strategy to reduce CO₂ emissions. Yet, it seems wholly unrealistic to expect these policies to be implemented in the near future.

International Policy Implications

There are some interesting implications for international climate policy implicit in IPCC 1995. This section highlights one such case using the following simple, yet persuasive, thought experiment. Consider the situation where 200,000 people might be killed due to climate impacts in 2045 (i.e., 50 years from now). For simplicity, we assume that the number of deaths are split between a developed country, USA for example, and a developing country like Bangladesh. The table below uses typical estimates of the value of lives lost and discount rates that have been cited in IPCC 1995. On the basis of these, the present value of future damage has been calculated for both cases.

Table II: Present Value of Future Damage					
	Year	# Killed	VOL ^a (\$)	Dscnt Rate ^b	PVFD (\$)
USA	2045	100,000	1.5 mill	3%	34.2 bill
Bangladesh	2045	100,000	0.15 mill	10%	0.128 bill
<p>Notes: VOL: value of one life</p> <p>Dscnt rate: discount rate</p> <p>PVFD: present value of future damage = $[100,000 \times \text{VOL}] / [1 + \text{dscnt rate}]^{50}$</p> <p>Sources: a and b -- IPCC 1995, chapters 6 (p 22), and 4 (p 13), respectively.</p>					

In addition, model results typically show that it is much cheaper to reduce a tonne of carbon emissions in a developing country than in a developed country (IPCC 1995, chap 9, especially pp 77, 88-89).

The import of the preceding discussion is that under a globally efficient climate policy, mitigation efforts should be undertaken in Bangladesh. However, the same policy would dictate that in the case of a climate catastrophe, relief efforts should be concentrated in the US! Such a strategy would be neither politically nor ethically defensible.

Conclusions: Where Do We Go From Here?

The draft 1995 report of the IPCC has done a commendable job by bringing together the results of a number of studies on the economic aspects of the climate change debate. Indeed, it is a comprehensive review of the current body of knowledge on the topic. The report highlights the principles that will impinge upon future decisions, and decision-making processes, as nations attempt to deal with global warming within the framework set out by the Climate Change Convention. In addition, a portfolio of possible technological and policy options has been identified, with an assessment of likely costs and benefits, thereof. Interregional and intercountry differences have been highlighted wherever possible.

At present, the literature is dominated by a focus on economically efficient strategies for dealing with climate change. It is necessary, however, to look beyond this rather narrow notion. Efficiency leads to an 'optimal' outcome for a given initial allocation of wealth. The initial allocation, however, is totally arbitrary with no moral justification (Rawls 1971).

Differences in abatement costs and the 'value of life', as they are usually calculated, reflect

current wealth and income differentials. Policies based on these estimates not only have an unjust starting point, they tend to enhance the existing disparities. For an outcome to be equitable, then, a redistribution of the initial allocation may be necessary. It is one thing to reach a predetermined goal in an economically efficient manner, but quite another to let efficiency determine what that goal is to be. The main reason why the choice of the SRTP for climate change has proved to be so controversial is because it synthesizes these two issues, viz., allocative efficiency and intergenerational equity, into a single numeral. On the one hand, the SRTP reflects the marginal productivity of capital; on the other hand, it embodies the values determining social decisions that affect the present and future state of the economy and the environment. As such, the choice of the appropriate rate to use is itself a public policy decision that depends not only on the merits of the underlying economic arguments, but also on the socio-economic implications of the possible outcomes. Thus, while philosophers and economists may debate the appropriateness of different rates, and while their arguments may be influential, the final choice is often determined by the relative political strengths of the various forces that support the different outcomes (Lind 1982a, p 5-8).

The solution to the climate change problem is not easy. It is inherently intertwined with the shape that economies and societies take in the future. The next step, therefore, is to place the climate change issue in perspective and to see it in the context of the notion of sustainable development. In other words, it is imperative to view global warming, not in isolation, but as part of a bigger problem with the evolution of modern human societies. Thus, while increased energy efficiency may be a part of the overall strategy to reduce GHG

emissions, this alone is not enough. To be effective, it must be complemented by changes in present, highly energy intensive lifestyles, and also reductions in population growth rates. These, in turn, will involve major changes in the current socio-economic paradigm.

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Figure 1: SDICE Optimal Control and Discount Rates

Source: Chapman et. al. (1995)

APPENDIX

Derivation of Ramsey-Keynes-Eckstein equation for social rate of time preference

Presented below is a simplified version of the Koopmans (1965, pp 275-276) derivation. Let

C_t : per capita consumption in period t (control variable)

Y_t : per capita output in period t

I_t : per capita investment in period t

K_t : per capita capital stock in period t (state variable)

$U(C_t)$: utility of per capita consumption in period t , such that $U'(C_t) > 0$ and $U''(C_t) < 0$

ρ : pure rate of time preference

Consider a perfectly competitive economy with perfectly functioning capital markets and a single consumer. Suppose the output in each period is determined by the capital stock, i.e.,

$$Y_t = f(K_t) \quad (1)$$

Then, the following identity holds:

$$Y_t \equiv C_t + I_t \quad (2)$$

$$\text{where } I_t = \partial K_t / \partial t \quad (3)$$

(for simplicity, we assume that there is no depreciation of capital)

Consider the following maximization problem:

$$\begin{aligned} \text{Max} \quad & \int_0^{\infty} U(C_t) e^{-\rho t} dt \\ \{C_t > 0\} \end{aligned} \quad (4)$$

subject to equations (1), (2) and (3).

The current value Hamiltonian for the above problem is:

$$H = U(C_t) + \mu_t [f(K_t) - C_t] \quad (5)$$

where $\mu_t > 0$ is the costate variable. The first order conditions for equilibrium are:

$$\partial H / \partial C_t = 0 \Rightarrow U'(C_t) - \mu_t = 0 \Rightarrow U'(C_t) = \mu_t \quad (6)$$

$$\mu - \rho\mu_t = -\partial H / \partial K_t = -\mu_t f'(K_t) \quad (7)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu_t K_t \rightarrow 0 \quad (8)$$

Rearranging equation (7) we get:

$$\mu = \mu_t [\rho - f'(K_t)] \quad (9)$$

Substituting equation (6) into equation (9):

$$\Rightarrow \frac{\partial [U'(C_t)]}{\partial t} \frac{1}{U'(C_t)} = \rho - f'(K_t) \quad (10)$$

$$\Rightarrow \frac{U''(C_t) C_t}{U'(C_t)} \frac{\partial C_t / \partial t}{C_t} = \rho - f'(K_t) \quad (11)$$

$$\Rightarrow f'(K_t) = \rho - \frac{U''(C_t) C_t}{U'(C_t)} \frac{\partial C_t / \partial t}{C_t} \quad (12)$$

$$\Rightarrow i = \rho + \theta g \quad (13)$$

where i = rate of interest

θ = elasticity of marginal utility with respect to per capita consumption

g = rate of growth of per capita consumption

Under the assumptions of perfect competition and perfectly capital markets, the marginal productivity of capital, i.e., $f'(K_t)$ is equal to the rate of interest, i . Furthermore, the latter captures the society's time preference (Eckstein 1957, p 67). That is, i is equivalent to SRTF in the text.

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