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Commodity Price Behavior: A Rational Expectations Storage Model of Corn

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Abstract: A structural model is developed to simulate the probability distributions of corn prices by month. The intent is to determine the relationship between model specifications, based on a rational expectations competitive storage framework, and the probability distributions of monthly prices. Specifically, can a structural model generate corn prices with characteristics that are consistent with those observed in the 1990s? The model in this paper produces cash prices that *inter alia* have positively skewed distributions where the mean and variance increase over the storage season. The model also generates futures prices as conditional expectations of spot prices at contract maturity. The variances of these futures prices have realistic time-to-maturity and seasonal effects. The model is solved and simulated so that the consequences of making the model increasingly complex can be determined. A "curse of dimensionality" is inevitable with the increased complexity, resulting in lengthy computing times, but the final specification generates plausible probability distributions. In contrast to other models in the literature, our specification does not depend on the unrealistic assumption of zero stock-levels to generate skewed price distributions and the Abackwardation@ commonly observed in prices between crop years. Non-linearity in the supply of storage is achieved by modeling convenience yield. The model can be used to depict price behavior conditional on varying levels of the state variables, e.g., for large or small stock levels. Having created realistic probability distributions of prices, a logical next step is to use the distributions to appraise marketing strategies to manage price risk for corn.

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Commodity price behavior is complex, making its modeling a daunting task. A typical price series is highly variable and autocorrelated. Commodity prices often have seasonal and cyclical components. Net of seasonality and cycles, these prices may be mean-reverting to some long-run average, associated with trends in the macro-economy, population growth, and technological changes. Occasional spikes are observed when prices jump abruptly and temporarily to a high level relative to its long-run average. Thus, distributions of prices are skewed to the right and often display kurtosis (e.g., Myers; Deaton and Laroque, 1992). Price changes are nonlinearly dependent—higher moments are correlated (e.g., Yang and Brorsen).

Many models have been developed to depict the systematic behavior of prices, but few have done so adequately (Tomek and Myers; Brorsen and Irwin). Both time-series and structural models require ad hoc assumptions to accommodate all time-series features of commodity prices. Standard models of financial assets do not account for seasonality (or cycles) in agricultural markets or for differences in the market characteristics among commodities. Moreover, little attention has been paid to the fundamental relationship between model parameters and parameters of the underlying probability distributions of prices.

A commodity model can be useful if it provides estimates of how prices are distributed on a given future date, perhaps conditional on known market variables. For effective intra-year marketing decisions, for example, conditional probability distributions of prices need to be available at least monthly. A major impediment to estimating the parameters of these distributions is that these markets undergo frequent structural changes, such as changes in government policy, and the number of observations generated by a constant structure is limited.

The objectives of this paper are to develop a structural model, which can simulate monthly prices, and to determine the relationship between the model's parameters and the parameters of probability distributions of prices. Our point of departure is the nonlinear rational expectations commodity storage model. This model emphasizes nonlinearity in storage—that aggregate storage cannot be negative (Gustafson)—and Muth's rational expectations hypothesis. Williams and Wright synthesize the "modern" theory of competitive storage, which appends supply, demand, and market clearing conditions to the intertemporal arbitrage equation of the classic model.

The resulting framework has simulated some of the time-series features of spot prices (e.g., Deaton and Laroque, 1992, 1996; Chambers and Bailey; Rui and Miranda; Routledge, Seppi, and Spatt). Deaton and Laroque (1992), for example, replicate the degree of skewness and kurtosis in observed price distributions, but fail to account for the degree of autocorrelation.[1] Such studies, however, typically apply a "generic" model to multiple commodities, and ignore commodity-specific characteristics such as the distinction between annual crops with storage and continuously produced, perishable commodities. Moreover, applications of the rational expectations storage framework to commodity-specific markets have focused on simulating policy scenarios (Miranda and Glauber; Miranda and Helmberger; Gardner and López; Lence and Hayes). Also, most research develops annual models, although a few quarterly models exist (Williams and Wright; Pirrong). Chambers and Bailey propose a monthly framework, but

[1] They used deflated series for observed price series, and deflating could introduce autocorrelation that did not exist in the nominal series.

assume a monthly harvest for seven commodities, including soybeans whose production is seasonal.[2]

This paper overcomes many of these limitations by specifying and simulating a model which allows price distributions to be recovered from the structural model. Given estimates of the distributions, the model can generate price series similar to those faced by agents in the industry, allowing us to overcome the small sample problem and thereby analyze long-run economic consequences of (say) risk management strategies. In order for the results to be useful, this paper models the U.S. corn market at a monthly frequency. Corn is a major crop that is storable and homogenous (relative to other commodities), satisfying the basic characteristics of a commodity for which the storage model was originally conceptualized. Moreover, the volume of trading for futures contracts for corn is the largest among agricultural commodity futures.

The paper is organized as follows. The next section discusses the price series to which the model is calibrated. Then, the conceptual model is developed, and the numerical model is specified. The subsequent section reports and examines the equilibrium solution and price behavior implied by the model. The results suggest a potential value of the rational expectations commodity storage framework in empirical price analysis. Monthly distributions of corn prices follow data-consistent seasonal trends in first and second moments. In addition to commonly observed price behavior, the model generates possible, but improbable seasonal price patterns in some "years," providing rich implications for understanding price behavior.

[2] Part of the validation of this model focuses on its prediction that prices follow a two-regime process depending on whether or not inventories are held (Deaton and Laroque, 1995, 1996; Chambers and Bailey; Ng; Michaelides and Ng). All studies have found some supporting evidence. Beck tests an implication of a nonnegative constraint on storage, i.e., the difference in asymmetry of price distributions between storable and non-storable commodities, and finds no significant difference.

Cash and Futures Price Data

Model calibration requires a sample of actual cash prices, but there is no such thing as "the price" of corn. Individual transaction prices differ by location, quality, and delivery terms. This diversity is illustrated by "cash prices at principal markets" published by the Grain and Feed Market News (USDA/AMS). Prices are reported as daily ranges of transaction prices at various locations, which can be averaged over a week's, month's, or year's period. Nonetheless, a published nominal cash price series for No.2 yellow corn at a Central Illinois market can be regarded as a representative example of corn prices. The focus on nominal prices is consistent with measuring risk in terms of deviations of nominal prices from expected (nominal) prices.

The sample period is the nine crop years from 1989/90 through 1997/98, i.e., 108 monthly observations from September 1989 through August 1998.[3] There is no obvious trend; the prices are variable with positive autocorrelation; and there is a prominent spike in early 1996 (Figure 1). Regarding the price series as an ARMA process, both the Akaike information and Schwartz's Bayesian criteria suggest a second-order autoregressive model with third- and fifth-order moving average terms. Over the sample period, the fitted equation is:

$$(1) \qquad P_m = \underset{(0.018)}{0.020} + \underset{(0.084)}{1.451}\, P_{m-1} - \underset{(0.085)}{0.530}\, P_{m-2} + e_m + \underset{(0.101)}{0.302}\, e_{m-3} - \underset{(0.102)}{0.238}\, e_{m-5}$$

where approximate standard errors are reported in parentheses, *e* is the error term, m is time in months, and observations in July and August 1989 are used for the initial lagged observations. According to Ljung-Box statistics, the null hypothesis of white residuals is not rejected (p value = 0.7936 for the first six lags).

[3] This is a relatively market-oriented time period. The change in the commodity loan program was initially enacted in 1985, and supply management programs for corn introduced planting flexibility in 1989 and 1990 (Westcott).

The prices can also be described by estimating monthly distributions, and to allow for skewness and non-zero kurtosis, a Gamma distribution is fitted to the sample by method-of-moments estimation.[4] These distributions are plotted in four separate panels; the September price distribution is plotted in all panels for ease of comparison (Figure 2). The estimates seem to provide sensible approximations. The modes increase over the storage period through April reflecting higher storage costs, and revert toward the harvest level during the growing season. The probability masses of prices during the harvest and immediate post-harvest periods are relatively centered. Prices become more dispersed as planting approaches, and this trend continues until August, when it is reversed in anticipation of the new crop. The distributions are positively skewed. These features of the estimated distributions should be reproduced by any useful model.

The prices of futures contracts reflect market expectations about supply and demand. Assuming no risk premium, the current futures contract price equals its expected value at contract maturity, and if there is no basis risk, futures and spot prices converge at maturity. In other words, futures prices for any individual contract in an efficient market are not mean reverting, since they are the expected values of a particular month's price.[5]

[4]Let X be a random variable having a Gamma distribution with parameters α and β. Then, $\mathrm{E}[X] = \alpha/\beta$ and $\mathrm{Var}[X] = \alpha/\beta^2$. Method-of-moment estimators are obtained by equating theoretical moments to its corresponding sample moments. Thus, $\hat{\beta} = \bar{X} \Big/ \frac{1}{(n-1)} \sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2$ and $\hat{\alpha} = \hat{\beta}\bar{X}$, where $\bar{X}$ is the sample average.

[5]A price risk premium, if it exists, is likely to be small, and the existence of a yield risk premium is debatable (see Wisner, Blue, and Baldwin; Zulauf and Irwin). Previous analyses, which report mean-reversion in futures prices, may suffer from statistical weaknesses (Irwin, Zulauf, and Jackson). Also, the results may be misleading due to constructing a sequence of prices from nearby futures contracts prices, i.e., using the current maturing contract prices until near the maturity date and then switching to the subsequent maturing contract prices.

The variance of futures prices for an individual contract is influenced mainly by the flow of information and its uncertainty.[6] Variability is expected to increase as contract maturity approaches (Samuelson), and is affected by the seasonality in the information flow in the underlying market. For a December corn futures contract, for example, the time-to-maturity effect means that price variability is larger in December than in May, ceteris paribus. Uncertainty about crop size during the growing season is a seasonal effect that coexists with the time-to-maturity effect, producing more price volatility in the summer than in December, ceteris paribus. Consequently, the May (for example) futures contract prices will be more variable than the December contract prices at their respective months of maturity.

The foregoing is illustrated by estimating monthly price distributions for selected futures contracts. Monthly price observations were constructed by averaging the settlement prices of the first and third Wednesdays (Thursday, if Wednesday was a holiday) of every month. December corn contract prices for the 12 months prior to maturity for the years 1990 through 1997 are fitted to a Gamma distribution by method-of-moment estimation (see footnote 3). The magnitude of the variance relative to the mean was, however, too small to estimate the distributions for December contract prices in December (a year prior to maturity) and January. The distributions are plotted in Figure 3. Similarly, distributions for the May contract prices are estimated from 1991 through 1998 (Figure 4).

For both contracts, the time-to-maturity effect is clearly observed—the longer the time-to-maturity, the less dispersed the price distribution, and the dispersion seems to increase

[6] Economic variables that describe supply and demand conditions, and market structure measures such as the ratio of speculators to hedgers are other factors that affect futures price volatility (Streeter and Tomek).

monotonically over time. This is consistent with findings in the literature (e.g., Fackler and Tian; Goodwin, Roberts, and Coble). For the December contract, the modes of the distributions in early months lie to the right of that of the maturity month's distribution, which may support the existence of a yield risk premium (as in Wisner, Blue, and Baldwin). But, the sample is too small to draw a definitive conclusion. Comparing the maturity-month distributions across the contracts, May contract prices have a larger dispersion than December contract prices, as expected.

Conceptual Model

The model incorporates minimum key features of the corn sector in the 1990s.[7] Namely, the crop is planted in April and harvested from September through November by producers who are assumed to be expected profit maximizers. The planting decision is conditional on a realized supply shock. Between planting and harvest, monthly crop estimates provide information on the expected new crop size, and during months preceding harvest, news arrives regarding how much of the annual crop will be harvested next month. A larger-than-average proportion of the annual crop may be harvested in September of an "early" year, or in November of a "late" year. Agents adjust their expectations accordingly. Available supply at the beginning of each month is either consumed or stored. Monthly demand is subject to shocks, and risk-neutral arbitrageurs make monthly storage decisions.[8]

[7] Supply management and governmental stocks programs have had major impacts on price variability by limiting producers' flexibility to respond to market signals and encouraging storage of grain beyond market incentives. The programs evolved in the farm legislation of 1985, 1990, and 1996 toward less governmental involvement; under the 1996 farm law, supply management programs no longer exist, and stocks programs no longer provide high price supports or storage subsidies that encouraged grain stockholding in the past (Westcott).

[8] Later, it is shown that this assumption is not restrictive.

All decisions depend upon the state of the world, defined by the month of the year, available supply, the realization of demand and supply shocks, expected crop size, and expected timing of harvest. Specifically, units of time are indexed by *t* and *m*, where *t* is calendar year and *m* is month (1 = January). To simplify notation, the index *t* is omitted whenever the implied indexes are unambiguous, and the index *m* appears as both subscripts and superscripts. Producers decide how large a crop to plant in April, but due to uncertain growing conditions, the planted crop size is an estimate of crop size at harvest. During the months between planting and harvest, the expected crop size, or crop estimate (*H*), follows a random walk:

(2) $$H_{m+1} = H_m + \varepsilon_{m+1}, \quad m = 5, \ldots, 10,$$

where ε_{m+1} is a mean-zero independent disturbance. In the model, planting is completed at the end of April, and the May crop estimate equals the planted crop size, i.e., $H_5 = H_4$.

During September and October, random proportions of the expected crop size are actually harvested; the remaining crop is harvested in November, revealing the true total crop size. Thus, the incoming harvest (*h*) is

(3) $$h_m = \alpha_m H_m, \qquad m = 9, 10, 11$$

where α_m is a random number bounded between 0 and 1 for *m* = 9 and 10, and α_{11} is a number between 0 and 1 such that $H_{11} = \sum_{m=9}^{11} h_m$. Expectations of these random proportions, $\hat{\alpha}_{m-1}$ (= E[α_m], where E[·] is the expectation operator), are revealed as news, indicating whether the majority of the crop is harvested early, late, or at average timing.

Producers, consumers, and storers form expectations and base their decisions on information available at the beginning of each month, which is represented by a vector of state variables $\boldsymbol{\theta}_m$. Thus, expectations given information at month m are conditional on $\boldsymbol{\theta}_m$, i.e., $\mathrm{E}_m[\cdot] \equiv \mathrm{E}[\cdot \mid \boldsymbol{\theta}_m]$.

Each producer is a price taker and makes the planting decision based on April's beginning inventory and supply shock, which defines April's state of the world. Supply shocks represent unsystematic changes in costs of production. Production technology is assumed to be fixed. The expected profit-maximizing condition implies a supply function where planted crop size (H_4) is an increasing function of expected (discounted) price at harvest. Expected price at harvest is an average of expected prices in September through November weighted by proportions of incoming harvest in respective months.[9] The resulting supply equation is

$$(4) \qquad H_4 = S_{\boldsymbol{\theta}_4}\left[\mathrm{E}_4\left[\sum_{m=9}^{11} \alpha_m \frac{P_m}{(1+r)^{m-4}} \right], y \right],$$

where $S_{\boldsymbol{\theta}_4}[\cdot]$ is the state-dependent aggregate supply function, y is random supply shock, and r is the monthly discount rate.

Monthly available supply (A) is equal to the carryover (s) from the previous month, except in months when the carryover is augmented by the incoming harvest (h):

$$(5) \qquad A_m = \begin{cases} s_{m-1}, & m = 1, \ldots, 8, 12 \\ s_{m-1} + h_m, & m = 9, 10, 11. \end{cases}$$

[9] The producer's problem is to maximize expected profit:

$$\max_{H_4} \mathrm{E}_4[\Pi] = \mathrm{E}_4\left[\sum_{m=9}^{11} \alpha_m \frac{H_m[H_4] P_m}{(1+r)^{m-4}} \right] - C_{\boldsymbol{\theta}_4}[H_4, y],$$

where $C_{\boldsymbol{\theta}_4}[\cdot]$ is the state-dependent total cost to plant and maintain a given amount of crop size, and r is the discount rate. Given that H_m is a random walk and producers are price takers, the first-order condition with respect to H_4 is:

$$\mathrm{E}_4\left[\sum_{m=9}^{11} \alpha_m \frac{P_m}{(1+r)^{m-4}} \right] = C'_{\boldsymbol{\theta}_4}[H_4, y].$$

Solving for H_4 yields the supply function (equation 4).

Quantity demanded (q) is a state-dependent function of current price and i.i.d. demand shock (u_m),

(6) $$q_m = D_{\boldsymbol{\theta}_m}\left[P_m, u_m\right], \qquad m = 1, \ldots, 12$$

where $\partial D_{\boldsymbol{\theta}_m} / \partial P_m < 0$ and $\lim_{q_m \to 0} D^{-1}_{\boldsymbol{\theta}_m}\left[q_m\right] = \infty$; i.e., consumption must be positive for the market to clear.

Storage is carried out by risk-neutral arbitrageurs, whose profit equals the expected appreciation in price less opportunity and carrying costs associated with storage. Assuming a constant monthly discount rate r and denoting the total carrying cost as $K_{\boldsymbol{\theta}_m}\left[s\right]$, the storer's maximization problem is:

$$\max_{s_m \geq 0} \mathrm{E}_m\left[\Pi_{m+1}\right] = \frac{\mathrm{E}_m\left[P_{m+1}\right]s_m}{(1+r)} - P_m s_m - K_{\boldsymbol{\theta}_m}\left[s_m\right], \quad m = 1, \ldots, 12 .$$

Carrying cost includes the physical cost of storage plus a possible financial risk premium minus the convenience yield of stocks (Brennan).[10] The marginal physical cost of storage is assumed to be constant per unit of storage and time, and the premium for financial risk is ignored, since it is very small or nonexistent (Leuthold, Junkus, and Cordier) and irrelevant to risk-neutral storers. Convenience yield is defined as the benefit from owning the physical commodity that is not obtained by holding a futures contract, such as the ability to profit from unexpected orders or the ability to keep an operation running. Convenience yield reflects the market's expectations concerning the future availability of the commodity; thus, it is state-

[10] This risk premium is associated with the risk from carrying stocks, such as financial loss from an unexpected fall in prices.

dependent and larger the greater the possibility of a shortage. Depreciation in quality due to storage is assumed to be unimportant.

The first-order conditions for the above maximization problem is the complementary slackness condition:

$$
(7)\qquad
\begin{aligned}
&\frac{E\left[P_{m+1} \mid \boldsymbol{\theta}_m\right]}{(1+r)} = P_m + K'_{\boldsymbol{\theta}_m}\left[s_m\right], \quad s_m > 0, \qquad m = 1, \ldots, 7, 11, 12,\\
&\frac{E\left[P_{m+1} \mid \boldsymbol{\theta}_m\right]}{(1+r)} \le P_m + K'_{\boldsymbol{\theta}_m}\left[s_m\right], \quad s_m \ge 0, \qquad m = 8, 9, 10
\end{aligned}
$$

where $K'_{\boldsymbol{\theta}_m}[\cdot]$ is the state-dependent marginal carrying cost per unit of storage. Note, this specification allows for risk-averse storage behavior despite the assumption of risk neutrality (e.g., Just).[11] These conditions guarantee no arbitrage opportunities in equilibrium. A positive amount is stored whenever expected appreciation in price is sufficient to cover the carrying cost. If nothing is carried over, the price expected for the following period may not be as high as the current price. Imposing positive consumption removes the possibility of stock-outs except in the months preceding harvest.

[11] The utility-maximization problem for a risk-averse storer can be expressed in terms of the certainty equivalent of a risky prospect, which is defined as the amount of profit for which the individual is indifferent between this certain amount and a risky prospect (Mas-Colell, Whinston, and Green, Ch. 6). Denoting the risk premium—the difference between the mean of the risky profit and its certainty equivalent—as ρ, and including the opportunity cost and constant marginal physical storage cost k in the profit, the certainty equivalent maximization problem is:

$$\max_{s_m \ge 0} \quad \frac{\mathrm{E}_m\left[P_{m+1}\right]s_m}{(1+r)} - P_m s_m - k s_m - \rho\left[s_m\right], \quad m = 1, \ldots, 12\,.$$

For a solution to exist at some $s_m^* > 0$, we must have $\mathrm{E}_m\left[P_{m+1}\right]/(1+r) - P_m - k - \rho'[\cdot] > 0$ at some $s_m \in (0,\ s_m^*)$. Since we observe $\mathrm{E}_m\left[P_{m+1}\right]/(1+r) - P_m - k < 0$ in some months empirically, we must have $\rho'[\cdot] < 0$ to guarantee a solution. Furthermore, the second-order condition implies a sufficient condition, $\rho''[\cdot] > 0$. Thus, the risk premium is a decreasing and convex function of storage. But, this is identical to the model assuming risk neutrality, replacing the carrying cost, $K_{\boldsymbol{\theta}}[s]$, with the sum of the physical storage cost and risk premium, $ks + \rho[s]$.

Equations (2) through (7) imply an equilibrium characterized by a rational expectations price function, $P = \Phi[\boldsymbol{\theta}]$, and a planting equation, $H_4 = \Psi[\boldsymbol{\theta}_4]$, which solve the following discrete-time, continuous-state, functional equations:

$$\text{(8a)} \qquad \Phi[\boldsymbol{\theta}_m] = \begin{cases} \mathrm{E}_m[\Phi[\boldsymbol{\theta}_{m+1}]]/(1+r) - K'_{\boldsymbol{\theta}_m}[s_m], & m = 1, \ldots, 7, 11, 12, \\ \max\{\mathrm{E}_m[\Phi[\boldsymbol{\theta}_{m+1}]]/(1+r) - K'_{\boldsymbol{\theta}_m}[s_m], D^{-1}_{\boldsymbol{\theta}_m}[s_{m-1}]\}, & m = 8, 9, 10 \end{cases}$$

$$\text{(8b)} \qquad D^{-1}_{\boldsymbol{\theta}_m}[A_m - s_m] = \Phi[\boldsymbol{\theta}_m], \qquad \forall m$$

$$\text{(8c)} \qquad \Psi[\boldsymbol{\theta}_4] = S_{\theta_4}\left[\hat{P}[\Psi[\boldsymbol{\theta}_4]]\right]$$

$$\text{(8d)} \qquad \hat{P}[H_4] = \mathrm{E}_4\left[\sum_{m=9}^{11} \alpha_m \Phi[\boldsymbol{\theta}_m] \Big/ (1+r)^{m-4} \,|\, H_4 \right]$$

where $\hat{P}[\cdot]$ represents the expected harvest price given a planted crop size of H_4, and the state variables in each month are:

$$\boldsymbol{\theta}_m = \begin{cases} \{m, A_m, u_m, y\}, & m = 4, \\ \{m, A_m, u_m, H_m\}, & m = 5, 6, 7, \\ \{m, A_m, u_m, H_m, \hat{\alpha}_m\}, & m = 8, 9, 10, \\ \{m, A_m, u_m\}, & \text{otherwise.} \end{cases}$$

Equation (8a) states that current price must equal discounted expected future price, less the marginal carrying cost K', except in months prior to harvest. During August through October, stock-outs, although improbable, could occur in which case the price equals the level implied by consuming all the carryover from the previous month, $D^{-1}_{\boldsymbol{\theta}_m}[s_{m-1}]$. In all months, the equilibrium price depends on inverse demand evaluated at available supply that was not stored, equation (8b). Equation (8c) is the planting function; the crop planted is the state-dependent supply function evaluated at expected harvest price. Expected harvest price, in turn, depends on planted crop size, because crop size influences price received at harvest (equation 8d).

The state variables $\boldsymbol{\theta}_m$ represent information available at the beginning of each month. Three states exist in all months: the month of the year, the available supply level, and the realized demand shock. In April, a supply shock conditions the planting decision; during May through October, the expected size of the incoming crop is relevant; and expected proportions of crop harvested in the subsequent month are revealed during August through October. In November, the information of the realized crop is incorporated into available supply. Storage and consumption are endogenous variables that are derivable from price, and hence are functions of the state variables.

The existence of price functions in a competitive storage model with seasonal production is proved in Chambers and Bailey. The assumption of rational, forward-looking behavior among the agents implies that stochastic dynamic programming or a similar recursive method must be employed to solve the model. The possibility that nonnegativity constraint on storage may bind adds further complications. Since the functions cannot be solved in closed form, the problem is solved numerically.

Numerical Model

Solution Method

The recent literature on numerical solution methods uses variations of a functional approximation method to replace the original functional equation problem with a finite-dimensional (discrete) problem (e.g., Williams and Wright; Rui and Miranda; Deaton and Laroque (1992)). Miranda has compared the accuracy and efficiency of different numerical strategies for computing approximate solutions to the nonlinear rational expectations commodity market model, and his results support the superiority of the cubic spline function collocation method over the traditional space discretization, linearization, and least-square curve-fitting methods.

In general, the solution to a functional equation problem is a function defined over a domain of multiple state variables, $Y = F[X_1, X_2, \ldots, X_\chi]$. For numerical solution methods, this specification can be simplified by discretizing all state variables but one, say X_1. Then, the solution may be regarded as a collection of single-variable functions, $Y = F^\delta[X_1]$, $\delta \in \Delta$, where each element of Δ corresponds to a combination of discrete values of the remaining state variables, $X_2, \ldots, X_\chi$, and the continuous variable X_1 can be regarded as the "principal" state.

This scheme is implemented by the cubic spline collocation method involving three steps. First, since the month of the year is a discrete state variable, the solution to the problem, an unknown price function $P[\cdot]$, is decomposed into 12 monthly component functions $P_m[\cdot]$, $m = 1, \ldots, 12$. Unknown price functions $P_m[\cdot]$, the planting function $\Psi[\cdot]$, and expected harvest price function $\hat{P}[\cdot]$ are approximated by finite linear combinations of known cubic b-spline functions, respectively. [12]

Second, for the price and planting functions, one of the state variables, available supply at the beginning of the month, is chosen as the "principal" state variable, and a finite number of points A_υ^m, $\upsilon = 1, \ldots N$, are selected within its range as the collocation nodes, where the function's approximant is required to hold precisely. Discrete levels of crop size H_υ^4, $\upsilon = 1, \ldots N_P$, are selected as the collocation nodes for the expected price function. The collocation nodes

[12] Cubic b-spline functions are:

$$\phi_\upsilon(A) = \begin{cases} \frac{2}{3}\left(1 - 6q^2(1-q)\right) & \text{if } q = \frac{|A - A_\upsilon|}{w} \le 1 \\ \frac{4}{3}(1-q)^3 & \text{if } 1 \le q = \frac{|A - A_\upsilon|}{w} \le 2 \\ 0 & \text{otherwise} \end{cases}$$

where $w = \left(\overline{A} - \underline{A}\right)/N$ and $A_\upsilon = \underline{A} + \upsilon w$. These piece-wise cubic functions have continuous first and second derivatives.

are equally spaced between the minimum and maximum of the approximation range. Thus, the function approximants are, in vector notation:

$$(9) \qquad \mathbf{P}_m[A_m] \approx \mathbf{\Phi}_m[A_m]\mathbf{c}_m, \quad \forall m$$

$$\mathbf{\Psi}[A_4] \approx \mathbf{\Phi}_S[A_4]\mathbf{c}_S$$

$$\hat{\mathbf{P}}[H_4] \approx \mathbf{\Phi}_P[H_4]\mathbf{c}_P$$

where $\mathbf{P}_m[\cdot]$ and $\mathbf{c}_m$ are N by Θ_m (the number of state variables in a given month less one), $\mathbf{\Phi}_m[\cdot]$ and $\mathbf{\Phi}_S[\cdot]$ are N by N, $\mathbf{\Psi}[\cdot]$ and $\mathbf{c}_S$ are N by J, $\hat{\mathbf{P}}[\cdot]$ and $\mathbf{c}_P$ are N_P by 1, and $\mathbf{\Phi}_P[\cdot]$ is N_P by N_P. Choosing the same number of basis functions and collocation nodes is not necessary, but it minimizes computational cost (Miranda and Fackler, 1999a).

Third, the continuous distributions of random state variables are replaced by approximating discrete distributions. Demand and supply shocks assume I_m-by-1 vector u_m and J-by-1 vector $\mathbf{y}$ with probability vectors $\mathbf{w}_{\mathrm{m}}$ and $\mathbf{x}$, respectively. Actual and expected proportions of crop harvested assume similar G_m-by-1 vectors $\boldsymbol{\alpha}_m$ and $\hat{\boldsymbol{\alpha}}_{m-1}$ with corresponding probability vector $\mathbf{z}_m$, so that, for example, actual proportions of crop harvested in September, α_9, have a distribution identical to their projections revealed in August, $\hat{\alpha}_8$.

The expected crop size (H) is discretized to an L-by-1 vector with elements that are equally spaced with increments ΔH, and a priori probability vector $\boldsymbol{v}$ is associated with it in May. For all adjacent months between May and November, the crop estimate is assumed to follow an L-state Markov chain with Markov transition matrixes with elements:

$$v_{k,l}^{m} = P\left\{H^{m+1} = H_k \,\middle|\, H^m = H_l\right\}, \qquad l, k = 1, \ldots, L, \quad m = 5, \ldots 10,$$

where the l-th crop size is denoted as H_l. Following Pirrong, the transition probability is calculated as:

$$v_{k,l}^{m} = \mathrm{N}(h_1) - \mathrm{N}(h_2)$$

where $h_1 = \left(\left(H_{k+1} + H_k\right)/2 - H_l\right)/\sigma_m$, $h_2 = \left(\left(H_k + H_{k-1}\right)/2 - H_l\right)/\sigma_m$, σ_m is the standard deviation of the error term ε_m in equation (2), and N(·) is the standard normal cumulative distribution function. In other words, the probability that the expected crop size will equal H_k in the next month, given that its current value is H_l, is equal to the probability that an i.i.d. random number drawn from the standard normal distribution will fall in an interval of length ΔH centered on $H_k - H_l$, standardized by the standard deviation of the monthly error term.

The above discretization implies that the original functional equation problem (8a)–(8d) is replaced by a finite-dimensional problem with an overall dimension of $2N\left(\sum_{m=11,12,1,2,3} I^m + \sum_{m=5,6,7} I^m L + \sum_{m=8,9,10} I^m L G^m + J\right) + NN^P$, where unknowns are collocation coefficients, storage levels, planted crop size, and expected harvest price. The discretized model is presented in Appendix 1.

The equilibrium conditions have a recursive structure in that Ψ does not enter the storer's arbitrage equation (8a). From the storer's point of view, harvest is exogenous, and information on expected crop size becomes available only after the crop has been planted and growing conditions become known. Hence, a solution can be obtained in a sequence of independent operations. In the first stage, equations (8a) and (8b) are solved to determine the equilibrium market price at all possible states. In the second stage, these price functions are used to determine expected harvest price ($\hat{P}$) from equation (8d), based on transition probabilities that link future states to the choice of planted crop size (H_4). Finally, the relationship between $\hat{P}$ and H_4 can be used to solve for the equilibrium planting equation in (8c).

To solve for price function coefficients and storage levels, Miranda proposes a two-step function iteration algorithm, where the solutions to monthly storage are found by holding the guesses for monthly price functions constant, and vice versa. Here, we combine his proposal and backward induction similar to that of dynamic programming.[13] Prior to the initial iteration, the monthly collocation nodes (the discretized available supply levels) and initial values for the price function coefficients $\mathbf{c}$ are assigned. The iteration begins by solving equation (8a) for October carryover levels $\mathbf{s}_{10}$ with the November price coefficients $\mathbf{c}_{11}$ fixed using Newton's method, which, given a function $f[x]: \Re^n \rightarrow \Re^n$, solves for an n-vector x that satisfies $f[x] = 0$. At each iteration, the function is approximated by its first-order Taylor series expansion about the latest guess and the root of the approximation is used as an improved guess for the root of f. The process is repeated until the guesses converge. Once such storage levels are found, equation (8b) is solved for the (updated) October price function coefficients $\hat{\mathbf{c}}_{10}$ using L-U factorization.[14]

Using these updated October price coefficients, equation (8a) is solved for September carryover levels $\mathbf{s}_9$, which is used to obtain new values of the September price coefficients $\hat{\mathbf{c}}_9$, and the procedure is repeated for the remaining ten months backwards in time. At the end of a 12-month iteration, the norm of the difference between the old and new November price function coefficients, $\mathbf{c}_{11}$ and $\hat{\mathbf{c}}_{11}$, is compared to a predetermined convergence level. If it is below the level, the new set of coefficients becomes a part of the solution and the algorithm stops; otherwise the iteration resumes with the new set of coefficients replacing the old guess.

[13] In fact, the equilibrium price conditions in equation (8a) match the optimal conditions from a dynamic programming problem that maximizes social welfare (or the discounted stream of expected future surplus) by choosing storage levels, given an exogenous harvest size.

[14] L-U factorization is an algorithm that solves a linear equation system $\mathbf{Ax} = \mathbf{b}$, by decomposing the $\mathbf{A}$ matrix into the product of lower and upper triangular matrices. In general, it is more efficient than computing the inverse of $\mathbf{A}$ (Mathworks, Inc.).

Using the equilibrium price functions, expected harvest price function is solved in the second stage. Prices during the harvest months are obtained from simulating the corn market from various availability levels and planted crop sizes in April through November for 5,000 times. Initial starting points are selected as combinations of the N collocation nodes used for April's price function and N^P levels of crop size between 5 and 15 billion bushels. Expected harvest price $(\hat{P})$ is calculated as the average (over the 5,000 simulations) of September, October, and November prices weighted by the ratio between crop harvested in each month and annual crop size. For each availability level, we can interpolate a functional relationship between crop size and expected harvest price over the range of crop size as:

$$\hat{\mathbf{P}}_n = \mathbf{\Phi}_P[\mathbf{H}_4]\mathbf{c}_{P,n} \qquad \forall A_n,\ n = 1, \ldots, N$$

solving for $\mathbf{c}_P$ using L-U factorization, which is equation (8d).

Lastly, analogously to the monthly price functions, Miranda's two-step algorithm can be applied to solve for the planting function defined in equation (8c). First,

$$\mathbf{\Phi}_S[\mathbf{A}_4]\mathbf{c}_S = S_{\boldsymbol{\theta}}[\mathbf{\Phi}_P[\mathbf{H}_4]\mathbf{c}_P]$$

is solved for planted crop size $\mathbf{H}_4$ with the supply coefficients $\mathbf{c}_S$ fixed using Newton's method. Once such crop size is found, the supply coefficients are updated according to:

$$\mathbf{H}_4 = \mathbf{\Phi}_S[\mathbf{A}_4]\mathbf{c}_S$$

via L-U factorization. The iteration is repeated until the coefficients converge.

Parameter Specification

Parameter specification is motivated by an effort to represent observed market conditions during the 1990s. Monthly bounds of available supply ($\underline{A}_m$ and $\overline{A}_m$, m = 1, …, 12) are imputed from quarterly total supply and ending stocks reported by the Economic Research Service at the U.S. Department of Agriculture, but collocation ranges are expanded to accommodate broader ranges of solutions (see Peterson for details). The expected crop size in May is discretized to L = 10 values, equally spaced values between the minimum and maximum annual harvested crop in the sample period (6.3 to 10.1 billion bushels). The values are associated with probabilities assuming a normal distribution with data-consistent mean and standard deviation. Subsequently, the crop estimate is revised at the beginning of each month from June through November (when, by assumption, the actual crop size is revealed)[15] with Markov transition probabilities. The standard deviations of the differences between the adjacent months' crop estimates are used as the standard deviations of the error terms in equation (2).

Proportions of crop harvested during September, October, and November are based on National Agricultural Statistics Service's crop progress report. Since 91.7 percent of the U.S. annual crop was harvested during the three months during 1989/90 and 1997/98, the percentages are adjusted so that the average of the three-month sum equals one. The adjusted September, October, and November proportions are 0.12, 0.55, and 0.32, respectively. The reported percentages indicate that individual September percentages are skewed to the right. Hence, normality is assumed for their logarithms. Because October percentages are not clearly skewed, they are assumed to be normally distributed. September and October proportions assume G_9 = G_{10} = 3 values; since the remaining crop is harvested in November, November proportions assume G_{11} = 9 values.

[15] According to Garcia et al., "errors [of USDA crop forecasts] are quite small by the time of November announcements (p.561)."

Monthly demands are specified as constant elasticity functions with multiplicative demand shocks, with the price elasticities set at –0.25 for the month.[16] The demand functions are calibrated to the mode of the estimated monthly price distributions and the mean of monthly total disappearance (quarterly total disappearance divided by three). Demand shocks (u_m) are specified as normal random variables with parameters that are calculated from the ratio between consumption and predicted consumption, assuming the constant elastic demand functions with calibrated coefficients. The shocks are discretized to assume $I^m = I = 5$ values for all months according to Gaussian quadrature principles, so that the discrete distribution possesses the same first $2I$–1 moments as the continuous distribution (Gerald and Wheatley).

The supply equation is also assumed to take a constant elasticity form with a multiplicative supply shock (y) with the supply elasticity is set at 0.2.[17] To obtain the constant, the function is calibrated to the sample-average harvest price discounted to April ($2.30 per bushel), the mean of annual production, and the mean of supply shock set to one.[18] Because expected harvest prices are not observed, the standard deviation of y is inferred from the sample. The calibrated supply curve (with $y = 1$) is evaluated at the minimum and maximum observed

[16] In the literature, the estimated annual demand elasticity varies from –0.54 to –0.73 (Holt, 1994; Holt and Johnson; Shonkwiler and Maddala). Using 1957 to 1975 data, Subotnik and Houck report the ranges of quarterly price elasticities of feed, of food, and of exports as –0.15 to –0.22, 0 to –0.034, and –0.71 to –2.0, respectively.

[17] The estimated acreage elasticity with respect to changes in expected price ranges from 0.05 to 1.04 (Holt, 1994, 1999; Chavas and Holt, 1990, 1996; Tegene, Huffman, and Miranowski; Lee and Helmberger), with an apparent consensus around 0.2. The estimated supply (production) elasticity with respect to price ranges from 0.28 to 0.39 (Shonkwiler and Maddala).

[18] Discounted harvest price is calculated for each crop year in the sample as a weighted average of September, October, and November prices, where each month's price is discounted to April and weighted by the percentages of crop harvested in that month. The monthly discount rate is based on an annual rate of 0.1 ($r = 0.0083 = 0.1/12$).

harvest prices and compared to the minimum and maximum observations of planted crop, respectively. The average of squared deviations between predicted and observed planting is used as an estimate of the variance of y. Assuming a normal distribution, J = 30 discrete levels of y are chosen by Gaussian quadrature.

The net carrying charge of storage is specified as the difference between current and expected prices, and consists of the physical storage costs and convenience yield. If the expected price in the next period differs from the current price only by the costs of storage, prices would increase indefinitely, on average, relative to the previous month. Yet, observed prices typically increase from October through May and then decrease. This puzzle of price backwardation is a long-standing issue (Frechette and Fackler), and in a conventional rational expectations storage model, aggregate stock-outs are used to account for price backwardation. But zero stocks of corn have never occurred in recorded history (though they were very small in 1934 and 1936).

Thus, we model convenience yield, despite the fact that it is unobservable, as a source of price backwardation. To be consistent with the observed price and inventory behavior, convenience yield is assumed to be a decreasing, convex function of storage level (s) that shifts according to levels of expected crop size (H).[19] Specifically, convenience yield is $CY_m = \omega_m s_m^{\xi_m} \exp\left[\zeta_m \left(H_m - \overline{H}\right)\right]$, where ω_m, $\xi_m < 0$, and ζ_m are parameters, and $\overline{H}$ is the historical average crop size. Setting the elasticity parameter ξ_m to –1, the constants ω_m are obtained by calibrating these functions through the monthly average convenience yield, storage level, and historical average crop size. Observations for convenience yield are calculated as the remainders of the non-arbitrage equation evaluated at monthly prices $CY_m = P_m + k - P_{m+1}/(1+r)$, where the monthly physical cost of storage (k) and the discount rate (r) are specified as 3 cents

[19] This is similar to Rui and Miranda, who specified a decreasing convex storage cost.

per bushel and 0.0083, respectively.[20] Convenience yield is assumed to be decreasing in crop size; the associated shift parameters ζ_m are calibrated so that the shift factors $\exp\left[\zeta_m\left(H_m - \overline{H}\right)\right]$ take on their maximum value when crop estimates are lowest.

For improved computational accuracy, quantities and prices are scaled so that one equals 10 billion bushels and $10 per bushel, respectively. Monotonicity is imposed on functional solutions during the solution process. The model is solved for a convergence tolerance level of 10^{-8}. Cubic spline interpolation and Gaussian quadrature methods are calculated by the computer routines developed by Miranda and Fackler (1999b); the algorithm for L-U factorization built in MATLAB version 5.3 is used.

Model Solutions

Equilibrium Spot Price Functions

The rational expectations model explains observed economic behavior through supply and demand shocks, but the combined effects are difficult to analyze. Thus, the model is solved in a series of nested cases with identical parameter values, starting with the case of certainty and exogenous supply (Case 1). Then, crop estimates, demand shocks, and harvest timing are introduced sequentially (Cases 2–4). Case 5 endogenizes the planting decision, and Case 6 adds uncertainty regarding production costs. The solution times reported are for a multiple-processor cluster of six servers running the Windows® 2000 operating system, where each server has four Intel Pentium® III Xeon 550 megahertz processors and 4 gigabytes of RAM.

[20] The monthly averages of convenience yield were close to zero from October through April and ranged from 9.5 to 21.2 cents per bushel during the remaining months; the convenience yield was set to zero for months October through April.

Certainty (Case 1). The states defined by demand shocks, expected crop size, and harvest-timing, respectively, are set to one ($I = L = G_m = 1$), and the states of these variables are set to the observed means. The only state variable is the quantity available at the beginning of every month. With the degree of interpolation (N) of 30, the dimension of the model is 720 (= $2 \times 12 \times N$) with 360 unknown price function coefficients **c** and storage levels, respectively. The model converges in 69.0 seconds.

The equilibrium price functions are monotone decreasing functions of available supply in all months (Figure 5). Theoretically, in months prior to harvest when stock-outs could occur, the equilibrium price coincides with the inverse consumption demand function at low supply levels, but lies above the consumption demand at higher supply levels. Thus, price functions are kinked (Williams and Wright). Kinks are not observed in Figure 5—the unevenness of the functions is due to MATLAB's graphics—since the empirical supply bounds do not include stock-outs, and the price functions are not defined for availability of, or close to, zero. But, the functions are graphed for unrealistically small stock levels in some months, and prices need to be evaluated relative to the realistic domains of availability. For instance, the November price line lies above the other lines, but November prices correspond to relatively large supplies. In contrast, July prices result from smaller supplies and are typically higher than November prices even though the November price line lies above the July line.

In months after harvest, prices are expected to appreciate to cover storage costs. Hence, the expected price in the subsequent month lies to the right of the current price. In months after planting but before harvest, net carrying cost becomes positive since convenience yield exceeds physical storage cost. In July, for example, the expected August price is lower than the current price.

Given a level of availability at the beginning of the month, the model carries over almost a fixed proportion to the next month. Storage is basically a linear function of availability for each month. The average ratio of carryover to available supply ranges from 0.77 in September through 0.87 in December, which are higher than the range of 0.42 to 0.73 of the sample-period (quarterly) data. Again, if the model allowed a possibility for stock-outs, the storage function would exhibit a kink as the non-negativity constraint binds.

Adding Uncertain Crop Size, Demand, and Harvest Timing (Cases 2–4). The crop-size estimate is introduced as an additional state variable in May through October in Case 2 (L = 10). The dimension of the model increases to 3,960 ($= 2\times(6\times N\times L + 6\times N)$), and the model takes 1901.9 seconds (31.7 minutes) to converge. By introducing demand uncertainty in Case 3 (I = 5), the dimension of the model is 19,800 ($= 2\times(6\times N\times I\times L + 6\times N\times I)$), and the model converges after 53070.8 seconds (14.75 hours). With varying timing of harvest in Case 4 ($G_8 = G_9 = 3$, $G_{10} = 9$), the dimension of the model reaches 55,800 ($= 2\times(N\times I\times L\times(G_8 + G_9 + G_{10}) + 3\times N\times I\times L + 6\times N\times I)$). It takes 449,182.65 seconds (5.2 days) for the model to converge.

With additional state variables, the number of lines representing price functions (similar to Figure 5) increases accordingly, and the functions exhibit more crossovers and larger ranges. The range of the price functions extends from \$1.05 to \$5.40 at 6 billion bushels of availability in Case 4. Equilibrium storage is similar to the certainty case, but multiple functions exist for months with additional states. The average proportions of availability that is carried over are unaffected from the certainty case.

The relationships among equilibrium price and state variables are intuitive. The numerical solution indicates that equilibrium prices are high when crop-size estimates are small, but as storage availability increases, this impact of crop estimates on prices declines.

Equilibrium prices are higher the larger the demand shock (i.e., smaller quantity), and this impact is stronger at lower availability levels. Similarly, prices are more responsive to the changes in demand shocks when smaller crops are expected.

When a small proportion is expected to be harvested in September, much of the uncertainty regarding total crop size is transferred to the remaining harvest months, causing August price to increase for a given level of availability, ceteris paribus. The impact is more apparent in the October price function. Since November is the terminal harvest month, a smaller expected proportion implies a smaller total crop for the year, shifting the October price function upward, with the largest impact at low availability levels. An analogous relationship exists among price, expected proportions harvested, and crop estimates. Demand shocks had no discernable effect on the relationship between price and the expected proportion harvested.

Endogenous Supply and Adding Uncertain Production Cost (Cases 5–6). Case 4 completes the first solution stage for the full model, where equilibrium monthly price functions that satisfy equations (8a) and (8b) are found. Here, the second and third stages are executed to endogenize the planting decision. The expected price function has a degree of interpolation (N_P) of 100 and is defined at $N = 30$ nodes of the planting function. In addition, the model must solve for the equilibrium planting level at these N nodes. Case 5 does not consider supply uncertainty ($J = 1$), but when it is incorporated in Case 6, there are $J = 30$ possible supply functions. The additional dimension for Case 6 (over Case 4) is 4800 ($= N \times N_P + 2N \times J$), where there are $N \times N_P$ unknown coefficients of the expected price function, $N \times J$ unknown planting function coefficients, and $N \times J$ unknown planting levels. The additional computing time is 150,061.5 seconds (41.7 hours), 99 percent of which was spent simulating the expected harvest price.

The upward-sloping lines in Figure 6 are supply functions with an elasticity of 0.2 with various supply shocks. A positive supply shock (i.e., lower cost of production) shifts the supply curve to the right and induces a larger crop to be planted. The supply curve with $y = 1$ corresponds to Case 5. The expected harvest price functions are analogous to inverse demand functions expected at harvest, and are the same for Cases 5 and 6. The highest of these functions corresponds to the lowest April availability, and vice-versa—smaller stocks in April increase expected harvest prices, ceteris paribus. The planting function is then derived from the intersections of the expected harvest price and supply curves to determine a relationship between availability and planted crop size. Across all supply states and April availability levels, the function ranges between 5 and 14.4 billion bushels of planted crop.

Time-Series Behavior and Long-Run Distributions

Equilibrium price and planting functions alone do not tell us much about the validity of the model, since they have no observable counterparts. Rather, the model is "good" if it generates equilibrium prices that are comparable to the sample of observed cash prices. Using the equilibrium functions, equilibria were sequentially generated for 10,000 "years" by drawing random disturbances consistent with the model. The simulation begins in January using the average ending stocks in December (during the sample period) as the initial starting point ("year" zero), and proceeds forward in time. At the beginning of each month, availability is realized as the amount carried over from the previous month plus any incoming harvest, and determines the current price through the price function. Then, the quantity consumed at that price is determined from the demand function, and the carryover to the next month is the difference between availability and consumption. For each monthly variable, basic statistics are calculated and compared with corresponding statistics from 1989/90–1997/98. The sample period provides a benchmark to evaluate the model, but it does not exhaust the range of possible outcomes. Prices are reported in dollars per bushel; storage, consumption, and crop estimates are in million bushels. The results are discussed in the same nested sequence as above.

Certainty (Case 1). It takes about 7 "years" for the model to adjust from the initial state to a steady state, where the price repeats a seasonal pattern with peaks in May and bottoms in October, consistent with the observed data. With certainty, the seasonal pattern is invariant from year to year. Long-run monthly price "distributions" are vertical lines at the respective monthly means. Hence, only the means of endogenous variables are reported in Table 1. Simulated price levels are slightly lower than the observed means in all months except in August.

Corresponding to the seasonal fluctuation in availability, storage declines steadily through the post-harvest season, since as noted earlier, an approximately constant proportion of available supply is carried over. Its average levels are larger than the sample observations by 300 to 400 million bushels. Consumption levels are consistent with the sample averages, but reflect the fact that quarterly observations were used to calibrate the demand functions—the consumption levels within a quarter are similar to actual levels.

Uncertain Crop Size (Case 2). Analogous to our earlier descriptions, the crop estimate evolves from May to the beginning of November, when the estimate is regarded as the "true" harvest size for the crop year. While the crop estimate is generated as a continuous variable in the simulation, distinct price functions are associated with the 10 (= L) discretized crop estimate levels. The price function that corresponds to the discretized level of crop estimate nearest the continuous realized value (i.e., minimum norm) is used. The amounts harvested in September and October are calculated as the historical average percentages of simulated total crop estimates. The amount harvested in November equals the simulated November crop estimate less what was harvested in the previous two months in the simulation.

Basic statistics of monthly prices closely follow those from the 1989/90-97/98 sample (Table 2). In particular, both means and standard deviations follow the observed seasonal patterns at similar magnitude, implying that uncertainty regarding crop size during growing season is an important factor influencing seasonal price behavior. Comparing simulated mean and median prices indicates positive skewness in the distributions, confirming Williams and Wright's claim that storage induces the skewness in price, although they use a different specification of supply shocks. Monthly maximum prices simulated by the model can exceed $10 per bushel, and minimum prices are lower than the observed lows. Hence, the model's price distributions are leptokurtic (peaked) relative to the data. The simulated price series is roughly comparable to the observed data (Figure 1), but when the model economy has only one source of uncertainty, simulated prices during the growing season are markedly more variable than the rest of the year.

The basic seasonal pattern of storage, consumption, and availability is similar to the certainty case, but varies from year to year (Figure 7). The model stores more, on average, than is observed in practice (Table 2). The main role of storage is to smooth out intertemporal consumption. In the sample period, the standard deviation of storage is 10 to 40 percent of the mean, while that of consumption is 8 to 10 percent. In the model, the standard deviation of storage ranges between 14 and 35 percent of the mean, and that of consumption ranges between 4.5 to 6 percent.

The statistics for simulated crop estimates and crop harvested are, in general, consistent with the sample.[21] The average size of the harvest is consistent with sample data, but the simulated and observed standard deviations of crop harvested differ significantly, suggesting additional sources of variability in reality.

Uncertain Demand and Harvest Timing (Cases 3–4). Demand shocks are added at the beginning of every month as independent normal random variables in Case 3. This increases price variability in the non-harvest months to more realistic levels, thereby remedying a problem with model simulation Case 2.

In Case 4, the states are distinguished according to harvest timing. At the beginning of August, a random variable is drawn from a standard normal distribution, and three states are equally likely to happen, "small," "large," or "average." The realized value is then used to calculate the actual percentage of crop harvested in September, which is assumed to be log-normally distributed. As previously noted, the normality or log-normality assumption is convenient, but places a part of the probability mass outside the range of realized values. Hence, the calculated percentage is truncated at the maximum observed percentage of crop harvested in September.

A similar procedure is used to estimate the October harvest, but using a standard normal distribution at the beginning of September. Given the August and September news regarding the incoming proportions of crop in September and October, the remaining proportion of the crop, that is expected to arrive in November, is known by the beginning of October. Accordingly, the

[21]The means of the USDA monthly crop estimates seem to decline, but the sample is too small to conclude that estimates are biased upward; the literature suggests little or no bias in longer time series (Sumner and Mueller; Garcia et al.).

price function that corresponds to the combination of harvest-timing states during the previous two months is used to determine the price in October.

The two additional state variables correct the variability in monthly consumption and harvest levels, while maintaining the simulated average price level and its seasonal pattern consistent with the data (Tables 3 and 4). The positive skewness and excess kurtosis noted in the previous cases are preserved as well. Statistics for other endogenous variables did not change notably, except storage levels are higher than previous cases. Despite the fact that percentages themselves are truncated, the simulated amounts of crop harvested assume wider ranges than the observed values.

<u>Endogenous Supply and Uncertain Production Cost (Cases 5–6)</u>. In Case 5, the planting decision is endogenized, assuming that the supply function is known with certainty. The planting equation generates a data-consistent average crop size, but one-fourth the observed variation (Table 5), implying that the specification lacks a source of supply variability. The small variation at planting translates into lower-than-observed variability in crop estimates throughout the growing season, ending in November with about half the variability observed in the sample. The variability of crop harvested, however, was mostly unchanged from the data-consistent magnitude, which suggests that its variability accrues mainly to uncertainty regarding timing of harvest rather than to its size. With too small variability in expected crop size, the price variability during the growing season stays at the same level as the rest of the year, contrary to the observed seasonal pattern. Storage variability decreased from the previous cases to a data-consistent level. Otherwise, seasonality in average prices levels, storage, and consumption behavior are unchanged from the previous cases.

A multiplicative production-related shock is realized as a normal random variable with mean of one and specified standard deviation in Case 6. The simulated values of planted crop are now consistent with observed variability and average levels (Table 6). Overall, this model is the most successful at reproducing the patterns in the historical data. Simulated average price levels are somewhat higher than observed in the sample (6 to 16 cents), but the seasonal pattern and skewness are data-consistent—the highest mean price in the simulation (which occurs in May) exceeds the lowest (in October) by 37 cents, corresponding exactly to the spread in the sample. There is seasonality in price variability, although the differences in variability are not as large as in the sample. The sample's standard deviations in May and July are more than double those in November, but the simulated May standard deviation is only 1.3 times larger than in November.

Histograms of simulated monthly prices (Figure 8) are long-run unconditional distributions of prices in a given month—or equivalently, distributions that are conditional on the month of the year—and are comparable to Figure 2 of the observed prices. For ease of comparison, September price distributions are plotted against the selected monthly distributions. As noted previously, positive skewness is observed in all months. The modes shift in a consistent manner from a low around harvest to higher levels in spring. The spread is small during harvest months and larger during the summer. The other endogenous variables are also largely data-consistent in levels and seasonal trends of means and standard deviations, although as before, the normality assumption leads to values outside the observed ranges.

The simulated prices are a complex aggregation of current and past information, but the model is still an abstraction that cannot capture all of the forces that affect prices. Together, these facts imply that disentangling the signal and noise is more difficult for observed prices than simulated ones. This can be illustrated by fitting the same ARMA model to the simulated series as used for the observed prices (equation (1)). The following equation was fitted to 120,000 (less 36) simulated months.

$$P_m = \underset{(0.007)}{0.313} + \underset{(0.003)}{0.857}\, P_{m\text{-}1} + \underset{(0.003)}{0.026}\, P_{m\text{-}2} + e_m + \underset{(0.003)}{0.005}\, e_{m\text{-}3} - \underset{(0.003)}{0.028}\, e_{m\text{-}5}$$

where approximate standard errors are reported in parentheses and e is the error term. Compared to equation (1), the autoregressive coefficients differ substantially, and the null hypothesis of white residuals is rejected (p value < 0.0001). When the complexity of the error term specification is increased (by adding various lags and combinations of moving-average factors), this discrepancy is partially resolved. For example,

$$P_m = \underset{(0.009)}{0.133} + \underset{(0.016)}{1.415}\, P_{m\text{-}1} - \underset{(0.014)}{0.465}\, P_{m\text{-}2} + (1 - \underset{(0.003)}{0.021}\, L^4 - \underset{(0.003)}{0.026}\, L^5)$$
$$(1 + \underset{(0.003)}{0.023}\, L^9 + \underset{(0.003)}{0.038}\, L^{10} + \underset{(0.003)}{0.050}\, L^{11})(1 - \underset{(0.015)}{0.604}\, L + \underset{(0.003)}{0.064}\, L^{12} + \underset{(0.002)}{0.046}\, L^{24})\, e_m$$

where L is the lag operator. The coefficients on lagged prices in this equation more closely resemble those from the sample-based regression, but the null of white residuals is still rejected (p value < 0.0001 for the first twelve lags).

The full simulation model allows for a variety of post-harvest price behaviors. In a typical year, prices are lowest immediately following harvest and then increase through the post-harvest season. But, the model permits post-harvest prices increases of various magnitudes; the largest simulated price increase from November to May is $2.23 per bushel. Also, prices do not necessarily increase monotonically, and it is possible (but improbable) to observe initially high prices in November followed by a seasonal decline. The largest simulated seasonal decrease is $2.98 per bushel, which happened once in the 10,000 simulations.

The model is tested for its robustness under various parameter specifications. While most of the parameters are calibrated to the data, five of them must be based on other sources: the interest rate, storage cost, and elasticity coefficients with respect to demand, supply, and convenience yield. The model was re-solved changing each parameter value individually, holding all others constant.[22] The results are summarized in the Appendix 2, each table corresponding to a parameter. The mean and standard deviation of each monthly endogenous variable are reported. Since the results suggest that the model is robust with respect to realistic changes in parameter values, and no major discrepancies exists between the model predictions and the sample data, the current model is used to simulate futures contract prices.

Conditional Price Distributions

Conditional or state-dependent price distributions can be derived from the rational expectations commodity storage model by simulating many time paths from a given initial state to a fixed contract maturity month. By taking the probability distribution of initial states into account, simulations can be used to generate a series of rational price expectations that is comparable to observed futures contract prices. Thus, the model is used to generate probability distributions of December and May futures prices, conditional on the state in each month starting from a year prior to maturity. Monthly state variables are discretized within respective ranges, and each combination of discrete state variables is regarded as an initial state. The model is simulated from all initial states to both December and May, for 10,000 times.

Only state variables that are possibly observable are considered, and the number of initial states in these simulations varies by month. From November to April, the initial states are the 30 discrete levels of monthly availability. During the growing season (May—July), 20 levels of

[22] The analysis here does not cover all possible combinations of parameter values.

expected crop size are specified in addition to the 30 availability levels, implying 600 discrete initial states. For August and October, the percentage of crop expected to arrive in the next month is discretized into 10 values, so that there are 6,000 initial states. In September, in addition to all the states already considered, the 10 levels of current harvest determine the expected percentage of crop arriving in November; including these as initial states implies there are a total of 60,000 September state combinations.

The simulated relationship between conditional expected prices and their conditioning factors are unsurprising. If stocks in February, for example, are abundant, both December and May expected prices are low and tightly distributed. As available supply decreases, conditional expected prices rise, and their distributions become more dispersed. Holding the inventory level constant, as planted crop sizes become larger in May, for example, median December and May (a year ahead) prices both decline, and the distributions become less dispersed.

Like the equilibrium spot price functions, these conditional price distributions do not have an observable counterpart, and cannot verify the model's ability to simulate futures prices. But, the mean of a conditional price distribution is a price expectation based on available information, and coincides with a futures price in an efficient market (assuming no basis risk). Hence, at each initial state, the averages of December and May conditional prices over the 10,000 simulations are comparable to prices of futures contracts with the same maturity dates. In the model, each initial state represents a unique set of available information at the time expectations are formed, so that means of conditional expected prices can be regarded as functions of state variables.

The means of conditional expected December and May prices (hereafter, futures prices) are plotted against their conditioning variable, February availability in Figure 9. In February, the

May futures price represents the expected value of the current year's crop, and the December futures price is the expected value of the new crop. The two prices cross at availability of about 11.1 billion bushels. If stocks are large in February, the stored crop is priced below the new crop, encouraging current consumption and discouraging production.

The relationships between state variables and futures expected prices are formalized by the collocation method, analogous to the spot price functions. Specifically, denoting *DFP* and *MFP* as December and May futures prices, respectively, both futures price functions are interpolated with cubic b-splines.

To obtain the frequency distributions for states over time, the model is again simulated for 10,000 "years," where states are realized endogenously. This procedure is identical to the long-run simulation for monthly spot prices and storage in Case 6, but also includes the interpolated futures price functions to generate a series of December and May futures prices. Hence, once the state is realized in a given month, *DFP*[·] and *MFP*[·] are evaluated to determine futures prices, in addition to other current endogenous variables.

Distributions of simulated monthly December and May futures prices are illustrated in Figures 10 and 11. The distributions of prices in the maturity months are plotted with circles. Since the futures price in maturity months coincides with the spot price under rational expectations, the distributions at maturity are for the simulated spot prices in December and May, respectively. The overall pattern through the lifetime of each contract resembles their observed counterpart in Figures 3 and 4. Namely, at the early stages of the contract, the price distributions are centered, and the spread widens as maturity approaches. The anomalous distributions in both figures are for October, which have a larger probability mass on price levels higher than reality.

Tables 7 and 8 report statistics of simulated December and May futures prices, respectively, along with their sample counterparts. The observed December futures prices averaged $2.57 per bushel one year prior to maturity, increase throughout spring, and then decline as maturity draws closer. June prices, for example, are on average 15 cents per bushel higher than at maturity, which as discussed earlier, appears to be inconsistent with an efficient market. The simulated means, in contrast, do not exhibit any trend, except the anomalous spikes in September and October. Standard deviations of observed December futures price are small in the early months of the contract, and the variability increases towards maturity, where the standard deviation at maturity is nearly three times as large than a year earlier. The seasonal increase is replicated in the simulated result, if September and October are ignored. The simulated variability at maturity is twice as large as the variability at a year prior.

The May futures price in the sample is about $2.80 per bushel, on average, a year prior to maturity, and declines to about $2.70 at harvest. The mean price level recovers after harvest to a high at maturity. Again, the seasonal pattern of the sample means seems to be inconsistent with an efficient market. The simulated means, although approximately six cents per bushel higher than the observed levels a year prior to maturity, maintain that level until maturity, again ignoring the anomalies in September and October. At maturity, the simulated and observed price levels are similar.

The observed variability of May futures contract remains low and increases after harvest until at maturity it is nearly double the prior year's standard deviation. Simulated variability follows the sample pattern, when prices in September and October are ignored, although the magnitude of the increase is not fully reproduced.

If September and October are ignored, the simulated futures prices are plausible representation of futures prices in an efficient market, exhibiting a time-to-maturity effect.[23] Moreover, the simulated futures prices replicate positive skewness indicated by observed futures prices.

Concluding Remarks

This paper approaches model building from a viewpoint that parameters of price distributions should be consistent with, and identifiable from, a structural model. Hence, a rational expectations commodity storage model was applied to the U.S. corn market, recognizing it as a potentially valuable framework for commodity price analysis. The model, calibrated to the 1989/90-1997/98 period, generates intra- and inter-year price series similar to those faced by market participants during the sample period, and the simulated long-run distributions of monthly prices are comparable to those estimated from the sample.

Consistent with the sample, the season-high prices occur in May and the season-low in November. Price variability is the smallest in November and highest during the growing season (May through August). Simulated December and May futures price distributions exhibit a time-to-maturity effect similar to those estimated from the sample period. Nonetheless, the distributions place some (small) probability mass on events outside the observed range during the 1989/90 to 1997/98 time-frame. The model solutions allow for various seasonal patterns, including unlikely ones, and the results are robust with respect to changes in parameter values.

[23] The likely factor causing the anomalies in the September and October distributions is the discretized specification of states regarding harvest timing. Despite careful calibration, it places a larger probability mass than in reality on those expected harvest proportions that implies high prices.

Overall, applications of a rational expectations commodity storage framework can be a useful tool to examine long-run implications of price behavior. Some limitations of the approach, however, need to be mentioned. A standard competitive storage model cannot fully replicate the observed seasonal price pattern, particularly the seasonal decline (or price backwardation) from May until harvest. The relationship between prices in adjacent periods is defined by the non-arbitrage condition, implying that expected prices appreciate by the carrying cost. In a standard model that assumes a constant physical storage cost, the only possible cause of price backwardation is stock-outs. In reality, an aggregate stock-out has never occurred for corn, but price backwardation occurs in most years. Because our application aims to replicate the observed market, stock-outs are not used as an explanation for price behavior. Instead, backwardation is modeled by including "convenience yield" as a component of the carrying cost, which encourages storage. Although its existence is controversial, this term can be viewed as a risk premium required by risk-averse storers. The specification is thus equivalent to relaxing the assumption of risk-neutrality. Alternatively, it represents the benefit of an implicit option value of the stocks. As inventories decline, the benefit increases, since the holders of inventory can meet unexpected demands.

A model, no matter how complex, is an abstraction from the reality. By including six state variables, the model generates seasonal probability distributions of prices that are comparable to those estimated from a short sample. But even in this case, the time-series properties of the simulated prices differ somewhat from the observed monthly prices. So many factors evidently combine to affect prices in reality that most of them cannot be disentangled from noise. Limited though the model may be, its complexity cannot be easily enhanced because of the “curse of dimensionality;” computational time increases almost exponentially as state variables are added.

Yet, the virtue of the approach is that both simulated cash and futures prices are consistent with the data and conform to conceptual expectations. For example, when the available supply is larger, the cash price is relatively low compared to the futures price, which encourages current consumption. The simulations, therefore, permit analysis of long-run impacts of economic decisions such as marketing or risk management practices. In such analyses, it should be noted that the framework cannot address basis risk because the basis always converges under rational expectations. Moreover, the rational expectations framework implies efficient markets—prices reflect all available information at a given point in time. Consequently, the model is incapable of examining hypotheses related to market inefficiency.

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Table 1. Simulated Mean Outcomes Under Certainty (Case 1)[a]

Month	Price (P) \$/bu	Storage (s) mil. bu.	Consumption (q) mil. bu.
January	2.53	5898	741
	(2.56)		(745)
February	2.58	5158	740
	(2.60)	(4740)	
March	2.63	4516	641
	(2.69)		
April	2.68	3879	637
	(2.74)		(644)
May	2.73	3247	632
	(2.77)	(2812)	
June	2.69	2730	517
	(2.72)		
July	2.64	2215	515
	(2.65)		(527)
August	2.58	1700	515
	(2.57)	(1234)	
September	2.46	1826	871
	(2.47)		
October	2.39	5576	870
	(2.40)		(876)
November	2.44	7381	867
	(2.46)	(6972)	
December	2.48	6639	742
	(2.51)		

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 2. Simulated Outcomes With Uncertain Crop Size (Case 2)[a]

	Price (P), $/bu.					Storage (s), mil. bu.					Consumption (q), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January	2.59	2.51	0.52	7.43	1.55	6422	6415	926	9339	3297	741	743	35	838	566
	(2.56)	(2.49)	(0.44)	(3.53)	(2.07)						(745)	(757)	(60)	(831)	(650)
February	2.64	2.56	0.52	7.51	1.60	5682	5673	893	8504	2730	740	742	34	835	567
	(2.60)	(2.58)	(0.48)	(3.71)	(2.05)	(4740)	(4789)	(631)	(5678)	(3800)					
March	2.69	2.61	0.52	7.58	1.64	5041	5031	863	7782	2238	641	643	29	722	492
	(2.69)	(2.59)	(0.51)	(3.92)	(2.16)										
April	2.74	2.66	0.53	7.66	1.68	4404	4392	835	7066	1748	637	638	29	716	490
	(2.74)	(2.50)	(0.67)	(4.47)	(2.23)						(644)	(657)	(54)	(727)	(547)
May	2.80	2.69	0.70	10.23	1.32	3771	3758	808	6418	1240	633	634	37	753	454
	(2.77)	(2.51)	(0.80)	(4.86)	(2.20)	(2812)	(2843)	(587)	(3709)	(1718)					
June	2.77	2.64	0.70	8.10	1.32	3253	3234	788	5889	822	518	520	30	618	393
	(2.72)	(2.59)	(0.78)	(4.74)	(2.09)										
July	2.71	2.59	0.69	9.56	1.29	2737	2719	768	5344	404	516	518	30	616	373
	(2.65)	(2.34)	(0.80)	(4.70)	(2.16)						(527)	(533)	(54)	(619)	(432)
August	2.66	2.52	0.71	7.47	1.29	2221	2205	751	4799	226	516	517	31	612	395
	(2.57)	(2.45)	(0.76)	(4.48)	(1.86)	(1234)	(1308)	(490)	(2113)	(426)					
September	2.53	2.43	0.58	6.56	1.32	2347	2329	716	4759	465	871	874	46	1018	681
	(2.47)	(2.35)	(0.42)	(3.39)	(2.08)										
October	2.44	2.37	0.48	5.41	1.36	6099	6085	818	9143	3662	871	872	40	1002	709
	(2.40)	(2.27)	(0.40)	(3.12)	(1.92)						(876)	(891)	(69)	(952)	(780)
November	2.50	2.42	0.51	7.29	1.47	7905	7901	996	11016	4429	867	869	42	984	659
	(2.46)	(2.41)	(0.38)	(3.22)	(2.02)	(6972)	(6940)	(724)	(8080)	(5937)					
December	2.54	2.46	0.51	7.35	1.51	7163	7158	961	10176	3863	742	744	35	840	566
	(2.51)	(2.42)	(0.40)	(3.36)	(2.06)										

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 2. (Continued)[a]

Month	Crop Estimate (*H*), mil. bu.					Harvest (*h*), mil. bu.				
	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
April	8291	8288	615	10511	6034					
May	8291	8288	615	10511	6034					
	(8649)	(8575)	(619)	(9840)	(7850)					
June	8290	8281	662	10475	5855					
	(8543)	(8500)	(634)	(9840)	(7850)					
July	8291	8284	710	10781	5442					
	(8386)	(8275)	(747)	(9700)	(7450)					
August	8290	8296	821	11120	4999					
	(8234)	(8122)	(775)	(9276)	(7348)					
September	8290	8297	836	11181	5124	996	997	100	1344	616
	(8210)	(8118)	(838)	(9268)	(7229)	(1107)	(864)	(696)	(2862)	(552)
October	8294	8298	863	11492	5115	4623	4625	481	6405	2851
	(8257)	(8022)	(966)	(9602)	(6962)	(4566)	(4404)	(1259)	(6073)	(2534)
November	8292	8302	903	11648	4720	2673	2673	396	4123	1229
	(8294)	(7934)	(1235)	(10051)	(6338)	(2720)	(2856)	(1404)	(4850)	(490)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 3. Simulated Outcomes With Uncertain Demand and Crop Size (Case 3)[a]

	Price (P), \$/bu.					Storage (s), mil. bu.					Consumption (q), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January	2.71	2.62	0.57	6.96	1.52	6245	6209	935	9485	3408	738	737	71	1019	453
	(2.56)	(2.49)	(0.44)	(3.53)	(2.07)						(745)	(757)	(60)	(831)	(650)
February	2.76	2.67	0.58	7.18	1.56	5508	5473	902	8702	2781	736	735	73	1045	452
	(2.60)	(2.58)	(0.48)	(3.71)	(2.05)	(4740)	(4789)	(631)	(5678)	(3800)					
March	2.81	2.72	0.58	7.74	1.57	4870	4834	874	8018	2242	639	638	69	912	420
	(2.69)	(2.59)	(0.51)	(3.92)	(2.16)										
April	2.86	2.78	0.59	7.70	1.60	4232	4199	846	7373	1735	638	637	77	912	365
	(2.74)	(2.50)	(0.67)	(4.47)	(2.23)						(644)	(657)	(54)	(727)	(547)
May	2.93	2.79	0.78	9.30	1.42	3593	3553	821	6753	1292	638	637	84	965	304
	(2.77)	(2.51)	(0.80)	(4.86)	(2.20)	(2812)	(2843)	(587)	(3709)	(1718)					
June	2.91	2.76	0.79	10.32	1.32	3075	3031	800	6232	843	519	519	52	746	339
	(2.72)	(2.59)	(0.78)	(4.74)	(2.09)										
July	2.85	2.70	0.76	9.91	1.35	2557	2514	781	5690	367	518	518	54	748	319
	(2.65)	(2.34)	(0.80)	(4.70)	(2.16)						(527)	(533)	(54)	(619)	(432)
August	2.79	2.63	0.79	10.52	1.30	2040	2000	762	5124	75	517	517	54	716	319
	(2.57)	(2.45)	(0.76)	(4.48)	(1.86)	(1234)	(1308)	(490)	(2113)	(426)					
September	2.64	2.53	0.63	7.61	1.33	2169	2130	731	4990	340	867	866	98	1291	490
	(2.47)	(2.35)	(0.42)	(3.39)	(2.08)										
October	2.54	2.47	0.52	5.84	1.31	5919	5881	831	9388	3366	867	865	98	1264	525
	(2.40)	(2.27)	(0.40)	(3.12)	(1.92)						(876)	(891)	(69)	(952)	(780)
November	2.60	2.52	0.56	7.21	1.45	7720	7689	1004	11138	4443	868	865	94	1303	539
	(2.46)	(2.41)	(0.38)	(3.22)	(2.02)	(6972)	(6940)	(724)	(8080)	(5937)					
December	2.65	2.57	0.56	7.09	1.50	6983	6951	968	10235	3939	737	736	71	1034	481
	(2.51)	(2.42)	(0.40)	(3.36)	(2.06)										

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 3. (Continued)[a]

	Crop Estimate (H), mil. bu.					Harvest (h), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
May	8278	8273	605	10853	5839					
	(8649)	(8575)	(619)	(9840)	(7850)					
June	8281	8285	650	10917	5888					
	(8543)	(8500)	(634)	(9840)	(7850)					
July	8281	8280	696	10917	5679					
	(8386)	(8275)	(747)	(9700)	(7450)					
August	8281	8283	804	11153	5205					
	(8234)	(8122)	(775)	(9276)	(7348)					
September	8282	8282	822	11280	5023	995	995	99	1356	604
	(8210)	(8118)	(838)	(9268)	(7229)	(1091)	(852)	(686)	(2824)	(544)
October	8283	8286	854	11510	5009	4617	4619	476	6415	2792
	(8257)	(8022)	(966)	(9602)	(6962)	(4502)	(4343)	(1241)	(5988)	(2499)
November	8281	8278	902	11731	4815	2669	2667	400	4237	1119
	(8294)	(7934)	(1235)	(10051)	(6338)	(2682)	(2816)	(1385)	(4782)	(483)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 4. Simulated Outcomes with Uncertain Harvest Timing, Demand, and Crop Size (Case 4)[a]

	Price (P), \$/bu.					Storage (s), mil. bu.					Consumption (q), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January	2.70	2.61	0.58	8.37	1.54	6406	6386	976	9644	3160	737	736	71	1011	440
	(2.56)	(2.49)	(0.44)	(3.53)	(2.07)						(745)	(757)	(60)	(831)	(650)
February	2.76	2.67	0.59	8.25	1.59	5669	5646	941	8739	2663	737	735	75	1007	463
	(2.60)	(2.58)	(0.48)	(3.71)	(2.05)	(4740)	(4789)	(631)	(5678)	(3800)					
March	2.81	2.72	0.59	8.65	1.62	5030	5008	911	8025	2124	640	638	70	890	389
	(2.69)	(2.59)	(0.51)	(3.92)	(2.16)										
April	2.86	2.77	0.60	8.71	1.65	4392	4370	883	7355	1667	638	636	76	947	375
	(2.74)	(2.50)	(0.67)	(4.47)	(2.23)						(644)	(657)	(54)	(727)	(547)
May	2.93	2.78	0.80	11.76	1.46	3753	3725	857	6586	1162	639	637	84	945	349
	(2.77)	(2.51)	(0.80)	(4.86)	(2.20)	(2812)	(2843)	(587)	(3709)	(1718)					
June	2.91	2.76	0.80	11.81	1.39	3235	3203	835	5978	789	519	517	52	714	321
	(2.72)	(2.59)	(0.78)	(4.74)	(2.09)										
July	2.86	2.70	0.78	11.34	1.39	2717	2680	815	5446	330	517	516	55	740	320
	(2.65)	(2.34)	(0.80)	(4.70)	(2.16)						(527)	(533)	(54)	(619)	(432)
August	2.81	2.64	0.82	11.10	1.32	2201	2169	795	4980	182	516	515	53	712	302
	(2.57)	(2.45)	(0.76)	(4.48)	(1.86)	(1234)	(1308)	(490)	(2113)	(426)					
September	2.66	2.52	0.78	10.15	1.04	2447	2388	936	6420	59	869	867	103	1262	497
	(2.47)	(2.35)	(0.42)	(3.39)	(2.08)										
October	2.57	2.45	0.68	10.51	1.19	6141	6103	1488	11015	1623	868	866	105	1297	527
	(2.40)	(2.27)	(0.40)	(3.12)	(1.92)						(876)	(891)	(69)	(952)	(780)
November	2.60	2.51	0.57	7.77	1.49	7880	7860	1045	11192	4355	869	868	94	1249	520
	(2.46)	(2.41)	(0.38)	(3.22)	(2.02)	(6972)	(6940)	(724)	(8080)	(5937)					
December	2.65	2.56	0.58	7.88	1.50	7143	7124	1010	10442	3782	737	736	71	1067	481
	(2.51)	(2.42)	(0.40)	(3.36)	(2.06)										

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 4. (Continued)[a]

	Crop Estimate (H), mil. bu.					Harvest (h), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
April	8294	8301	618	10737	5992					
May	8294	8301	618	10737	5992					
	(8649)	(8575)	(619)	(9840)	(7850)					
June	8290	8292	661	11053	5565					
	(8543)	(8500)	(634)	(9840)	(7850)					
July	8288	8292	711	11484	5225					
	(8386)	(8275)	(747)	(9700)	(7450)					
August	8284	8290	821	11583	5041					
	(8234)	(8122)	(775)	(9276)	(7348)					
September	8284	8283	836	11531	5075	1114	986	580	4245	123
	(8210)	(8118)	(838)	(9268)	(7229)	(1091)	(852)	(686)	(2824)	(544)
October	8281	8281	865	11690	5125	4562	4554	1216	8355	1572
	(8257)	(8022)	(966)	(9602)	(6962)	(4502)	(4343)	(1241)	(5988)	(2499)
November	8279	8281	911	11707	4813	2609	2555	1302	6893	0
	(8294)	(7934)	(1235)	(10051)	(6338)	(2682)	(2816)	(1385)	(4782)	(483)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 5. Simulated Outcomes with Endogenous Supply and Uncertain Harvest Timing, Demand, and Crop Size (Case 5)[a]

	Price (P), $/bu.					Storage (s), mil. bu.					Consumption (q), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January	2.64	2.61	0.36	5.20	1.63	6411	6394	664	9221	3966	739	738	66	971	518
	(2.56)	(2.49)	(0.44)	(3.53)	(2.07)						(745)	(757)	(60)	(831)	(650)
February	2.69	2.66	0.37	5.17	1.65	5674	5655	643	8479	3352	737	737	68	1010	449
	(2.60)	(2.58)	(0.48)	(3.71)	(2.05)	(4740)	(4789)	(631)	(5678)	(3800)	(745)	(757)	(60)	(831)	(650)
March	2.75	2.71	0.37	5.35	1.69	5033	5020	625	7712	2780	641	640	66	883	410
	(2.69)	(2.59)	(0.51)	(3.92)	(2.16)						(644)	(657)	(54)	(727)	(547)
April	2.80	2.76	0.38	5.51	1.75	4393	4380	608	7035	2255	640	639	73	941	381
	(2.74)	(2.50)	(0.67)	(4.47)	(2.23)						(644)	(657)	(54)	(727)	(547)
May	2.82	2.83	0.29	4.94	1.99	3755	3745	597	6394	1593	639	638	76	925	336
	(2.77)	(2.51)	(0.80)	(4.86)	(2.20)	(2812)	(2843)	(587)	(3709)	(1718)	(644)	(657)	(54)	(727)	(547)
June	2.77	2.74	0.33	4.73	1.83	3233	3223	586	5819	1105	522	522	45	703	354
	(2.72)	(2.59)	(0.78)	(4.74)	(2.09)						(527)	(533)	(54)	(619)	(432)
July	2.72	2.69	0.36	5.20	1.78	2713	2701	575	5205	634	520	520	48	710	343
	(2.65)	(2.34)	(0.80)	(4.70)	(2.16)						(527)	(533)	(54)	(619)	(432)
August	2.68	2.63	0.44	5.68	1.61	2194	2178	537	4670	237	519	519	47	727	339
	(2.57)	(2.45)	(0.76)	(4.48)	(1.86)	(1234)	(1308)	(490)	(2113)	(426)	(527)	(533)	(54)	(619)	(432)
September	2.55	2.49	0.49	6.53	1.13	2466	2402	747	6261	369	872	871	94	1240	521
	(2.47)	(2.35)	(0.42)	(3.39)	(2.08)						(876)	(891)	(69)	(952)	(780)
October	2.49	2.41	0.49	9.07	1.26	6172	6192	1318	10954	2074	872	872	98	1245	469
	(2.40)	(2.27)	(0.40)	(3.12)	(1.92)						(876)	(891)	(69)	(952)	(780)
November	2.54	2.51	0.35	4.90	1.53	7890	7877	707	10898	5302	870	869	89	1242	523
	(2.46)	(2.41)	(0.38)	(3.22)	(2.02)	(6972)	(6940)	(724)	(8080)	(5937)	(876)	(891)	(69)	(952)	(780)
December	2.59	2.56	0.36	5.11	1.58	7150	7132	685	10062	4600	740	739	66	999	521
	(2.51)	(2.42)	(0.40)	(3.36)	(2.06)						(745)	(757)	(60)	(831)	(650)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 5. (Continued)[a]

Month	Crop Estimate (H), mil. bu.					Harvest (h), mil. bu.				
	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
May	8307	8308	122	8769	7818					
	(8649)	(8575)	(619)	(9840)	(7850)					
June	8304	8300	264	9267	7308					
	(8543)	(8500)	(634)	(9840)	(7850)					
July	8306	8306	376	9917	6944					
	(8386)	(8275)	(747)	(9700)	(7450)					
August	8304	8300	549	10362	6193					
	(8234)	(8122)	(775)	(9276)	(7348)					
September	8306	8299	573	10520	6162	1144	989	548	3817	490
	(8210)	(8118)	(838)	(9268)	(7229)	(1091)	(852)	(686)	(2824)	(544)
October	8306	8296	615	10932	6124	4578	4598	1179	7899	1742
	(8257)	(8022)	(966)	(9602)	(6962)	(4502)	(4343)	(1241)	(5988)	(2499)
November	8302	8298	672	11023	5773	2588	2550	1278	6763	0
	(8294)	(7934)	(1235)	(10051)	(6338)	(2682)	(2816)	(1385)	(4782)	(483)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 6. Simulated Outcomes with Endogenous Supply, Uncertain Production Cost, Harvest Timing, Demand, and Crop Size[a]

	Price (P), $/bu.					Storage (s), mil. bu.					Consumption (q), mil. bu.				
Month	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January	2.67	2.59	0.51	8.30	1.57	6428	6422	865	9453	3187	739	739	70	1051	462
	(2.56)	(2.49)	(0.44)	(3.53)	(2.07)						(745)	(757)	(60)	(831)	(650)
February	2.72	2.64	0.51	8.34	1.59	5691	5687	836	8684	2617	737	736	71	992	474
	(2.60)	(2.58)	(0.48)	(3.71)	(2.05)	(4740)	(4789)	(631)	(5678)	(3800)	(745)	(757)	(60)	(831)	(650)
March	2.77	2.69	0.52	8.53	1.64	5052	5048	811	7977	2113	639	638	68	910	389
	(2.69)	(2.59)	(0.51)	(3.92)	(2.16)						(644)	(657)	(54)	(727)	(547)
April	2.83	2.75	0.53	8.78	1.66	4412	4401	788	7333	1608	640	638	76	921	315
	(2.74)	(2.50)	(0.67)	(4.47)	(2.23)						(644)	(657)	(54)	(727)	(547)
May	2.87	2.72	0.62	8.31	1.54	3772	3764	772	6658	1137	639	638	83	1039	344
	(2.77)	(2.51)	(0.80)	(4.86)	(2.20)	(2812)	(2843)	(587)	(3709)	(1718)	(644)	(657)	(54)	(727)	(547)
June	2.84	2.73	0.63	9.18	1.54	3251	3241	758	6072	687	522	520	49	745	345
	(2.72)	(2.59)	(0.78)	(4.74)	(2.09)						(527)	(533)	(54)	(619)	(432)
July	2.79	2.67	0.63	9.86	1.45	2732	2718	744	5530	323	519	517	53	721	341
	(2.65)	(2.34)	(0.80)	(4.70)	(2.16)						(527)	(533)	(54)	(619)	(432)
August	2.75	2.63	0.66	9.29	1.44	2215	2203	732	5035	135	517	517	52	700	321
	(2.57)	(2.45)	(0.76)	(4.48)	(1.86)	(1234)	(1308)	(490)	(2113)	(426)	(527)	(533)	(54)	(619)	(432)
September	2.59	2.49	0.64	7.67	1.16	2489	2435	861	6048	84	873	871	100	1321	524
	(2.47)	(2.35)	(0.42)	(3.39)	(2.08)						(876)	(891)	(69)	(952)	(780)
October	2.51	2.41	0.59	6.94	1.22	6206	6198	1412	11125	1894	872	869	100	1261	537
	(2.40)	(2.27)	(0.40)	(3.12)	(1.92)						(876)	(891)	(69)	(952)	(780)
November	2.57	2.49	0.50	8.38	1.49	7907	7902	926	11116	4254	870	870	92	1184	566
	(2.46)	(2.41)	(0.38)	(3.22)	(2.02)	(6972)	(6940)	(724)	(8080)	(5937)	(876)	(891)	(69)	(952)	(780)
December	2.62	2.54	0.50	8.34	1.53	7168	7161	895	10289	3730	739	739	69	1001	484
	(2.51)	(2.42)	(0.40)	(3.36)	(2.06)						(745)	(757)	(60)	(831)	(650)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 6. (Continued)[a]

Month	Crop Estimate (H), mil. bu. Mean	Median	Std. Dev.	Max.	Min.	Harvest (h), mil. bu. Mean	Median	Std. Dev.	Max.	Min.
April	8294	8300	632	10685	5562					
May	8294	8300	632	10685	5562					
	(8649)	(8575)	(619)	(9840)	(7850)					
June	8297	8304	674	10831	5403					
	(8543)	(8500)	(634)	(9840)	(7850)					
July	8299	8307	723	11047	5013					
	(8386)	(8275)	(747)	(9700)	(7450)					
August	8297	8297	830	11506	4825					
	(8234)	(8122)	(775)	(9276)	(7348)					
September	8298	8296	846	11487	4875	1147	989	556	3947	385
	(8210)	(8118)	(838)	(9268)	(7229)	(1091)	(852)	(686)	(2824)	(544)
October	8296	8301	876	11928	4731	4589	4605	1219	8546	1565
	(8257)	(8022)	(966)	(9602)	(6962)	(4502)	(4343)	(1241)	(5988)	(2499)
November	8300	8304	922	11827	4771	2571	2492	1303	6923	0
	(8294)	(7934)	(1235)	(10051)	(6338)	(2682)	(2816)	(1385)	(4782)	(483)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 7. Simulated December Futures Prices[a]

Month	Mean	Median	Std. Dev.	Max.	Min.
December	2.60	2.59	0.24	3.44	1.88
	(2.57)	(2.54)	(0.14)	(2.88)	(2.43)
January	2.61	2.59	0.24	3.44	1.86
	(2.60)	(2.57)	(0.16)	(2.92)	(2.42)
February	2.60	2.59	0.24	3.43	1.85
	(2.65)	(2.63)	(0.20)	(3.07)	(2.41)
March	2.61	2.59	0.24	3.44	1.86
	(2.69)	(2.64)	(0.22)	(3.13)	(2.43)
April	2.61	2.60	0.25	3.45	1.87
	(2.70)	(2.62)	(0.26)	(3.27)	(2.46)
May	2.61	2.56	0.36	4.42	1.65
	(2.69)	(2.62)	(0.34)	(3.48)	(2.38)
June	2.61	2.56	0.37	5.27	1.63
	(2.71)	(2.70)	(0.35)	(3.45)	(2.29)
July	2.60	2.56	0.39	4.94	1.62
	(2.65)	(2.51)	(0.43)	(3.60)	(2.25)
August	2.60	2.54	0.42	5.40	1.57
	(2.57)	(2.48)	(0.35)	(3.29)	(2.20)
September	2.73	2.64	0.55	8.09	1.57
	(2.57)	(2.46)	(0.38)	(3.25)	(2.21)
October	3.46	3.29	0.91	12.40	1.69
	(2.54)	(2.48)	(0.37)	(3.18)	(2.09)
November	2.60	2.53	0.49	8.58	1.56
	(2.55)	(2.55)	(0.39)	(3.29)	(2.12)
December	2.60	2.53	0.49	8.30	1.49
	(2.56)	(2.56)	(0.41)	(3.35)	(2.12)

[a] In $/bu. Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 8. Simulated May Futures Contract Prices[a]

Month	Mean	Median	Std. Dev.	Max.	Min.
May	2.86	2.83	0.28	4.25	2.07
	(2.79)	(2.73)	(0.33)	(3.55)	(2.49)
June	2.85	2.83	0.29	4.93	2.05
	(2.81)	(2.80)	(0.34)	(3.54)	(2.42)
July	2.86	2.83	0.30	4.65	2.05
	(2.76)	(2.63)	(0.41)	(3.69)	(2.39)
August	2.85	2.81	0.32	4.96	2.00
	(2.70)	(2.62)	(0.34)	(3.42)	(2.35)
September	2.95	2.89	0.42	7.19	1.99
	(2.71)	(2.59)	(0.37)	(3.39)	(2.36)
October	3.50	3.37	0.70	10.66	2.10
	(2.67)	(2.62)	(0.35)	(3.25)	(2.24)
November	2.85	2.80	0.37	7.57	1.98
	(2.69)	(2.67)	(0.35)	(3.35)	(2.28)
December	2.85	2.80	0.37	7.60	1.95
	(2.70)	(2.63)	(0.39)	(3.50)	(2.28)
January	2.85	2.80	0.37	7.42	1.93
	(2.73)	(2.65)	(0.45)	(3.67)	(2.26)
February	2.85	2.80	0.38	7.31	1.91
	(2.76)	(2.75)	(0.46)	(3.73)	(2.21)
March	2.85	2.80	0.38	7.21	1.94
	(2.80)	(2.74)	(0.49)	(3.84)	(2.22)
April	2.86	2.81	0.38	7.33	1.95
	(2.81)	(2.56)	(0.66)	(4.36)	(2.30)
May	2.85	2.71	0.62	6.90	1.35
	(2.84)	(2.59)	(0.86)	(4.94)	(2.25)

[a] In $/bu. Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

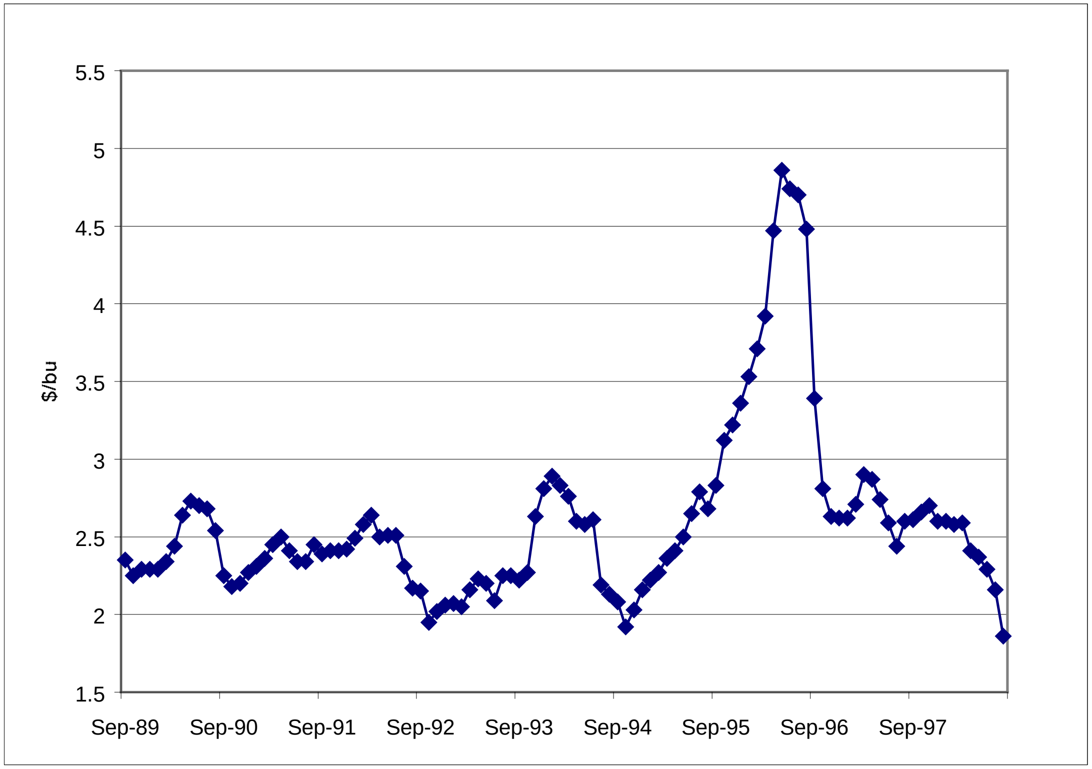


Figure 1. Monthly Cash Prices of No. 2, Yellow Corn, Central Illinois.

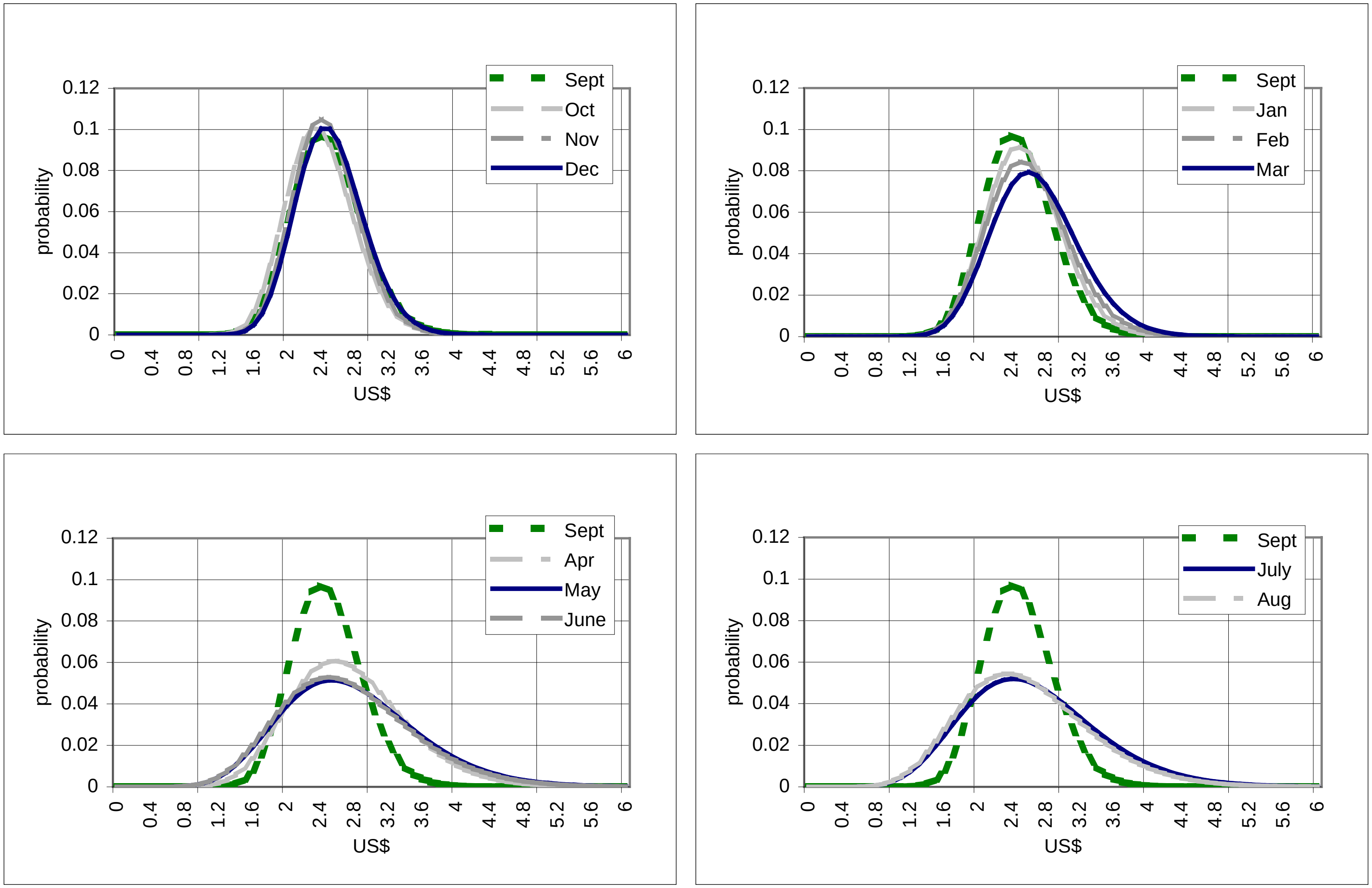


Figure 2. Estimated Monthly Price Distributions for Corn, 1989-90/1997-98.

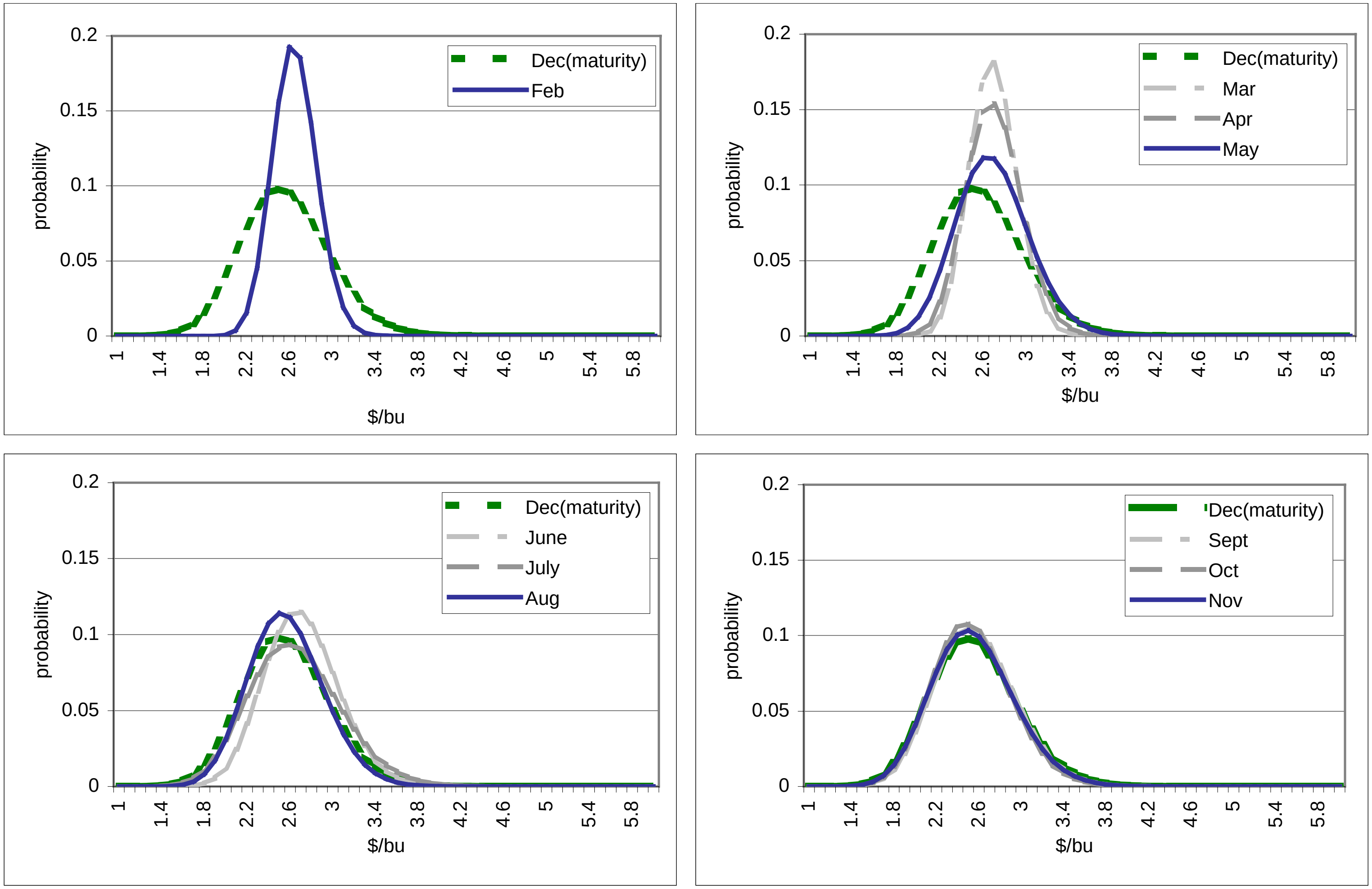


Figure 3. Estimated Distributions of Prices of December Futures Contract, by Month.

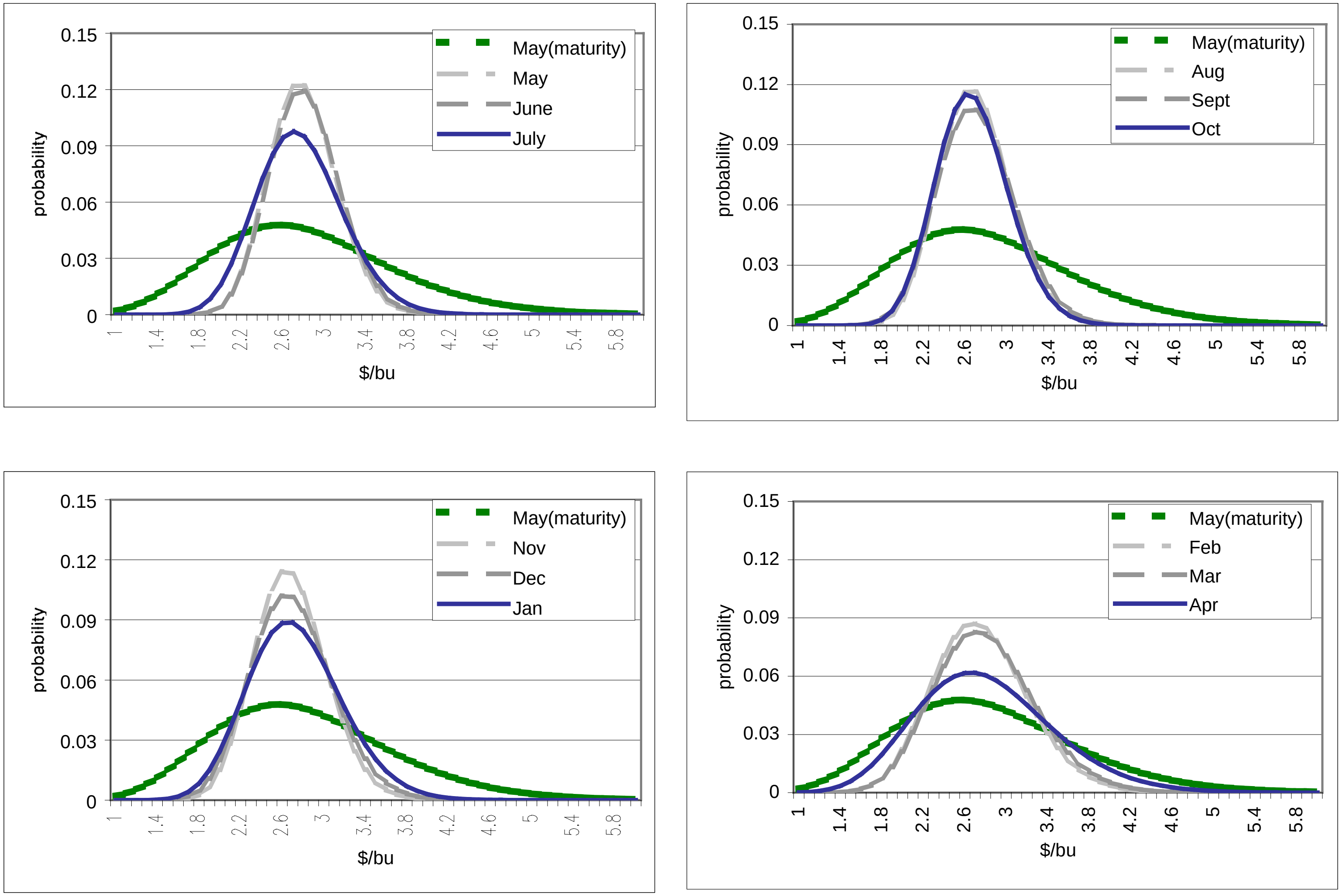


Figure 4. Estimated Distributions of Prices of May Futures, by Month.

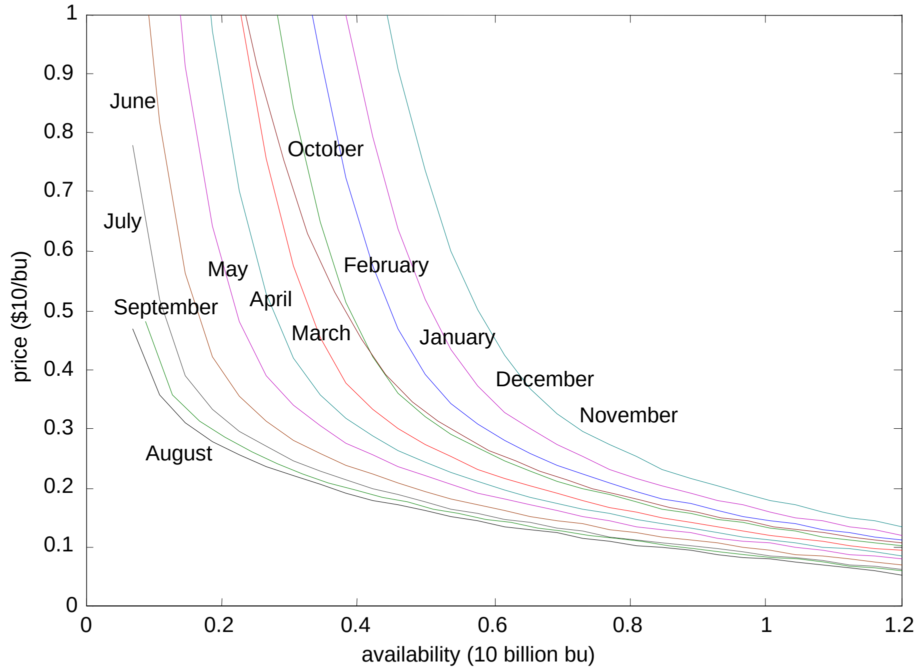


Figure 5. Equilibrium Price Function under Certainty (Case 1).

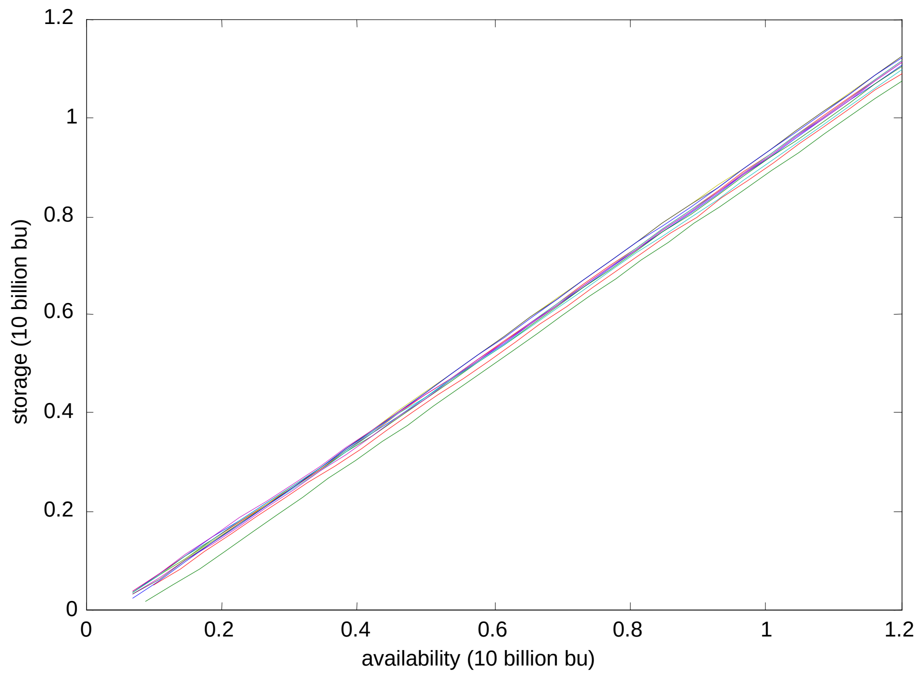


Figure 6. Equilibrium Storage under Certainty (Case 1).

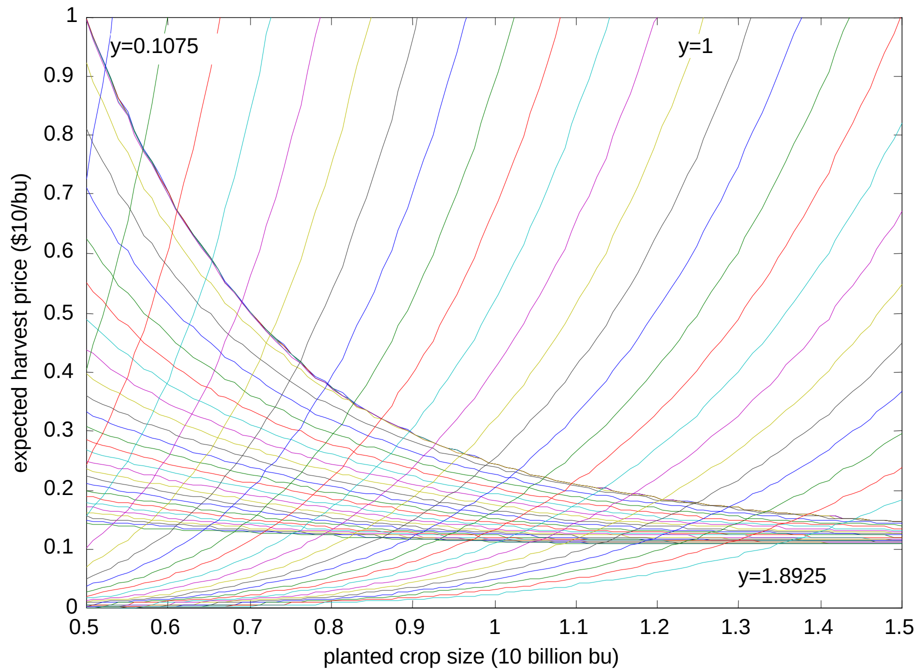


Figure 7. .Harvest-Time Demand and Supply Functions (Case 6).

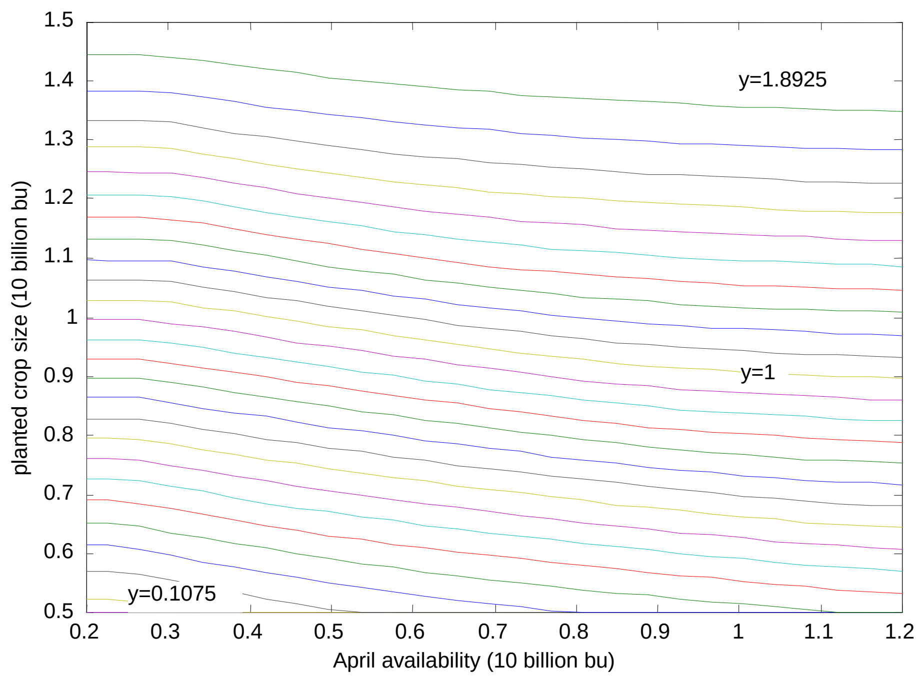


Figure 8. Equilibrium Planting Function (Case 6).

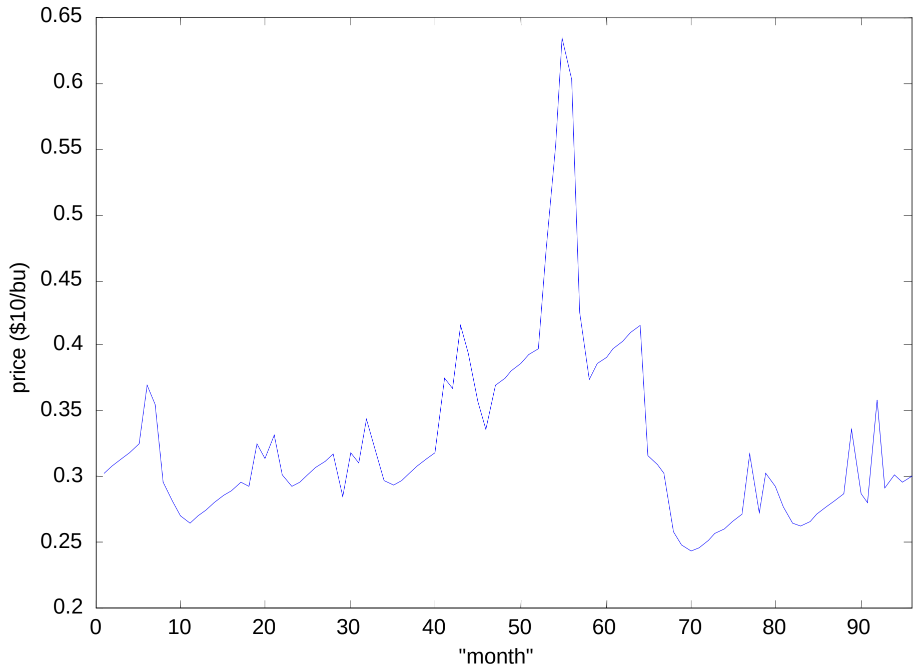


Figure 9. Simulated Prices under Uncertain Crop Size (Case 2).

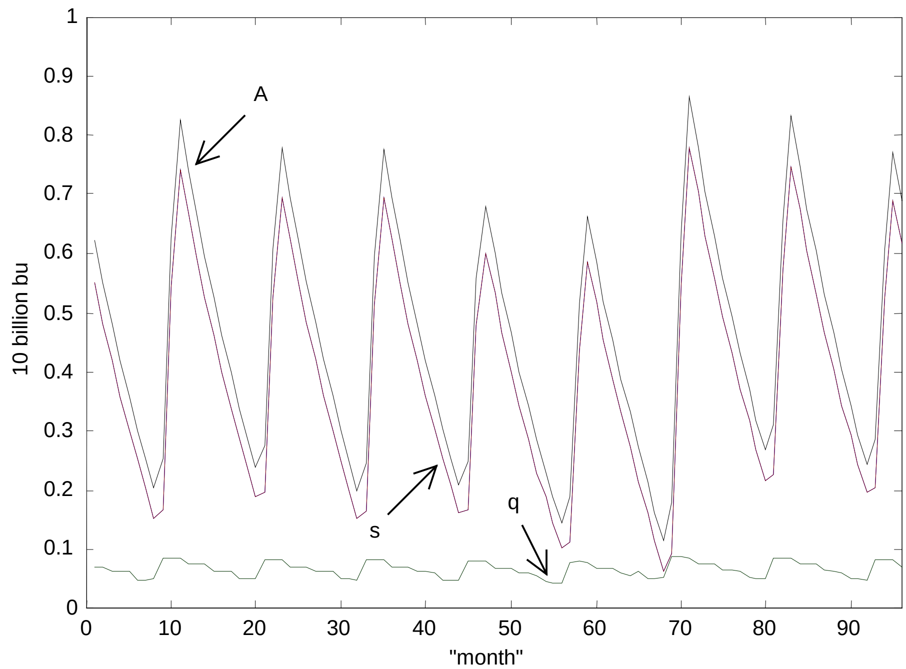


Figure 10. Simulated Storage (*s*), Consumption (*q*), and Availability (*A*) under Uncertain Crop Size (Case 2).

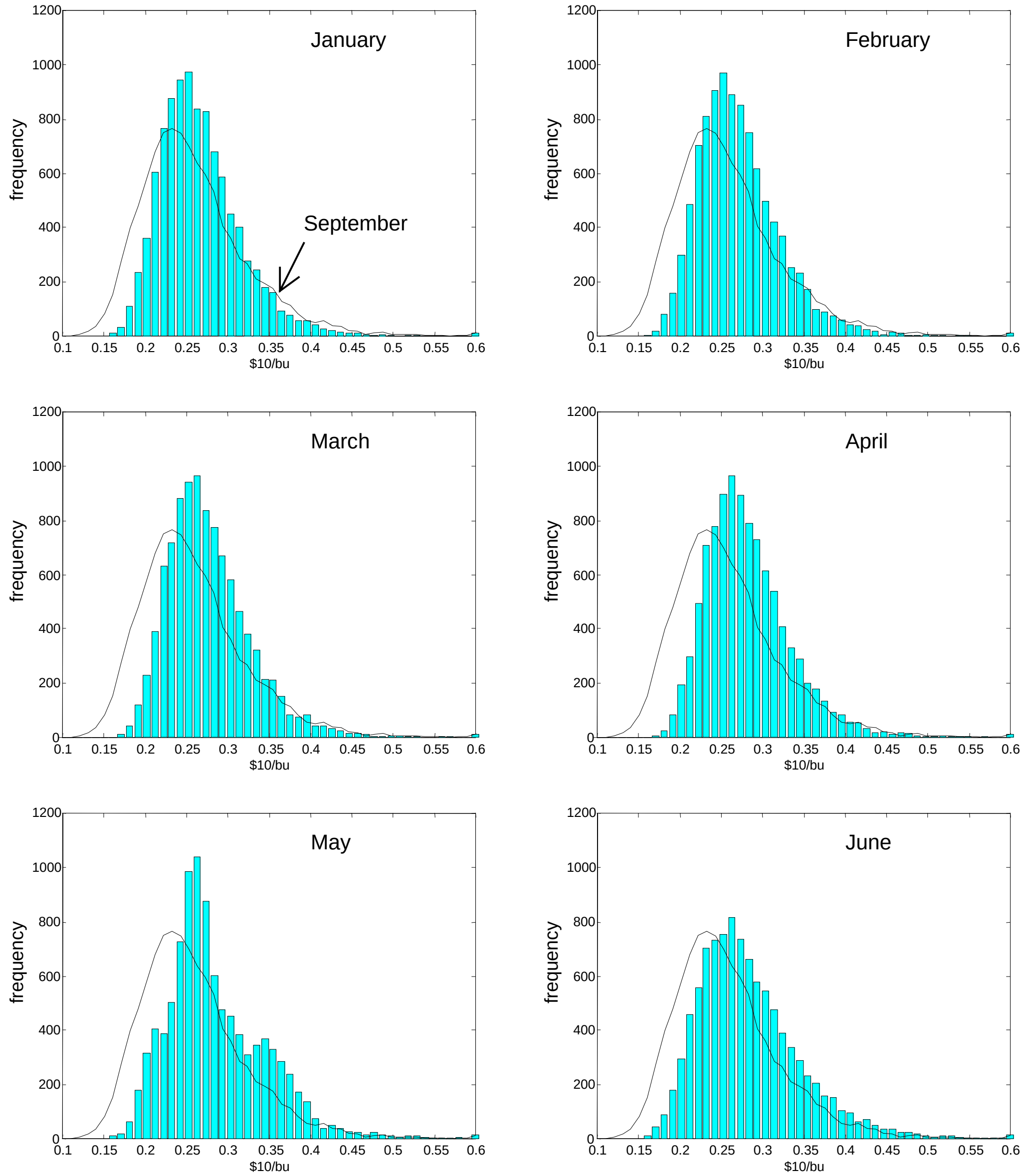
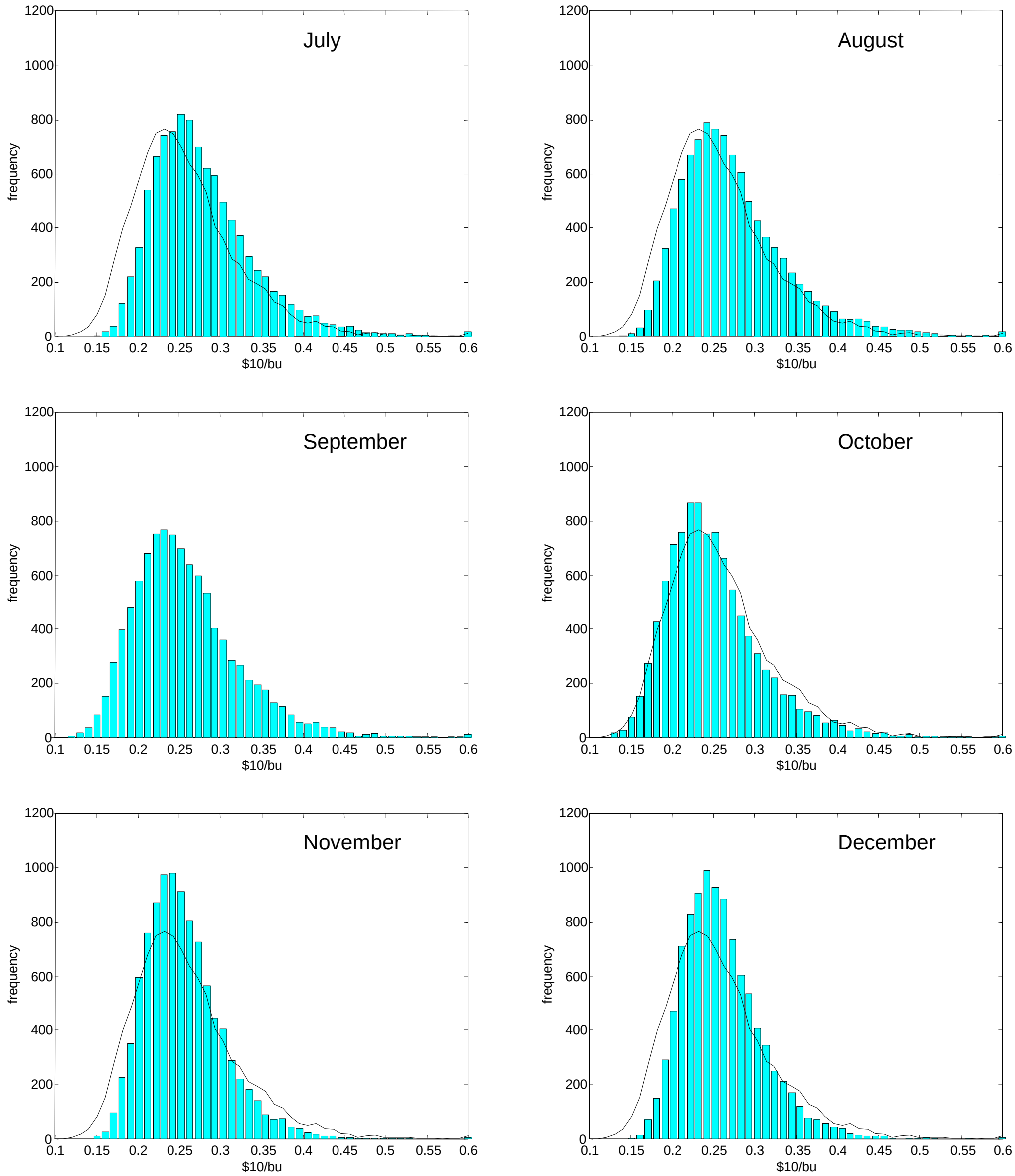


Figure 11. Distributions of Simulated Monthly Prices (Case 6).

Figure 11 (Continued)



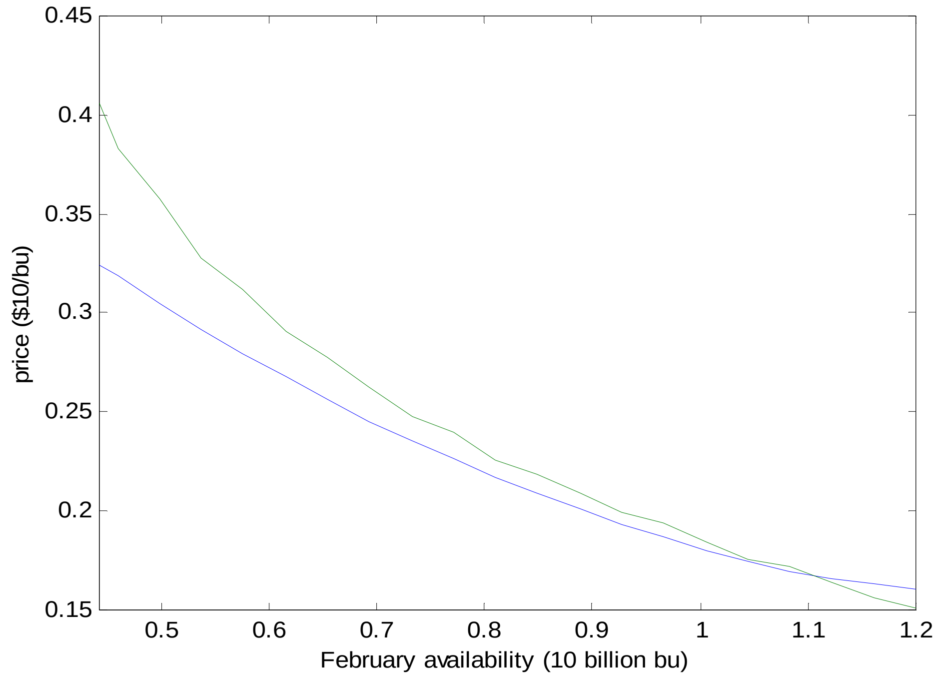


Figure 12. Means of the December and May Price Distributions Conditional on February Availability.

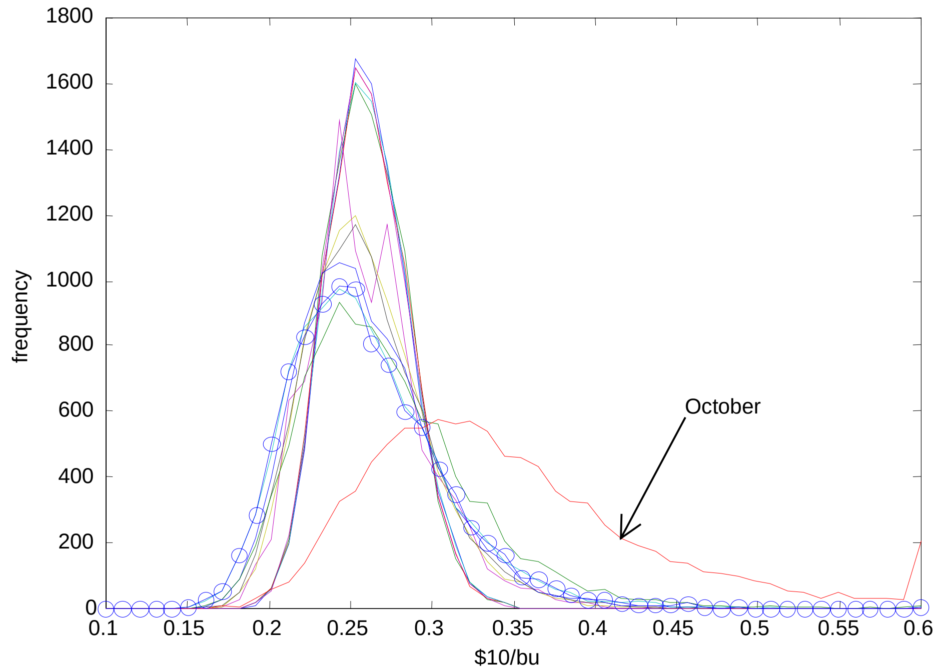


Figure 13. Monthly Distributions of December Futures Price.

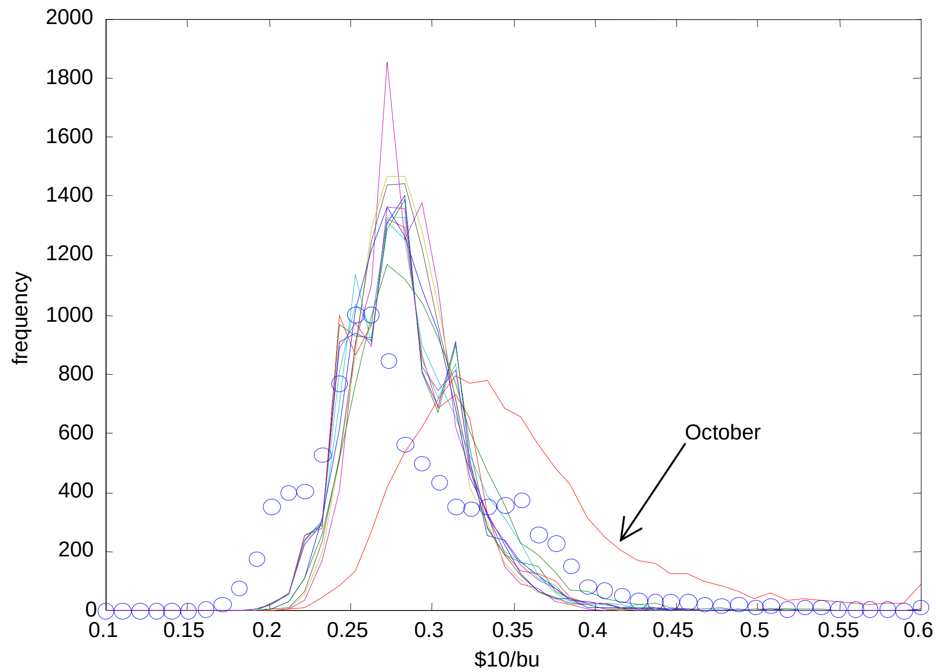


Figure 14. Monthly Distributions of May Futures Price.

Appendix 1. Numerical Model

The discretization explained in the text implies that the original functional equation problem (8a)–(8d) is replaced by a finite-dimensional problem. Corresponding to equation (8a) is a set of monthly arbitrage equations:

$$(8a)' \quad \frac{\sum_{i'=1}^{I'}\sum_{n'=1}^{N'} w_{i'} c_{n',i'}^{m'} \phi_{n'}^{m'}\left[s_{n,i}^{m}\right]}{1+r} - D_{m,i}^{-1}\left[A_n^m - s_{n,i}^m\right] - K'\left[s_{n,i}^m\right] = 0, \qquad m = 1, 2, 3, 11, 12, \forall n, i$$

$$\frac{\sum_{l'=1}^{L'}\sum_{i'=1}^{I'}\sum_{n'=1}^{N'} w_{i'} v_{l'} c_{n',i',l'}^{m'} \phi_{n'}^{m'}\left[s_{n,i}^{m}\right]}{1+r} - D_{m,i}^{-1}\left[A_n^m - s_{n,i}^m\right] - K'\left[s_{n,i}^m\right] = 0, \qquad m = 4, \forall n, i$$

$$\frac{\sum_{l'=1}^{L'}\sum_{i'=1}^{I'}\sum_{n'=1}^{N'} w_{i'} v_{l',l} c_{n',i',l'}^{m'} \phi_{n'}^{m'}\left[s_{n,i,l}^{m}\right]}{1+r} - D_{m,i}^{-1}\left[A_n^m - s_{n,i,l}^m\right] - K'\left[s_{n,i,l}^m\right] = 0, \qquad m = 5, 6, \forall n, i, l$$

$$\frac{\sum_{g'=1}^{G'}\sum_{l'=1}^{L'}\sum_{i'=1}^{I'}\sum_{n'=1}^{N'} w_{i'} v_{l',l} z_{g'} c_{n',i',l',g'}^{m'} \phi_{n'}^{m'}\left[s_{n,i,l}^{m}\right]}{1+r} - D_{m,i}^{-1}\left[A_n^m - s_{n,i,l}^m\right] - K'\left[s_{n,i,l}^m\right] = 0, \qquad m = 7, \forall n, i, l$$

$$\frac{\sum_{g'=1}^{G'}\sum_{l'=1}^{L'}\sum_{i'=1}^{I'}\sum_{n'=1}^{N'} w_{i'} v_{l',l} z_{g'} c_{n',i',l',g'}^{m'} \phi_{n'}^{m'}\left[s_{n,i,l,g}^{m} + \hat{\alpha}_g H_{l'}\right]}{1+r} - D_{m,i}^{-1}\left[A_n^m - s_{n,i,l,g}^m\right] - K'\left[s_{n,i,l,g}^m\right] \le 0,$$

$$m = 8, 9, \forall n, i, l, g$$

$$\frac{\sum_{l'=1}^{L'}\sum_{i'=1}^{I'}\sum_{n'=1}^{N'} w_{i'} v_{l',l} c_{n',i'}^{m'} \phi_{n'}^{m'}\left[s_{n,i,l,g}^{m} + \hat{\alpha}_g H_{l'}\right]}{1+r} - D_{m,i}^{-1}\left[A_n^m - s_{n,i,l,g}^m\right] - K'\left[s_{n,i,l,g}^m\right] = 0,$$

$$m = 10, \forall n, i, l, g$$

where right primes (´) denote next month values.

Like (8a), each of the statements in (10a) requires that the current price equals the discounted expected price in the following month less carrying cost. Statements differ by month because there are different state variables. In post-harvest, pre-planting months (November–March), the only unknown future states are the demand shocks. Hence, the expectations are taken with respect to demand shocks in the following month, $i' = 1, \ldots, I'$. In April, the expectation is calculated with respect to possible states in May, which depend on demand shocks and planted crop size, and so forth.

Parallel to equation (8b), equation (8b)′ requires the monthly price function approximants to equal the price on the demand function, and this relationship must hold exactly at the collocation nodes, A_j^m, $j = 1, \ldots, N$.

(8b)′ $$\sum_{n'=1}^{N'} c_{n',i,l,g}^m \phi_{n'}^m \left[A_j^m\right] = D_{m,i}^{-1}\left[A_n^m - s_{n,i,l,g}^m\right], \qquad \forall n, m, i, l, g\ .$$

Equation (8c), which is the equilibrium condition for planted crop size, can be re-stated as:

(8c)′ $$\sum_{n''=1}^{N''} c_{n'',j}^S \phi_{n''}^S \left[A_n^4\right] = S_j\left[\sum_{n'=1}^{N^P} c_{n',n}^P \phi_{n'}^P \left[H_{n,j}^4\right]\right], \qquad \forall n, j\ .$$

The left-hand side is the planted crop size as a function of April availability; the right-hand side is the supply function evaluated at the expected price at harvest, associated with the planted crop size.

Finally, the expected price and planting functions must be defined so that the function approximants are exact at all collocation nodes:

(8d)′ $$\hat{P}_{n,j} = \sum_{n'=1}^{N^P} c_{n',n}^P \phi_{n'}^P \left[H_{n,j}^4\right] \qquad \forall n, j\ ,$$

(8e)′ $$H_{n,j}^4 = \sum_{n'=1}^{N'} c_{n',j}^S \phi_{n'}^S \left[A_n^4\right], \qquad \forall n, j\ .$$

The expected prices $\hat{P}$ in equation (8d)′ are determined according to equation (8d); they are the expected harvest price, conditional on the planted crop size, based on the given transition probabilities for states and the monthly price functions from equation (8a).

Appendix 2.

Table A1. Mean Simulated Outcomes, by Annualized Interest Rate Parameter (R)[a,b]

	Price (P), \$/bu.			**Storage (s), mil. bu.**			**Consumption (q), mil. bu.**			**Crop Estimate (H), mil.bu.**			**Harvest (h), mil. bu.**		
Month	R=0.05	R=0.1[c]	R=0.15	R=0.05	R=0.1[c]	R=0.15	R=0.05	R=0.1[c]	R=0.15	R=0.05	R=0.1[c]	R=0.15	R=0.05	R=0.1[c]	R=0.15
January	2.76	2.67	2.71	6839	6428	5956	732	739	736						
	(0.47)	(0.51)	(0.56)	(934)	(865)	(798)	(69)	(70)	(70)						
February	2.80	2.72	2.78	6105	5691	5221	733	737	736						
	(0.48)	(0.51)	(0.57)	(906)	(836)	(767)	(70)	(71)	(71)						
March	2.84	2.77	2.84	5469	5052	4584	637	639	637						
	(0.48)	(0.52)	(0.58)	(883)	(811)	(739)	(68)	(68)	(68)						
April	2.88	2.83	2.91	4833	4412	3949	636	640	635	8230	8294	8261			
	(0.48)	(0.53)	(0.59)	(859)	(788)	(714)	(76)	(76)	(75)	(673)	(632)	(646)			
May	2.96	2.87	2.98	4201	3772	3314	632	639	635	8230	8294	8261			
	(0.59)	(0.62)	(0.72)	(845)	(772)	(696)	(81)	(83)	(82)	(673)	(632)	(646)			
June	2.93	2.84	2.95	3684	3251	2798	517	522	516	8232	8297	8263			
	(0.60)	(0.63)	(0.74)	(831)	(758)	(681)	(48)	(49)	(51)	(718)	(674)	(690)			
July	2.88	2.79	2.90	3169	2732	2283	515	519	515	8233	8299	8263			
	(0.58)	(0.63)	(0.74)	(819)	(744)	(667)	(50)	(53)	(54)	(767)	(723)	(738)			
August	2.84	2.75	2.84	2656	2215	1769	513	517	515	8227	8297	8269			
	(0.63)	(0.66)	(0.80)	(807)	(732)	(654)	(49)	(52)	(53)	(866)	(830)	(844)			
September	2.69	2.59	2.64	2937	2489	2039	862	873	871	8226	8298	8268	1144	1147	1141
	(0.60)	(0.64)	(0.75)	(933)	(861)	(794)	(96)	(100)	(103)	(883)	(846)	(858)	(562)	(556)	(559)
October	2.62	2.51	2.53	6620	6206	5738	861	872	870	8225	8296	8267	4544	4589	4569
	(0.55)	(0.59)	(0.67)	(1458)	(1412)	(1362)	(98)	(100)	(105)	(913)	(876)	(885)	(1229)	(1219)	(1220)
November	2.67	2.57	2.58	8303	7907	7431	863	870	869	8226	8300	8264	2547	2571	2562
	(0.47)	(0.50)	(0.54)	(990)	(926)	(863)	(92)	(92)	(93)	(958)	(922)	(926)	(1304)	(1303)	(1288)
December	2.71	2.62	2.65	7571	7168	6692	732	739	739						
	(0.47)	(0.50)	(0.55)	(962)	(895)	(831)	(67)	(69)	(70)						

[a] Numbers in parentheses are standard deviations. [b] The paramater in the model is $r = R/12$. [c] Base model, from Table 6.

Table A2. Mean Simulated Outcomes, by Storage Cost Parameter (k)[a]

	Price (P), \$/bu.			Storage (s), mil. bu.			Consumption (q), mil. bu.			Crop Estimate (H), mil.bu.			Harvest (h), mil. bu.		
Month	k=0.02	k=0.03[b]	k=0.04	k=0.02	k=0.03[b]	k=0.04	k=0.02	k=0.03[b]	k=0.04	k=0.02	k=0.03[b]	k=0.04	k=0.02	k=0.03[b]	k=0.04
January	2.64	2.67	2.71	7040	6428	5981	740	739	736						
	(0.44)	(0.51)	(0.57)	(971)	(865)	(800)	(69)	(70)	(71)						
February	2.69	2.72	2.78	6301	5691	5246	740	737	735						
	(0.44)	(0.51)	(0.58)	(946)	(836)	(767)	(70)	(71)	(73)						
March	2.73	2.77	2.84	5658	5052	4609	643	639	637						
	(0.45)	(0.52)	(0.58)	(923)	(811)	(739)	(68)	(68)	(69)						
April	2.77	2.83	2.90	5016	4412	3974	642	640	635	8314	8294	8283			
	(0.45)	(0.53)	(0.59)	(902)	(788)	(712)	(74)	(76)	(76)	(698)	(632)	(651)			
May	2.81	2.87	2.96	4374	3772	3338	642	639	636	8314	8294	8283			
	(0.54)	(0.62)	(0.72)	(886)	(772)	(695)	(82)	(83)	(83)	(698)	(632)	(651)			
June	2.78	2.84	2.93	3851	3251	2820	523	522	517	8314	8297	8282			
	(0.56)	(0.63)	(0.75)	(873)	(758)	(680)	(48)	(49)	(51)	(737)	(674)	(695)			
July	2.74	2.79	2.88	3330	2732	2304	521	519	516	8311	8299	8285			
	(0.55)	(0.63)	(0.74)	(860)	(744)	(667)	(51)	(53)	(54)	(783)	(723)	(746)			
August	2.70	2.75	2.83	2811	2215	1790	519	517	514	8318	8297	8281			
	(0.57)	(0.66)	(0.79)	(849)	(732)	(655)	(50)	(52)	(54)	(885)	(830)	(856)			
September	2.58	2.59	2.63	3081	2489	2063	872	873	873	8318	8298	8279	1142	1147	1145
	(0.55)	(0.64)	(0.76)	(959)	(861)	(799)	(96)	(100)	(103)	(897)	(846)	(875)	(557)	(556)	(560)
October	2.52	2.51	2.52	6797	6206	5775	869	872	874	8316	8296	8279	4587	4589	4586
	(0.51)	(0.59)	(0.69)	(1484)	(1412)	(1349)	(98)	(100)	(105)	(921)	(876)	(903)	(1240)	(1219)	(1227)
November	2.56	2.57	2.59	8520	7907	7455	870	870	869	8316	8300	8274	2595	2571	2550
	(0.43)	(0.50)	(0.56)	(1025)	(926)	(867)	(91)	(92)	(94)	(962)	(922)	(944)	(1301)	(1303)	(1286)
December	2.60	2.62	2.65	7781	7168	6717	739	739	738						
	(0.43)	(0.50)	(0.57)	(998)	(895)	(834)	(68)	(69)	(70)						

[a] Numbers in parentheses are standard deviations. [b] Base model, from Table 6.

Table A3. Mean Simulated Outcomes, by Demand Elasticity Parameter (κ)[a]

	Price (P), \$/bu.			**Storage (s), mil. bu.**			**Consumption (q), mil. bu.**			**Crop Estimate (H), mil.bu.**			**Harvest (h), mil. bu.**		
Month	κ=-0.15	κ=-0.25[b]	κ=-0.35	κ=-0.15	κ=-0.25[b]	κ=-0.35	κ=-0.15	κ=-0.25[b]	κ=-0.35	κ=-0.15	κ=-0.25[b]	κ=-0.35	κ=-0.15	κ=-0.25[b]	κ=-0.35
January	2.72	2.67	2.63	6722	6428	6326	740	739	739						
	(0.66)	(0.51)	(0.39)	(882)	(865)	(818)	(64)	(70)	(76)						
February	2.77	2.72	2.68	5982	5691	5588	740	737	738						
	(0.66)	(0.51)	(0.39)	(860)	(836)	(785)	(65)	(71)	(79)						
March	2.82	2.77	2.73	5341	5052	4947	641	639	641						
	(0.67)	(0.52)	(0.40)	(841)	(811)	(758)	(61)	(68)	(76)						
April	2.88	2.83	2.78	4700	4412	4308	641	640	639	8317	8294	8290			
	(0.68)	(0.53)	(0.40)	(822)	(788)	(732)	(65)	(76)	(87)	(615)	(632)	(654)			
May	2.87	2.87	2.84	4058	3772	3668	642	639	640	8317	8294	8290			
	(0.67)	(0.62)	(0.54)	(811)	(772)	(715)	(68)	(83)	(97)	(615)	(632)	(654)			
June	2.85	2.84	2.81	3535	3251	3148	523	522	520	8317	8297	8290			
	(0.70)	(0.63)	(0.54)	(802)	(758)	(699)	(48)	(49)	(53)	(663)	(674)	(693)			
July	2.81	2.79	2.76	3013	2732	2631	522	519	517	8315	8299	8291			
	(0.71)	(0.63)	(0.53)	(792)	(744)	(685)	(50)	(53)	(56)	(715)	(723)	(745)			
August	2.78	2.75	2.71	2493	2215	2115	520	517	516	8319	8297	8293			
	(0.78)	(0.66)	(0.56)	(783)	(732)	(673)	(49)	(52)	(56)	(823)	(830)	(849)			
September	2.63	2.59	2.55	2772	2489	2393	873	873	873	8317	8298	8296	1151	1147	1151
	(0.82)	(0.64)	(0.52)	(906)	(861)	(820)	(86)	(100)	(111)	(840)	(846)	(865)	(562)	(556)	(564)
October	2.56	2.51	2.46	6499	6206	6119	873	872	873	8315	8296	8297	4600	4589	4600
	(0.79)	(0.59)	(0.46)	(1423)	(1412)	(1371)	(89)	(100)	(115)	(869)	(876)	(889)	(1218)	(1219)	(1215)
November	2.61	2.57	2.52	8201	7907	7804	871	870	871	8317	8300	8298	2574	2571	2556
	(0.65)	(0.50)	(0.38)	(930)	(926)	(886)	(82)	(92)	(102)	(912)	(922)	(930)	(1286)	(1303)	(1278)
December	2.66	2.62	2.58	7462	7168	7065	739	739	739						
	(0.65)	(0.50)	(0.38)	(905)	(895)	(852)	(64)	(69)	(76)						

[a] Numbers in parentheses are standard deviations. [b] Base model, from Table 6.

Table A4. Mean Simulated Outcomes, by Convenience Yield Elasticity Parameter (ξ)[a]

	Price (*P*), $/bu.			**Storage (*s*), mil. bu.**			**Consumption (*q*), mil. bu.**			**Crop Estimate (*H*), mil.bu.**			**Harvest (*h*), mil. bu.**		
Month	ξ=-0.8	ξ=-1[b]	ξ=-1.2	ξ=-0.8	ξ=-1[b]	ξ=-1.2	ξ=-0.8	ξ=-1[b]	ξ=-1.2	ξ=-0.8	ξ=-1[b]	ξ=-1.2	ξ=-0.8	ξ=-1[b]	ξ=-1.2
January	2.66	2.67	2.67	5827	6428	6985	739	739	737						
	(0.52)	(0.51)	(0.48)	(832)	(865)	(872)	(70)	(70)	(70)						
February	2.71	2.72	2.73	5087	5691	6246	740	737	738						
	(0.53)	(0.51)	(0.49)	(802)	(836)	(843)	(71)	(71)	(71)						
March	2.77	2.77	2.78	4445	5052	5606	643	639	640						
	(0.53)	(0.52)	(0.50)	(776)	(811)	(819)	(70)	(68)	(68)						
April	2.82	2.83	2.83	3805	4412	4967	640	640	640	8301	8294	8297			
	(0.54)	(0.53)	(0.50)	(751)	(788)	(796)	(76)	(76)	(75)	(665)	(632)	(669)			
May	2.86	2.87	2.89	3165	3772	4327	640	639	640	8301	8294	8297			
	(0.64)	(0.62)	(0.61)	(735)	(772)	(781)	(83)	(83)	(82)	(665)	(632)	(669)			
June	2.84	2.84	2.85	2644	3251	3807	521	522	520	8301	8297	8295			
	(0.66)	(0.63)	(0.63)	(720)	(758)	(769)	(50)	(49)	(49)	(705)	(674)	(711)			
July	2.80	2.79	2.80	2125	2732	3289	519	519	518	8301	8299	8301			
	(0.65)	(0.63)	(0.61)	(707)	(744)	(756)	(53)	(53)	(52)	(750)	(723)	(757)			
August	2.75	2.75	2.75	1610	2215	2773	517	517	516	8303	8297	8297			
	(0.69)	(0.66)	(0.65)	(692)	(732)	(745)	(52)	(52)	(51)	(851)	(830)	(855)			
September	2.59	2.59	2.59	1878	2489	3046	873	873	872	8302	8298	8297	1141	1147	1146
	(0.68)	(0.64)	(0.62)	(821)	(861)	(872)	(101)	(100)	(99)	(866)	(846)	(871)	(556)	(556)	(561)
October	2.51	2.51	2.51	5620	6206	6761	872	872	872	8303	8296	8299	4614	4589	4587
	(0.63)	(0.59)	(0.56)	(1387)	(1412)	(1403)	(102)	(100)	(101)	(896)	(876)	(900)	(1234)	(1219)	(1230)
November	2.56	2.57	2.57	7306	7907	8460	872	870	870	8305	8300	8298	2558	2571	2571
	(0.51)	(0.50)	(0.47)	(893)	(926)	(930)	(92)	(92)	(92)	(939)	(922)	(942)	(1293)	(1303)	(1286)
December	2.61	2.62	2.62	6566	7168	7722	740	739	739						
	(0.51)	(0.50)	(0.48)	(863)	(895)	(900)	(70)	(69)	(70)						

[a] Numbers in parentheses are standard deviations. [b] Base model, from Table 6.

Table A5 Mean Simulated Outcomes, by Supply Elasticity Parameter (η)[a]

	Price (*P*), $/bu.			**Storage (*s*), mil. bu.**			**Consumption (*q*), mil. bu.**			**Crop Estimate (*H*), mil.bu.**			**Harvest (*h*), mil. bu.**		
Month	η=0.1	η=0.2[b]	η=0.3	η=0.1	η=0.2[b]	η=0.3	η=0.1	η=0.2[b]	η=0.3	η=0.1	η=0.2[b]	η=0.3	η=0.1	η=0.2[b]	η=0.3
January	2.71	2.67	2.65	6441	6428	6432	740	739	738						
	(0.66)	(0.51)	(0.42)	(1059)	(865)	(756)	(74)	(70)	(67)						
February	2.76	2.72	2.70	5704	5691	5695	737	737	737						
	(0.67)	(0.51)	(0.42)	(1021)	(836)	(732)	(76)	(71)	(69)						
March	2.81	2.77	2.75	5063	5052	5053	641	639	642						
	(0.68)	(0.52)	(0.43)	(988)	(811)	(712)	(72)	(68)	(66)						
April	2.87	2.83	2.80	4425	4412	4414	638	640	639	8296	8294	8298			
	(0.69)	(0.53)	(0.44)	(958)	(788)	(692)	(78)	(76)	(74)	(902)	(632)	(497)			
May	2.98	2.87	2.82	3788	3772	3774	638	639	640	8296	8294	8298			
	(0.95)	(0.62)	(0.45)	(933)	(772)	(680)	(87)	(83)	(79)	(902)	(632)	(497)			
June	2.94	2.84	2.79	3269	3251	3253	519	522	521	8295	8297	8298			
	(0.94)	(0.63)	(0.47)	(913)	(758)	(670)	(55)	(49)	(47)	(931)	(674)	(551)			
July	2.88	2.79	2.75	2751	2732	2733	517	519	520	8290	8299	8299			
	(0.90)	(0.63)	(0.48)	(895)	(744)	(660)	(57)	(53)	(50)	(967)	(723)	(611)			
August	2.83	2.75	2.71	2234	2215	2214	518	517	519	8293	8297	8301			
	(0.92)	(0.66)	(0.54)	(877)	(732)	(651)	(56)	(52)	(49)	(1049)	(830)	(737)			
September	2.65	2.59	2.56	2502	2489	2488	870	873	875	8294	8298	8304	1138	1147	1148
	(0.81)	(0.64)	(0.56)	(980)	(861)	(804)	(105)	(100)	(97)	(1063)	(846)	(753)	(561)	(556)	(566)
October	2.55	2.51	2.49	6213	6206	6202	870	872	871	8292	8296	8303	4581	4589	4586
	(0.72)	(0.59)	(0.53)	(1509)	(1412)	(1359)	(105)	(100)	(100)	(1088)	(876)	(786)	(1263)	(1219)	(1211)
November	2.60	2.57	2.54	7920	7907	7910	870	870	870	8290	8300	8303	2579	2571	2578
	(0.65)	(0.50)	(0.41)	(1136)	(926)	(808)	(96)	(92)	(90)	(1121)	(922)	(830)	(1319)	(1303)	(1288)
December	2.66	2.62	2.59	7181	7168	7170	739	739	740						
	(0.66)	(0.50)	(0.41)	(1097)	(895)	(782)	(73)	(69)	(68)						

[a] Numbers in parentheses are standard deviations. [b] Base model, from Table 6.