OPTIMAL EQUITY RECOVERY FOR A COOPERATIVE FINANCIAL INSTITUTION

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Introduction

Losses of $2.7 billion in 1985 and $968 million during the first six months of 1986 suffered by the nation's leading farm lender, the Farm Credit System (FCS), questions the financial stability of this cooperative institution (Freshwater; Lins; Webster). This represents a sharp contrast to the previous decade when the FCS was viewed as one of the safest segments of the financial community. Rising asset values provided lenders with more than adequate security, while borrowers profited from the depreciating real value of their debt. This combination in the midst of rising farm income expectations led to a dramatic increase in the agricultural sector's level of debt. The FCS was able to capture a larger share of this growing market largely due to its lower rates, based on average cost loan pricing during a period of rising interest rates.

The prosperity of the FCS and other agricultural lenders changed with the financial health of the sector they service. Farm income levels dropped, and with the additional indebtedness assumed in the previous period, debt servicing problems were accentuated. Farm asset values consequently dropped, providing insufficient security for the loans held against them. This process has forced lenders to increase loan loss provisions and chargeoffs, which in turn has resulted in a reduction of their institutions' net worth.

The Farm Credit Banks are funded principally through the sale of securities that are backed by the resources of all 37 FCS Banks. They are not guaranteed against default by the government and, as a result, equity erosion of the FCS has raised the risk perceived by bondholders. In October 1985, this reaction cultimated with the basis spread between FCS bonds and comparable Treasury bond issues reaching an unprecedented level of 110 points.

The problems confronting the FCS by the third quarter of 1985 led to the implementation of a new Farm Credit Act. By giving the System a more central focus, and by providing the framework for a government line of credit, investor confidence has been restored. The new legislation has narrowed the basis point spread to approximately 20 points without requiring an increase in equity.

However, the present set of circumstances resulting in the narrow spread is unlikely to continue. Since the future outlook for farm income remains pessimistic, FCS equity may continue to erode.

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Federal help will be provided when mounting bad debts strain the System's resources to the point that its viability as a lender is questioned. But any delay in this support will lead to a major credit curtailment and result in the loss of both investor and borrower confidence. Even if the process was prompt in such a situation, funds may be difficult or even impossible to obtain in view of Gramm-Rudman deficit-cutting legislation. Either way, the FCS has a large incentive to prevent further depletion of its equity. An increase in equity is necessary to keep the cost of FCS funds competitive and thus ensure the long-term stability of the System.

With this in mind, troubled FCS districts began raising interest rates to solvent borrowers last year by adding a surcharge in order to rebuild equity lost from loan losses to distressed borrowers. Since then, this surcharge has been implicitly incorporated into the prevailing FCS rates which have remained relatively constant despite a general decline in the level of other interest rates. The equity buildup, however, has been tempered by the exodus of financially sound borrowers who can obtain credit from competing institutions at a lower rate. The loss of quality borrowers by the imposition of a surcharge will leave the System with a smaller loan base and proportionately less equity capital. The loss of borrowers does not directly diminish System earned equity, but their contributed equity (class A stock) is removed. The loss of loan volume will, however, directly retard equity restoration. The System would also be left with a riskier loan portfolio and higher default rate. The impact of these events would be to eventually reverse the equity restoration process. Bondholder confidence would again be reduced and the resulting higher cost of borrowed funds, which the surcharge had been originally designed to curtail, would accentuate the above process.

The problem is to determine an optimal recovery strategy for lost equity in view of the System's dynamics. The use of a surcharge to increase equity and thus the ability of the System to sell securities in the bond market must be balanced against the competitive ability to lend money to financially sound borrowers. The purpose of this paper is to demonstrate how dynamic optimization can be utilized to solve this problem for a cooperative financial institution. Although discussion has referred to the Farm Credit System thus far, the mathematical model used to characterize the problem and demonstrate the technique is hypothetical. The procedure can be applied to any cooperative financial institution, including the FCS, with necessary model modifications and estimation of parameters.

Dynamic Optimization Model

The mathematical solution technique for the problem is dynamic optimization using nonlinear mathematical programming. Other optimization techniques are available, including calculus of variations and optimal control, but given that discrete results (quarterly or annual) would be desired rather than continuous results, the following model is formulated in discrete form (Kamien and Schwartz). The time period can be altered which, as demonstrated later, will change the results. The implication is that a borrower with longer term objectives may wish his bank to operate differently than one with short-term objectives.

$$\underset{s_i}{\text{Minimize}} \sum_{i=1}^{n} (c_i + s_i + r_i ((l_i + \Delta l_i)/E_i)^2 + k_i/(l_i + \Delta l_i)) \qquad (1)$$

subject to

$$\Delta E_i = s_i(l_i + \Delta l_i) - d_i l_i \qquad i=1, \ldots, n \qquad (2)$$

$$E_{i+1} = E_i + \Delta E_i \qquad i=1, \ldots, n \qquad (3)$$

$$\Delta l_i = w_i(b_i - c_i - s_i - r_i ((l_i + \Delta l_i)/E_i)^2 - k_i/(l_i+\Delta l_i)) - d_i l_i + y_i \qquad i=1, \ldots, n \qquad (4)$$

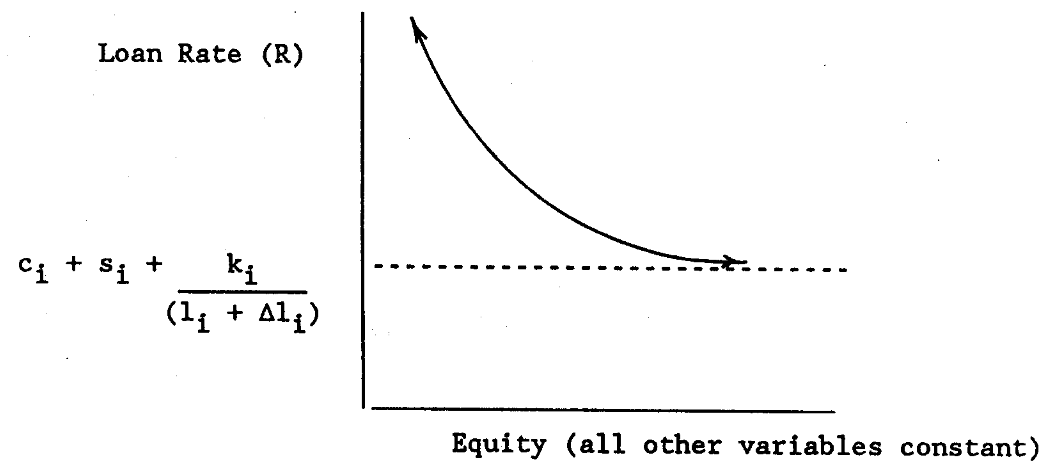
$$l_{i+1} = l_i + \Delta l_i \qquad i=1, \ldots, n \qquad (5)$$

c_i = fund cost (.08)
s_i = surcharge
r_i = risk factor (.00004)
l_i = loans
Δl_i = change in loans
E_i = equity (retained earnings)
k_i = fixed cost ($.90 B)
ΔE_i = change in equity
d_i = default rate (.015)
w_i = loan change due to rate difference ($500 B)
b_i = competitor's rate (.115)
y_i = exogenous change in loan volume ($2 B)
n = number of years

Initial conditions
l_o = $60 B
E_o = $ 3 B

The objective function in equation 1 seeks to minimize the cost of a member borrowing from the bank over time. Since the financial institution we are examining is a cooperative, this objective is consistent with its prescribed mandate. Alternative objectives could be specified, including multivalued functions. The interest rate charge on a dollar loan consists of the cost of funds c_i, a surcharge to be determined each year s_i, a risk cost of funds factor based upon capitalization, and a fixed operating cost spread over the volume of loans.[1] The risk cost capitalization factor is written as loans divided by equity (retained earnings), the ratio squared, all multiplied by r_i, which here was set at .00004 for all periods. So if the bank has $60 billion in loans and $3 billion in equity, the ratio is 20. Twenty squared and multiplied by .00004 is 160 basis points. If equity was $6 billion rather than $3 billion, the risk cost factor would only be 40 basis points, which is about normal for agency debt.[2] This relationship between equity and the rate charged is depicted by the hyperbolic curve in Figure 1. Holding all other variables constant, if equity approaches zero, the risk cost of funds would approach infinity and consequently so would the rate charged to borrowers. As equity increases, the perceived risk of bondholders will fall and the associated risk premium required by investors will decline. In this situation the institution loan rate will gradually approach a level consisting of the cost of risk-free funds, the operating cost, plus any surcharge assessed.

Figure 1. Effect of Equity on Institution Loan Rate



$$\frac{\partial R}{\partial E} = \frac{-.00008\, l^2}{E^3} < 0 \quad \forall\ l, E > 0 \qquad \text{where } l = (l_i + \Delta l_i)$$

$$\frac{\partial^2 R}{\partial E^2} = \frac{.00024\, l^2}{E^4} > 0 \quad \forall\ l, E$$

∴ curve is decreasing and strictly convex.

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[1] The projected cost of funds could be minimized subject to cost risk by a model developed previously (Tauer and Boehlje). Interest costs are not discounted to the present. Using a high discount rate may be appropriate for a farmer currently facing financial difficulty.

[2] Equity defined here is strictly retained earnings. Borrower contributed equity could be included but its impact on lowering risk cost would probably be insignificant since many view it as "soft" equity.

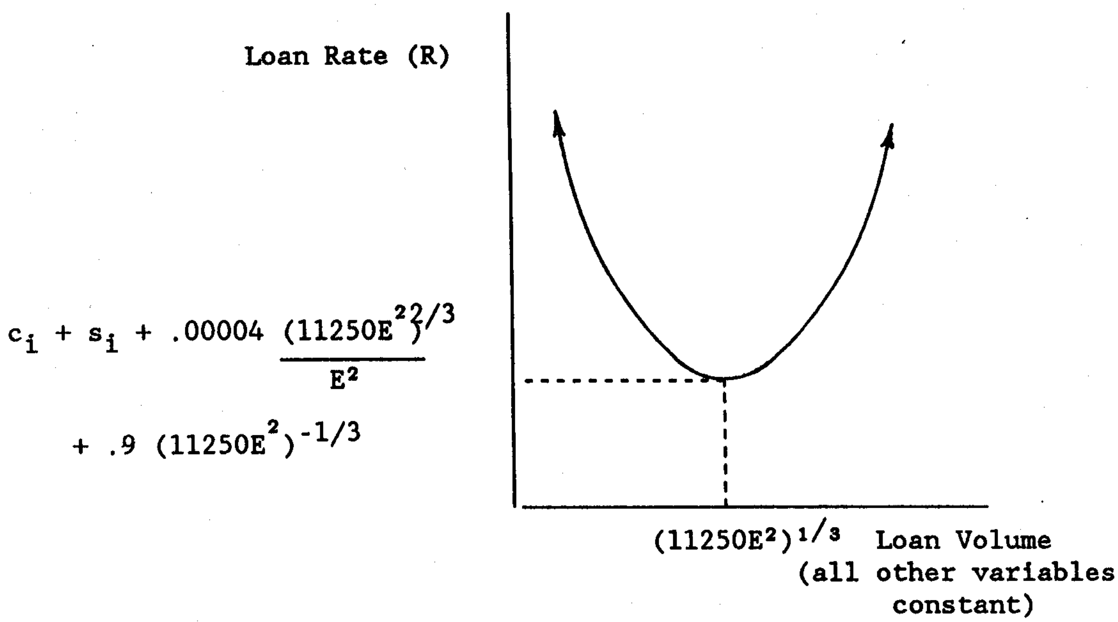
Along with equity, the other variable affecting the risk cost of funds is the loans for a period as computed by $l_i + \Delta l_i$, or loans at the beginning of a period plus the change in loans during that interval. The assumption is that any loan change will occur instantaneously at the beginning of the period after a new interest rate is announced, although a more gradual change could have been modeled. We have also assumed that loan volume has a positive effect on the risk cost factor of funds. In the initial example above where equity was $3 billion, a decline in loan volume to $45 billion from the present $60 billion would lower the risk premium 70 basis points to 90. However, it should be noted that the impact of loan volume is dependent upon equity level, or in other words, the institution's capitalization rate. If equity were to increase to $6 billion but loan volume doubled at the same time, the risk cost factor would remain at 160 points rather than the 40 points predicted if loans were constant.

Loan volume also affects the cost of operations through the following linear functional form: $o_i = k_i + a_i l_i$. Dividing by l_i produces $k_i/l_i + a_i$, but for simplicity a_i has been added to the cost of funds c_i, and k_i has been set at .90. Thus, the fixed cost per dollar of loan is 150 basis points if loan volume is $60 billion. By increasing the loan volume to $90 billion, the fixed cost would decrease to 100 basis points.

The effect of loan volume on the interest rate charged to borrowers through the preceding relationships is modeled in Figure 2. Holding all other variables constant, a strictly convex function results. An initial increase in volume from a small base will lower fixed cost per dollar loan, but it will have little impact on the risk factor due to the strong capitalization rate. However, as loan volume increases, so too will the risk premium required by investors. The initial decline in loan interest will thus reverse itself as the increase in the risk factor is no longer offset by the smaller declines in the fixed cost component.

Equity and loan volume are the variables used to describe the financial position of the cooperative institution and thus are referred to as state variables. These variables are altered through the imposition of a surcharge onto the loan rate assessed to borrowers. Since the institution can freely choose the level of surcharge, and thus its financial status, it can minimize the objective function in equation 1 through the proper selection of this control variable. If the surcharge had no impact on the state variables, borrower interest rate would be minimized by selecting a surcharge equal to zero. (We have assumed that the surcharge cannot be negative, which would imply the institution is giving a subsidy.) However, this is not the case, and the objective function is minimized subject to the constraint equations (2), (3), (4), and (5), which describe the transition of the state variables over time. For n years there are n equations of each type which are influenced by the selection of the control variable in that period.

Figure 2. Effect of Loan Volume on Institution Loan Rate



$$\frac{\partial R}{\partial l} = \frac{.00008\, l}{E^2} - \frac{.9}{l^2} \gtreqless 0 \quad \text{where } l = (l_i + \Delta l_i)$$

$$\frac{\partial^2 R}{\partial l^2} = \frac{.00008}{E^2} + \frac{.18}{l^3} > 0 \quad \forall\, l,\ E>0$$

∴ curve is strictly convex.

Equation (2) states that the change in equity during a year is equal to the surcharge times the loan volume, minus the default rate times beginning loan volume. Profit above the competitive equilibrium is obtained by assessing a surcharge on outstanding loans. The resulting increase in retained earnings is partially offset by the proportion of loans which are not recovered because of default. A linear functional form has been assumed so that the increase in equity resulting from a surcharge may be slightly overstated as the relationship does not account for the deteriorating loan portfolio resulting from the imposition of a surcharge as discussed earlier. It has also been assumed that Δl_i, either lost loans or new loans added, will not be subject to default so the default rate chosen is applicable to beginning loans. Members that receive loans from competitors are assumed to leave with good credit ratings. Similarly, the cooperative would not assume unsound new loans. A default rate of .015 was used. Again, any loan change is assumed to happen instantly and the surcharge is only collected on the loan volume during the period. All other interest rate charge components go to bond holders and for operating costs and thus are not available to rebuild equity. Equation (3) simply states that equity at the end of the period is equal to its initial level plus the change during a period.

The change in net loans outstanding for any period is given by equation (4). It is equal to the loans applied for in a period, which is a function of the institution's competitive position, minus the loans lost through default. An exogenous loan change y_i is added to reflect a general increase or decline in loan demand for all lenders. It has been set here at $2 billion. However, the key force in the equation is the spread between the competitor's rate, b_i, minus the rate charged members. As discussed earlier, the imposition of a surcharge effectively raises the interest rate assessed to the institution's borrowers. The predicted exodus of financially sound members is tempered if the institution enjoys a competitive advantage over other competing lenders. The interest rate spread is multiplied by a factor, w_i, which was set at 500 to determine the net change in loans. For example, if the competitor's rate is 200 basis points greater than the bank's rate, then $10 billion in loans will be gained. On the other hand, if the situation is reversed and the competitor's rate is say 100 points less, then $5 billion in loans will be lost. The factor w_i is used to denote the price sensitivity of the cooperative members. If loyalty to the cooperative is conditional only on low cost of borrowed funds, then the factor w_i will be larger. If w_i was raised to 700, $7 billion in loans would be lost if competitors had a 100 basis point advantage over the cooperative financial institution. Finally, equation (5) states that the loan volume at the end of the period is equal to beginning loan volume plus change in loans.

Results

The dynamic model formulated in the previous section was solved by a nonlinear programming technique (Murtagh and Saunders).[3] The optimal surcharge was determined for each time period over the horizon of n years as were the resulting values of the state variables, equity and loan volume. The solution values are summarized in Tables 1 through 5 which represent 5 different possible dynamic and competitive conditions facing the cooperative institution. Unless otherwise specified, the parameter estimates used in the derivation are those expressed in the original statements of the model.

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[3] Although all constraints are equality constraints, some variables are required to be non-negative. The objective function is convex and the constraints are concave. The constraint qualification is satisfied. Thus, the Kuhn-Tucker conditions are necessary and sufficient for a minimum.

Tables 1 and 2 results are from these original parameters estimates. The only difference between them is the length of period over which the interest rate is to be minimized. In Table 1, the minimization period was 6 years. The results show that a surcharge of .031 should be applied per dollar of loan in the first year of the 6-year interval. Equity is expanded the first year by $.589 billion as a result of the surcharge which also causes the loan volume to drop by $12.6 billion. The changes are not proportional, however, and the improved capitalization rate allows the risk cost to fall, which in turn lowers the overall interest rate. After the first period, the level of surcharge drops successively until the second to last period of the horizon, 1991, where no surcharge will be applied. Up to that year, the decrease in surcharge slows the reduction in loan level until it eventually reverses itself. Equity erodes sharply in the last period which in turn forces up the interest rate. This myopic result occurs because the member does not care what happens beyond the last period, and is content to let the financial situation of the cooperative institution deteriorate.

This pattern of results compares closely to Table 2 where the model is identical except the optimization is over 9 periods rather than 6. After an initial surcharge is levied to rebuild equity, it is gradually lowered until it becomes nonexistent in the second to last period, or in this case, the eighth year. At that point in time, loan volume starts to increase again after successive years of decline. The pattern of results for both models is similar although exact results are different since the time period is different. However, it is interesting to note that the optimal conditions are almost identical for both the 6-year and 9-year borrower during the initial two years. The surcharge for those two periods should be .031 and .018 (or .019). This demonstrates the robust results generated by this model since the critical results are from the first few periods which are consistent for both models.[4]

Table 3 shows the results if competitors charge a rate of .13 rather than .115. With this improved competitive position, the cooperative is able to impose a higher initial surcharge and keep it at a relatively high level throughout much of the period. The surcharge is used to build up equity in order to support the higher loan volume. Loan volume increases every year since the interest rate being charged members is below other lenders' rate of .13 except for the first period. The results are again myopic and total interest cost reaches a minimum in the eigth year at .0974. This rate is below the minimum level attained in the previous two models. By enjoying a comparative advantage, the cooperative is able to build up equity through the surcharge, yet remain competitive. The resulting strong financial position allows the cooperative to offer the borrower low interest rates in the last periods.

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[4] If the model is used for management purposes, it should be solved before the start of each period using current parameters to obtain optimal conditions for the next period.

Table 4 shows the results that increasing the competitiveness has on the change in loan volume and consequently the performance of the cooperative. By increasing the spread factor (w_1) to 700 from 500 we are suggesting that current and future cooperative members are even more price sensitive. The result is a smaller initial surcharge during the first year and a rapid decline in succeeding years. However, loan volume responds differently than in previous models and falls throughout the period. Total interest cost is lower during the initial years in comparison to the counterpart in Table 2, but is higher in the final years, and is never quite able to match the competitors' rate of .115. The end result is a cooperative with approximately half the market share it started with. Due to the price sensitivity of its members, the cooperative is able to rebuild its financial position only through liquidation of loans and downsizing, and not through the imposition of a surcharge.

Table 5 demonstrates the situation of a well capitalized financial cooperative. Starting with an equity position of $6 billion rather than $3 billion, the institution starts off with a total interest cost that matches the competitive rate of .115. The rate then falls throughout the period and allows loan volume to rise correspondingly. The initial surcharge rate of .016 is lower than in any other model. It is kept at approximately that level both to pay for defaults and to increase equity until the final 3 years, which are myopic.

Table 1. Optimal Values to Minimize the Cost of Borrowing $1 from 1987 through 1992

	Billions of $		Cost of $1 of Loan				
Year	Loans	Equity	Fund Cost	Risk Cost	Fixed Cost	Surcharge Cost	Total Cost
1987	$60.000	$3.000	$.08	$.0160	$.0150	$.031	$.1420
1988	47.389	3.589	.08	.0070	.0190	.018	.1240
1989	44.314	3.665	.08	.0058	.0203	.013	.1192
1990	43.688	3.558	.08	.0060	.0206	.005	.1116
1991	46.487	3.157	.08	.0087	.0194	.000	.1080
1992	51.273	2.460	.08	.0174	.0176	.000	.1149

Table 2. Optimal Values to Minimize the Cost of Borrowing $1 from 1987 through 1995

	Billions of $		Cost of $1 of Loan				
Year	Loans	Equity	Fund Cost	Risk Cost	Fixed Cost	Surcharge Cost	Total Cost
1987	$60.000	$3.000	$.08	$.0160	$.0150	$.031	$.1420
1988	47.437	3.586	.08	.0070	.0190	.019	.1250
1989	43.891	3.695	.08	.0056	.0205	.015	.1211
1990	42.254	3.662	.08	.0053	.0213	.013	.1196
1991	41.207	3.573	.08	.0053	.0218	.012	.1192
1992	40.449	3.445	.08	.0055	.0223	.010	.1178
1993	40.265	3.256	.08	.0061	.0224	.004	.1125
1994	42.750	2.838	.08	.0091	.0211	.000	.1101
1995	46.545	2.197	.08	.0180	.0193	.000	.1173

Table 3. Optimal Values to Minimize the Cost of Borrowing $1 from 1987 through 1995 if Competitor's Rate is 13%

	Billions of $		Cost of $1 of Loan				
Year	Loans	Equity	Fund Cost	Risk Cost	Fixed Cost	Surcharge Cost	Total Cost
1987	$60.000	$3.000	$.08	$.0160	$.0150	$.034	$.1450
1988	53.486	3.931	.08	.0074	.0168	.024	.1282
1989	55.719	4.449	.08	.0063	.0162	.022	.1244
1990	59.613	4.931	.08	.0058	.0151	.022	.1229
1991	64.272	5.448	.08	.0056	.0140	.022	.1216
1992	69.603	6.004	.08	.0054	.0129	.021	.1193
1993	76.070	6.532	.08	.0054	.0118	.013	.1103
1994	86.672	6.540	.08	.0070	.0104	.000	.0974
1995	103.668	5.240	.08	.0157	.0087	.000	.1043

Table 4. Optimal Values to Minimize the Cost of Borrowing \$1 from 1987 through 1995 if the Spread Factor w_i is 700

Year	Billions of \$ Loans	Billions of \$ Equity	Cost of \$1 of Loan: Fund Cost	Risk Cost	Fixed Cost	Surcharge Cost	Total Cost
1987	\$60.000	\$3.000	\$.08	\$.0160	\$.0150	\$.026	\$.1370
1988	45.573	3.293	.08	.0077	.0197	.016	.1234
1989	40.858	3.272	.08	.0062	.0220	.013	.1213
1990	38.197	3.137	.08	.0059	.0236	.011	.1205
1991	36.066	2.946	.08	.0060	.0250	.009	.1199
1992	34.101	2.710	.08	.0063	.0264	.007	.1197
1993	32.564	2.413	.08	.0073	.0276	.000	.1149
1994	33.489	1.955	.08	.0117	.0269	.000	.1186
1995	32.463	1.453	.08	.0200	.0277	.000	.1277

Table 5. Optimal Values to Minimize the Cost of Borrowing \$1 from 1987 through 1995 if Beginning Equity is \$6 Billion

Year	Billions of \$ Loans	Billions of \$ Equity	Cost of \$1 of Loan: Fund Cost	Risk Cost	Fixed Cost	Surcharge Cost	Total Cost
1987	\$60.000	\$6.000	\$.08	\$.0040	\$.0150	\$.016	\$.1150
1988	61.251	6.062	.08	.0041	.0147	.016	.1148
1989	62.364	6.152	.08	.0041	.0144	.016	.1145
1990	63.458	6.256	.08	.0041	.0142	.016	.1143
1991	64.723	6.357	.08	.0041	.0139	.016	.1141
1992	66.418	6.423	.08	.0043	.0136	.013	.1108
1993	69.411	6.342	.08	.0048	.0130	.003	.1008
1994	77.660	5.508	.08	.0080	.0116	.000	.0095
1995	86.224	4.343	.08	.0158	.0104	.000	.1062

Conclusions

This model demonstrates the usefulness of dynamic optimization in deriving optimal equity recovery strategies. Although the problems of the Farm Credit System were the basis for this research, the dynamic model was formulated for a hypothetical cooperative financial institution. The technique can be applied to the Farm Credit System or to any individual bank given that the appropriate coefficients and parameters are estimated. Once such a model is designed, it can be used on an operational basis with periodic reestimation of the coefficients.

Despite being a general model under the present parameters, some conclusions and policy implications regarding the current plight of the FCS can be deduced. It appears that the imposition of a surcharge may be an appropriate and viable method to rebuild equity for some districts. However, the current FCS practice of inflexibility in allowing districts to meet competing rates will have a severe negative impact on districts where members are more price sensitive. As general interest rates continue to fall, maintaining FCS rates at current levels will force the exodus of financially sound borrowers. This is already being reported by some districts where this competitive disadvantage exists. As shown by our results, this inflexibility could be devastating to individual district banks. Although a surcharge is still necessary, the FCS must be careful not to price itself out of the market. Therefore, the System must look at each district individually when selecting an appropriate surcharge.

A viable alternative is suggested by Table 5. If the government were to inject funds into the present system now, it could avoid a much larger outlay that appears will be required in the future. As Model 5 shows, a modest injection of $3 billion into our hypothetical financial cooperative permits it to price loans competitively and to serve agriculture by maintaining its loan volume. Under current policy, government aid will only come when the System's self-help resources have been stretched to the limit. This could be too late. By helping the System get back on its feet earlier, the government could restore the confidence of both investor and borrower. The stronger financial position, in combination with the restructuring legislated by the new Farm Credit Act and from the past lessons learned, would allow the FCS to remain a viable farm lender through a difficult upcoming period.

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