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A Preliminary Analysis

Robert A. Collins & Peter J. Barry

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Department of Agribusiness Economics
Southern Illinois University at Carbondale
Carbondale, IL 62901

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The current system of loss sharing in the Farm Credit System appears to create a classic free-rider problem. Since healthy districts are required to contribute reserves to distressed districts, the possibility exists for one district to impose external costs on others. If one district does not monitor the quality of their assets adequately, or if they are imprudent with their capital structure, the loss sharing system imposes the potential costs of these actions on the system as a whole. Since these potential costs are external to the negligent district, the incentive to maintain asset quality and adequate capitalization may be diluted. If, however, a central entity could evaluate the riskiness of each district and adjust the cost of funds to each district so the cost of funds reflected the external costs, the externality would be internalized and, the potential for a free-rider would be eliminated. This paper suggests a logical framework for creating such a system.

The system could resemble a F.D.I.C. for the institutions in the farm credit system, only rates would be scaled according to the unique risks associated with each institution. Even though a program of this type may present some very complex problems to implement in practice, the basic conceptual structure is straightforward. If the central entity could add a risk premium to each district's cost of funds that reflected the liklihood of having to make a capital infusion to each district, the individual districts would have a direct incentive to maintain their financial integrity. On the other hand, if a particular district felt the need to acquire some risky assets or draw down their reserves, they could feel free (within regulatory limits) to do so. If the system were actuarially sound, it would also be able to provide the necessary reserves when a district realizes extraordinary losses. Therefore, such a system would allow more flexibility in meeting regional financial needs and would eliminate the practice of transferring reserves from prudent districts to imprudent districts.

The remaining problem is to demonstrate how such a risk premium could be determined. The primary assumption is that each district must have at least some positive minimum level of reserves (r_m) to operate. As long as the actual reserve level (r) exceeds r_m, no

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The authors are Associate Professor, University of Arkansas and Professor, University of Illinois.

infusion of reserves is necessary. If r drops below r_m, however, an infusion of $r_I = r_m - r$ must be made to keep the institution viable. The characteristics of the probability density function (p.d.f) of the reserve level at the end of each period $[f(r_{t+1})]$ depend on the districts' asset quality, capital structure and earnings. One would expect this p.d.f. to be highly skewed (see fig.1) since there is some positive probability that all assets could be lost leaving r=-D where D denotes debt, while there is there is no probability that reserves could increase by more than what would occur if no assets were lost and all interest was paid by borrowers.

The likelihood of a required infusion of reserves may be viewed as a binominal event. Where $\bar{r}_I$ represents the expected infusion conditioned on the event that an infusion occurs, the expected reserve infusion $E(r_I)$ is:

$$E(r_I) = \bar{r}_I \{P(r<r_m)\} + 0 \{P(r \geq r_m)\}. \qquad (1)$$

The conditional expected infusion is:

$$\bar{r}_I = \int_{-D}^{r_m} (r_m - r)\, f(r)\, dr \Big/ \int_{-D}^{r_m} f(r)\, dr. \qquad (2)$$

Therefore, the denominator cancels, and the expected infusion of reserves is:

$$E(r_I) = \int_{-D}^{r_m} (r_m - r) f(r)\, dr. \qquad (3)$$

The central entity must collect a risk premium equal to the expected infusion in order to be actuarity neutral, and to internalize the externality. The total risk premium spread over the total debt of the district would create a risk premium per dollar of debt of $E(r_I)/D$. Therefore, denoting the bond rate for FCS bonds as K_F, the total cost of funds for district i (K_i) would be:

$$K_i = K_F + \{E(r_I)/D\}_i.$$

The utilization of this method requires a probability distribution for reserves. Once the distribution is determined, it may be numerically integrated to calculate the expected reserve infusion and the risk premium per dollar of debt. The distribution of next periods' equity for a particular bank depends on their capital structure, asset quality and operating surplus. Where α denotes the rate of return on assets from operations and γ denotes the proportion of assets lost through loan losses, next periods' reserves are:

$$r_{t+1} = r_t + (\alpha - \gamma) A_t. \qquad (4)$$

By redefining $(\alpha-\gamma)$ as θ, and by factoring out assets, next period's reserves may be expressed:

$$r_{t+1} = A_t \{r_t/A_t + \theta\} \quad (5)$$

This form of the relationship highlights the dependence of r_{t+1} on capital structure and asset management. Capital structure is reflected by the ratio of reserves to assets. Loan security and profitability are reflected in θ. If the asset management parameter (θ) is larger negatively than the capital structure parameter $[r_t/A_t]$ next period's equity will decline. If losses are severe enough, a reserve infusion is required. Asset management and capital structure are substitutes for avoiding reserve infusions, more reserves allow larger losses and vice versa.

Regarding θ as a random variable reflecting both the inherent business and financial risk of the region's agriculture, and the institution's ability to manage it $\tilde{r}_{t+1}$ may be expressed:

$$\tilde{r}_{t+1}=r_t+\tilde{\theta}A_t \quad (6)$$

If the p.d.f. of θ is $f(\theta)$, the p.d.f. of r_{t+1}, $g(r_{t+1})$ may be found by a transformation of random variables. Since the linear function provides a one-to-one mapping, $g(r_{t+1})$ is:

$$g(r_{t+1})=f\ \{(r_{t+1}-r_t)/A_t\}\{1/A_t\} \quad (7)$$

Therefore, once the distribution of the net rate of return on assets is determined $\{f(\theta)\}$ the expected infusion of reserves is:

$$E(r_I)=\int_{-\infty}^{r_m} f\ \{(r_{t+1}-r_t)/A_t\}\{1/A_t\}dr_{t+1} \quad (8)$$

and the appropriate cost of debt would be:

$$K=K_F+\int_{-\infty}^{r_m} f\ \{(r_{t+1}-r_t)/A_t\}\ \{1/A_t\}\ dr_{t+1}/D_t \quad (9)$$

Therefore, the calculation of a cost of funds that internalizes the external costs of risk management for each district and creates an actuarially neutral "insurance" fund is easy to compute once the form of $f(\theta)$ (the p.d.f. of the rate of return on assets) is known for each district. The crucial problem for implementing such a system in practice is estimating the p.d.f. $f(\theta)$. This deserves a considerable amount of thought and further research. It appears that there are two general strategies for estimating such a distribution; the first would be simple to implement in practice but backward looking and conceptually incorrect, while the second would be forward looking and correct, but very complex to implement into practice.

The simple method would be to simply derive an empirical p.d.f. from the historical net rate of return on assets for the past N years. This would have the implied feature of payment in arrears, i.e., if a district had heavy losses, it would cause the cost of their funds to rise for the next N years until they "paid it back". Although this may have intuitive appeal, it would impose the risk management incentive when it is too late, and creates additional operating costs when a distressed district would be least able to pay them. Therefore, although this alternative is so simple that it could literally be done by a clerk with a piece of graph paper, it would fail to accomplish the primary objectives.

The second approach would be forward looking and would require "payment in advance" for risks as they are taken. This approach would be much more desirable from an incentive point of view, but would create serious data analysis problems. First, the rate of return on assets from operating surpluses would have to be estimated. Operating costs could probably be viewed as known without loss of realism, and assets could be catagorized into three categories, current (all payments are likely to be made in a timely fashion), carried over (no payments expected this period but are likely to be made up in the future), and default (no payments expected and losses may occur on foreclosure). This could be very likely accomplished by a loan screening model with a very high degree of accuracy. A trinominal logit model would be the standard tool, and research has shown them to be quite effective at this task when they are properly specified. This approach would have the strength of using the economic attributes of the individual loans to classify them, rather than arbitrary judgements, but would have the weakness of not having a classification for loans that make partial payments. This may not present a serious problem for forecasting operating surpluses, which would be simple once loans are classified and costs are known.

The difficult problem appears to be modeling the distribution of future loan losses. One would expect the distribution of loss from the loans in default to have a gamma type p.d.f. where the parameters of the distribution are a function of the economic characteristics of the loan. A substantial research effort would be required to determine how this estimation task could be accomplished. Even if the loss distribution could be accurately estimated for individual loans, the aggregation of correlated gamma distributions into an overall p.d.f. for losses would not be an easy task.

If these problems could be overcome and an accurate method of estimating the p.d.f. $f(\theta)$ could be demonstrated, it appears that a workable, forward looking system could exist. If these problems could not be resolved, the trade offs of the backward looking procedure would have to be evaluated to determine if it would produce a workable and effective system. In either case, more research is clearly needed.

FIGURE ONE