Liquidity Management Under Uncertainty:
Theoretical Foundations and Empirical Tests

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I. Introduction

It has been widely observed that Decision Makers (DMs) tend to hold liquidity reserves in the form of cash and unused credit to counter risk in their economic environment.[1] Several empirical studies[2] have also shown that farm DMs appear to place positive liquidity value on their reserves of cash and credit, with the level of these values depending on their attitudes towards risk. Finally, farm decision models[3] incorporating the concepts of management of liquidity in the form of cash and credit reserves and based on somewhat ad hoc although plausible assumptions about the shapes of liquidity value curves have produced encouraging results when used to study farm decision making under uncertainty.

This paper is essentially an attempt to formalize these concepts in a rigorous theoretical framework. In Section II, we develop a theoretical model, in an expected utility maximization framework, that explains the demand for cash and credit reserves as primarily a risk response and also provides insights into the shape and structure of the liquidity value curves. In section III, this theoretical model is extended to rigorously formulate a "Liquidity Management Hypothesis" according to which a risk-averse DM, in order to maximize his expected utility, acts to allocate his resources in such a way that at the optimum he holds a least-cost combination of his liquidity reserves in the form of cash and unused credit. In Section IV, farm level data on Illinois cash grain farmers are used to test various hypotheses generated by the theoretical analysis about the shapes and behavior of their liquidity value curves for credit reserves. In the final section, we summarize our theoretical and empirical findings and offer some comments about their relevance to the current financial stress among the U.S. farmers.

II. Liquidity Value of Cash and Credit Reserves

We use the analytical tools of expected utility theory and mean-variance approach to develop a theoretical model that explains the maintenance of liquidity reserves in the form of cash and unused credit by a Decision Maker (DM) faced with uncertainty in his economic environment as primarily a risk response and thus provides a rigorous analytical foundation for the hypothesis of

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liquidity management. We first develop the model with cash reserves as the main source of liquidity and later extend it to incorporate the DM's credit reserves as his primary source of liquidity to counter risk. The theoretical results are then critically analyzed in terms of their major implications for the liquidity management hypothesis.

The Model with Cash Reserves

The conceptual framework underlying the model is essentially the portfolio decision problem faced by a risk-averse DM who wishes to so allocate his wealth between a set of available risky assets and a riskless asset or cash as to maximize his expected utility of the terminal portfolio value. As part of his portfolio decisions, the DM must also consider his random cash needs that must be met. The solution of the model provides explicit expression for the 'reservation price' or the liquidity value of the cash reserves. These values are shown to depend on the DM's level of risk-aversion, liquidity structure of his assets, and the pattern of his cash needs. Thus, the model clearly brings out the role and significance of liquidity reserves as a risk response.

The various components of the model and the notations used to represent them are as follows:

W_i = i^{th} DM's initial wealth or equity.

B_i = The market value of the i^{th} DM's holding of the riskless liquid asset or part of the DM's equity reserved as cash.

A_i = (n x 1) vector of the i^{th} DM's investment in the n risky assets;

$A'_i = (A_{i1}, A_{i2}, \ldots, A_{in})$ where A_{ij} is the market value of the i^{th} DM's holding of the j^{th} risky asset.

R_f = 1 + riskless rate of return.

$E(\tilde{R}_j)$= 1 + expected rate of return on the j^{th} asset.

μ = (n x 1) vector of the expected return on risky assets;

$\mu' = [E(\tilde{R}_1), E(\tilde{R}_2), \ldots, E(\tilde{R}_n)]$.

Σ = $[\sigma_{jk}]$, (n x n) covariance matrix of the returns on risky assets; $\sigma_{jk} = \text{cov}(\tilde{R}_j, \tilde{R}_k)$.

$\tilde{j}_i$ = i^{th} DM's random cash needs;

$f(\tilde{j}_i)$ - the probability density function, and

$F(\tilde{j}_i)$ - the distribution function of $\tilde{j}_i$.

ρ = Proportional liquidation cost of risky assets, a constant averaged over all risky assets.

$E(\tilde{\phi}_i), V(\tilde{\phi}_i)$ = Expected value and variance, respectively, of the penalty cost function.

Σ_i^{ϕ} = (n x 1) vector of covariances between risky asset returns and i^{th} DM's penalty cost function.;

$\Sigma_i^{\phi'} = [cov(\hat{R}_1, \tilde{\phi}_i), \ldots, cov\ (\hat{R}_n, \tilde{\phi}_i)]$.

I = (nx1) column vector of ones

O = (nx1) column vector of zeroes.

In order to keep the analysis mathematically tractable and especially to derive clear testable propositions, we make the following simplifying assumptions which incidentally are also commonly made in standard portfolio theory:

1. All DMs are risk-averse and single-period expected utility maximizers in a mean-variance framework.
2. Markets are assumed to be perfect to the extent that all DMs are price takers, pay no taxes, and have costless access to information. In addition, all assets are perfectly divisible. However, transaction costs are non-zero.
3. No liquidation cost is incurred as long as the riskless liquid asset or cash is used to meet cash demands. However, risky assets can be used only at a penalty incurred as a result of non-zero liquidation cost.
4. DMs are assumed to have homogeneous expectations regarding the probability distributions of risky asset returns and cash demands and these distributions are multivariate normal.

In making portfolio decisions at the beginning of the period, each DM must take into account his stochastic cash demands that must be met at the end of the planning period. Shortage of cash reserves to meet cash demands entails penalty cost arising from the necessity of having to liquidate risky assets at a positive but constant liquidation cost. This can be formally incorporated into the model with the help of a technique widely used in inventory theory, namely, by defining a penalty cost function as follows:

$$\phi_i = \begin{cases} \rho(\tilde{j}_i - B_i R_f) & , \tilde{j}_i > B_i R_f \\ 0 & , \tilde{j}_i \leq B_i R_f \end{cases} \qquad (2.1)$$

We can then express the i^{th} DM's end-of-the-period portfolio value net of penalty costs as,

$$\tilde{W}_i = \sum_{j=1}^{n} A_{ij}\hat{R}_j + B_i R_f - \tilde{\phi}_i \qquad (2.2)$$

The expected value and the variance of the net ending-portfolio value are then given by,

$$e_i = E(\tilde{W}_i) = A_i'\mu + B_i R_f - E(\tilde{\phi}_i) \qquad (2.3)$$

and, $$v_i = var(\tilde{W}_i) = A'_i \Sigma A_i + V(\tilde{\phi}_i) - 2 A_i' \Sigma_i^{\phi} \qquad (2.4)$$

The i^{th} DM's portfolio decision problem can now be written as,

$$\text{Max } U^i(e_i, v_i)$$
$$\text{s.t, } A'_i I + B_i = W_i \qquad (2.5)$$

The solution of this portfolio decision problem leads to the following result,

$$\frac{\partial e_i}{\partial W_i} = R_f + [-\frac{\partial E(\tilde{\phi}_i)}{\partial B_i} - \eta_i \frac{\partial V(\tilde{\phi}_i)}{\partial B_i} + 2 \eta_i A'_i \frac{\partial \Sigma \phi_i}{\partial B_i}]$$

$$\text{where } \eta_i = \frac{de_i}{dv_i} \qquad (2.6)$$

Now equation (2.6) can be interpreted as saying that the DM will choose the optimal level of the riskless liquid asset or cash reserve by equating at the margin its return to the expected return from his wealth. The return on cash reserve comprises two distinct parts: (1) an explicit return equal to the riskless interest rate and (2) an implicit return arising from the liquidity services performed by the cash reserve in offsetting the adverse impact of the unforeseen cash demands on the portfolio return. Accordingly, the expression within brackets in (2.6) gives the reservation price or liquidity value of cash reserve. As is shown later, each term in this bracketed expression is positive for a risk-averse DM which indicates that a risk-averse DM would place positive liquidity value on his cash reserves.

The Model with Credit Reserves

We now assume that the credit reserve is the primary source of liquidity available to the DM. We further assume that depending on his initial wealth or equity (W_i) and other characteristics that might affect his lender's evaluation, the DM has or expects to have a certain maximum credit limit Z_i up to which he can borrow to raise debt capital (D_i). He uses part of his credit to obtain loans and uses them along with his equity to invest in n risky enterprises or assets; moreover, he reserves the remaining part of the scarce credit, i.e. maintains credit reserves (C_i), to meet his random cash needs $\tilde{J}_i$ which, if they exceed the reserved credit, will have to be met by liquidating the risky assets at a constant liquidation cost of ρ. Following the procedure used earlier for cash reserve, we get the corresponding optimality condition for credit reserve,

$$\frac{\partial e_i}{\partial Z_i} = i_d + [- \frac{\partial E(\tilde{\phi}_i)}{\partial C_i} - \eta_i \frac{\partial V(\tilde{\phi}_i)}{\partial C_i} + 2 \eta_i A'_i \frac{\partial \Sigma \phi_i}{\partial C_i}] \quad (2.7)$$

Equation (2.7) states that the DM chooses an optimal level of credit reserve by equating at the margin its return to the marginal expected return from his credit. The right-hand side of the equation thus gives the total return that accrues to the DM when he increases his investment in credit reserve by one dollar by reducing his debt. The first term i_d represents explicit return in terms of interest rate while the second term represents implicit return in terms of liquidity services provided by the credit reserve. Accordingly, the bracketed expression gives the reservation price or liquidity value of credit reserve.

Interpretations

In this section, we closely examine the theoretical results obtained in the preceding two sections and further explore their implications for the liquidity management as a risk response. We start by reconsidering the first-order optimality conditions for cash and credit reserves i.e., equations (2.6) and (2.7), respectively,

$$\frac{\partial e_i}{\partial W_i} = R_f + i_{c\ell} \quad (2.8)$$

where $i_{c\ell}$, the liquidity value of cash reserve, is given by,

$$i_{c\ell} = - \frac{\partial E(\tilde{\phi}_i)}{\partial B_i} - \eta_i \frac{\partial V(\hat{\phi}_i)}{\partial B_i} + 2\eta_i \sum_{k=1}^{n} A_{ik} \frac{\partial \, cov(\tilde{R}_k, \tilde{\phi}_i)}{\partial B_i} \quad (2.9)$$

and,

$$\frac{\partial e_i}{\partial Z_i} = i_d + i_{d\ell} \quad (2.10)$$

where $i_{d\ell}$, the liquidity value of credit reserves, is given by,

$$i_{d\ell} = - \frac{\partial E(\tilde{\phi}_i)}{\partial C_i} - \eta_i \frac{\partial V(\tilde{\phi}_i)}{\partial C_i} + 2\eta_i \sum_{k=1}^{n} A_{ik} \frac{\partial \, cov(\tilde{R}_k, \tilde{\phi}_i)}{\partial C_i} \quad (2.11)$$

Note from (2.9) and (2.11) that the liquidity value curves for both cash and credit reserves have identical structure; we will therefore examine the credit reserve liquidity curve given by (2.11) in detail and point out the similarity of results for cash reserves. Now, assuming multivariate normal distributions for risky returns and cash needs, the following results can be easily established:

$$\left.\begin{aligned}
&\frac{\partial E(\tilde{\phi}_i)}{\partial C_i} = -\rho r[1 - F(rC_i)] \qquad < 0\\
&\frac{\partial V(\tilde{\phi}_i)}{\partial C_i} = -2\rho r\, E(\tilde{\phi}_i)\, F(rC_i) \qquad < 0\\
&\frac{\partial\, cov(\tilde{R}_k, \tilde{\phi}_i)}{\partial C_i} = -\rho r\, cov(\tilde{R}_k, \tilde{j}_i)\, f(rC_i) \quad \lesseqgtr 0,\\
&\qquad\qquad \text{if } cov(\tilde{R}_k, \tilde{j}_i) \gtreqless 0.
\end{aligned}\right| \qquad (2.12)$$

Also note that the factor η_i in (2.11) represents the DM's level of risk-aversion; for example, for a DM with a negative exponential utility function i.e., $u(x) = -e^{-\lambda_i x}$,

$$\eta_i = \frac{\lambda_i}{2}, \qquad (2.13)$$

where λ_i is the Arrow-Pratt measure of absolute risk-aversion.

We begin analyzing the structure of the liquidity value curve by first trying to interpret the various terms in equation (2.11). Using the results in (2.12), we can interpret each term as follows:

$\frac{\partial E(\tilde{\phi}_i)}{\partial C_i}$ - represents the liquidity service provided by the credit reserve in the form of marginal reduction in the DM's expected penalty cost.

$\frac{\partial V(\tilde{\phi}_i)}{\partial C_i}$ - represents the liquidity service provided by the credit reserve in the form of marginal reduction in the variance of the DM's expected penalty cost.

$\dfrac{\partial\ cov(\hat{R}_k, \hat{\phi})}{\partial C_i}$ - represents the liquidity service provided by the credit reserve in the form of marginal reduction in the liquidity risk of the DM's portfolio.

These interpretations clearly indicate that for a risk-averse DM, the value of credit reserve arises from the various liquidity services it provides in countering the adverse effect of random cash needs.

In order to identify the various determinants of the liquidity value of credit reserve, we substitute results from (2.12) and (2.13) into (2.11) to obtain,

$$i_{d\ell} = \rho r[1-F(rC_i)] + \lambda_i \rho r\ E(\hat{\phi}_i)\ F(rC_i) - \lambda_i \rho r\ f(rC_i)\ cov\ (\hat{R}_{pi}, \hat{j}_i) \qquad (2.14)$$

Note that we have written here,

$$cov(\hat{R}_{pi}, \hat{j}_i) = \sum_{k=1}^{n} A_{ik}\ cov(\hat{R}_k, \hat{j}_i) = cov(\Sigma A_{ik}\hat{R}_k, \hat{j}_i)$$

where $\hat{R}_{pi}$ is the i^{th} DM's risky portfolio return and $cov(\hat{R}_{pi}, \hat{j}_i)$ represents the liquidity risk of his portfolio. Also, since a risky portfolio is normally liquidity-averse, we have $cov(\hat{R}_{pi}, \hat{j}_i) < 0$.

Equation (2.14) suggests that a DM's liquidity value for credit reserves will mainly depend upon the level of his risk-aversion represented by λ_i, the level of his credit reserves C_i, the pattern of his random cash demands represented by $F(\cdot)$, the liquidity structure of the risky earning assets as represented by the average liquidation cost ρ, and the portfolio liquidity-risk $cov(\hat{R}_{pi}, \hat{j}_i)$. The following characteristics of a liquidity value curve for credit reserve are readily seen to be implied by (2.14):

1. We observe that if $\lambda_i > 0$, each term in (2.14) is positive; this suggests that a risk-averse DM will place a positive liquidity value on his credit reserves. It should also be noted that if we set $\lambda_i = 0$, the first term survives; accordingly, credit reserve will appear to have a positive liquidity value even to a risk-neutral DM.

2. The liquidity value curve is non-linear and the form and extent of non-linearity are jointly determined by the shape of the DM's underlying utility function as embodied in the function λ_i and by the pattern of his cash demands as embodied in the functions $F(\cdot)$ and $f(\cdot)$. Moreover, this curve will for the most part be a decreasing function of credit reserves.

3. It is obvious from (2.14) that the height and slope of the liquidity value curve will depend on the DM's level of risk-aversion, the curve being higher and presumably steeper for the more risk-averse DM (i.e., one with a larger λ_i).

4. We further note from (2.14) that if $\rho = 0$, then $i_{d\ell} = 0$ which suggests the intuitively plausible result that if a DM's risky assets are as liquid as cash or credit reserves, he will have no reason to put any value on the latter as sources of liquidity. Another interesting implication is that other things being equal, a DM whose risky assets are more liquid (i.e., with a lower liquidation cost ρ) will tend to place lower liquidity value on credit reserves.

5. We also note that for a DM whose production organization is such that the income streams ($\hat{R}_{pi}$) are more closely matched with the expenditure streams ($\hat{j}_i$) i.e., $\mathrm{cov}(\hat{R}_{pi}, \hat{j}_i)$ is less negative or positive, credit reserves will have a smaller liquidity value.

6. Finally, ρ in (2.14) can be interpreted as the excess liquidation cost of risky assets over that of cash or credit reserves. Now, it is reasonable to assume that liquidation cost of cash reserve will be lower than that of credit reserve; accordingly, (2.14) suggests that for a risk-averse DM, the liquidity value curve for cash reserve will be higher and presumable steeper than that for credit reserve.

III. Liquidity Management Hypothesis

In this section, we propose a hypothesis of Liquidity Management and use the analytical framework developed in the previous section to provide a rigorous theoretical foundation for this hypothesis. The proposed hypothesis has the following elements:

1. Reserves of cash and credit are usually the most flexible, least costly, and least disruptive sources of liquidity available to a DM to counter unanticipated variations in his cash demands;
2. A risk-averse, expected utility maximizing DM allocates his resources in such a way that at the optimum he holds a least-cost combination of his liquidity reserves in the form of cash and unused credit. Thus, cash and credit reserves are important choice variables for a risk-averse DM in his optimal allocation of resources; and,

3. Management of liquidity in the form of cash and credit reserves is often the dominant response of a risk-averse DM to counter uncertainty in his economic environment; however, to the extent that other strategies to manage risk are available and used, the liquidity response will tend to be less pronounced.

A rigorous theoretical formulation of the Liquidity Management Hypothesis requires that we establish that (1) a risk-averse DM will hold, at the optimum, a least-cost combination of cash and credit reserves, and (2) the efficient frontier containing optimal combinations of cash and credit reserves will be concave and the DM's indifference curves will be convex in the cash-credit reserve space.

We consider the portfolio decision problem of a risk-averse DM who uses part of his credit ($\overline{Z}$) to obtain loans (D) and allocates them along with his equity ($\overline{E}$) between a risky asset (A) with random returns $\tilde{r}$ and a risk-free asset or cash (C) with a non-random rate of return i_c. The remaining part of credit or credit reserve (L) serves along with cash as a primary source of liquidity to cope with unanticipated variations in cash demands. The costs of illiquidity i.e., the possibility that cash and credit reserves might prove to be insufficient to cover cash demands, can be incorporated into the model by defining a penalty cost function as,

$$\tilde{\phi} = \begin{cases} \rho[\tilde{j} - (I_c C + IL)] & , \tilde{j} > (I_c C + IL) \\ 0 & , \tilde{j} \leq (I_c C + IL) \end{cases}$$

$$\text{where} \quad I_c = 1 + i_c ; \; I = 1 + i_d \tag{3.1}$$

The return on the DM's ending portfolio net of penalty costs is given by,

$$\tilde{\pi} = \tilde{r}\,A + i_c\,C - i_d D - \tilde{\phi}$$

Substituting $D = \overline{Z} - L$ and rearranging, we get,

$$\tilde{\pi} = \tilde{r}\,A + i_c C + i_d L - i_d \overline{Z} - \tilde{\phi} \tag{3.2}$$

The mean and variance are,

$$e = E(\tilde{\pi}) = \overline{r}A + i_c C + i_d L - i_d \overline{Z} - \overline{\phi} \tag{3.3}$$

$$v = \text{var}(\tilde{\pi}) = \sigma_r^{\,2} A^2 + \sigma_\phi^{\,2} - 2A\sigma_{r\phi} \tag{3.4}$$

The DM's portfolio choice problem can now be written as,

$$\text{Max } G(e,v) = e - \frac{\lambda}{2} v$$

$$\text{s.t., } A + C + L = \bar{E} + \bar{Z} \equiv \bar{K} \qquad (3.5)$$

Note that for convenience we are assuming a negative exponential utility function for the DM so that given normal distributions for returns, his expected utility can be written in the above form. Also, λ here represents the Arrow-Pratt coefficient of absolute risk aversion and $\bar{K}$ is the total capital that is available to the DM for investments.

From (3.5) we can write the Lagrangian function as,

$$\Psi = (e - \frac{\lambda}{2} v) + \Theta(\bar{K} - A - C - L) \qquad (3.6)$$

The first-order conditions for optimality are,

$$\Psi_A = \frac{\partial \Psi}{\partial A} = \bar{r} - \lambda[\sigma_r^2 A - \sigma_{r\phi}] - \Theta = 0 \qquad (3.7)$$

$$\Psi_C = \frac{\partial \Psi}{\partial C} = i_c + [- \frac{\partial \bar{\phi}}{\partial C} - \frac{\lambda}{2} \frac{\partial \sigma_\phi^2}{\partial C} + \lambda A \frac{\partial \sigma_{r\phi}}{\partial C}] - \Theta = 0 \qquad (3.8)$$

$$\Psi_L = \frac{\partial \Psi}{\partial L} = i_d + [- \frac{\partial \bar{\phi}}{\partial L} - \frac{\lambda}{2} \frac{\partial \sigma_\phi^2}{\partial L} + \lambda A \frac{\partial \sigma_{r\phi}}{\partial L}] - \Theta = 0 \qquad (3.9)$$

$$\Psi_\Theta = \frac{\partial \Psi}{\partial \Theta} = \bar{K} - A - C - L = 0 \qquad (3.10)$$

From (3.8) and (3.9), we obtain,

$$i_c + i_{c\ell} = i_d + i_{d\ell} \qquad (3.11)$$

where, $i_{c\ell}$ and $i_{d\ell}$ are the liquidity values of cash and credit reserves, respectively. The implication of (3.11) is that a risk-averse DM will maintain cash and credit reserves at levels where their total marginal cost--both explicit and implicit--will be the same. Furthermore, since rate on borrowing (i_d) is usually greater than rate on lending (i_c), (3.11) implies that the liquidity value of cash reserves ($i_{c\ell}$) is greater than that of credit reserve ($i_{d\ell}$). In other words, the liquidity value curve for cash reserve will in general be higher than the liquidity value curve for credit reserve. It may also be noted that if the markets are perfect, the liquidity values of cash and credit reserves will be equal since the rates on lending and borrowing are the same in perfect markets. This has the important implication that the more efficient the financial markets are, the better substitute credit reserve will be for cash reserve as a source of liquidity.

Now, from the first-order conditions (3.8) and (3.9), we have the result,

$$i_c + i_{c\ell} = i_d + i_{d\ell} \equiv \Theta$$

which when substituted into (3.7) gives,

$$E(\tilde{r}) = (i_c + i_{c_\ell}) + \lambda[\sigma_r{}^2 A - \sigma_{r\phi}] \qquad (3.12)$$

and,

$$E(\tilde{r}) = (i_d + i_{d\ell}) + \lambda[\sigma_r{}^2 A - \sigma_{r\phi}] \qquad (3.13)$$

In order to further explore the conditions for an optimal mix of cash and credit reserves and the resulting trade-offs, let us consider the following first-order optimality conditions,

$$E(\tilde{r}) = (i_d + i_{d\ell}) + \lambda[\sigma_r{}^2 A - \sigma_{r\phi}] \qquad (3.13)$$

$$A + C + L = \overline{K} \qquad (3.10)$$

Equation (3.13) can be solved for A and substituted into the budget constraint (3.10) to get an equation involving cash C and credit reserves L. We assume that this equation can be solved to give C in terms of L i.e.,

$$C = f(L) \qquad (3.14)$$

Now, the relationship in (3.14) clearly represents optimal combinations of cash reserves C and credit reserves L characterized by the equality of their total marginal cost i.e.,

$$i_c + i_{c\ell} = i_d + i_{d\ell} \qquad (3.15)$$

The function in (3.14) can be thought of as defining an efficient frontier containing all the optimal combinations of C and L. It can be shown that this frontier is concave in the cash-credit reserve space.[4] It can also be shown that the indifference curves for a risk-averse DM will be convex in this space.[4] These results imply that a risk-averse DM will choose his optimal mix of cash and credit reserves at the point where his indifference curve is tangent to the efficient frontier and that this combination will be uniquely determined by his risk preferences and the overall riskiness of his portfolio.

In order to examine the cash-credit reserve tradeoffs, given by the tangent line to the efficient frontier, we differentiate the budget constraint w.r.t L and rearrange to get,

$$\frac{\partial A}{\partial L} = -\frac{\partial C}{\partial L} - 1 = -(1 + \frac{\partial C}{\partial L}) \tag{3.16}$$

Again, differentiating the first-order condition (3.13) w.r.t. L we get,

$$-\lambda \sigma_r^2 (\frac{\partial A}{\partial L}) + \lambda \frac{\partial \sigma_{r\phi}}{\partial L} = \frac{\partial i_{d\ell}}{\partial L}$$

substituting for $\frac{\partial A}{\partial L}$ from (3.16) and rearranging, this gives,

$$\frac{\partial C}{\partial L} = -1 + \frac{1}{\lambda \sigma_r^2} (\frac{\partial i_{d\ell}}{\partial L}) - \frac{1}{\sigma_r^2} \frac{\partial \sigma_{r\phi}}{\partial L} \tag{3.17}$$

We have proved elsewhere[4] that $\frac{\partial \sigma_{r\phi}}{\partial L} \geq 0$; thus, if the liquidity value of credit reserve is a declining function i.e., $\frac{\partial i_{d\ell}}{\partial L} \leq 0$, all the three terms on the right hand side of (3.17) are negative implying that $\frac{\partial C}{\partial L} < 0$ i.e.,there is generally a negative trade-off between cash and credit reserves, as one would expect. Now, notice that the second term in (3.17) is divided by λ, usually a very small quantity; as a result, the third term, usually a small quantity by itself, is likely to be relatively very small as compared with the first and second terms and can thus be ignored. We can therefore write from (3.17)

$$\frac{\partial C}{\partial L} = -1 + \frac{1}{\lambda \sigma_r^2} (\frac{\partial i_{d\ell}}{\partial L}) \tag{3.18}$$

This relationship clearly suggests that the cash-credit reserve trade-off depends on the DM's level of risk-aversion, the variance or riskiness of his portfolio, and the slope of his liquidity value curve for credit reserves. Now, equation (3.18) gives a relationship for cash reserves in terms of credit reserves; however, since credit reserve is not an easily observable quantity, the usefulness of this relation for empirical work is severely limited. We can, however, reformulate (3.18) in terms of outstanding debt level D as follows:

Since, $L = \bar{Z} - D$, $\frac{\partial}{\partial L} \equiv -\frac{\partial}{\partial D}$. Using this in (3.18), we can write,

$$\frac{\partial C}{\partial D} = 1 + \frac{1}{\lambda \sigma_r^2} \left(\frac{\partial i_{d\ell}}{\partial D}\right) \qquad (3.19)$$

Equation (3.19) now expresses a relationship between cash reserves and debt levels, both of which are easily measurable quantities. Thus, equation (3.19) can be used to test various hypotheses regarding the shape of the credit-reserve liquidity value curve. Issues relating to the empirical testing of (3.19) are discussed in detail in the next section.

IV. Empirical Tests

We undertake in this section empirical testing of various hypotheses suggested by our theoretical analysis regarding the shape of the liquidity value curves for credit reserve. This empirical analysis is based on the following theoretical result, derived in the previous section, which specifies the tradeoffs between cash reserve and debt levels for a risk-averse DM,

$$\frac{\partial C}{\partial D} = 1 + \frac{1}{\lambda \sigma_p^2} \left(\frac{\partial i_\ell}{\partial D}\right) \qquad (4.1)$$

where C and D are DM's cash and debt levels, respectively, λ is the coefficient of risk aversion, σ_p^2 is the portfolio variance and $\frac{\partial i_\ell}{\partial D}$ is the slope of the DM's liquidity value curve for credit reserve. Now it is clear that for a particlar functional form of the liquidity value curve $i_\ell(D)$, (4.1) will yield a relationship between C and D which can be econometrically estimated, given the data on the DM's cash reserve and debt levels. Alternatively, given an estimated relationship between C and D, (4.1) can be used to test hypotheses regarding the shape of the underlying liquidity value curve. This is seen as follows:

Assume that a functional relationship between C and D has been estimated using the available data on the DMs' cash reserve and debt levels. Let it be represented by,

$$C = f(D) \qquad (4.2)$$

From (4.1) and (4.2), we can write,

$$\frac{\partial C}{\partial D} = f'(D) = 1 + k \left(\frac{\partial i_\ell}{\partial D}\right) \qquad (4.3)$$

where we have written, $k = 1/\lambda\sigma_p^2$ (4.4)

Clearly, for a risk-averse or risk-neutral DM, $k > 0$. From (4.3), we can further write,

$$k\frac{\partial i_\ell}{\partial D} = f'(D) - 1 \qquad (4.5)$$

and, $$k\frac{\partial^2 i\ell}{\partial D^2} = f''(D) \qquad (4.6)$$

Thus, given the estimated function f(D) from (4.2), we can use (4.5) and (4.6) to test hypotheses regarding the signs of $\frac{\partial i_\ell}{\partial D}$ and $\frac{\partial^2 i\ell}{\partial^2 D}$ which together determine the shape of the underlying liquidity value curve. However, it is perhaps appropriate at this stage to take note of one important limitation imposed on the scope of the empirical analysis by the above theoretical framework. To see this, consider (4.5) which allows us to write,

$$\frac{\partial i_\ell}{\partial D} = \frac{1}{k}\,[f'(D) - 1] \equiv F(D)$$

We can now integrate both sides to obtain,

$$i_\ell = \int F(D)\, dD + \eta \qquad (4.7)$$

where η is the constant of integration which cannot be determined unless we have an independent estimate of the liquidity value i_ℓ at at least one level of debt D. Since the model does not provide such an estimate, the constant of integration η in (4.7) is an unknown quantity. The implication of this result is obvious: the econometric model used here does not permit testing of any hypotheses regarding either the sign and magnitude of the liquidity value i_ℓ or the height of the liquidity value curves; as noted above, only the hypotheses relating to the shape of the liquidity value curves can be tested within this framework.

Now, in order to test the hypotheses regarding the shape of the liquidity value curves, we need to estimate the functional relationship between the cash reserves C and the debt levels D. Our theoretical analysis as well as other empirical evidence strongly suggests a polynomial or an exponential relationship between C and D. Accordingly, we have used the available firm-level data on C and D to estimate a total of nine alternative polynomial and exponential functional forms. The choice of a functional form giving the "best fit" was based on adjusted R^2, randomness of residuals and t-tests. Another important empirical issue relates to the possibility of simultaneous equation bias in the estimation of this functional relationship. Although not reported here, we used Hausman's Specification Test (1978) to test for this bias; the results, however, seemed to rule out the presence of any such bias.[4]

The FBFM Data on Illinois Farmers

The empirical analysis undertaken here is based on data obtained from the Illinois Farm Business Records for 1983 maintained at the Department of Agricultural Economics, University of Illinois, in collaboration with the Illinois Farm Business Farm Management (FBFM) Association. Currently, around 8000 Illinois farmers are enrolled as members of the FBFM association and every year extensive though mostly non-financial data are collected from these farmers and used for farm-business analysis, research and educational programs. For a smaller sample of about 200 member farmers, financial data are also collected annually under this scheme and it is this data set that has been used in the empirical analysis reported in the following pages.

The cash holding C used in the empirical analysis has been calculated as a simple average of a farmer's reported bank balance at the beginning and the end of the year; similarly, the debt level D has been calculated as a simple average of a farmer's liabilities at the beginning and the end of the year. We should point out here that calculating the variables C and D from data available at only two points of time in the year is not very satisfactory. For one thing, there might be considerable variations in the magnitudes of these two variables during the year depending on different business characteristics and these will not be reflected in the simple averages calculated from beginning-of-the-year and end-of-the-year figures; also, many farmers tend to "manipulate" the year-end figures in their financial statements for tax and other purposes. As a result, the computed levels of the variables C and D used in this analysis may not accurately reflect a farmer's true behavior regarding his demand for reserves of cash and credit as sources of liquidity and to the extent that they do not, the reliability of the empirical results might be impaired. Clearly, this aspect of the data-set may constitute a significant limitation of the empirical analysis attempted here and its results should therefore be interpreted with care.

Empirical Findings

The FBFM data set contains the relevant data on cash reserves C and debt levels D for 216 farmers, most of them cashgrain and livestock types. We have noted earlier the desirability of using relatively homogeneous samples of farmers for the purposes of the empirical analysis. Accordingly, the entire sample is first divided into two subgroups of cashgrain and livestock farmers and each subgroup is further divided into high and low leverage categories. Samples in these groups are used to estimate functional relationships between the cash reserves C and the debt levels D for the respective group of farmers. Results of these estimations, reported here only for cashgrain farmers, and their implications for the shape of the underlying liquidity value curves are described and discussed in the following pages.

Cashgrain Farmers by Leverage Category

(i) Low-leverage: The debt/asset ratios of the cashgrain farmers included in this subsample vary from 0-60 percent with a mean ratio of about 38 percent. Thus, the farmers in this group have relatively low degree of leverage. The estimation results for the cash-debt functional relationship, given in column 1 of Table 1, suggest that,

$$k\frac{\partial i_{\ell}}{\partial D} = -1 + \frac{\partial C}{\partial D} = -1 + 8.04 \times 10^{-6} D \qquad (4.8)$$

$$k\frac{\partial^2 i_{\ell}}{\partial D^2} = 8.04 \times 10^{-6} \qquad (4.9)$$

Since these equations hold for a range of debt D = \$300 to \$720,000, we can write,

$$\left.\begin{aligned} &k\frac{\partial i_{\ell}}{\partial D} \leq 0 \ , \ D = 300\text{-}120{,}000 \\ &k\frac{\partial i_{\ell}}{\partial D} > 0 \ , \ D = 120{,}000\text{-}720{,}000 \\ &k\frac{\partial^2 i_{\ell}}{\partial D^2} > 0, \ D = 300\text{-}720{,}00 \end{aligned}\right| \qquad (4.10)$$

Clearly, (4.8) and (4.10) imply a liquidity value curve which is of quadratic form and which first decreases and then increases with the debt-level D at an increasing rate. Figure 1a contains a rough sketch of the shape of the curve implied by these results. Clearly, given the fact that farmers in this group have low degree of leverage, the shape of their liquidity value curve as shown in Figure 1a appears to be fairly consistent with the theoretical predictions.

(ii) High-leverage: The cashgrain farmers included in this subsample are highly leveraged with their debt/asset ratios varying from 60-100 percent and a mean debt/asset ratio of 90 percent. The estimation results of the cash-debt functional relationship, given in the second column of Table 1, indicate a linear relationship of the form,

$$C = 4.53 \times 10^{-2} D$$

which in turn implies that,

$$k\frac{\partial i_\ell}{\partial D} = -1 + \frac{\partial C}{\partial D} = -0.95 \qquad (4.11)$$

and,

$$k\frac{\partial^2 i_\ell}{\partial D^2} = 0 \qquad (4.12)$$

Clearly, these results suggest a liquidity value curve which is constantly declining at a constant rate as shown in Figure 1b.

(iii) Technically-Insolvent: The cashgrain farmers in this subsample have debt/asset ratios that exceed 100 percent, i.e., they can be regarded as technically insolvent. The relevant cash-debt functional form estimation results given in the last column of Table 1 suggest that,

$$\frac{\partial C}{\partial D} = 0 \qquad (4.13)$$

which in turn implies that,

$$k\frac{\partial i_\ell}{\partial D} = -1 \qquad (4.14)$$

$$k\frac{\partial^2 i_\ell}{\partial D^2} = 0 \qquad (4.15)$$

Equation (4.13) states that the marginal rate of substitution of cash for debt is zero. In other words, these highly-leveraged farmers put so little value on their credit reserve that they will not substitute any cash to offset its loss. (Note that an increase in debt-level D means a decrease in the level of credit reserve.) Furthermore, equations (4.14) and (4.15) imply a liquidity value curve that is linear and constantly declining as illustrated in Figure 1c.

A comparison of the three liquidity value curves displayed in Figures 1a, b and c serves to illustrate the possible effect of the degree of leverage on the shape of the liquidity value curves. The liquidity value curve in Figure 1a refers to low-leverage farmers while the curves in Figures 1b and 1c refer to highly-leveraged farmers. We note that the curve for low-leverage farmers is non-linear and shows an increasing trend over most of the range of D. By contrast, the liquidity value curves for highly leveraged farmers are decreasing over the entire range of D and moreover, they appear to lose their non-linearity and tend to become linear. (Note that we cannot determine whether the liquidity value curves for highly-leveraged farmers represent higher or lower liquidity values in absolute terms than those represented by the liquidity value curves for low-leverage farmers.) We also observe here that this is consistent with our theoretical analysis which explains these trend in terms of the fact that for highly-leveraged farmers the cost of converting their credit reserve into loans becomes very large.

In conclusion, it is perhaps fair to say that despite certain difficulties with the quality of the data used, the results of the empirical analysis appear to strongly support most of the theoretical predictions that were tested. Moreover, the empirical analysis appears to provide many valuable insights into the behavior of the liquidity value curves with important implications for the Theory of Liquidity Management. We have observed that the liquidity value curves for the highly-leveraged U.S. farmers are constantly declining, indicating that credit reserve rapidly loses its liquidity value for financially-strapped farmers who will therefore have to find alternative means of coping with risk. We have analytically demonstrated elsewhere[4] that liquidity costs of credit have significant impact on the input and output choices made by farmers. In view of this, persistent financial stress among farmers is likely to significantly affect the production organization and the structure of U.S. agriculture.

V. Summary and Conclusions

This paper explores the role of liquidity in decision making under uncertainty. Expected Utility Maximization hypothesis is used to develop a rigorous theoretical model that explains the demand for liquidity in the form of cash and credit reserves (defined as unused credit or borrowing power) as primarily a risk response. The model also shows that the reservation price or the liquidity value that a risk-averse decision maker (DM) places on his cash or credit reserves mainly depends upon his degree of risk aversion, the liquidity and liquidity-risk characteristics of his portfolio, and the pattern of his random cash demands. Finally, the theoretical framework is extended to rigorously formulate a "Liquidity Management Hypothesis" according to which a risk-averse DM maximizes his expected utility by allocating his resources in such a way that at the optimum he holds a least-cost combination of his liquidity reserves in the form of cash and credit.

Firm-level financial data on Illinois cashgrain farmers are used to test various hypotheses about the shape and behavior of their liquidity value curves for credit reserve. Despite certain difficulties with the quality of the data used, the empirical results are fairly consistent with the theoretical predictions. Furthermore, the empirical analysis appears to provide many useful insights into the structure and behavior of the liquidity value curves with important implications for the theory and practice of liquidity management. In particular, the liquidity value curves for the highly-leveraged group of U.S. farmers are found to be constantly declining with increasing debt levels indicating that credit reserve rapidly loses its liquidity value for financially strapped farmers. As a result, farmers in financial distress apparently cannot rely on credit reserves as an effective means of coping with risk and must find alternative risk management strategies to survive. Moreover, since liquidity costs of credit are an important factor in production decision, persistent financial stress among farmers is likely to significantly affect the production, organization and structure of agriculture.

The theoretical analysis presented here has undoubtedly enhanced our knowledge and understanding of the structure and behavior of the liquidity value curves which is likely to be very useful in their empirical applications to study decision making under uncertainty. The main implication of the theory, however, is that the structure of the liquidity value curves appears to be surprisingly complex and that relatively simplistic assumptions about their shape and behavior might lead to misleading results.

Finally, we should mention that the theoretical framework outlined in this paper has been extensively used to analytically explore the role of inflation, credit risks, and insurance in the management of liquidity, as well as the role of credit in production under uncertainty. For details on these aspects, the interested reader is referred to Chhikara [1986].

Footnotes

[1] See previous studies by Goldsmith [1954], Hirschleifer [1964], Heady and Swanson [1952], and Hesser and Jensen [1960].

[2] See Barry and Baker [1971] and Harris [1985].

[3] See Baker and Bhargava [1968] and Kamajou and Baker [1980].

[4] See Chhikara [1986] for details.

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Table 1 : Estimated Cash-Debt Relationship For Cashgrain Farmers By Leverage Category

Debt/Asset Ratio					
0-60 Percent		60-100 Percent		Over 100 Percent	
Range of Debt-level: \$300-\$720,000		Range of Debt-level: \$10,000-\$1,300,000		Range of Debt-level: \$40,000 \$3,400,000	
Functional Form: quadratic[1]		Functional Form: cubic[1]		Functional Form: cubic[1]	
Sample Size: 77		Sample Size: 56		Sample Size: 37	
Adjusted R^2: 0.22		Adjusted R^2: 0.16		Adjusted R^2: 0.08	
Parameter Estimate	T-value	Parameter Estimate	T-value	Parameter Estimate	T-value
$\hat{\alpha}_o = 3276.15$	3.60*	$\hat{\alpha}_o = -238.86$	-0.09	$\hat{\alpha}_o = -1754.17$	-0.34
$\hat{\alpha}_1 = -1.37 \times 10^{-2}$	-1.61	$\hat{\alpha}_1 = 4.53 \times 10^{-2}$	-1.82	$\hat{\alpha}_1 = 6.94 \times 10^{-2}$	1.48
$\hat{\alpha}_2 = 4.02 \times 10^{-6}$	2.84*	$\hat{\alpha}_2 = -5.95 \times 10^{-8}$	-1.02	$\hat{\alpha}_2 = -1.17 \times 10^{-7}$	-1.23
		$\hat{\alpha}_3 = 2.02 \times 10^{-14}$	0.60	$\hat{\alpha}_3 = 2.91 \times 10^{-14}$	1.19

1 The cubic form is represented as $C = \alpha_o + \alpha_1 D + \alpha_2 D^2 + \alpha_3 D^3$ while the quadratic form is represented as $C = \alpha_o + \alpha_1 D + \alpha_2 D^2$, where C denotes the cash holding and D denotes the debt level.

* denotes significance at the 5% level.

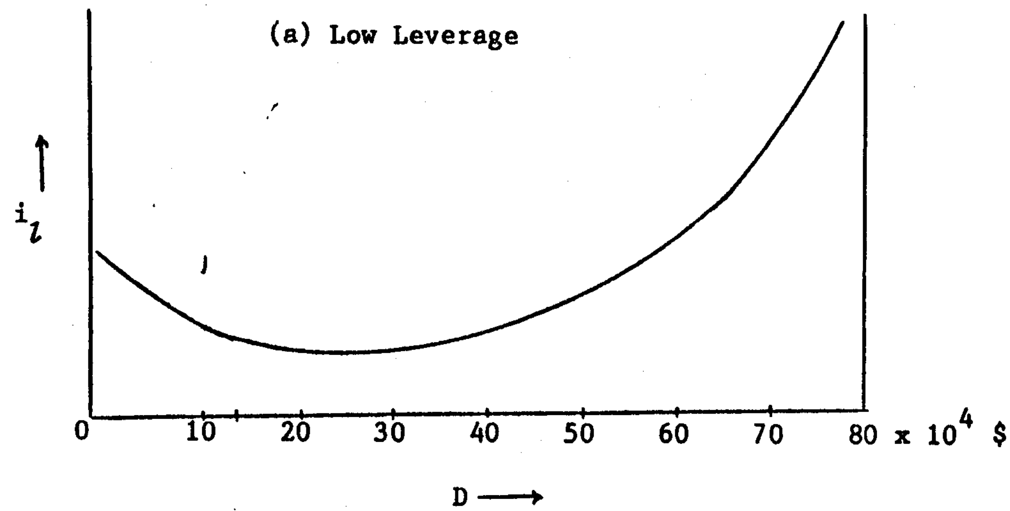
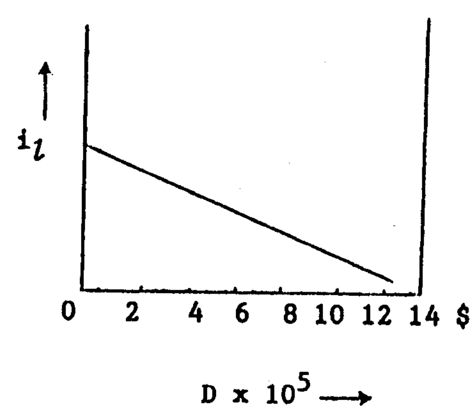
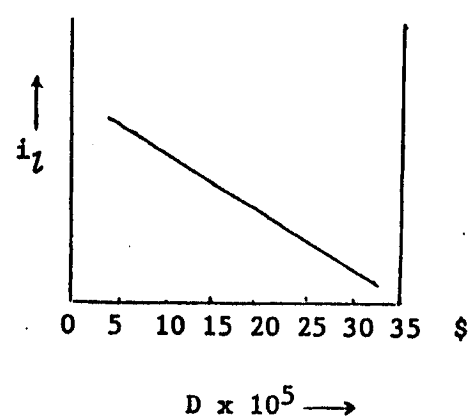


Fig. 1a : Implied Shape of the Liquidity Value Curve for Low-leveraged Cashgrain Farmers



(b) High-Leverage



(c) Technically Insolvent

Fig. 1b,c : Implied Shape of the Liquidity Value Curve for Highly-leveraged Cashgrain Farmers