# Improvement of a bio-economic mathematical programming model in the case of on-farm source inputs and outputs

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***Selected Paper prepared for presentation at the International Association of Agricultural Economists (IAAE) Triennial Conference, Foz do Iguaçu, Brazil, 18-24 August, 2012.***

### Abstract

A significant part of crop productions is not marketed and can be totally (fodders and pastures) or partly (cereals) used on-farm. Regarding inputs, Nitrogen fertilizers are also not only provided by the market, but they are significantly on-farm provided by livestock. When outputs and inputs are related through technical "dose-response functions", like Nitrogen ($N$)-yield functions, the optimal gross margin is easily computed at the plot level, when input and output are marketed. In this case, market prices are required.

The question addressed in this article is how to assess the optimal values when one crop production (respectively one input) is entirely on-farm used (respectively on-farm produced). We propose a simple iterative computation method aiming at replacing market prices by shadow costs. We apply this method to a large set of the bio-economic agricultural supply model AROPAj, which covers a large range of agricultural activities over the European Union. This method allows us to keep efficient the basic linear framework of the model even when the yield functions make it non linear.

**Keywords:** Bio-economic farm model; Mathematical programming; On-farm source products; Shadow prices

**Abbreviations**CAP - Common Agricultural Policy; FADN - Farm Accountancy Data Network; AROPAj: annual economic farm model; EU - European Union; FG - farm-group

***JEL Classification:*** Q10; Q15; Q50; Q57

## 1 Introduction

European agriculture is the main land user (43% of the EU-27 area), while arable land account for over half of this sector (60%) and grassland occupy an important part (one-third) which constitute an important economic and environmental resource [20, 28, 12]. A large part of these agricultural productions are not traded in markets and no market prices are available to reflect their economic value. Therefore, alternative ways are required to estimate their monetary values.

The present paper deals with these issues by using a bio-economic farm model as a tool to calculate the values of non-marketable productions and to integrate them in the farming costs. The bio-economic farm model calls for formulation describing farmers' resource management decisions that represent current and alternative production possibilities in terms of required inputs to produce outputs, both yields and environmental effects. Such a model is developed to enable assessment of policy changes and technological innovations, for specific categories of farming systems [18].

We are particularly interested in fodder, pastures and totally on-farm used crops, as well as organic nitrogen. As presented by Oenema (2005) [21], livestock production systems convert plant protein into animal protein. Depending on animal species, ration and management, between 55% and 95% of the nitrogen ($N$) present in the proteins of the plant is excreted via urine and feces [21], which can be used as a nutrient source for plant growth and can thus reduce the use of mineral fertilizers. Therefore, it is important to take into account these outputs, which depend on farming management could represent a valuable input or otherwise a source of pollutant.

In case of on-farm recycling of product or input, the market price ratio has to be replaced by the shadow price ratio. We show how close (or far) is the proxy solution based on market prices compared to the optimal solution. And we suggest a simple method to get the solution in almost all cases. The test is conducted on 1074 AROPAj farm groups across the EU-15.

The remainder of this paper is structured as follows. *Section 2* examines the various interpretations of the concept of shadow price and abatement costs. *Section 3* is devoted to the methodology used. The results are discussed in *Section 4* and *Section 5* concludes.

## 2 Theoretical underpinnings

In this section, we introduce different studies done to calculate the shadow price of goods (desirable) and bad (undesirable) outputs and their impacts on farming systems. The shadow prices can be described as the true value of prices for the scare resource in an observed situation [16].

Several studies use econometric tools to assess the impacts of nitrogen pollution on the water, as well as determining the implicit cost associated with the externalities related to animals waste treatment. Hadley (1988) applies the output distance function methodology to evaluate the shadow prices for nitrate emissions to groundwater from a sample of UK dairy farms [15] and shows that the marginal cost of abatement for producers with high levels of undesirable emissions is significantly lower than for those with low level of nitrate emissions. Fare et al (2006), used a quadratic functional form for the directional output distance function to derive estimates of production inefficiency, shadow prices of polluting outputs, and also the associated pollution costs in the U.S. agricultural sector. They find that the significant pollution costs (shadow values) from the runoff and leaching of pesticides represent on average 6% of annual crop and animal revenues [10].

Reinhard et al. (1999) study a panel of Dutch dairy systems in the Netherlands, mentioned that it is possible to use the value of the shadow price to generate an adjustment index of the evolution of productivity or the marginal cost of nitrogen pollution abatement which deals with the estimation of indicators of technical efficiency and environmental [24]. The authors find a positive correlation between the level of intensification of farming systems and their environmental effectiveness, thus the shadow price of one kilogram of nitrogen surplus is equal to 3.14 guilders ($\simeq$ 1.42€), which decreased with the size of the property used. In another article, Shaik et al. (2002) calculate the shadow prices directly or indirectly, by considering pollution as a common input or as an output side [26]. Their results of agriculture analysis in the State of Nebraska, show that it is better to assume environmental pollution as a desirable output production, as by doing so, we can identify both the economic and non-economic dimensions which include environmental pollution. They estimate the average shadow prices of 1.73$ and 1.95$ for the reduction of nitrogen pollution as a direct and indirect approach respectively.

Piot-Lepetit et al. (1998) work on organic nitrogen derived from the pig farms in France from the analysis of technical inefficiency in the pig sector under the assumption of weak and strong disposability [23], when considering the removal of organic nitrogen assuming strong disposability of shadow price obtained is 0.84 French franc/kg ($\simeq$ 0.13€/kg), and show that producers have an increase in their profits due to the application of nitrogen, which in this case represents the internalization of environmental damage. Moreover, they find the shadow price of -5.97 French franc/kg ($\simeq -0.9$€/kg) that represents additional costs to production in the case when one takes into account the assumption of weak disposability.

To study the behavior of the balance between supply and demand of fertilizers, some researchers are interested in analyzing the change in nitrogen prices and the associated response according to the change in nitrogen losses (leakage or leaching). A small positive change in the price of nitrogen fertilizer can result in a decrease in the nitrogen surplus, which can achieve optimal (efficient nitrogen production), if the shadow prices increased by 12.5%, that is the case study by Reinhard et al. (2000) for dairy farming in Holland [25]. In addition to the estimated costs associated with pollution, there are alternative measures that the subject of study of several works, such as effluent treatment may be partially subsidized rights marketing spreading between farmers and herders, the implementation of the polluter pays principle or the reduction of livestock. Comparing with a model that simulates various scenarios for managing livestock manures (pigs, poultry and cattle) in the department of Cotes d'Armor in France, either short- or long- term, affirm that reduction of livestock number is the least expensive in the long term. In the case of raising pigs, chickens and poultry meat, it represent respectively a decline of 20%, 40% and 30% in numbers of animals [9].

Shaik and Helmersy's (1999) paper exploits the duality theory between the output (input) distance function and the revenue (cost) function in retrieving shadow prices, as the prices of environmental bads from the producer perspective are seldom available [27]. The authors show higher shadow prices when the environmental bad is treated as an input compared to an undesirable output. The shadow price computed from the output distance function represents the opportunity cost to the producer to reduce pollution along with increasing agriculture production given the level of inputs. In contrast higher shadow price estimates from the input distance function reflects the value of production forgone if nitrate and pesticide are not applied or if agriculture land is lost to wetlands conservation. De Ridder et al. (1985) show that the shadow price of pasture might be measured in terms of livestock production from that land [8].

The next section aims at presenting our methodology used to calculate values of non-marketable productions such as fodder, pastures and totally on-farm used crop.

# 3 Methodological Framework

## 3.1 Agricultural supply economic model (AROPAj)

The AROPAj model is a mathematical linear mixed programming model (MLIP), developed by INRA (the French National Institute for Agricultural Research). The model is dedicated to the simulation of European Union (EU) farming systems behavior facing change in economics issues (CAP, European Directives regarding environment, climate change and bio-energy) [6, 5, 19, 4, 7, 11]. For a given economic situation (i.e. a set of prices, taxes and policy measures, etc.), it provides an assessment of the type and amount of the agricultural products delivered on the markets at different scales from the farm to the EU levels. Data are mainly provided by the Farm Accountancy Data Network (FADN [1]), which provides economic information (animal numbers, crop yield, price, areas, etc.) for a sample of thousands surveyed farmers across the EU.

AROPAj consists of a set of independent mixed integer and linear-programming models. Each model describes the individual annual supply choices of an agent, representative of the behavior of "real farmers" (hereafter referred as "farm-group" and denoted by "$k$"), in terms of crop area and output, land allocation, animal production (milk and meat), intra-consumption or/and purchased animal feed. The farm-group representation makes possible to account for the wide diversity of technical constraints faced by European farmers. Each farm-group $k$ is assumed to choose the supply level and the input demand ($x_k$) in order to maximize total gross margin ($\pi_k$). The generic model for farm-group $k$ can be written as follows :

$$\begin{aligned} \max_{x_k} \pi_x(x_k) &= g_k.x_k \\ \text{s.c.} \quad A_k.x_k &\leq z_k \\ \forall k \ x_k &\geq 0 \end{aligned}$$

Where $x_k$, $g_k$, $A_k$ respectively denote the activity of farm-group, gross margin and the coefficient matrix of input-output. $z_k$ represent the vector of resources.

Farm-groups used in AROPAj result from a classification of observed farms within each FADN region. This classification is based on 3 characteristics: (i). *Altitude*, which corresponds to the average altitude of holding (<300m, 300-600m, >600m), (ii). *Economic size* that is defined by the concept of standard gross margin and is expressed in terms of European size unit, and (iii). *Type of farming*, 13 types of farming proposed in the FADN classification are considered and characterize farms specialized in particular field crops, livestock, farm mixing crops and/or livestock.

AROPAj also includes a greenhouse gases (GHG) calculation module inventorying about 20 sources of $CH_4$ and $N_2O$ from livestock and arable farming, based on the IPCC Tier 1 guidelines [17]. The model assumes that the most important factors behind GHG emissions are assumed to be livestock size (for $CH_4$ and $N_2O$), and nitrogen fertilizer use (for $N_2O$) [4, 22]. Manure can be either applied to crops, deposited directly on soil by grazing animals, or stored/treated using different management systems. The total production of manure-related nitrogen is computed as the product of nitrogen content of manure defined for each animal category and the corresponding animal numbers.

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[1] http://ec.europa.eu/agriculture/rica/index en.cfm

### 3.2 Related activities and constraints

As explained above, a farming system is assumed to adopt the standard economic rationale summarized by the gross margin maximization. Gross margin is based on the valuation of a set of activities regarding crop and livestock outputs, and of inputs such as fertilizers and animal feeding. Maximization has to take account of technical (agronomic bindings, nutriment requirement, herd composition) and economic constraints (nitrogen spread limit, regulation tools implementation, mainly due to the CAP).

The mathematical programming framework stands here for a one-year period model. The agricultural sector model belongs to the economic supply-side category. Aiming at assessing the economic value of on-farm recycled products, we focus on variables and constraints (furthermore renamed as "dual variables") related to $N$-intakes (marketed mineral fertilizers and manure) and to animal feeding (on-farm use of cereals and fodders, pasture and marketed feed).

The $i$ and $j$ indices respectively refer to crops and animals. Enclosing the relevant parameters, the (primal and dual) variable subset are listed in annex A. Let us set the equations including the intermediate primal variables which allow to keep linear the optimization program presented below:

$$\begin{aligned} N_j^{min} &= n_j^{min} S_j \\ N_{ij}^{org} &= n_{ij}^{org} S_j \\ F_i &= f_i A_i \\ \forall i : \quad \sum_j N_{ij}^{org} - a_i A_i &\leq 0(\eta_i) \end{aligned}$$

In the above equation, $N_j^{min}$ denotes the bought mineral nitrogen used by the $j$-crop, $n_j^{min}$ the bought mineral nitrogen used by the $j$-crop per area unit, $S_j$ is the crop and fodder area, $N_{ij}^{org}$ represents on-farm nitrogen from $i$-origin used by the $j$-crop, $n_{ij}^{org}$ is the on-farm nitrogen from $i$-origin used by the $j$-crop per area unit. Generic feed requirement per livestock unite (LU) [2] is denoted by $F_i$ and $f_i$ represents the generic feed requirement per LU. Quasi-fix livestock is represented by $A_i$ and $a_i$ is nitrogen output per LU.

The $N$-yield and feed-product functions are assumed to be monotonously increasing and concave.

AROPAj includes a bio-technical block which is based on $N$-yields functions calibrated for a wide set of crops and for all individual "farm groups" (see among others [11]). The $N$-yields functions are calibrated thanks to the STICS crop model [1]. When the related functions are activated in the model, the initially linear programming model becomes non linear. But when crop production is sold on the market (at least partly) and when a part of fertilizer is bought, a two-step optimization procedure allows us to keep the linear basic structure of the model (it allows us to keep friendly-user aspects of linear or mixed programming). The first step leads to use the market price ratio to estimate the yield and

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[2] Livestock Unit (LU) is a unit based on the feeding demand of the animals, which used to compare livestock size of different species or category of animals

the $N$-input related to each crop. The second step calls for the usual linear programme in which first-step yields and inputs replace the default estimates.

In order to study the impact of using fodder, pasture and organic nitrogen fertilizers in land allocation, farm management and economic markets, we use the methodology developed by Godard [13, 14]. which model the relationship between the yield and the nitrogen fertilization for the major agricultural crops of EU, by coupling an agronomic approach (STICS [2, 1, 3]) to an economic model of EU agriculture (AROPAj), in order to increase the sensitivity of the economic model to different kinds of agricultural policy scenarios. This coupling leads to transform the AROPAj model, based on linear programming, to a non-linear programming model, by replacing the average yield of each crop with the response function of nitrogen. The modeling curve proposed to link the yield and the nitrogen fertilization has an exponential form, which can meet both agronomic and economic interpretations [3], and could offer the best estimation of the economic optimum fertilizer rates.

$$r(N) = B \ - \ (B - A) \ \exp(-t \ N)$$

In this equation, $r(N)$ represents the crop yield, $B$ and $A$ indicate the maximum and minimum yield respectively. The parameter $t$ characterizes the curvature and $N$ is the nitrogen fertilization amount.

After getting several potential curve from the STICS model, one single curve is selected, which is the best curve fitting the economic criterion of plot gross margin maximisation. First, selected curves are compared with the level of reference yield ($r_0$) provided by AROPAj. Curves that are below this yield are eliminated, and other curves are tested by the marginal condition of the use of nitrogen :

$$\frac{dr}{dN} = \frac{w}{p_j}$$
$$\texttt{when } N > 0$$
$$\texttt{else } N = 0$$

The marginal productivity of nitrogen $dr/dN$ (the slope of the yield function at the intersection point) has to be compared to the price ratio in which $p_j$ represents the selling price of the crop-$j$, and $w$ the price of nitrogen fertilizer. The curve that minimizes the difference between the price ratio and the tangent of the curve near the observed yield (FADN reference) is selected.

This method is correct for the major European crop productions, when crop and nitrogen prices are available and when part of crop and part of fertilizer are marketed. There is an interesting problem arising when production is fully consumed on-farm. We can show that in this case the market price -when it exists- is not the adequate price. Instead, we need the shadow price which is at the same time provided by the optimization program.

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[3] From an economic point of view, it is important that the chosen $N$-curve is concave, strictly increasing, with a finite limit when $N$ is infinitely increasing. From an agronomic point of view, the curve also had to be increasing, with a finite positive value in zero and a finite positive limit in the infinite

In this case, we do the convergence test to estimate the optimal solution, based on the substitution of market prices by the shadow prices of several crops, and finally instead of using mineral fertilizers, we develop using nitrogen organic via animal production and by calculating the shadow price of organic nitrogen, we do the substitution of mineral fertilizers price by the organic one (see Section 3.3, Equations 1 and 2).

## 3.3 Optimal shadow prices

The maximization program ($P$) given in annex B. When $f_i$ and $n_j$ are given, the seminal $P$ program is linear. The related linear program is denoted by ($LP$). Livestock is assumed to be strictly positive ($\forall i \; A_i > 0$). The Lagrangian of the problem ($P$) is presented in annex C.

In the subsequent text, the optimal solution is denoted by unchanged characters (i.e. $S_j$ presents the optimal area devoted to the $j$-crop). When it exists, the optimal solution has to satisfy the $1^{st}$ order and exclusion conditions (applying the Kuhn and Tucker theorem). The partial derivative of function $f(x)$ with respect to the $x_i$ component will be denoted by $f_{x_i}$. The (total) derivatives of the functions $r_j$ and $g_i$ will be denoted respectively by $r'_j$ and $g'_i$. The last general consideration relates to the optimal shadow prices which are positive ($\beta^X \geq 0$).

Let us consider the "interesting" case of the produced $j$-crop (i.e. $S_j > 0$ which implies $\beta^S_j = 0$) and the use of on-farm nitrogen ($n_j > 0$, $\beta^n_j = 0$). Focus first on the optimality equation subset $\{\mathcal{L}_{n_j} = 0; \mathcal{L}_{N^{min}_j} = 0 \; ; \mathcal{L}_{N^{org}_j} = 0\}$ , rewritten as:

$$\begin{aligned} \mathcal{L}_{n_j} &= \left[r'_j(n_j)\rho_j - \xi_j\right] S_j + \beta^n_j = 0 \\ \mathcal{L}_{N^{min}_j} &= -w + \xi_j + \beta^{N^{min}}_j = 0 \\ \forall i \;:\; \mathcal{L}_{N^{org}_{ij}} &= \xi_j - \eta_i - \epsilon_j + \beta^{N^{org}}_{ij} = 0 \end{aligned}$$

The subset of $j-$crop product variables and the related $1^{st}$ order conditions lead to the equations:

$$\begin{aligned} \mathcal{L}_{Y_j} &= p_j - \rho_j - \psi^C_j + \beta^Y_j = 0 \\ \forall i \;:\; \mathcal{L}_{C_{ij}} &= -\rho_j + c_{ij}\gamma_i + \beta^C_{ij} = 0 \end{aligned}$$

As a basic result, the $j-$crop yield productivity is equal to the shadow price ratio:

$$r'_j(n_j) = \frac{\xi_j}{\rho_j} \tag{1}$$

- When at least a part of the $j$ product is marketed ($Y_j > 0$ and $\beta^Y_j = 0$), the shadow price $\rho_j$ is less or equal to the market price $p_j$. There is strict equality when the (eventual) $j$ quota is not bounded
- When the total $j$ product is on-farm consummed, $\rho_j$ is strictly greater than the market price, reflecting that the product is better valuated on the farm than on the market ($\beta^Y_j > 0$, and $\psi^C_j = 0$, thanks to $0 = Y_j < Q^C_j$)

- Regarding the $N$-input, the shadow price ($\xi_j$) is strictly equal to the market price ($w$) when the $N$-input is partly or totally bought ($N_j^{min} > 0$ and $\beta_j^{N^{min}} = 0$)
- When the $N$-input is entirely provided by manure ($N_j^{min} = 0$ and $\beta_j^{N^{min}} > 0$), the shadow price is strictly lower than the market price

Animal production analysis leads to consider the $i$-indiced equation subset which follows (basically, $f_i > 0$ and $G_i > 0$ which implies $\beta_i^G = 0$):

$$\begin{aligned}
\mathcal{L}_{f_i} &= \left[g_i'(f_i)\alpha_i - \gamma_i\right] A_i = 0 \\
\mathcal{L}_{G_i} &= v_i - \alpha_i - \psi_i^A + \beta_i^G = 0 \\
\mathcal{L}_{B_i} &= -q + b_i\gamma_i + \beta_i^B = 0 \\
\forall j \ : \ \mathcal{L}_{C_{ij}} &= -\rho_j + c_{ij}\gamma_i + \beta_{ij}^C = 0
\end{aligned}$$

The livestock productivity with respect to feed is equal to the shadow price ratio:

$$g_i'(f_i) = \frac{\gamma_i}{\alpha_i} \tag{2}$$

- When the animal product $i$ is not bounded by any quota, the $i$ product shadow price is equal to the market price ($\alpha_i = v_i$)
- When the product is bounded by a quota ($\psi_i^A > 0$), the shadow price is lower than the market price
- When the optimal solution leads to the use of marketed feed to feed the $i$ animal ($\beta_i^B = 0$), the shadow price of the $i$ feed is equal to the feed market price with respect to the $i$ animal requirement ($\gamma_i = \frac{q}{b_i}$)
- When no market feed is used for the $i$ animal, production is more efficiently obtained thanks to on-farm use of cereals or fodders. In this case, the $i$ feed shadow price is lower than the market price with respect to $i$ requirement ($\gamma_i < \frac{q}{b_i}$)

Qualitative results related to the ($\mathcal{P}$) program can be summarized on the table 1. It should be noticed that solving the linear program with market prices instead of shadow prices in Equations (1) and (2) lead to a sub-optimal optimum except when optimal marketed quantities are strictly positive (on the output side and on the input side).

In this last case, i.e. assuming that the optimal marketed quantities are strictly positive, the two-step process which leads to the optimum is subsequently summarized:

1. Solving Equations (1) and (2) when shadow prices are replaced by market prices
2. Solving the linear program ($LP$)

In other cases, i.e. when there is at least one crop $j$ for which (i) production is strictly positive, and (ii) marketed $j$ nitrogen input is equal to 0 or marketed $j$ destination is equal

|  | bounded output quota or bounded used input | marketed output $> 0$ and marketed input $> 0$ | totally on-farm used or totally on-farm used |
|---|---|---|---|
| crop $j$ | $\frac{\xi_j}{\rho_j} < \frac{w}{p_j}$ | $\frac{\xi_j}{\rho_j} = \frac{w}{p_j}$ | $\frac{\xi_j}{\rho_j} > \frac{w}{p_j}$ |
| livestock $i$ | $\frac{\gamma_i}{\alpha_i} < \frac{q}{b_i v_i}$ | $\frac{\gamma_i}{\alpha_i} = \frac{q}{b_i v_i}$ | $\frac{\gamma_i}{\alpha_i} > \frac{q}{b_i v_i}$ |

Table 1: Comparison of shadow price ratio and market price ratio regarding the input-output functions related to crops and livestock.

to 0, $j$ shadow prices and market prices differ. Shadow prices are provided by solving the basic $(\mathcal{P})$. In these cases, replacing shadow prices by market prices does not lead to the exact optimal solution. Theoretically there is place for distorting cumulative effects, namely the intensive effect and the extensive effect. The intensive effect occurs when the slope of the tangent and the ratio of shadow prices differ. Change in intensive effect can affect the land allocation among crops.

Here we briefly present the computation method limited to $T$ iterations. To sum up :

1. $t = 0$ : $1^{st}$ AROPAj run when price ratio is computed thanks to the FG market prices (always provided by the FADN)
2. $t = t + 1$
3. Extraction of shadow prices related to the production destination constraints at the iteration $t - 1$.
4. $t^{st}$ AROPAj run when $N$-yield productivity is estimated through the last computed shadow price ratio
5. While $t \leq T$ goto step 2.

This methodology has been used at both fram-group types and European scale, to examine when the optimal solution will be achieved. The results are presented in the next section.

# 4 Results and Discussion

This section aims at presenting and analysing the results of applying shadow prices in the AROPAj model for EU-15. Our study is based on substitution of market prices by the shadow prices, when crop productions are partly or totally used on-farm. We limit the analysis to crops for which the crop model STICS provide us with $N$-yield functions, i.e. in the case of "grandes cultures" (such as wheat, rice, corn, barley, sugar beet, potatoes, rapeseed, sunflower). There is an ongoing work devoted to fodders, but it is not operational yet). In the model, for these crops, the shadow price problem arises only for cereals which are possibly marketed or on-farm used. Among them, it has to be noted that $N$-yield functions exist for durum wheat, soft wheat, barley and corn.

We focus on the farm gross margin, as one of the objectives of this study is to determine whether these changes can benefit farmers and increase their gross margin. We hypothesize that it is the case when crop shadow prices are shown to be greater than market prices. The effect of substituting the market prices by the shadow prices is weak (+0.07% for EU-15) but significant and leads to an increase in farm gross margin (Figure 1). This is not significant at the national scale, when a lot of farmers sell all their production and when other farmers sell a part of their production (in this case, we recall that optimal shadow prices are equal to the market prices).

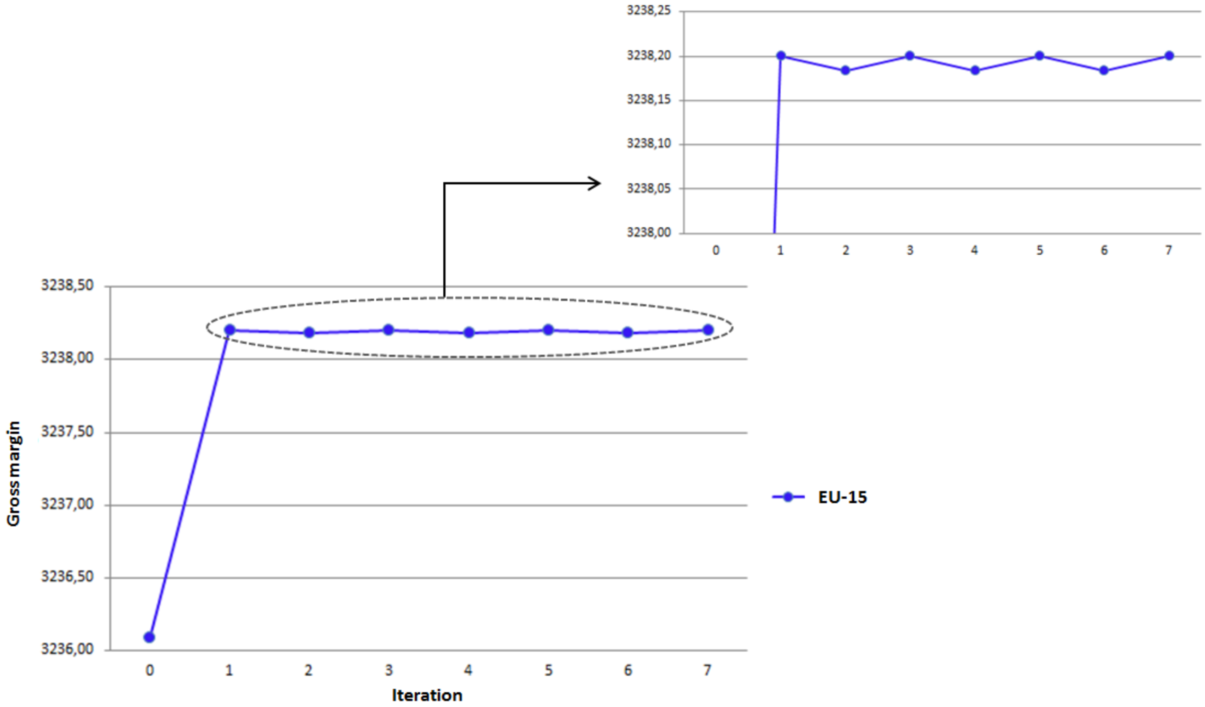

Figure 1: Gross margin variation (€$/ha$) at the European scale

Let us focus at the farm-group scale (FG). The EU-15 are represented by 101 regions and 1074 FG (with respect to the FADN region classification). It should be notice that all simulations realized for the 1074 FG lead to strictly positive amount of purchased fertilizer by +1.5% at EU level.

The optimal solution is achieved at the first iteration for 947 farm groups, for which shadow prices are equal to the market prices. Most of the rest of other (49) farm groups reach the optimal solution at the second iteration (Figure 2). Therefore, the optimal solution is provided after 3 runs, except for nine FG for which the LP solution follows a 2-period diagram (see annexe D).

We present different example of the behavior at different FG to introduction of our method. These nine FG are characterized as follows :

1. *Baden-Württemberg region in Southern Germany*: Applying shadow prices leads to a decreased bought feed. Farmers are more interested by on-farm use of crop than

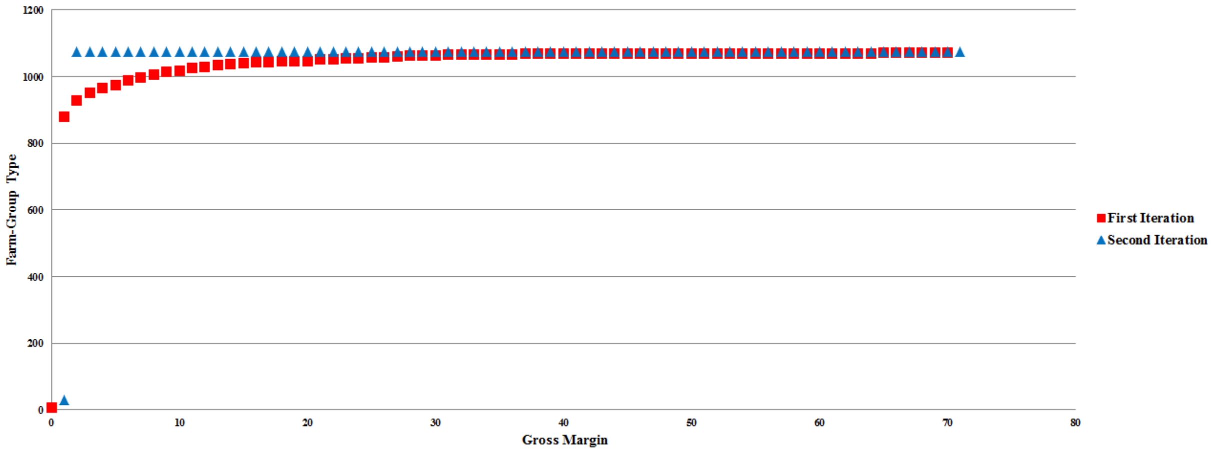

Figure 2: Cumulative distribution of gross margin (€/*ha*) deviation between the market price solution and the shadow price solution, among the relevant farm group subset - when at least one of the cereal production is entirely on-farm used

by sale. Compared to the initial run, increase use of on-farm cereal productions such as barley, oats and grain corn is recorded.

2. *Limousin region in the Central-West France*: Price modifications lead to a change of on-farm use of cereals such as barely, oats and grain corn and at the same time leads to a decreased barely marketed part.

3. *Austria region*: Implementation of shadow prices relates to an increase of grassland area and to a decrease of cereal and fodder areas.

4. *Galicia region in North-Western Spain*: On-farm use of grain corn increased by applying shadow prices. Farmers are more interested by on-farm use of crop.

5. *Murcia region in Southern Spain*: Price modifications decrease total concentrated feed and leads to increased on-farm use of barley.

6. *Extremadura region in the East of Spain*: Applying shadow prices results in an increase in on-farm use of durum wheat.

7. *Alentejo e do Algarve regions in the South of Potugal*: Application of shadow prices increased on-farm use of cereal productions such as soft wheat and durum wheat and at the same time leads to decreased concentrated feed.

8. *Etela-Suomi region in the South of Finland*: We observe price changes and an increase in on-farm use of cereals such as rye.

In all the cases, no changes in livestock number are recorded. As shown in annexe D, on average gross margin changes account for less than 0.2€ per hectare. The reason for this unsteady state lies in the changes brought by the on-farm and marketed crop productions.

# 5 Conclusion

In this paper we have developed and illustrated an analytical method aiming at evaluating the value of on-farm recycled outputs and inputs. The theoretical base of the method is underpinned by a general approach including crop and livestock production, and nitrogen mineral and organic fertilizers.

This method is applied to an existing supply side LP model devoted to the European agriculture, in which a bio-economic block integrates nitrogen-yield functions. The method keeps operational the basic linear programming structure of the model, when optimal yields and inputs are estimated, thanks to market price ratios. It is applied in the case of $N$-yield functions calibrated for cereals. When at least one crop is entirely on-farm used (e.g. for feed), the key point is to replace market prices by shadow prices. But shadow prices are endogenously computed.

A simple operational method consists in promoting a few iteration procedure in which shadow prices computed at an iteration are used for the next one. Applied results highlight that shadow prices application enhances the on-farm production and increases the farm gross margin for most farm groups. But gross margin (like other LP variables indeed) impacts could be considered as weak for the major part of farm groups. In any case, the optimal solution is found in less than 4 iterations for most of the FG, except for 9 FG.

For future development, regarding fodder and pasture for which $N$-yield function have still to be calibrated, the challenge will be harder, when no market prices exist.

This work will require a improved finer algorithm, for a better understanding of the effects of applying shadow prices in farm systems, that could be costly in terms of computing time, which is one of the problems of the operational research.

## Acknowledgments

The authors would like to gratefully acknowledge the financial support brought by the French Environment and Energy Management Agency (ADEME) and the French National Institute for Agricultural Research (INRA). The research leading to these results has been conducted as part of the AnimalChange project which received funding from the European Community's Seventh Framework Programme (FP7/ 2007-2013) under the grant agreement n° 266018, this work has been also carried out as a part of the ANR program, "Validate" project (n° ANR-07-VULN-11-011).

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## A Annex: Set of primal and dual variables, and parameters

We define the elements of interest for our problem. These elements are model subsets respectively for activities (i.e. primal variables), constraints (i.e. identified to diual variables), and parameters.

- **Primal variables**
  - $S_j$ $(ha)$ : crop and fodder area ($j$-index)
  - $Y_j$ $(t)$ : marketed part of the crop production
  - $n_j$ $(tN^{min}/ha)$ : generic nitrogen input related to the $j$ product per unit of area
  - $n_j^{min}$ $(tN^{min}/ha)$ : bought mineral nitrogen used by the $j$-crop per area unit
  - $n_{ij}^{org}$ $(tN^{org}/ha)$ : on-farm nitrogen from $i$-origin used by the $j$-crop per area unit
    (only one manure category considered in this simplified model)
  - $N_j^{min}$ $(tN^{min})$ : bought mineral nitrogen used by the $j$-crop

- $N_{ij}^{org}$ $(tN^{org})$ : on-farm nitrogen from $i$-origin used by the $j$-crop
- $A_i$ $(LU)$ : quasi-fix livestock ($i$-index)
- $B_i$ $(t)$ : bought feed
- $C_{ij}$ $(t)$ : on-farm use of $j$-product for $i$-animal
- $f_i$ $(t/LU)$ : generic feed requirement per LU
- $F_i$ $(t)$ : generic feed requirement
- $G_i$ $(t)$ : animal product

- **Dual variables (shadow prices) viewed through the resource limit**
  - $\lambda$ $(k€/ha)$ : land allocation limited by the Utilized Agricultural Area (UAA) : $\lambda$ $(k€/ha)$
  - $\rho_j$ $(k€/t)$ : crop product use
  - $\xi_j$ $(k€/tN^{min})$ : minimal nitrogen requirement for crops
  - $\eta_i$ $(k€/tN^{org})$ : organic $N$-availibility due to livestock
  - $\gamma_i$ $(k€/LU)$ : feed requirement
  - $alpha_i$ $(k€/t)$ : animal product use
  - $\psi_j^C$ $(k€/t)$ : crop quota (sugar beet)
  - $\psi_i^A$ $(k€/t)$ : animal production quota (milk)
  - $\sigma_j$ $(k€/ha)$ : area limit (rotation)
  - $\epsilon_j$ $(k€/ha)$ : manure spreading limit
  - $\delta^+$ , $\delta^-$ $(k€/LU)$ : range of variation of livestock
  - $\beta^X$ (k€$/unit_X$) : generic dual variable related to the generic $X$ primal variable

- **Parameters and functions**
  - $SAU$ (ha) : Utilized Agricultural Area
  - $p_j$ $(k€/t)$ : crop prices
  - $w$ $(k€/tN)$ : nitrogen price
  - $x_j$ $(k€/ha)$ : other variable charges
  - $v_i$ $(k€/LU)$ : animal product value
  - $q$ $(k€/t)$ : feed price
  - $SAU$ $(ha)$ : utilized agricultural area
  - $c_{ij}$ $(LU/t)$ : on-farm feed nutriment

- $b_i$ $(LU/t)$ : bought feed nutriment
- $a_i$ $(tN^{org}/LU)$ : nitrogen output per livestock unit
- $Q_j^C$ $(t)$ : (possible) crop quota
- $Q_j^A$ $(t)$ : (possible) animal production quota
- $s_j$ $(-)$ : crop area limit
- $e_j$ $(tN^{org}/ha)$ : manure spread limit
- $r_j(n)$ $(t/ha)$ : $N$-yield function
- $g_i(f)$ $(t/LU)$ : feed-product function
- $d$ $(-)$ : livestock adjustment ratio
- $A_i^0$ $(LU)$ : reference livestock

# B Annex: The maximization program (P)

$$
(\mathcal{P})\left|\begin{array}{llll}
max_{A,B,C,F,N,S,Y} & \sum_j (p_jY_j - wN_j^{min} - x_jS_j) + \sum_i (v_iG_i - qB_i) & & \\
s.t. & \sum_j S_j & \leq SAU & (\lambda) \\
\forall j : & Y_j + \sum_i C_{ij} - r_j(n_j)S_j & \leq 0 & (\rho_j) \\
\forall j : & n_jS_j - N_j^{min} - \sum_i N_{ij}^{org} & \leq 0 & (\xi_j) \\
\forall i : & \sum_j N_{ij}^{org} - a_iA_i & \leq 0 & (\eta_i) \\
\forall i : & f_iA_i - \sum_j c_{ij}C_{ij} - b_iB_i & \leq 0 & (\gamma_i) \\
\forall i : & G_i - g_i(f_i)A_i & \leq 0 & (\alpha_i) \\
\forall j : & Y_j & \leq Q_j^C & (\psi_j^C) \\
\forall i : & G_i & \leq Q_i^A & (\psi_i^A) \\
\forall j : & \sum_i N_{ij}^{org} - e_jS_j & \leq 0 & (\epsilon_j) \\
\forall j : & S_j - s_j \sum_k S_k & \leq 0 & (\sigma_j) \\
\forall i : & A_i & \leq (1+d)A_i^0 & (\delta_i^+) \\
\forall i : & -A_i & \leq -(1-d)A_i^0 & (\delta_i^-) \\
\text{positivity} : & X_k^Y & \geq 0 & (\beta_j^{X^Y})
\end{array}\right. \qquad ((\mathcal{P}))
$$

# C Annex: The Lagrangian of the problem (P)

$$
\begin{aligned}
\mathcal{L} = & \sum_j (p_j Y_j - w N_j^{min} - x_j S_j) + \sum_i (v_i G_i - q B_i) - \left[ \sum_j S_j - SAU \right] \lambda \\
& - \sum_j \left[ Y_j + \sum_i C_{ij} - r_j(n_j) S_j \right] \rho_j - \sum_j \left[ n_j S_j - N_j^{min} - \sum_i N_{ij}^{org} \right] \xi_j \\
& - \sum_i \left[ \sum_j N_{ij}^{org} - a_i A_i \right] \eta_i - \sum_i \left[ f_i A_i - \sum_j c_{ij} C_{ij} - b_i B_i \right] \gamma_i \\
& - \sum_i \left[ G_i - g_i(f_i) A_i \right] \alpha_i - \sum_j \left[ \sum_i N_{ij}^{org} - e_j S_j \right] \epsilon_j - \sum_j \left[ S_j - s_j \sum_k S_k \right] \sigma_j \\
& - \sum_i \left[ A_i - (1+d) A_i^0 \right] \delta_i^+ - \sum_i \left[ -A_i + (1-d) A_i^0 \right] \delta_i^- \\
& - \sum_j \left[ Y_j - Q_j^C \right] \psi_j^C - \sum_i \left[ G_i - Q_i^A \right] \psi_i^A \\
& + \sum_j S_j \beta_j^S + \sum_j Y_j \beta_j^Y + \sum_{ij} N_{ij}^{org} \beta_{ij}^{N^{org}} + \sum_j N_j^{min} \beta_j^{N^{min}} + \sum_j n_j \beta_j^n \\
& + \sum_i B_i \beta_i^B + \sum_i G_i \beta_i^G + \sum_{ij} C_{ij} \beta_{ij}^C
\end{aligned}
$$

# D Annex: Gross margin variation (€/$ha$)