A Dynamic Optimal Control Model of Crop Thinning

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Abstract:
We develop a general optimal control model of crop thinning applicable to settings in agriculture, horticulture and viticulture. Using a simple optimal control model for the land owner’s profit maximization problem, relevant scenarios are discussed using phase diagram analysis: (1) when the initial crop quantity is sufficiently small, crop thinning is not optimal, (2) when the initial crop quantity is high, it is optimal to thin the crop from the beginning of the relevant planning horizon and to reduce it over time until the stock of the crop has arrived at its optimum. From thereon, crop quantity evolves solely according to natural growth and/or decay. Depending on the model's parameters, this “stopping time” is reached sooner or later.

Key words: Crop thinning, yield management, optimal control
JEL Codes: Q10, C61,

Introduction

Crop thinning or more general yield management are standard instruments to maximize economic returns derived from arable land use. Different methods of crop thinning and yield management applications can be found in agriculture, horticulture, silviculture and viticulture. In viticulture for example, it is well known that high yields will have a negative impact on grape and wine quality (Ough and Nagaoka, 1984, Sun et al. 2011). Because vines have limited productive capacities, excess grapes are removed from the vine in order to raise the quality of the grapes harvested. The vast majority of high quality wines around the world are produced from low yielding vineyards. Similarly, in horticulture excessive fruit (e.g. apples, peaches) is removed from trees in order to raise the overall quality of the remaining fruit and thus economic returns from an orchard (Martín et al. 2010). Thinning and pruning is also used in silviculture in order to optimize the growth rates and wood quality of the remaining trees (Jozsa and Brix 1989). In agriculture, sugar beets are thinned in order to provide enough space for the remaining beets to flourish. For wheat, the literature reports postive thinning effects on kernel weight up to 20% (Fischer and Laing, 1976).

Site specific characteristics such as soil fertility and weather conditions will of course also have a significant impact on yields. In viticulture for example, steep vineyard sites (e.g. on the Mosel river in Germany) are known to have very little organic material. Such vineyard sites often have very low yields but their grapes develop extreme flavors, tannins, and levels of sugar and acidity which turn into high quality crafted wines. More fertile soils can also produce excellent wines, but vineyard managers often have to engage in a number of corrective measures to make it happen. Besides pruning new shoots and excess leaves to guarantee proper exposure to sunlight, it is often necessary to also cut excess grapes during the growing season in an effort to correct the yield and to harvest higher quality grapes later.

From an economic point of view, crop thinning (or yield management) can be regarded as trading-off quantity for quality with substantial economic implications. Higher crop yields may produce quantities at lower qualities which can also be economical within specific price ranges. However, producers may need to reach higher quality products at least partially due to lower levels of production. Producers must find the correct balance between quantity, quality and the ensuing prices obainable in the market.

Literature Review

A large number of applications related to optimal thinning can be found in the forest literature. An early application of optimal control (calculus of variations) to the issue of optimal thinning and rotation length for a forest was done by Schreuder (1971). Medhurst et al. (2003) found that "more frequent, less severe pruning of Acacia melanoxylon (a highly adaptive tree species) may be perferable in terms of maintaining Acacia melanoxylon vigour", and that "with careful management, a nurse-crop system can be manipulated to improve growth and form of Acacia melanoxylon. However, intensive management is required to maximise Acacia melanoxylon growth and optimise stem form." Recently, Sourcy (2011), focussing on forest stands, points out that thinning is often asssumed to improve overall stand profitability. Lu and Gong (2003) develop a model of optimal stocking level and harvest decisions for even-aged trees under uncertain future timber prices. They show that optimal thinning depends on the stand age and on other factors, and that there is a optimal time of final harvest.

In horticulture, crop thinning is primarily used to raise overall fruit quality. Martín et al. (2010) found that thinning of peach crops may be necessary to obtain high yields, and that there is an optimal balance between yield and fruit size, which may depend on factors like the initial crop load, fruit size potential, time of thinning, date of harvest, and on the price of the crop. They also found that crop thinning costs may be considerable and depending on the thinning technique (hand thinning, mechanical thinning or chemical thinning). Meland (2009) analyzed thinning at first bloom and at 20mm fruit size comparing fruit growth and color of Elstar apples in Norway and found that thinning at first bloom resulted in significantly higher fruit weights and improved background fruit color. Lang and Ophardt (2000) report that flower bud thinning in cherries prior to bloom although decreasing total yield up to 25% increased average fruit size up to 43%.

In viticulture, there is also a link between crop load, grape quality and the resulting wine quality, which will have significant implications for wine prices and overall returns from a vineyard. Ough and Nagaoka (1984) found that the quality of California Cabernet Sauvignon was slightly higher when the crop was thinned in at least two out of three years. In a recent study, Sun et al. (2011) found that wine growers implementing specific crop thinning practices would have to receive higher bottle prices (between \$0.02–\$0.41) to compensate for additional labor costs and lost yield in order to maintain their economic welfare.

When examining the literature, we find that there is a quite a lot of empirical field research done to show that thinning is very useful in order to improve the overall quality of specific crops and that it can optimize the economic returns pertaining to the growers. This practical research is available for a variety of crops in agriculture, horticulture, and viticulture. However, to the best of our knowledge there is little formal work on a general model analyzing the relationship between crop load, quality and optimal economic outcomes using a dynamic optimal control approach. This paper proposes a model to close this gap in the literature.

The Model

To study how crop thinning can be done in an optimal way, we apply a dynamic model, based on optimal control theory. We would like to point out that our model differs from thinning models applied to forestry problems. Unlike in the case of timber, our model assumes that the planning horizon is fixed by nature (i.e. the problem of optimal rotation length does not emerge), there are no different aged crops in the field and the crop load which is thinned is not marketable but has to be thrown away. For a more detailed discussion of the forestry literature on harvesting and rotation lengths, see Newman (2002). Moreover, for the sake of simplicity we will assume that there is no uncertainty about demand conditions at terminal time T. Our model is therefore deterministic.

The land owner's problem is to maximize the profit from selling the crop, grown on a given and fixed area of land, at the end of the growing period, facing a downward sloping demand curve for crop. In other words, the crop owner is supposed to have some monopoly power. Reasons for this monopoly power can be that the crop owner is able to influence the quality perception of his product making it a particular varietal expression of the crop, which can be distiguished from other producers (monopolistic competition). For example, such a market is certainly given for high quality wine grapes or regional origin denominated horticultural products. The demand for the crop, x, depends negatively on the price but positively on the crops' quality. x measures the quantity of crop (i. e. the number of crop), not its size, weight or quality. The inverse demand function for the crop is given as

$$p(x,q); \quad p_x < 0, p_q > 0$$

Crop quality is a function of crop quantity, and we reasonably assume that after some amount of crop, the more crop is grown on the given area of land, the lower the crop's quality. Formally, we introduce the crop quality function

$$q = f(x) \quad f'(x) \gtrless 0, f''(x) < 0$$

The owner has the possibility to cut crop in an attempt to increase its quality. Denoting s the rate at which crop is cut, and δ the rate of natural decay of crop, the stock (quantity) of crop, x, evolves according to

$$\dot{x} = -(s+\delta)x$$

Cutting crop and maintaining the crop field causes costs. The cost function is given as

$$C(x,s); \quad C_x > 0, C_s > 0$$

At the end of the planning/crop growing period, achieved at time T, the crop is sold at the market. The crop owner's problem to maximize profits Π by choosing the time path of the cutting rate s, i. e.

$$\Pi = p(x, f(x))x(T)e^{-rT} - \int_0^T C(x,s)e^{-rt}\,dt, \tag{1}$$

where r is the interest rate applied to discount future costs and revenues, subject to

$$\dot{x} = -(s+\delta)x \tag{2a}$$

$$x(t) \geq 0 \tag{2b}$$

$$s(t) \geq 0 \tag{2c}$$

$$x(0) = x_0 \tag{2d}$$

The inequalities (2b) and (2c) state that both the stock of crop on the field and the cutting rate cannot become negative. Equation (2d) is the initial condition, that is the amount of crop given at time 0.

To be able to perform a qualitative analysis, we apply the following functional forms for the inverse demand function, the crop quality function and the cost function

$$p(x, f(x)) = f(x) - bx;\ b > 0 \tag{3a}$$

$$f(x) = k + mx - nx^2; k > 0,\ m > 0,\ n > 0 \tag{3b}$$

$$C(x,s) = \frac{\alpha}{2}x^2 + \frac{\beta}{2}xs^2; \alpha > 0, \beta > 0 \tag{3c}$$

where α, β, b, k, m, and n are positive constants. The inverse demand function (3a) has an ordinate intercept (i. e. a prohibitive price) which depends on crop quality $f(x)$. Crop quality (3b) is modeled as a concave quadratic function. Crop quality first increases with the amount of crop and eventually decreases when crop quantity is above a certain threshold $\breve{x} = \frac{m}{2n}$. Finally, the cost function is quadratic in the amount of crop grown on the given area of land, and the cutting costs depend on crop quantity and increase quadratically with the cutting rate, s.[1]

Using equations (3a) and (3b), we can calculate the crop owner's marginal revenue as

$$MR(x) = k + 2(m-b)x - 3nx^2 \tag{4}$$

Marginal revenue is a concave quadratic function of crop quantity. Using the functional forms (3), the Hamiltonian of the crop owner is

$$H \equiv -\frac{\alpha}{2}x^2 - \frac{\beta}{2}xs^2 - \lambda(s+\delta)x + \gamma_1 x + \gamma_2 s$$

where λ is the shadow value of crop, and γ_1, γ_2 are the Lagrange multipliers attached to the non-negativity constraints on the stock of crop on the field and on the cutting rate, respectively. The first order conditions of the crop owner's problem are

$$-\beta xs - \lambda x + \gamma_2 = 0 \tag{5a}$$

$$\dot{\lambda} - r\lambda = \alpha x + \frac{\beta}{2}s^2 + \lambda(s+\delta) - \gamma_1 \tag{5b}$$

[1] Martín et al. (2010) found a close relationship between cost and field capacity, which, in our model, is proxied with crop quantity, *x*.

together with the complementary slackness conditions

$$x \geq 0, \gamma_1 \geq 0, \gamma_1 x = 0; \quad s \geq 0, \gamma_2 \geq 0, \gamma_2 s = 0$$

and the terminal condition

$$\lambda(T) = MR(x(T)) = k + 2(m-b)x(T) - 3nx(T)^2 \tag{5c}$$

Note that as long as the cutting rate s and the stock of crop x are positive, the multipliers γ_1, γ_2 are zero. Equation (5a) equates the marginal cutting cost to the marginal value of cutting, γ_2. Equation (5b) is a dynamic no-arbitrage condition, requiring that along an optimal path the rate of return on crop has to be equal to the interest rate.[2]

Optimal Policy

Assume that the cutting rate s is initially positive (otherwise the problem would degenerate). Hence, $\gamma_2 = 0$. Also, let the initial stock of crop be positive, so $\gamma_1 = 0$, too. In this case, we can solve equation (5a) for the cutting rate

$$s = -\frac{\lambda}{\beta} \tag{6}$$

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[2] This can be seen by rewriting (5b) as $\frac{\dot{\lambda}}{\lambda} - \frac{1}{\lambda}\left(ax + \frac{\beta}{2}s^2 - \gamma_1\right) - (s+\delta) = r$ The left hand side represents the rate of return on crop, where the first term is the „capital gain", the second term is a "dividend yield" and the third

and insert this into (5b), yielding the equation of motion for the shadow value

$$\dot{\lambda} = \alpha x + \lambda(s+\delta) - \frac{1}{2\beta}\lambda^2 \tag{7a}$$

Substituting the cutting rate (6) into the equation of motion for crop (2a) gives

$$\dot{x} = \left(\frac{\lambda}{\beta} - \delta\right)x \tag{7b}$$

Equations (7a) and (7b) form the dynamic system, given the assumption that both the stock of crop and the cutting rate are positive. System (7) can be shown to have three steady-states, the values of which are denoted by a tilde:

$$\tilde{x} = 0, \tilde{\lambda} = 0 \tag{8a}$$

$$\tilde{x} = 0, \tilde{\lambda} = 2\beta(r+\delta) \tag{8b}$$

$$\tilde{x} = -\beta\delta(r+0.5\delta)/\alpha, \tilde{\lambda} = \beta\delta \tag{8c}$$

none of which, of course, will be actually achieved, if one follows the optimal policy, because a zero terminal stock of crop would imply zero revenues, and a negative terminal stock of crop would violate the non-negativity condition on crop, equation (2b).

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term a loss due to cutting and decay. Note that as long as the stock of crop is positive, $\gamma_1 = 0$.

In case that, say during transition, the cutting rate, *s*, becomes zero, while the stock of crop, *x*, remains positive (implying $\gamma_1 = 0$), equations (5a) and (5b) become

$$\lambda x = \gamma_2 \tag{9}$$

$$\dot{\lambda} = \alpha x + \lambda(r + \delta) \tag{10a}$$

and the equation of motion for the stock of crop is simply

$$\dot{x} = -\delta x \tag{10b}$$

implying an exponential decay of crop. Then equation (9) simply determines the shadow value of cutting, γ_2. Equations (10a) and (10b) are a system of linear differential equations, the roots of which are $\mu_1 = -\delta < 0,\ \mu_2 = r + \delta > 0$. Hence the unique steady-state

$$\tilde{x} = 0,\ \tilde{\lambda} = 0 \tag{11}$$

is saddle point stable. Putting the information provided by system (7) and system (10) together, we can construct a phase diagram for the relevant space, shown in figure 1.[3]

Insert figure 1 here

[3] For the sake of clarity, we have not drawn in the other two steady-states (8b) and (8c), because they are irrelevant.

The red arrowed curves are the stable and unstable branches of systems (7) and (10). We have drawn some trajectories. Any starting point located below the stable saddle going through the origin[4] is unfeasible, as the shadow value of crop remains negative. Thus, a candidate for a starting point, given the initial stock of crop $x(0) = x_0$ has to be located above the stable branch.[5] Hence, we have to determine the initial value of the jump variable λ, and thus we have to figure out which path the crop owner should follow, we have to implement the terminal condition (5c), requiring that the shadow value of crop equals its marginal revenue. Adding the marginal revenue curve to Figure 1 results in Figure 2.

Insert figure 2 here

The terminal condition (5c) requires that at terminal time T the trajectory has to meet exactly the marginal revenue curve. Thus, from figure 2 we can identify two possible outcomes:

First, if the intial quantity of crop is already sufficiently small, as represented by the intial value x_0^2, then it is optimal to not cut crop, but simply to let the crop evolve over time and to harvest and market it at time T. In this case, the shadow value of crop has to be positive right at time zero, and the stock of crop simply evolves according to equation (10b) and decayes exponentially at rate δ. The correct initial shadow value $\lambda(0)$ is determined by the condition that the trajectory has to meet the marginal revenue curve at terminal time T. Not cutting crop in turn keeps the crop owner's costs as low as possible, thus contributing to his intertemporal profit.

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[4] Note that as long as there is some cutting $(s > 0)$, the shadow value λ has to be negative (see equation (6)). Hence, in that case the system's relevant steady state is given by equation (8a), which is the origin.

[5] Note that a position on the stable branch is also not optimal, as the goal is not to achieve the steady-state with a zero stock of crop.

Second, if the initial stock of crop is high, for example, $x(0) = x_0^1$ then the owner's optimal policy has two stages. In the first stage, at the beginning of the planning horizon, it is optimal to cut crop, however at a decreasing rate, as the trajectory confirms. Together with natural decay, the stock of crop shrinks over time and its shadow value becomes less negative. At some point in time, the trajectory crosses the horizontal axis, and the shadow value of crop becomes zero. At that point, the second stage begins. It is now optimal to stop cutting and let the crop simply shrink according to nature. The shadow value of crop becomes positive and increases over time. Eventually, at terminal time T, the trajectory reaches the marginal revenue curve. The crop is then harvested and sold on the market.

There is also a third, but less interesting possibility. It may turn out that both of the above policies, even though they maximize the crop owner's intertemporal profit, result in an overall loss. In this case, the crop owner will simply shut down leaving the land idle. In this case, intertemporal profits would be zero.

Conclusion

Using a simple optimal control model for the crop owner's profit maximization problem, we found that are three possible outcomes for the decision if crop should be thinned. First, if the initial crop quantity is sufficiently small, then it is optimal not to thin the crop, but simply to let it evolve over time and to harvest and market them at time T. Second, if initial crop quantity is high, then it is optimal to cut crop at the beginning of the planning horizon and to reduce cutting (i. e. the cutting rate s falls) over time until the stock of crop has arrived at its optimum. Depending on the model's parameters, this "stopping time" is reached sooner or later. From thereon, crop quantity evolves solely according to natural decay. Third, if the intertemporal profit is negative, it is optimal to shut down.

In this paper we have developed a general optimal control model of crop thinning applicable to a relevant settings in agriculture, horticulture and viticulture. We stress that our model is different from timber harvest models. However, it fits observed behavior for high quality wine production as well as for horticultural products (e.g. apples, peaches, cherries). Comparing scenario 2 with the result obtained by Chapman et al. (2004) who "conclude that Cabernet Sauvignon aromas and flavors respond to yield manipulation, but do so significantly only when yield is altered early in fruit development" underscores the practical relevance of our model.

A limitation of the model is that it is in continuous time rather than in discrete time. Nevertheless, its general conclusion to reduce thinning over time to arrive at an optimal stock of the crop would remain valid. In horticulture, multiple treatments Another limitation of the model is its deterministic nature. Introducing uncertainty would be an interesting extention to the model. Finally, the model implicitly imposes the requirement that all of the exisiting crop at terminal time T has to be sold on the market. A possible extension would be to allow for the possibility to sell only part of the crop and throw away the remaining crop, aiming at "riding the demand curve".

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Figure 1: Phase Diagram with Paths

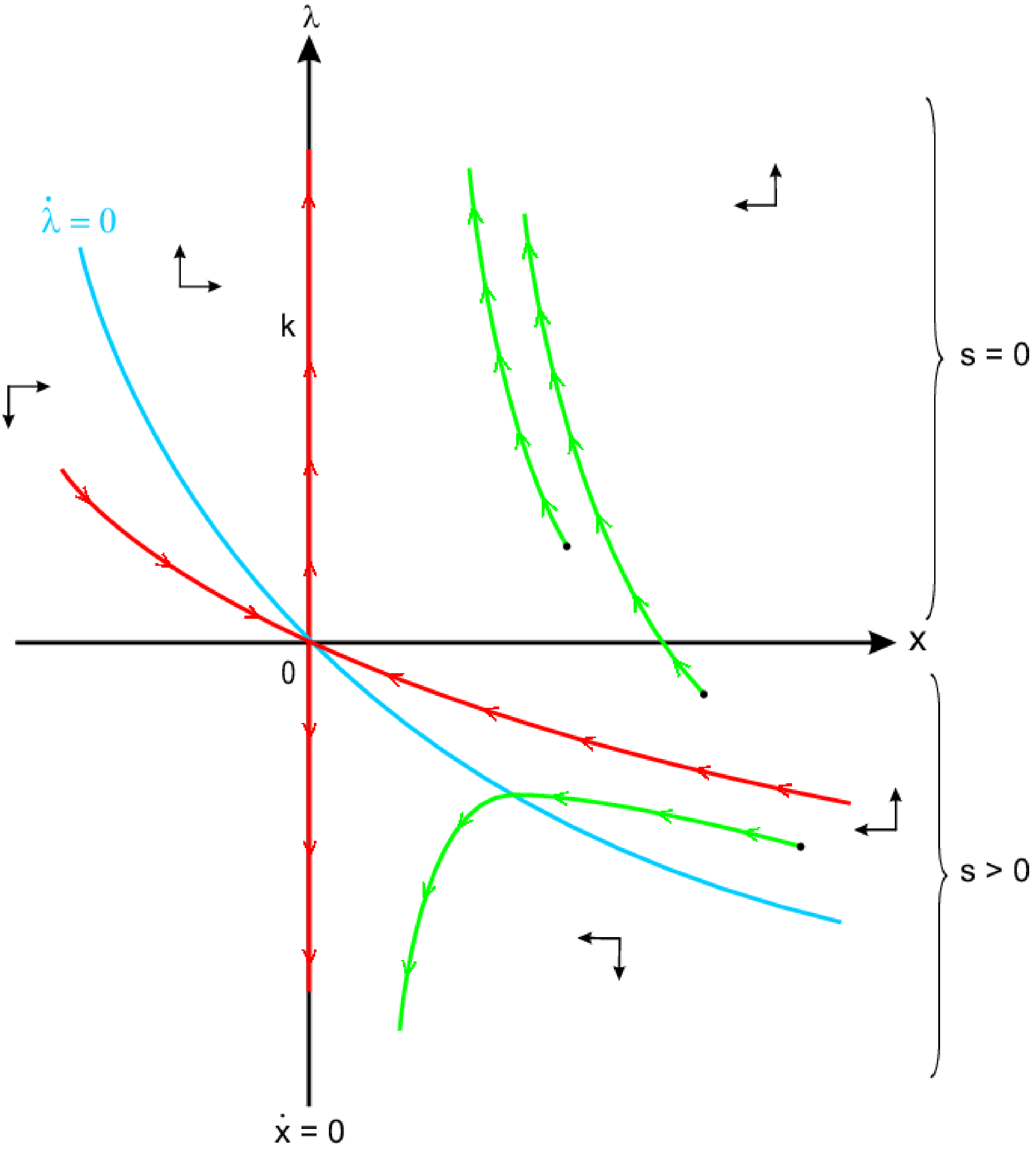

Figure 2: Phase Diagram with MR