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On Market Equilibrium Analysis

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Abstract: The paper develops the implications of competitive market equilibrium for production and household behavior when some prices are endogenously determined. The properties of market equilibrium functions are explored, including the effects of pricing policy. A Slutsky-type equation relating compensated and uncompensated market equilibrium functions is derived. Implications for multi-market welfare analysis are presented, focusing on the effects of pricing policy and technical change.

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On Market Equilibrium Analysis

I. Introduction

Competitive market equilibrium analysis has been the subject of much research. Examples include the study of the effects of pricing policy (e.g., Floyd; Gardner, 1979), of technical change (e.g., Jones), of government regulatory policy (e.g., Hazilla and Kopp), and of the determination of the marketing margin in vertically linked markets (e.g., Gardner, 1975; Wohlgenant). Market equilibrium analysis has become even more relevant over the last decade given the increased reliance on trade and market mechanisms in resource allocation. In this context, it is widely recognized that economic effects in one market may have effects in other markets, which in turn feed back into the market in question. Market equilibrium analysis also plays a central role in the welfare evaluation of technical change and of government pricing or regulatory policy: general equilibrium effects must be considered to properly evaluate the welfare implications of a particular change.

A common approach used in applied general equilibrium analysis is to start from a set of partial equilibrium aggregate supply-demand functions and to use market clearing equations to solve for the market equilibrium prices. The partial equilibrium behavior of competitive firms or households (which treat prices as exogenous) is well established in the literature. Given information concerning partial equilibrium behavior, market equilibrium prices and quantities can then be derived from the market clearing equations using appropriate numerical methods.

An alternative approach is to investigate directly the market equilibrium prices and quantities. The corresponding market equilibrium functions are of interest since they measure the net effect on the relevant variables (e.g., price distortion or technological change) on the general allocation of resources, allowing for economic adjustments in related markets (e.g., Just et al., p. 200-213; Thurman and Wohlgenant; Thurman; Thurman and Easley). The properties of such market equilibrium functions have been investigated by Arrow and Hahn, Diewert, Heiner, and Braulke (1984, 1987). Arrow and Hahn found it difficult to establish these properties in general without imposing a priori restrictions on economic behavior. Diewert and Heiner derived a number of results under the assumption that the partial equilibrium output demand schedules are “normal”, i.e., falling with respect to price. Braulke (1984, 1987) generalized Heiner's results to multi-product industries involved in markets exhibiting less than infinitely elastic output demand or input supply schedules. In the context of final demand, Braulke's "normal condition" is that the matrix of price effects on consumer demand is symmetric and negative semi-definite. While this condition holds for partial equilibrium Hicksian (compensated) demand functions, it does not hold in general for Marshallian (uncompensated) demand functions (i.e., the matrix of Marshallian price effects need not be symmetric negative semi-definite). This suggests that general properties of industry behavior under competitive market equilibrium need further elaboration.

The objective of this paper is to refine the economic and welfare implications of competitive market equilibrium, where some prices are endogenously determined. For example, in a small open economy, while the prices for internationally traded goods are exogenous, prices for non-tradable goods are endogenous. Alternatively, in the analysis of pricing policy, price subsidies and/or taxes can affect prices and quantities in related markets (e.g., Floyd; Gardner, 1979). Finally, technical change in an industry (e.g., agriculture) is expected to affect prices in related markets (e.g., retail food prices). Given that induced price adjustments are likely to be found throughout the economy, this underlies the importance of approaching economic analysis and welfare evaluation from a market equilibrium perspective. In this context, we investigate the effects of changing exogenous factors and price policy instruments (i.e., taxes and subsidies) on resource allocation in all relevant markets. We focus on multi-input/multi-output industries and a household sector, and analyze several issues that have apparently not been addressed previously in the literature. We derive a Slutsky-like equation that illustrates how income effects influence market equilibrium supply-demand functions. This provides new insights on the relationships between compensated and uncompensated behavior in a market equilibrium context (Thurman). We also analyze the market equilibrium effects of technical change.

Building on the work of Just et al., Thurman and Wohlgenant, Thurman, we investigate the implications of our approach for general equilibrium welfare measurements. Our results provide a basis for an empirical evaluation of pricing policy, technical change or government regulatory policy. Contrary to Bullock's findings, we show that the areas behind equilibrium supply-demand curves have welfare significance under fairly general conditions. This can provide simple measures of the net effects of alternative pricing policies on all sectors of the economy. We also investigate how partial equilibrium welfare measures (taking prices as given) differ from their market equilibrium counterparts (allowing for induced price adjustments) (see Thurman and Easley). Our results indicate that the neglect of induced price adjustments provides an upward biased estimate in welfare change. This provides additional insights and clarifications for the analysis of pricing policy. Finally, we propose a simple measure of the welfare effects of technical change in a market equilibrium framework. Martin and Alston have expressed the need to evaluate the welfare consequences of technical change in interrelated markets. However, their approach relies on a partial equilibrium analysis, treating all prices as exogenous. By taking into consideration induced price adjustments throughout the economy, our approach provides more general insights in the analysis and welfare measurement of technical change.

The plan of the paper is as follows. Section II develops the notation and characterizes the market equilibrium comprised of competitive multi-output/multi-input firms and households. Sections III and IV examine the properties of the associated compensated and uncompensated market equilibrium supply-demand functions. Implications of the analysis for multi-market welfare and policy analysis are presented in Section V.

II. The Characterization of Market Equilibrium

Consider an economy constituted of firms and households facing competitive markets.

A - Firms:

Denote by J the set of firms marketing a vector of commodities purchased and sold in the competitive markets. Associate with each commodity an index m = 1, 2, ..., and denote by M the set of these indexes, M = {1, 2, ...}. Denote the associated price vector by $p = \{p_m: m \in M\}$, where p_m is the competitive market price of the m-th commodity.

Consider a particular firm, say the j-th firm, producing the netput vector y_j and facing a production technology represented by a non-empty, closed, convex production possibility set $T_j(\alpha_j)$, $y_j \in T_j(\alpha_j)$, where α_j is a vector of technology parameters, $j \in J$. This allows for heterogeneity across firms, as each firm may face a different technology. We have $y_j = \{y_{mj}: m \in M\}$, where y_{mj} is the quantity of the m-th netput either used or produced by the j-th firm. We use the netput notation where positive elements of y_j denote outputs while negative elements denote inputs, $j \in J$. The case where the j-th firm specializes in one or a few activities would imply that some elements of the vector y_j are zero, corresponding to the commodities not used nor produced by the j-th firm.

Assuming that economic decisions in the j-th firm are made to maximize profit, we have

$$\pi_j(p, \alpha_j) = p' \, y_j^*(p, \alpha_j) = \operatorname*{Max}_{y_j} \{p' \, y_j: \; y_j \in T_j(\alpha_j)\} \tag{1}$$

where $y_j^*(p, \alpha_j)$ is the profit maximizing netput decision vector, and $\pi_j(p, \alpha_j)$ is the indirect profit function or quasi-rent for the j-th firm, $j \in J$. Expression (1) defines a partial equilibrium model of production where decisions depend on relevant market prices which are treated as exogenous. The economic implications of model (1) are well known (e.g., Lau; Fuss and McFadden). The indirect profit function $\pi_j(p, \alpha_j)$ is linear homogeneous and convex in prices. Under differentiability, it satisfies Hotelling's lemma:

$\partial\pi_j(p, \alpha_j)/\partial p_j = y_j^*(p, \alpha_j)$. The choice functions $y_j^*(p, \alpha_j)$ are homogenous of degree zero in prices p and, under differentiability, the matrix $(\partial y_j^*/\partial p)$ is symmetric, positive semi-definite.

B - Households:

While some of the commodities produced by the firms are intermediate products used in the production of other goods, others may be final products purchased by households. Also, the households sell labor to the producing firms.[1] Letting H be the set of households, consider a particular household, say the h-th household, $h \in H$. Denote by y_h the netput quantity vector of final products consumed and labor supplied by the h-th household. We have $y_h = \{y_{mh}: m \in M\}$, where y_{mh} is the quantity of the m-th netput either purchased or sold by the h-th household. By convention, we define the elements of the vector y_h to be positive for purchased consumer goods and negative for commodities sold (e.g., labor supply). The case where the h-th household does not purchase or sell some commodities would imply that the corresponding elements of the vector y_h are zero.

Assume that consumption-labor decisions are made in a way consistent with utility maximization subject to a budget constraint. Let $u_h(y_h, \alpha_h^u)$ be the (direct) utility function of the h-th household, α_h^u being a vector of preference parameters. Denote exogenous non-labor household income by x_h, and the feasible set for y_h by $T_h(\alpha_h^t)$, $y_h \in T_h(\alpha_h^t)$, where α_h^t is a vector of parameters reflecting possible constraints (e.g., rationing) facing the h-th household. This allows for heterogeneity across households, as each household may face different income x_h or parameters (α_h^u, α_h^t). Then, the h-th household decisions can be represented by

$$w_h(p, x_h, \alpha_h) = u_h(y_h^*(p, x_h, \alpha_h), \alpha_h^u) = \max_{y_h} \{u_h(y_h, \alpha_h^u): p' y_h \leq x_h;\ y_h \in T_h(\alpha_h^t)\}, \tag{2}$$

where $\alpha_h = (\alpha_h^u, \alpha_h^t)$, $y_h^*(p, x_h, \alpha_h)$ are Marshallian choice functions, and $w_h(p, x_h, \alpha_h)$ is the indirect utility function, $h \in H$. Expression (2) defines a partial equilibrium model where consumption-labor decisions depend on exogenous non-labor income x_h and prices p treated as exogenous variables. The economic implications of model (2) are well known (e.g., Deaton and Muellbauer). Under non-satiation with respect to income, the indirect utility function $w_h(p, x_h, \alpha_h)$ is homogenous of degree zero and quasi-convex in (p, x_h). Also, the choice functions $y_h^*(p, x_h, \alpha_h)$ are homogeneous of degree zero in (p, x_h).

Additional properties of $y_h^*(p, x_h, \alpha_h)$ are associated with the function

$$e_h(p, U_h, \alpha_h) = p'\, y_h^c(p, U_h, \alpha_h) = \min_{y_h}\{p'\, y_h: U_h \leq u(y_h, \alpha_h^u);\ y_h \in T_h(\alpha_h^t)\}, \tag{3}$$

where $\alpha_h = (\alpha_h^u, \alpha_h^t)$, $y_h^c(p, U_h, \alpha_h)$ are compensated Hicksian choice functions holding utility constant at level U_h, and $e_h(p, U_h, \alpha_h)$ is the expenditure function for the h-th household, $h \in H$. Throughout the paper, the superscript “c” will be used to denote “compensated” functions, holding consumer welfare constant. The functions $w_h(p, x_h, \alpha_h)$ and $e_h(p, U_h, \alpha_h)$ are dual: they are inverse functions of each other as $w_h(p, e_h(p, U_h, \alpha_h), \alpha_h) = u_h$, or $e_h(p, w_h(p, x_h, \alpha_h), \alpha_h) = x_h$, $h \in H$. The function $e_h(p, U_h, \alpha_h)$ is linear homogeneous and concave in p (e.g., Deaton and Muellbauer). Under differentiability, it satisfies Shephard’s lemma: $\partial e_h(p, U_h, \alpha_h)/\partial p = y_h^c(p, U_h, \alpha_h)$. The Hicksian choice functions $y_h^c(p, U_h, \alpha_h)$ are homogeneous of degree zero in prices p and, by duality, satisfy $y_h^c(p, U_h, \alpha_h) = y_h^*(p, e_h(p, U_h, \alpha_h), \alpha_h)$. Under differentiability, this generates the Slutsky equation: $\partial y_h^c/\partial p = \partial y_h^*/\partial p + (\partial y_h^*/\partial x_h)\, y_h^*$ = a symmetric, negative semi-definite matrix.

C - Market Equilibrium:

Consider partitioning the set of commodities M into two subsets: M = (K, R), the subset K being associated with "endogenous prices" determined through market equilibrium, and the subset R being associated with "exogenous prices". Let $p = (p_K, p_R)$, where $p_K = \{p_m: m \in K\}$ is the price vector for all netputs in K, and $p_R = \{p_m: m \in R\}$ is the price vector for all netputs in R. This can be interpreted to represent a small open economy, where the commodities in K are non-tradable goods (with prices p_K

determined on domestic market), while the commodities in R are internationally traded goods (with "exogenous" prices p_R determined in the world market). Alternatively, the "large country" case would correspond to a situation where all prices are endogenously determined through market equilibrium, i.e. where M = K. We focus our attention here on the price determination for p_K under competitive market conditions.

Allowing for government pricing policy, we consider the case where possible taxes (subsidies) are paid (received) by a target group of firms and/or households. The exact nature of the target group typically depends on the policy context. For example, deficiency payments in agriculture are subsidies targeted to domestic producers, while import taxes affect both domestic producers and domestic consumers. To simplify the presentation, throughout the paper, we will take the target group for taxes or subsidies as given. In this context, we evaluate the effects of pricing policy on resource allocation and welfare. Denote by $s = \{s_k: k \in K\}$ the vector of taxes/subsidies associated with the prices p_K, where s_k is the price subsidy (or tax if negative) for the k-th commodity, $k \in K$. Assume that firms or households outside the target group face the price p_K, while firms or household members in the target group face the prices $(p_K + s)$. This means that the vector s represents pricing policy generating price wedges for the commodities K between the agents in the target group and those outside the target group. Relative to the agents within the target group, $s_k > 0$ is a price subsidy to the producers of the k-th commodity, and/or a tax to the consumers of the k-th commodity. Alternatively, $s_k < 0$ is a price tax to the producers of the k-th commodity, and/or a subsidy to the consumers of the k-th commodity. The absence of pricing policy is a special case where s = 0, corresponding to standard competitive market conditions for the commodities K.

Let $\delta_i = 1$ if the i-th firm or household is a member of the target group (i.e., if it faces prices (p_K+s) for commodities in K), and $\delta_i = 0$ otherwise (i.e., if it faces prices p_K), for $i \in$ (J or H). Then, the uncompensated supply functions from the j-th firm is $y_j^*(p_K+\delta_j s, p_R, \alpha_j) = (y_{Kj}^*, y_{Rj}^*)$, $j \in J$, while the uncompensated demand functions from the h-th household is $y_h^*(p_K+\delta_h s, p_R, x_h, \alpha_h) = (y_{Kh}^*, y_{Rh}^*)$. And the compensated demand functions from the h-th household is $y_h^c(p_K+\delta_h s, p_R, U_h, \alpha_h) = (y_{Kh}^c, y_{Rh}^c)$. These functions are all <u>partial equilibrium</u> functions treating the prices $p = (p_K, p_R)$ as exogenous.

At the aggregate level, we will be interested in the <u>aggregate net quantities supplied</u> from all agents. This will be denoted by the vector $N = (N_K, N_R) = \sum_{j\in J} y_j - \sum_{h\in H} y_h$, which measures the aggregate quantities of all commodities (K, R) supplied by all firms ($\sum_{j\in J} y_j$) minus the aggregate quantities demanded by all households ($\sum_{h\in H} y_h$). We will also consider the vector of aggregate net quantities supplied from the agents targeted by pricing policy (and thus facing prices p_K+s). This will be denoted by the vector $N_T = (N_{KT}, N_{RT}) = \sum_{j\in J} \delta_j y_j - \sum_{h\in H} \delta_h y_h$, which measures the aggregate quantities of all commodities (K, R) produced by the firms facing prices (p_K+s), minus the aggregate quantities demanded from the corresponding households. For example, we denote the vector of aggregate partial equilibrium net supply functions from all agents by

$$N^*(p_K, p_R, s, x, \alpha) \equiv (N_K^*, N_R^*) \equiv \sum_{j\in J} y_j^*(p_K+\delta_j s, p_R, \alpha_j) - \sum_{h\in H} y_h^*(p_K+\delta_h s, p_R, x_h, \alpha_h),$$

where $x = \{x_h: h \in H\}$ denotes the distribution of income across all households, $\alpha = (\alpha_J, \alpha_H)$, $\alpha_J = \{\alpha_j: j \in J\}$ is the vector of technology parameters, and $\alpha_H = \{\alpha_h: h \in H\}$ is the vector of household preference shifters. Similarly, we define the vector of aggregate partial equilibrium net supply functions from the firms and households targeted by pricing policy (and thus facing prices p_K+s) as

$$N_T^*(p_K, p_R, s, x, \alpha) \equiv (N_{KT}^*, N_{RT}^*) \equiv \sum_{j\in J} \delta_j\, y_j^*(p_K+s, p_R, \alpha_j) - \sum_{h\in H} \delta_h\, y_h^*(p_K+s, p_R, x_h, \alpha_h).$$

Now, consider the market determination for the prices p_K. <u>Uncompensated market equilibrium</u> for the commodities in the set K is characterized by

$$N_K^*(p_K, p_R, s, \alpha) \equiv \sum_{j\in J} y_{Kj}^*(p_K+\delta_j s, p_R, \alpha_j) - \sum_{h\in H} y_{Kh}^*(p_K+\delta_h s, p_R, x, \alpha_h) = 0, \qquad (4a)$$

where y_{Kj}^* is the vector of the j-th firm supply functions for commodities in K, while y_{Kh}^* is the vector of the h-th household Marshallian demand functions for the commodities in K. Equation (4a) simply states that, under market equilibrium, excess demand is zero for all the commodities in K. Note that equation (4a) allows for products with different uses in different firms or households. For example, this is typically the case for household labor which is allocated among the producing firms. Implicitly, solving (4a) for the vector of endogenous prices p_K (and assuming that a unique solution exists) yields the uncompensated market equilibrium price functions $p_K^*(p_R, s, x, \alpha)$. When $s \neq 0$, this means that, after the markets clear, agents targeted by pricing policy face prices (p_K^*+s) while others face prices p_K^*. This is illustrated in Figure 1, representing a subsidy $s_k > 0$ paid to the producers of the k-th commodity.

Following Braulke (1987), p_K can be interpreted as the "endogenous component" of prices, while s is the "exogenous component" of prices for the commodities in K. In this context, under a "small country" assumption, the prices p_R can be interpreted as exogenous prices reflecting "world market conditions" for commodities in R. And while the prices p_K are endogenously determined by market equilibrium conditions, they are influenced by the tax/subsidy vector s reflecting pricing policy targeted toward some target group (e.g., domestic producers benefiting from "deficiency payments"; domestic producers and consumers affected by import tax; etc.). Note that our approach is very general in the sense that it can handle a full general equilibrium approach under a "large country" assumption where all markets clear. This a special case of our analysis where $p = p_K$ and all prices are endogenously determined through market equilibrium (as influenced by the pricing policy instruments s and the associated target group).

Turning to compensated behavior, we denote the vector of aggregate partial equilibrium compensated net supply functions from all agents by

$$N^c(p_K, p_R, s, U, \alpha) \equiv (N_K^c, N_R^c) \equiv \sum_{j\in J} y_j^*(p_K+\delta_j s, p_R, \alpha_j) - \sum_{h\in H} y_h^c(p_K+\delta_h s, p_R, U_h\ \alpha_h),$$

where $U = \{U_h: h \in H\}$. Similarly, we define the vector of aggregate partial equilibrium net supply functions from the firms and households targeted by pricing policy (and thus facing prices p_K+s) as

$$N_T^c(p_K, p_R, s, U, \alpha) \equiv (N_{KT}^c, N_{RT}^c) \equiv \sum_{j\in J} \delta_j\, y_j^*(p_K+s, p_R, \alpha_j) - \sum_{h\in H} \delta_h\, yh^c(p_K+s, p_R, U_h, \alpha_h).$$

In a way similar to uncompensated equilibrium, we define compensated market equilibrium for the commodities in the set K as follows

$$N_K^c(p_K, p_R, s, U, \alpha) \equiv \sum_{j\in J} y_{Kj}^*(p_K+\delta_j s, p_R, \alpha_j) - \sum_{h\in H} y_{Kh}^c(p_K+\delta_h s, p_R, U_h, \alpha_h) = 0, \tag{4b}$$

where y_{Kh}^c is the vector of compensated (Hicksian) demand functions from the h-th household for the commodities in K. Implicitly, solving (4b) for the vector of endogenous prices p_K (and assuming that a unique solution exists) yields the compensated market equilibrium price functions $p_K^c(p_R, s, U, \alpha)$. When $s \neq 0$, this means that, after the markets clear, agents targeted by pricing policy would face prices (p_K^c+s), while others face prices p_K^c.

This allows the following definitions of firm (or household) level market equilibrium choice functions for all commodities (netputs) in M:

$$y_j^{e*}(p_R, s, x, \alpha) = y_j^*(p_K^*(p_R, s, x, \alpha)+\delta_j s, p_R, \alpha_j), \; j \in J, \tag{5a}$$

$$= y_j^*(p_K^*(p_R, s, x, \alpha)+\delta_j s, p_R, x_j, \alpha_j), \; j \in H, \tag{5b}$$

and

$$y_j^{ec}(p_R, s, U, \alpha) = y_j^*(p_K^c(p_R, s, U, \alpha)+\delta_j s, p_R, \alpha_j), \; j \in J, \tag{6a}$$

$$= y_j^c(p_K^c(p_R, s, U, \alpha)+\delta_j s, p_R, U_j, \alpha_j), \; j \in H. \tag{6b}$$

Expressions (5a) and (5b) are uncompensated market equilibrium choice functions, while (6a) and (6b) are compensated market equilibrium choice functions holding households' utility constant.[2] In either case, the firm (or household) level functions do not depend on p_K as the prices p_K now endogenously adjust to changing market conditions given market equilibrium (4a) or (4b). Throughout the paper, we use the superscript "e" to denote market equilibrium functions.

Using (5) and (6), we can define the corresponding aggregate market equilibrium supply-demand functions. From (5a) and (5b), aggregate uncompensated net supply functions for all agents are

$$N^{e*}(p_R, s, x, \alpha) \equiv (N_K^{e*}, N_R^{e*}) \equiv \Sigma_{j\in J}\, y_j^{e*}(p_R, s, x, \alpha) - \Sigma_{h\in H}\, y_h^{e*}(p_R, s, x, \alpha)$$

$$= N^*(p_K^*(p_R, s, x, \alpha), p_R, s, x, \alpha).$$

Also, $N_T^{e*}(p_R, s, x, \alpha) \equiv \Sigma_{j\in J}\, \delta_j\, y_j^{e*}(p_R, s, x, \alpha) - \Sigma_{h\in H}\, \delta_h\, y_h^{e}(p_R, s, x, \alpha)$ will denote the vector of aggregate market equilibrium uncompensated net supply functions for all agents targeted by pricing policy (i.e., facing prices p_K+s).

In the same fashion, we can define aggregate market equilibrium compensated functions. From (6a) and (6b), aggregate compensated net supply functions for all agents are

$$N^{ec}(p_R, s, U, \alpha) \equiv (N_K^{ec}, N_R^{ec}) \equiv \Sigma_{j\in J}\, y_j^{ec}(p_R, s, U, \alpha) - \Sigma_{h\in H}\, y_h^{ec}(p_R, s, U, \alpha)$$

$$= N^*(p_K^c(p_R, s, U, \alpha), p_R, s, \alpha).$$

Finally, we denote $N_T^{ec}(p_R, s, U, \alpha) = \Sigma_{j\in J}\, \delta_j\, y_j^{ec}(p_R, s, U, \alpha) - \Sigma_{h\in H}\, \delta_j\, y_h^{ec}(p_R, s, U, \alpha)$ as the aggregate market equilibrium compensated net supply functions for all firms and households targeted by pricing policy (i.e., facing prices p_K+s). Throughout the paper, we assume that these functions are differentiable.[3] Their properties are analyzed in the following sections.

We will also be interested in aggregate (market level) profit and expenditure functions. In a partial equilibrium framework, the aggregate profit function across all firms is denoted by $\Pi(p, s, \alpha_J) = \Sigma_{j\in J}\, \pi_j(p_K+\delta_j s, p_R, \alpha_j)$, and the aggregate expenditure function across all households by $E(p, s, U, \alpha_H) = \Sigma_{h\in H}\, e_h(p_K+\delta_h s, p_R, U_h, \alpha_h)$. Then, aggregate willingness-to-pay can be measured as aggregate profit net of consumer expenditure:

$$V(p, s, U, \alpha) = \Pi(p, s, \alpha_J) - E(p, s, U, \alpha_H), \tag{7}$$

where $\alpha = (\alpha_J, \alpha_H)$. The function $V(p, s, U, \alpha)$ in (7) is a partial equilibrium aggregate welfare measure that takes all prices $p = (p_K, p_R)$ as exogenous.

Using the compensated market equilibrium price function $p_K^c(p_R, s, U, \alpha)$ obtained from (4b), the aggregate profit and expenditure functions can also be defined in a market equilibrium context. First, the market equilibrium aggregate profit function is $\Pi^e(p_R, s, U, \alpha) = \Pi(p_K^c(p_R, s, U, \alpha), p_R, s, \alpha_J)$. Second, the market equilibrium expenditure function is $E^e(p_R, s, U, \alpha) = E(p_K^c(p_R, s, U, \alpha), p_R, s, U, \alpha_H)$. These results can be combined to obtain a market equilibrium aggregate willingness-to-pay:

$$V^e(p_R, s, U, \alpha) = \Pi^e(p_R, s, U, \alpha) - E^e(p_R, s, U, \alpha) \tag{8a}$$

$$= V(p_K^c(p_R, s, U, \alpha), p_R, s, U, \alpha). \tag{8b}$$

Expression (8a) is the sum of the profits across all firms, minus the sum of households' expenditures (holding utility constant), letting the prices p_K adjust to changing market conditions. This general equilibrium measure will be of interest in multi-market welfare analysis (see section V below). The properties of these functions are discussed next.

III. Compensated Market Equilibrium

In this section, we analyze the properties of compensated market equilibrium functions, at both the micro level (i.e., firm or household) and the aggregate level. First, the compensated market equilibrium prices $p_K^c(p_R, s, U, \alpha)$ are obtained from (4b). Since y_j^* and y_h^c are homogeneous of degree zero in prices, it follows that the functions $p_K^c(p_R, s, .)$ are linear homogeneous in (p_R, s): a proportional change in all prices has no real effect on resource allocation. Also, given $p = (p_K, p_R)$ and using the implicit function theorem in (4b), the equilibrium price functions $p_K^c(p_R, s, U, \alpha)$ satisfy

$$\partial p_K{}^c/\partial\gamma \;=\; -[\partial N_K{}^c/\partial p_K]^{-1}\, \partial N_K{}^c/\partial\gamma, \tag{9}$$

where $N_K{}^c$ is the vector of (partial equilibrium) aggregate net supply functions for commodities K from all firms and households, $\gamma = (p_R, s, U, \alpha)$, and the matrix $\partial N_K{}^c/\partial p_K$ is assumed non-singular.[4] It follows that the firm (or household) compensated market equilibrium functions $y_j{}^{ec}(p_R, .)$ in (6a) and (6b) are homogeneous of degree zero in (p_R, s) and satisfy

$$\partial y_j{}^{ec}/\partial(s, p_R) = \partial y_j{}^*/\partial(s, p_R) - \partial y_j{}^*/\partial p_K\, [\partial N_K{}^c/\partial p_K]^{-1}\, \partial N_K{}^c/\partial(s, p_R),\;\; j \in J, \tag{10a}$$

$$\partial y_j{}^{ec}/\partial(s, p_R) = \partial y_j{}^c/\partial(s, p_R) - \partial y_j{}^c/\partial p_K\, [\partial N_K{}^c/\partial p_K]^{-1}\, \partial N_K{}^c/\partial(s, p_R),\;\; j \in H. \tag{10b}$$

The behavioral properties associated with the (compensated) <u>aggregate net supplies</u> $N = (N_K, N_R) = (\Sigma_{j\in J}\, y_j - \Sigma_{h\in H}\, y_h)$ and $N_T = (N_{KT}, N_{RT}) = (\Sigma_{j\in J}\, \delta_j\, y_j - \Sigma_{h\in H}\, \delta_h\, y_h)$ have been investigated previously by Diewert, Heiner, and Braulke (1984, 1987). Extending these earlier results, these properties are presented next. (See the proof in the Appendix).

<u>Proposition 1</u>:

$$\partial(N_{KT}{}^c, N_R{}^c)/\partial(s, p_R) = \text{a symmetric, positive semi-definite matrix}, \tag{11a}$$

$$\partial(N_{KT}{}^{ec}, N_R{}^{ec})/\partial(s, p_R) = \text{a symmetric, positive semi-definite matrix}, \tag{11b}$$

and

$$[\partial(N_{KT}{}^c, N_R{}^c)/\partial(s, p_R) - \partial(N_{KT}{}^{ec}, N_R{}^{ec})/\partial(s, p_R)] = \text{symmetric, positive semi-definite}, \tag{11c}$$

where $N_R{}^c = (\Sigma_{j\in J}\, y_j{}^* - \Sigma_{h\in H}\, y_h{}^c)$ are aggregate compensated <u>partial equilibrium</u> (taking prices $p = (p_K, p_R)$ as exogenous) net supply functions for the commodities in R from all firms and households, $N_{KT}{}^c = (\Sigma_{j\in J}\, \delta_j\, y_{Kj}{}^* - \Sigma_{h\in H}\, \delta_h\, y_{Kh}{}^c)$ are similar aggregate net supply functions for the commodities in K from the agents targeted by pricing policy, $N_R{}^{ec} = (\Sigma_{j\in J}\, y_{Rj}{}^{ec} - \Sigma_{h\in H}\, y_{Rh}{}^{ec})$ are aggregate compensated <u>market equilibrium</u> (letting prices p_K adjust) net supply functions for the commodities in R from all firms and households, and $N_{KT}{}^{ec} = (\Sigma_{j\in J}\, \delta_j\, y_{Kj}{}^{ec} - \Sigma_{h\in H}\, \delta_h\, y_{Kh}{}^{ec})$ are similar aggregate net supply functions for the commodities in K from the agents targeted by pricing policy.

Expression (11a) states that the matrix of partial equilibrium effects of exogenous prices (s, p_R) on aggregate compensated net supply functions (N_{KT}^c, N_R^c) is symmetric, positive semi-definite.[5] This follows from the symmetry and positive semi-definiteness of $\partial y_j^*/\partial p$ for the j-th firm, $j \in J$, and the symmetry and negative semi-definiteness of $\partial y_h^c/\partial p$ for the h-th household, $h \in H$. It implies the well known result that partial equilibrium aggregate compensated net supply functions are necessarily upward sloping.

Expression (11b) establishes that a similar result also holds in a market equilibrium framework for aggregate compensated net supply functions. This extends earlier results proved by Diewert, Heiner and Braulke (1984, 1987). Diewert, Heiner, and Braulke (1984) derived the properties of N_R^c and N_R^{ec} with respect to p_R. And Braulke (1987) derived the properties of N_{KT}^{ec} with respect to s. Equation (11b) goes beyond previous literature by presenting the joint effects of (s, p_R) on (N_{KT}^{ec}, N_R^{ec}). It relies on the condition that $\partial(\Sigma_{h\in H}\, y_{Kh}^c)/\partial p_K$ is a symmetric, negative semi-definite matrix. This has been called the "normal condition" by Heiner and Braulke (1984, 1987). This condition is satisfied in proposition 1 because of the symmetry, negative semi-definiteness of the Slutsky matrix ($\partial y_h^c/\partial p$) for the h-th household, $h \in H$. Equation (11b) implies that, like their partial equilibrium counterpart, the aggregate compensated market equilibrium net supply functions (N_{KT}^{ec}, N_R^{ec}) are necessarily upward sloping with respect to the "exogenous" prices (s, p_R).

Expression (11c) establishes a general relationship between the properties of partial equilibrium and market equilibrium compensated aggregate net supply functions. Again, this generalizes previous results obtained by Diewert, Heiner and Braulke (1984) under the "normal condition". Diewert, Heiner and Braulke (1984) proved that [$\partial N_R^c/\partial p_R - \partial N_R^{ec}/\partial p_R$] is a symmetric, positive semi-definite matrix.

Expression (11c) extends their analysis to the joint effects of (s, p_R) on the difference between the partial equilibrium aggregate net supply functions (N_{KT}^c, N_R^c) and their market equilibrium counterparts (N_{KT}^{ec}, N_R^{ec}). It shows that price adjustments through market equilibrium tend to reduce the magnitude of adjustments in aggregate compensated quantities (N_{KT}^c, N_R^c). In other words, (11c) of proposition 1 states that letting p_K adjust tends to reduce the aggregate compensated net supply elasticities (or the absolute value of net demand elasticities) in the sense that: $0 \leq \partial(N_{KT}^{ec}, N_R^{ec})/\partial(s, p_R) \leq \partial(N_{KT}^c, N_R^c)/\partial(s, p_R)$.

Next consider the aggregate willingness-to-pay functions $V(p, s, U, \alpha)$ in (7) and $V^e(p_R, s, U, \alpha)$ in (8). Some properties of these functions are presented next. (See the proof in the Appendix).

Proposition 2: The partial equilibrium aggregate willingness-to-pay function $V(p, s, U, \alpha)$ satisfies

$$\partial V/\partial s = N_{KT}^c(p, s, U, \alpha) = \Sigma_{j\in J}\, \delta_j\, y_{Kj}^*(p, s, \alpha_j) - \Sigma_{h\in H}\, \delta_h\, y_{Kh}^c(p, s, U_h, \alpha_h), \tag{12a}$$

$$\partial V/\partial p = N^c(p, s, U, \alpha) = \sum_{j\in J} y_j^*(p, s, \alpha_j) - \sum_{h\in H} y_h^c(p, s, U_h, \alpha_h), \tag{12b}$$

$$\partial V/\partial \alpha = p'\, \partial N^c(p, s, U, \alpha)/\partial \alpha, \tag{12c}$$

where $\alpha = (\alpha_J, \alpha_H)$, and $p = (p_K, p_R)$. Alternatively, treating prices p_K as endogenously determined through market equilibrium and prices p_R as exogenous, the market equilibrium aggregate willingness-to-pay function $V^e(p_R, s, U, \alpha)$ satisfies

$$\partial V^e/\partial s = N_{KT}^{ec}(p_R, s, U, \alpha)$$

$$= \Sigma_{j\in J}\, \delta_j\, y_{kj}^{ec}(p_R, s, U, \alpha) - \Sigma_{h\in H}\, \delta_h\, y_{kh}^{ec}(p_R, s, U, \alpha), \tag{13a}$$

$$\partial V^e/\partial p_R = N_R^{ec}(p_R, s, U, \alpha)$$

$$= \sum_{j\in J} y_{Rj}^{ec}(p_R, s, U, \alpha) - \sum_{h\in H} y_{Rh}^{ec}(p_R, s, U, \alpha), \tag{13b}$$

$$\partial V^e/\partial \alpha = p'\, \partial N^c(p, s, \alpha)/\partial \alpha, \text{ evaluated at } p_K = p_K^c(p_R, s, U, \alpha), \tag{13c}$$

$$= p_R'\, \partial N_R^{ec}(p_R, s, U, \alpha)/\partial \alpha. \tag{13d}$$

where $\alpha = (\alpha_J, \alpha_H)$.

Proposition 2 states that the market equilibrium function $V^e(.)$ has properties similar to its partial equilibrium counterpart V(.). In particular, expression (12a) is a well-known envelope result applied to partial equilibrium analysis at the aggregate level: the derivative of the compensated willingness-to-pay V(p, .) with respect to price vector p is equal to the vector of compensated net supply functions. It derives directly from Hotelling's lemma and Shephard's lemma. Equations (13a) and (13b) show that a similar envelope result applies in a market equilibrium framework. In particular, in (13a), the derivative of market equilibrium aggregate willingness-to-pay V^e with respect to the subsidy vector s generates $N_{KT}{}^{ec}(p_R, s, U, \alpha)$, the aggregate market equilibrium net supply function for commodities K from all firms and households targeted for pricing policy. And in (13b), the derivative of market equilibrium aggregate willingness-to-pay V^e with respect to p_R generates $N_R{}^{ec}(p_R, s, U, \alpha)$, the aggregate market equilibrium compensated net supply functions for the commodities in R from all firms and households. The usefulness of this result will be further discussed below.

The impact of technical change or preference shifts (as measured by $\alpha = (\alpha_J, \alpha_H)$) on the aggregate willingness-to-pay functions V and V^e is given in equations (12c), (13c) and (13d). While equation (12c) is trivial, equations (13c) and (13d) appear to be new. They are "envelope-type" results applied to market equilibrium functions. They present alternative measures of the marginal effect of a change in α on the market equilibrium aggregate willingness-to-pay V^e. The usefulness of (13c) and (13d) in the welfare analysis of technical change or government regulations will be further explored in section V below.

Combining the results obtained in propositions 1 and 2, the properties of and relationship between the aggregate, partial equilibrium willingness-to-pay, V, and its market equilibrium counterpart, V^e, are given by the following proposition. (See the proof in the Appendix).

Proposition 3: The partial equilibrium aggregate willingness-to-pay function $V(p, s, U, \alpha)$ is linear homogeneous and convex in prices (p_K, p_R, s). This implies that $\partial^2V/\partial p^2 = \partial N^c/\partial p$ is a symmetric, positive semi-definite matrix where $p = (p_K, p_R)$, and that $\partial^2V/\partial(s, p_R)^2 = \partial(N_{KT}{}^c, N_R{}^c)/\partial(s, p_R)$ is a symmetric, positive semi-definite matrix. Alternatively, the market equilibrium aggregate willingness-to-pay function $V^e(p_R, s, U, \alpha)$ is linear homogeneous and convex in prices (p_R, s), implying that $\partial^2V^e/\partial(s, p_R)^2$ is a symmetric, positive semi-definite matrix. Finally, the partial equilibrium function $V(p_K, p_R, s, U, \alpha)$ is more convex in (s, p_R) than its market equilibrium counterpart $V^e(p_R, s, U, \alpha)$ in the sense that $[\partial^2V/\partial(s, p_R)^2 - \partial^2V^e/\partial(s, p_R)^2]$ is a symmetric, positive semi-definite matrix.

Again, proposition 3 indicates that the two willingness-to-pay functions V(.) and V^e(.) have similar properties: they are both linear homogeneous and convex in prices. However, in general, the partial equilibrium function $V(p_K, p_R, s, U, \alpha)$ is more convex in prices (s, p_R) than its market equilibrium counterpart $V^e(p_R, s, U, \alpha)$. As seen in equation (11c), this is because allowing for induced price adjustments in p_K through market equilibrium tends to reduce the effects of prices (s, p_R).

IV. Uncompensated Market Equilibrium:

The previous section has discussed some general results concerning the properties of compensated market equilibrium functions. Unfortunately, compensated behavior is generally not observed in the real world, as household utility is rarely held constant in a changing economy. Instead, uncompensated behavior is typically observed. This suggests a need to characterize uncompensated market equilibrium behavior and to understand its relationship with compensated behavior. This section focuses on the properties of uncompensated market equilibrium functions, both at the micro level (i.e., firm and household) and at the aggregate level. Special attention is given to income effects.

First, the uncompensated equilibrium market prices $p_K^*(p_R, s, x, \alpha)$ are obtained (implicitly) as a solution from (4a). Since y_j^* and y_h^* are homogeneous of degree zero in (p, s, x_h), it follows that the functions $p_K^*(p_R, s, x, \alpha)$ are linear homogeneous in (p_R, s, x): proportional changes in all prices and incomes have no real effect on resource allocation. Also, given $p = (p_K, p_R)$ and using the implicit function theorem in (4a), the equilibrium price functions $p_K^*(p_R, s, x, \alpha)$ satisfy

$$\partial p_K^*/\partial\beta = - [\partial N_K^*/\partial p_K]^{-1}\, \partial N_K^*/\partial\beta\ , \tag{14}$$

where $\beta = (p_R, s, x, \alpha)$, and the matrix $\partial N_K^*/\partial p_K$ is assumed non-singular. Finally, from duality, the following relationship holds between compensated and uncompensated price functions

$$p_K^c(p_R, s, U, \alpha) = p_K^*(p_R, s, e(p_R, p_K^c(p_R, s, U, \alpha), s, U, \alpha_H), \alpha). \tag{15}$$

where $e(p, s, U, \alpha_H) = \{e_h(p, s, U_h, \alpha_h): h \in H\}$,

The properties of the micro level (i.e., firm or household level) uncompensated market equilibrium functions $y_j^{e*}(p_R, s, x, \alpha)$ defined in (5a) and (5b) are presented next. (See the proof in the Appendix).

<u>Proposition 4</u>: The firm (or household) uncompensated market equilibrium functions $y_j^{e*}(p_R, s, x, \alpha)$ are homogeneous of degree zero in (p_R, s, x) and satisfy

$$y_j^{ec}(p_R, s, U, \alpha) \;=\; y_j^{e*}(p_R, s, e(p_R, p_K^c(p_R, s, U, \alpha), s, U, \alpha),\; \alpha),\; j \in (J \text{ or } H), \tag{16}$$

implying the Slutsky-like equations

$$\partial y_j^{ec}/\partial s \;=\; \partial y_j^{e*}/\partial s \;+\; \Sigma_{h\in H}\, \{\partial y_j^{e*}/\partial x_h\, [\delta_h\, y_{Kh}^{e*} + y_{Kh}^{e*\prime}\, (\partial p_K^c/\partial s)]\},\; j \in (J \text{ or } H), \tag{17a}$$

$$\partial y_j^{ec}/\partial p_R \;=\; \partial y_j^{e*}/\partial p_R \;+\; \Sigma_{h\in H}\, \{\partial y_j^{e*}/\partial x_h\, [y_{Rh}^{e*} + y_{Kh}^{e*\prime}\, (\partial p_K^c/\partial p_R)]\},\; j \in (J \text{ or } H). \tag{17b}$$

Equation (16) presents the relationship between compensated and uncompensated market equilibrium supply-demand functions. The Slutsky-like equations (17a) and (17b) are obtained simply by differentiating (16) with respect to (s, p_R). When aggregated to the market level, proposition 4 generates the following result.

Proposition 5: The aggregate uncompensated market equilibrium net supply functions N^e are homogeneous of degree zero in (p_R, s, x) and satisfy

$$\partial(N_{KT}^{ec}, N_R^{ec})/\partial s = \partial(N_{KT}^e, N_R^e)/\partial s + \Sigma_{h\in H}\{\partial(N_{KT}^e, N_R^e)/\partial x_h\, [\delta_h\, y_{Kh}^{e*} + y_{Kh}^{e*\prime}\, (\partial p_K^c/\partial s)]\}, \quad (18a)$$

$$\partial(N_{KT}^{ec}, N_R^{ec})/\partial p_R = \partial(N_{KT}^e, N_R^e)/\partial p_R + \Sigma_{h\in H}\{\partial(N_{KT}^e, N_R^e)/\partial x_h\, [y_{Rh}^{e*} + y_{Kh}^{e*\prime}\, (\partial p_K^c/\partial p_R)]\}. \quad (18b)$$

With the explicit incorporation of a household sector in a market equilibrium context, expressions (17) and (18) relate compensated market equilibrium price effects (the left-hand side in (17) and (18)) to its uncompensated counterpart (the first expression on the right-hand side of (17) and (18)) and an "income effect" (the second expression on the right-hand side of (17) and (18)). In other words, uncompensated market equilibrium price effects can be decomposed into two parts: compensated price effects, and income effects. This is a market equilibrium analogy to the classical Slutsky equation from partial equilibrium consumer theory, relating Marshallian (uncompensated) and Hicksian (compensated) price response. It illustrates how income effects influence market equilibrium behavior.

To clarify this result, consider the uncompensated matrix $\partial(N_{KT}^e, N_R^e)/\partial(s, p_R)$ in (18a) and (18b), where (N_{KT}^e, N_R^e) are aggregate uncompensated market equilibrium net supply functions. While the aggregate compensated matrix $(\partial N_{KJ}^{ec}, N_R^{ec})/\partial(s, p_R)$ is always symmetric, positive semi-definite (from proposition 1), expressions (18a) and (18b) imply that the aggregate uncompensated matrix $(\partial N_{KT}^e, N_R^e)/\partial(s, p_R)$ is in general neither symmetric nor positive semi-definite when $\partial N^e/\partial x \neq 0$. This is of interest given the prevalence of significant income effects in many empirical situations. The Slutsky-like equations (17) and (18) illustrate how non-zero income effects $\partial y_j^{e*}/\partial x$ influence the uncompensated price responses of the market equilibrium functions y_j^{e*} and their aggregate counterparts N^e.[6]

The above results provide a formal relationship between compensated and uncompensated market equilibrium functions (Thurman). This relationship can be useful in applied economic analyses of market equilibrium. For example, it may be empirically attractive to estimate directly the uncompensated equilibrium functions $p_k(p_R, s, x, \alpha)$, $N_{KT}^e(p_R, s, x, \alpha)$, and $N_R^e(p_R, s, x, \alpha)$. Indeed, estimating $p_k(p_R, s, x, \alpha)$ or $N_R^e(p_R, s, x, \alpha)$ does not require information about the quantities y_K. Similarly, estimating $N_{KT}^e(p_R, s, x, \alpha)$ or $N_R^e(p_R, s, x, \alpha)$ does not require information about the prices p_K. Compared to a partial equilibrium approach, this may also help reduce collinearity problems by justifying the exclusion of p_K as explanatory variables. Finally, the functions $p_k(p_R, s, x, \alpha)$, $N_{KT}^e(p_R, s, x, \alpha)$, and $N_R^e(p_R, s, x, \alpha)$ are reduced form equations: their estimation avoids potential simultaneous equation bias and may be less affected by model misspecifications.

Among the reasons this reduced form approach is not commonly used in empirical work is that its linkage with economic theory has apparently not been well understood. The above propositions help make this linkage more explicit and suggest the usefulness of our results. For example, the Slutsky-like decomposition of price effects in (18) generates a testable implication of the theory: from proposition 1, the matrix of compensated price effects (on the left-hand side of (18)) is symmetric, positive semi-definite. Taking the theory as maintained and using (18), the symmetry of the compensated price effects can be either imposed or tested in the estimation of $N_{KT}^e(p_R, s, x, \alpha)$ and $N_R^e(p_R, s, x, \alpha)$.

V. Welfare Implications

In this section, we explore the implications of our results for welfare analysis. Recall from (8) that $V^e(p_R, U, \alpha)$ is an aggregate welfare measure across all firms and households given induced adjustments in the price vector p_K through market supply-demand equilibrium. Hence, it provides a basis for conducting market equilibrium welfare compensation tests. To see this, let $\theta = (p_R, s, \alpha)$ and consider a change in the parameters θ from θ^0 to θ^1. Using U as a reference level of utility, the change in V^e associated with the change in θ is given by

$$\Delta V^e = V^e(\theta^1, U) - V^e(\theta^0, U)$$

$$= \int_{\theta^0}^{\theta^1} [\partial V^e(\theta, U)/\partial\theta]\, d\theta, \qquad (19)$$

where ΔV^e is the aggregate willingness-to-pay for the change across all firms and households, allowing for induced adjustments in p_K. To the extent that the welfare analysis is limited to all firms in J and all households in H, then the following interpretations hold.[7] If U represents the utility levels before the change and $\Delta V^e \geq 0$, then the change in θ passes the potential Pareto improvement test in the sense that aggregate welfare is increasing: the gainers can compensate the losers so that (potentially) no one is made worse off. Alternatively, if $\Delta V^e < 0$, it would follow that the change in θ fails the Pareto improvement test in the sense that the gainers cannot compensate the losers and at least one firm or household in the economy is necessarily made worse off.

Perhaps more importantly, note that the results presented in section 3 have relevant implications for the empirical measurement of ΔV^e in (19). This may be of particular interest when $\theta = (s, p_R)$ where (s, p_R) are policy variables (e.g., price support program, deficiency payments, import tax, etc.; see Floyd; Gardner, 1979). In the case of an exogenous change in the prices (s, p_R), equation (13a) and (13b) imply that (19) takes the form

$$\Delta V^e = \int_{s^0}^{s^1} N_{KT}^{ec}(p_R, s, U, \alpha)\, ds, \qquad (20a)$$

and

$$\Delta V^e = \int_{p_R^0}^{p_R^1} N_R^{ec}(p_R, s, U, \alpha)\, dp_{R.} \tag{20b}$$

Expressions (20a) and (20b) measures ΔV^e by the changes in the areas between aggregate compensated market equilibrium net supply functions and the corresponding prices (s, p_R). These areas are the traditional producer and consumer surplus measures, except that they are measured from market equilibrium (rather than partial equilibrium) functions. They measure economy-wide welfare impacts of changes in the price vector p_R. These results are consistent with those obtained by Just et al., Just and Hueth, Thurman, Thurman and Wohlgenant, and Bullock. They provide a simple and practical way of evaluating the welfare impact of exogenous price changes (e.g., due to government intervention) on all the industries and households affected by the change. Of particular interest is the implication from (20a) and (20b) that economy-wide welfare measures can be obtained from knowing market equilibrium net supply functions for only a subset of commodities.

To illustrate, consider the case where "s" is a government subsidy (e.g., deficiency payments) to the producers of commodities K (see Figure 1). The government cost of this pricing policy is (s' N_{KT}^{ec}), suggesting that the "deadweight loss" to society of this pricing policy is $L = \Delta V^e$ - s' N_{KT}^{ec}. From (11b) and (20a), it is clear that $\Delta V^e \leq$ s' N_{KT}^{ec}, yielding the well-known result that the deadweight loss of government price distortions is necessarily non-negative: $L \geq 0$. Note that this evaluation of the economy-wide welfare effect of this pricing policy (including its indirect market-equilibrium effects in the markets for the commodities R) involves only knowing the net supply function N_{KT}^{ec}. This shows how equations (20a) and (20b) can prove useful in evaluating the welfare effects of government pricing policy.[8]

Given that only uncompensated behavior is typically observable, one may ask what error would be made if uncompensated market equilibrium functions N^e were used in (20) (instead of their compensated counterparts N^{ec}). From (17) and (18), the error would be zero in the absence of income effects ($\partial y_j^e/\partial x_h$ = 0), but in general non-zero in the presence of income effects ($\partial y_j^e/\partial x_h \neq 0$). As in the partial equilibrium case (e.g., Willig), we can expect the error to be relatively small as long as the income effects are also small.

Equations (20a) and (20b) are also of interest when interpreted in the light of proposition 1. What error would be made if the measurements in (20a) and (20b) neglected the induced price adjustments in p_K? Equation (11c) in proposition 1 implies that producer and consumer surplus obtained from partial equilibrium functions (N^c) would overstate the true aggregate willingness to pay ΔV^e stated in (20a) and (20b). In other words, our results indicate that a neglect of the induced price adjustments in related markets would provide a systematically upward biased estimate of welfare change. This reinforces the need for a careful evaluation of market adjustments throughout the economy.

Finally, when $\theta = \alpha$, then equation (19) provides a basis for investigating the welfare impact of technical change (or a change in consumer preferences). This may be of particular interest when technical change is generated by government regulations (e.g., Hazilla and Kopp). Given $\theta = \alpha$, equations (13c) and (13d) imply that (19) takes the form

$$\Delta V^e = \int_{\alpha^0}^{\alpha^1} [p' \, \partial N^c(p, s, U, \alpha)/\partial\alpha: p_K = p_K{}^c(p_R, s, U, \alpha)] \, d\alpha, \tag{21a}$$

$$= \int_{\alpha^0}^{\alpha^1} [p_R' \, \partial N_R{}^{ec}(p, s, U, \alpha)/\partial\alpha] \, d\alpha. \tag{21b}$$

Equation (21a) appears to be new in the literature: it is obtained from equation (13c), which is a form of "envelope theorem" result under market equilibrium. Equations (21) measure the economy-wide welfare impact of change in the parameters α, allowing for induced adjustments in the price vector p_K.

They provide a simple and practical way of evaluating the total welfare impact of technical change or government regulation on all the firms and households affected by the change in a market equilibrium context. The need to evaluate the welfare effects of technical change in a multi-market framework has been stressed by Martin and Alston. However, Martin and Alston relied on a partial equilibrium analysis, treating all prices as exogenous. By taking into consideration induced price adjustments throughout the economy, our approach provides additional insights in the analysis and welfare measurement of technical change in a more general market equilibrium framework.

VI. Concluding Remarks

This paper has examined the properties of market equilibrium supply-demand functions when prices are allowed to adjust through competitive market equilibrium. We consider the general situation where some prices may be exogenous (as in the "small country" case) while others are endogenously determined through market equilibrium. Pricing policy is incorporated in this market equilibrium analysis through price taxes or subsidies that create a price wedge reflecting the difference between prices faced by "targeted agents" (e.g., producers of a given commodity) and others. This generates market distortions that can affect every sector of the economy, either directly or indirectly (through price effects in related markets). In this context, we derive the general properties of market equilibrium functions.

We expand on previous research by Diewert, Heiner, and Braulke (1984, 1987) deriving the joint effects of exogenous prices and taxes/subsidies on aggregate net supply functions. Examining the relationship between partial equilibrium and market equilibrium functions, we show that allowing price adjustments tends to reduce aggregate (compensated) supply response. Our approach provides a unified analytical framework and additional clarification for analyzing the joint impact of exogenous prices and taxes/subsidies on aggregate net supply in a market equilibrium context.

With an explicit consideration of households, we investigate the implications of possible income effects associated with the changing prices of consumer goods. In this context, we derived a Slutsky-like equation which formalizes the role of income effects in market equilibrium functions. Finally, we illustrate the usefulness of the analysis for multi-market welfare evaluation letting prices adjust. Contrary to Bullock's findings, we show that aggregate market equilibrium functions have useful welfare significance under a wide range of policy relevant contexts. Building on previous work by Just et al., Thurman, and Thurman and Wohlgenant, we investigate simple market equilibrium measurements of aggregate welfare effects associated with exogenous price changes. This includes the welfare effects of pricing policy as reflected by price taxes and/or subsidies. We show that neglecting induced price adjustments tends to provide an upward-biased estimate of the welfare effects of pricing policy. We also present some new results on the welfare measurement of market equilibrium effects of technical change. These measures capture the direct as well as indirect (through induced price adjustments) impacts of technical change throughout the economy.

We hope that our results will appear useful in the empirical analysis of pricing policy and technical change, and that they will help stimulate further research on market equilibrium allocations.

Appendix A

Proof of Proposition 1:

Note that $\partial(N_{KT}^c, N_R^c)/\partial(s, p_R) = \partial(N_{KT}^c, N_{RT}^c)/\partial(s, p_R) + \partial[0_K, \sum_{j\in H} (1-\delta_j)y_{Rj}^* - \sum_{h\in H} (1-\delta_h)y_{Rh}^c]/\partial(0_K, p_R)$.

But both right-hand side matrices are symmetric, positive semi-definite, due to the symmetry, positive semi-definiteness of $\partial y_j^*/\partial p$, $j \in J$, and the symmetry negative semi-definiteness of $\partial y_h^c/\partial p$, $h \in H$, where $p = (p_K, p_R)$. This proves (11a).

Following similar steps, it can be easily shown that the matrices $\partial(N_K^c, N_R^c)/\partial(p_K, p_R)$, $\partial(N_{KT}^c, N_{RT}^c)/\partial(s, p_R)$ and $[\partial(N_K^c, N_R^c)/\partial(p_K, p_R) - \partial(N_{KT}^c, N_{RT}^c)/\partial(s, p_R)]$ are each symmetric, positive semi-definite, and that $(\partial N_R^c/\partial s)' = \partial N_{KT}^c/\partial p_R$, and that $\partial N^c/\partial s = \partial N_T^c/\partial p_K$. It follows that the matrix

$$A = \begin{bmatrix} \partial(N_K^c, N_R^c)/\partial(p_K, p_R) & \partial(N_{KT}^c, N_{RT}^c)/\partial p_K \\ [\partial(N_{KT}^c, N_{RT}^c)/\partial s]' & \partial N_{KT}^c/\partial p_K \end{bmatrix}$$

is symmetric, positive semi-definite. Consider the matrix

$$Q = \begin{bmatrix} -(\partial N_{KT}^c/\partial p_K)(\partial N_K^c/\partial p_K)^{-1} & 0 & I_K \\ -(\partial N_R^c/\partial p_K)(\partial N_K^c/\partial p_K)^{-1} & I_R & 0 \end{bmatrix}.$$

This means that the matrix [Q A Q'] is also symmetric, positive semi-definite. Note that (10a) and (10b) imply that

$$\partial(N_{KT}^{ec}, N_R^{ec})/\partial(s, p_R) = \partial(N_{KT}^c, N_R^c)/\partial(s, p_R) - \partial(N_{KT}^c, N_R^c)/\partial p_K\, [\partial N_K^c/\partial p_K]^{-1}\, \partial N_K^c/\partial(s, p_R). \tag{A1}$$

But the right-hand side of (A1) equals [Q A Q']. This proves (11b).

Since $\partial N_K^c/\partial p_K$ is a symmetric, positive semi-definite matrix, and $[\partial(N_{KT}^c, N_R^c)/\partial p_K]' = \partial N_K^c/\partial(s, p_R)$, it follows that $\partial(N_{KT}^c, N_R^c)/\partial p_K\, [\partial N_K^c/\partial p_K]^{-1}\, \partial N_K^c/\partial(s, p_R)$ is a symmetric, positive semi-definite matrix. Using (A1), this proves (11c).

Proof of Proposition 2:

Equation (12a) and (12b) follow directly from differentiating the partial equilibrium willingness-to-pay $V(p, s, U, \alpha)$ in (7) with respect to s or p, and applying Hotelling's lemma and Shephard's lemma.

Note that the partial equilibrium willingness-to-pay $V(p, s, U, \alpha)$ in (7) can be written as

$$V(p, s, U, \alpha) = p' N^c(p, s, U, \alpha).$$

Differentiating this expression with respect to $\alpha = (\alpha_J, \alpha_H)$ yields (12c).

Differentiating (8b) with respect to (s, p_R) yields

$$\partial V^e/\partial(s, p_R) = \partial V/\partial(s, p_R) + \partial V/\partial p_K (\partial p_K{}^c/\partial(s, p_R)).$$

But, from (12b) and (4b), $\partial V/\partial p_K = N_K{}^c = 0$. Noting from (12b) that $\partial V/\partial p_R = N_R{}^c$ and using (6a) and (6b), this proves (13a).

Using the aggregate willingness-to-pay defined in (8b), differentiating V^e with respect to α gives

$$\partial V^e/\partial\alpha = \partial V/\partial\alpha + \partial V/\partial p_K (\partial p_K{}^c/\partial\alpha).$$

But $\partial V/\partial p_K = N_K{}^c = 0$ from (12b) and (4b). Using (12c), this proves (13c).

Alternatively, V^e in (8) can be written as $V^e = p' N^{ec}$, or using (4b) as $V^e = p_R' N_R{}^{ec}$. Differentiating this last expression with respect to α yields (13d).

Proof of Proposition 3:

The linear homogeneity of $V(p, s, .)$ follows from (7) and the linear homogeneity of the indirect profit functions $\pi_j(p, s, .)$, $j \in J$, and expenditure functions $e_h(p, s, .)$, $h \in H$. The convexity of $V(p, .)$ is implied by the convexity of $\pi_j(p, .)$, $j \in J$, and the concavity of $e_h(p, .)$, $h \in H$.

The linear homogeneity of $V^e(s, p_R, .)$ follows from (8b) and the linear homogeneity of $V(p_K, p_R, s, .)$ and $p_K^c(p_R, s, .)$. The convexity of $V^e(p_R, s, .)$ is implied by (13a), (13b) and (11b). Finally, the positive semi-definiteness of $[\partial^2 V/\partial(s, p_R)^2 - \partial^2 V^e/\partial(s, p_R)^2]$ follows from (12a), (12b), (13a), (13b) and (11c).

Proof of Proposition 4:

The homogeneity of degree zero of $y_j^{e*}(p_R, s, x, .)$ follows from (5a) and (5b), from the homogeneity of degree zero of $y_j^*(p_K, p_R, s, x, .)$, and from the linear homogeneity of $p_K^*(p_R, s, x, .)$.

Equation (16) follows from duality. Equation (17) is obtained by differentiating equation (16) with respect to p_R, and using Shephard's lemma.

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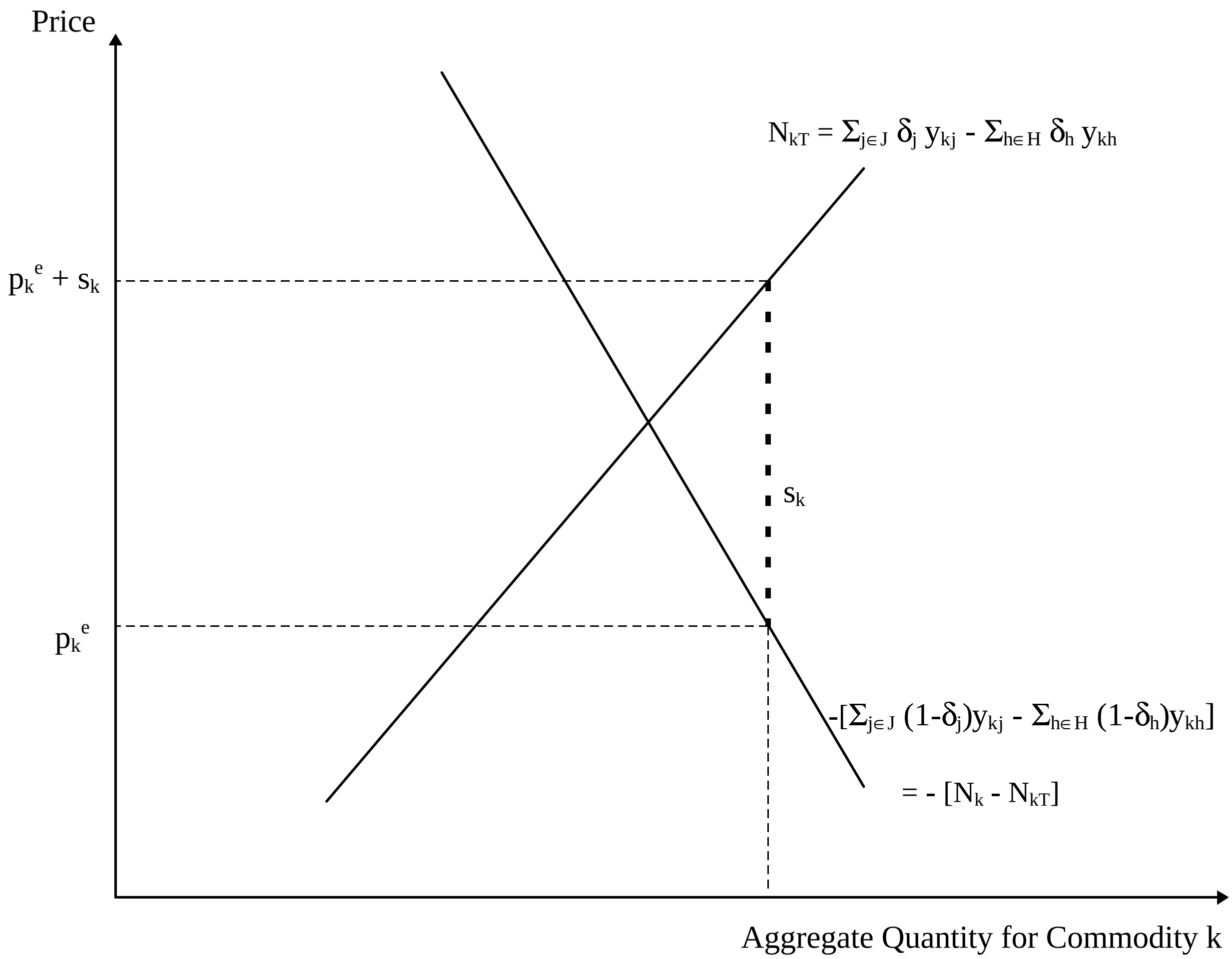
Figure 1- Pricing Policy and Market Equilibrium



FOOTNOTES

[1] If the households are also involved directly in the production of outputs that are marketed, then the arguments presented below can be easily modified in the context of household production theory (e.g., see Deaton and Muellbauer, Chapter 10).

[2] Note that, for firms ($j \in J$), the compensated market equilibrium functions y_j^{ec} in (6a) are defined using the profit maximizing functions y_j^* because of the absence of income effects for firms. In this case, the effects of a compensation on y_j^{ec} are only through the compensated price equilibrium function p_K^c.

[3] Although the differentiability assumption is convenient for deriving our results, it could be relaxed (e.g. see Braulke, 1987).

[4] Note that $\partial N_K^c/\partial s = \partial N_{KT}^c/\partial p_K$ in equation (9).

[5] In addition to (11a), aggregate net supply functions exhibit the following properties: the matrices $\partial N^c/\partial(p_K, p_R)$ and $\partial N_T^c/\partial(s, p_R)$ are each symmetric, positive semi-definite. Again, such properties follow directly from the symmetry, positive semi-definiteness of $\partial y_j^*/\partial p$, $j \in J$, and the symmetry, negative semi-definiteness of $\partial y_h^c/\partial p$, $h \in H$.

[6] In empirical situations, it may be convenient to represent income distribution $\{x_h: h \in H\}$ by sufficient statistics of this distribution. This would help make the empirical estimation of aggregate income effects (the second expression on the right-hand side of (18)) easier. Alternatively, assuming linear market equilibrium Engel curves (where $\partial y_j^e/\partial x_h$ = a constant) would greatly simplify the empirical tractability of income effects in (18).

[7] Note that, typically, welfare analysis also needs to consider explicitly government cost or changes in the balance of payments. Such considerations can be incorporated in the analysis by adding (subtracting) the associated benefits (costs) to our welfare measures. See below.

[8] Note that, while equations (20a) and (20b) evaluate aggregate net welfare effects, they do not provide information on distributional effects across firms or households. Evaluating these distributional effects requires additional information on both the direct (partial equilibrium) effects and the indirect (market equilibrium) effects across agents throughout the economy.