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## A PRIMAL-DUAL APPROACH TO NONPARAMETRIC PRODUCTIVITY ANALYSIS: THE CASE OF U.S. AGRICULTURE

by

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**ABSTRACT:** Nonparametric methods for measuring productivity indexes based on bounds for the underlying production technology are presented. Following Banker and Maindiratta, the lower bound is obtained from a primal approach while the upper bound corresponds to a dual approach to nonparametric production analysis. These nonparametric bounds are then used to estimate input-based and output-based distance functions. These radial measures provide the basis for measuring productivity indexes. Application to times series data on U.S. agriculture indicates a large gap between the primal lower bound and the dual upper bound. This generates striking differences between the primal and dual nonparametric productivity indexes.

**KEYWORDS:** index numbers, productivity, nonparametric.

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# A PRIMAL-DUAL APPROACH TO NONPARAMETRIC PRODUCTIVITY ANALYSIS: THE CASE OF U.S. AGRICULTURE

## I) Introduction.

A considerable amount of research has centered on the measurement of productivity. Over the last two decades, much interest has focused on the total factor productivity index proposed by Christensen and Jorgenson (CJ) (1970). Diewert (1976) has shown that the CJ index can be derived from a constant return to scale translog transformation function that is separable in inputs and outputs. Caves et al. (1982a) have relaxed this separability requirement and shown that the CJ productivity index can be obtained from a translog transformation function exhibiting constant return to scale and constant second order coefficients. The CJ productivity index appears attractive because it is empirically tractable while being derived from a flexible multioutput-multifactor representation of the production technology (e.g. Ball (1985)). Finally, Caves et al. (1982b) have generalized the CJ index to handle variable return to scale under the translog functional form.

Caves et al. (1982b) have made explicit the relationship between productivity indexes and distance functions. The input-based and output-based distance functions have been studied in detail by Shephard (1970). They provide a general representation of the production technology that is particularly convenient for productivity analysis (Moorsteen (1961); Malmquist (1953)). This raises the question of the empirical characterization of these distance functions. If the distance functions are translog and exhibit constant second order coefficients, then Caves et al. (1982b) have derived empirically tractable productivity indexes. But what if the second order coefficients are not constant? Or what if the underlying technology is not translog? Typically, economists do not have good a priori information about the true functional form representing technology. Even if we restrict our attention to flexible functional forms, there is a very large number of possibilities for choosing a parametric functional structure.

This suggests a need to develop nonparametric methodologies for productivity analysis that would not depend on the choice of a particular functional form (Diewert (1980); Diewert and Parkan (1983)). Nonparametric representations of production technologies have been developed over the last two decades. Using a primal approach, Afriat (1972), Färe, Grosskopf and Lovell (1985), Färe, Grabowski and Grosskopf (1985) and Banker et al. (1984) have developed the nonparametric approach to production frontier estimation. This has been called Data Envelopment Analysis (DEA) in the management science literature (Banker et al. (1984)). Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983), Varian (1984), Chavas and Cox (1988, 1990) and Cox and Chavas (1990) have developed a dual approach to nonparametric production analysis under cost minimization or profit maximization. More recently, Banker and Maindiratta (1988) have established relationships between the primal and the dual nonparametric approaches. Specifically, they have shown that the primal and dual representations can provide nonparametric bounds on the underlying production technology.

The objective of this paper is to explore the use of both the primal and the dual nonparametric approaches in the analysis of productivity. We focus on the general multiproduct-multifactor case with a finite number of observations on the associated prices and quantities. As in previous nonparametric analysis, the approach is not restricted by any particular functional form. Following Banker and Maindiratta (1988), a primal lower bound and a dual upper bound representation of the underlying technology can be obtained. Using these representations, we propose to estimate the associated distance functions. This is done simply by solving linear programming problems formulated using either just the quantity information (primal approach), or the price and quantity information (dual approach). These distance functions are then used to construct nonparametric productivity indexes. The methodology is applied to time series data on U.S. agriculture.

The results show that the gap between the primal lower bound and the dual upper bound can be quite large, generating very different productivity indexes for U.S. agriculture. This suggests that,

while the data can likely support a variety of functional forms as representations of the underlying technology, there may not be any good basis for discriminating among them. It is found that the CJ productivity index for U.S. agriculture falls in between our upper bound and lower bound productivity indexes, although it is closer to the dual upper bound. The findings also illustrate how an input-based productivity index can differ from an output-based index, reflecting a departure from constant return to scale.

## II) Productivity Measurement.

Based on the earlier work of Moorsteen (1961) or Malmquist(1953), Diewert (1976), Caves et al. (1982a, 1982b) and others have stimulated renewed interest on the index number approach to productivity measurement. The common starting point for this work is Shephard's distance functions, which provide a convenient and flexible representation of the underlying technology for productivity measurement purposes.

Consider a  $(m \times 1)$  input vector  $x = (x_1, x_2, \dots, x_m)' \geq 0$  used in the production of a  $(s \times 1)$  output vector  $y = (y_1, y_2, \dots, y_s)' \geq 0$ . Characterize the underlying technology by the production possibility set  $T$ , where  $(y, -x) \in T$ . We assume throughout the paper that the set  $T$  is non-empty, closed, convex and negative monotonic.<sup>1</sup>

Shephard (1970, p. 64-78) defines the input distance function as

$$D_T(y, x) = \sup_{\delta} \{ \delta : (y, -x/\delta) \in T \}. \quad (1)$$

The input distance function in (1) can be used to generate the input requirement set  $IR_T(y) = \{x: D_T(y, x) \geq 1\}$  as well as the frontier isoquant of a production set  $IS_T(y) = \{x: D_T(y, x) = 1\}$  (Shephard (1970), p. 67). Note that the input distance function completely characterizes the technology  $T$ . It measures the proportional (or radial) reduction in all inputs  $x$  that would bring the firm to the frontier isoquant

$IS_T(y)$ .

The input distance function has been of great interest in efficiency analysis. It is the reciprocal of the Farrell (1957) measure of technical efficiency, where  $1/D_T(y, x) = 1$  corresponds to technical efficiency while  $1/D_T(y, x) < 1$  identifies technical inefficiency. In this latter case,  $1/D_T(y, x)$  measures the proportional (or radial) rescaling of all inputs that would bring the firm to the frontier isoquant  $IS_T(y)$  (Färe, Grosskopf and Lovell, 1985). Similarly,  $[1 - 1/D_T(y, x)]$  can be interpreted as the proportional reduction in production cost that can be achieved by moving to the frontier isoquant.

Shephard (1970, p. 206-212) alternatively defines the output distance function as

$$F_T(y, x) = \inf_{\delta} \{ \delta : (y/\delta, -x) \in T \}. \quad (2)$$

The output distance function can be used to generate the production correspondence  $PC_T(x) = \{y: F_T(y, x) \leq 1\}$  and the frontier correspondence  $FC_T(x) = \{y: F_T(y, x) = 1\}$  (Shephard (1970), p. 209). It follows that  $F_T(y, x)$  in (2) defines the substitution alternatives among the outputs  $y$ , given inputs  $x$ . As with the input distance function in (1), note that the output distance function in (2) provides a complete characterization of the underlying technology. Also, note that  $1/F_T(y, x)$  measures the proportional rescaling of all outputs  $y$  that would bring the firm to the frontier production correspondence  $FC_T(x)$ . Then,  $[1/F_T(y, x) - 1]$  can be interpreted as the proportional increase in revenue that can be achieved by moving to the frontier correspondence.

The properties of the two distance functions  $D_T(y, x)$  and  $F_T(y, x)$  have been analyzed in detail by Shephard (1970, p. 207-208). Note that there exists a simple relationship between these two functions under constant return to scale. Indeed,  $(y/k, -x) \in T$  implies  $(y, -kx) \in T$  for any  $k > 0$  under constant return to scale. From (1) and (2), this implies that  $D_T(y, x) = 1/F_T(y, x)$ . In other words, each distance function in (1) or (2) is a reciprocal function of the other under constant returns to scale. However, under a variable return to scale technology,  $D_T(y, x) \neq 1/F_T(y, x)$  in general.

The distance functions (1) and (2) provide a basis for measuring productivity. From (1) under technology  $T$ , the Malmquist input-based productivity index for observation  $(y, x)$  relative to observation  $(y^T, x^T) \in T$  is

$$IM = D_T(y^T, x^T)/D_T(y, x).$$

If observation  $(y^T, x^T) \in T$  is technically efficient, then  $D_T(y^T, x^T) = 1$ . This generates the input-based productivity index proposed by Caves et al. (1982b)

$$IP = 1/D_T(y, x)$$

It measures the radial inflation factor for all inputs such that the inflated inputs  $(IP \ x) = x/D_T(y, x)$  lie on the frontier isoquant  $IR_T(y)$  generated by technology  $T$  (Caves et al. (1982b), p. 1407). In this context, a firm choosing  $(y, x)$  has a higher (lower) productivity than the reference technology  $T$  if  $IP > 1$  ( $< 1$ ).

Alternatively, from (2) under technology  $T$ , the Malmquist output-based productivity index for observation  $(y, x)$  relative to observation  $(y^T, x^T) \in T$  is

$$OM = F_T(y, x)/F_T(y^T, x^T).$$

If observation  $(y^T, x^T) \in T$  is technically efficient, then  $F_T(y^T, x^T) = 1$ . This generates the output based productivity index proposed by Caves et al. (1982b)

$$OP = F_T(y, x)$$

which measures the radial deflation factor for all outputs by which the deflated outputs  $(y/OP) = y/F_T(y, x)$  lie on the frontier correspondence  $FC_T(x)$  generated by technology  $T$  (Caves et al. (1982b), p. 1402). Thus, a firm choosing  $(y, x)$  has a higher (lower) productivity than the reference technology  $T$  if  $OP > 1$

( $< 1$ ).

Under constant returns to scale, we have seen that the two distance functions (1) and (2) are reciprocal to each other. It follows that the two productivity indexes, IP and OP, are identical under constant returns to scale (Caves et al. (1982b), p. 1408). Correspondingly, these measures will differ from each other under variable returns to scale.

The above productivity indexes can help evaluate the rate of technical change in an industry. Their use typically depends on the nature of the data available. With cross-section data, firm level information is available only for a given time period. In such a situation, productivity indexes are basically undistinguishable from radial technical efficiency indexes. Yet, if the industry is affected by technical progress, these indexes can reflect different adoption rates of new technology across firms as the production possibility set expands. With time series data, the productivity indexes allow a measurement of the rate of shift of frontier technology over time. And if firm level data are available both across firms and over time (e.g. the case of panel data), then it becomes possible to distinguish between efficiency and productivity: the cross section information across firms provides a basis for estimating technical efficiency indexes within each period; and the time series information allows the estimation of productivity indexes across periods.<sup>2</sup>

In the analysis of time series data, IP and OP based on observing  $(y, x)$  at time  $t$  are productivity indexes for time  $t$  given the reference technology  $T$ . Two approaches can be distinguished depending on the choice of the reference technology  $T$ : 1/ sequential analysis; and 2/ intertemporal analysis. A sequential productivity index for time  $t$  involves a reference technology  $T$  corresponding to current and previous observations but not subsequent ones. An intertemporal productivity index for time  $t$  involves a reference technology  $T$  evaluated on the basis of observations from all time periods.

Diewert (1976), and Caves et al. (1982a, 1982b) propose productivity indexes associated with a flexible translog transformation function.<sup>3</sup> Caves et al. (1982b) show that such indexes can be built

from the commonly used Törnquist-Theil indexes for inputs and outputs, corrected by scale factors. Although their approach is quite appealing, it is restricted by two underlying assumptions:<sup>4</sup> 1/ the technology must be represented by a translog production transformation; and 2/ the second order coefficients of the translog distance function must be the same between the firm being evaluated and the reference technology (Caves et al. (1982b), theorems 3 and 4).

This raises several questions. What if the underlying production transformation is not translog? Or, what if the second order coefficients of the translog representation are not constant? In these cases, the productivity measurements proposed by Caves et al. (1982b) may not be appropriate. In an attempt to answer these questions, we investigate some alternative productivity measurements in a general nonparametric framework that requires less a priori structure on the underlying technologies.

### III) The Nonparametric Approach.

In order to make productivity indexes empirically tractable, we need to obtain some representation of the reference technology  $T$ . In this section, we explore how nonparametric methods can be used for that purpose. We will limit our discussion to the analysis of productivity based on time series data.<sup>5</sup>

Consider a set  $I = \{1, 2, \dots, n\}$  of  $n$  observations on  $(y, x)$  in a given industry. Assume that the output-input vectors  $(y_i, -x_i) \in T$  and the corresponding price vectors  $(p_i, r_i)$  are observed for each observation  $i \in I$ . Here,  $y_i \geq 0$  and  $p_i \geq 0$  are  $(s \times 1)$  vectors of output quantities and prices, and  $x_i \geq 0$  and  $r_i \geq 0$  are  $(m \times 1)$  vectors of input quantities and prices. Assuming that the set  $I$  involves time series data, this information can be used to measure productivity over time. In a sequential analysis of productivity for time  $t$ , the reference technology  $T$  is evaluated based on the set  $I$  of observations made up to time  $t$ . Alternatively, in an intertemporal analysis, the reference set  $T$  is evaluated based on the observations for all time periods.

Consider the maintained hypothesis of profit maximization:

$$\text{Max}_{y,x} [p_i'y_i - r_i'x_i : (y_i, -x_i) \in T], \quad (3)$$

for each  $i \in I$ . Let  $y_i^* = y_i(p_i, r_i)$  and  $x_i^* = x_i(p_i, r_i)$  denote the profit maximizing output supply and input demand functions corresponding to (3) for firm  $i \in I$ . Then, by definition of the maximization problem in (3), profit maximizing behavior must satisfy the following set of inequalities:

$$(p_i'y_i^* - r_i'x_i^*) - (p_i'y_j^* - r_i'x_j^*) \geq 0 \quad (4)$$

for all  $i \in I$  and all  $j \in I$ . Expression (4) corresponds to Varian's Weak Axiom of Profit Maximization: it is a necessary as well as a sufficient condition for profit maximization given the  $n$  observations on production behavior (Varian (1984), p. 584).

However, all the observations on the  $n$  firms may not satisfy (4) for all  $i \in I$  and all  $j \in I$ . For example, this could happen if the  $n$  firms do not all have access to or use the same technology. In this case, it is useful to consider the subset of firms  $E \subseteq I$  that do satisfy (4). For this purpose, define the function  $G_i$  for each  $i \in I$ :

$$G_i = \{\max_j [(p_i'y_j - r_i'x_j) - (p_i'y_i - r_i'x_i) : j \in I]\} \geq 0.$$

Clearly, finding  $G_i = 0$  for some  $i \in I$  implies there does not exist a  $j \in I$  that would satisfy  $(p_i'y_j - r_i'x_j) > (p_i'y_i - r_i'x_i)$  (i.e. that would violate the inequalities in (4)). This generates the following definition of the subset  $E$ :

$$E = \{i : G_i = 0, i \in I\} \subseteq I.$$

Any observation  $i \notin E$  unambiguously deviates from profit maximizing behavior for any production possibility set containing all observations. Alternatively, any firm  $i \in E$  is necessarily consistent with

profit maximization given the observations on all  $n$  firms.

On this basis, we are interested in the following class of admissible sets A:

$$A = \{(y, -x): p_i' y_i - r_i' x_i \geq p_i' y - r_i' x, i \in E, (y, -x) \in T\}.$$

In other words, an admissible possibility set is any closed, convex and negative monotonic production possibility set relative to which the performance of each firm in the set  $E$  remains consistent with profit maximization.

Following Banker and Maindiratta (1988), we propose to use the admissible set  $A$  as a reference technology  $T$  for the evaluation of productivity. The question then is how to construct production possibility sets within  $A$ . In general, the choice of a production possibility set in  $A$  is not unique; there will be many such sets that are consistent with a finite number of observations on production behavior (Varian (1984), p. 591). However, it is possible to bound all these possibilities within the class  $A$  of admissible sets. This can be done by constructing two families of production possibility sets,  $S$  and  $L$ , that provide a lower and upper bound on the reference technology  $T$ . Banker and Maindiratta ((1988), p. 1321) propose the set  $S$

$$S \equiv [(y, -x): y \leq \sum_{i \in I} \lambda_i y_i, x \geq \sum_{i \in I} \lambda_i x_i, \sum_{i \in I} \lambda_i = 1, \\ y \geq 0, x \geq 0, \lambda_i \geq 0 ], \quad (5)$$

and the set  $L$

$$L \equiv [(y, -x): p_i' y - r_i' x \leq p_i' y_i - r_i' x_i, i \in E, \\ y \geq 0, x \geq 0 ]. \quad (6)$$

Each set  $S$  or  $L$  is convex, closed, negative monotonic and admissible, and corresponds to a general variable-return-to-scale technology. Banker and Maindiratta ((1988), p. 1321) have shown that,

for any admissible production possibility set  $T \in A$ ,

$$S \subseteq T \subseteq L.$$

In other words,  $S$  in (5) is the tightest lower bound while  $L$  in (6) is the tightest upper bound for any set  $T \in A$ . This establishes a basis for investigating the nonparametric bounds of the production possibility set that can be generated by a finite number of observations on production behavior.

Note that the inner bound  $S$  in (5) requires only quantity information on  $y$  and  $x$ . Hence, it corresponds to a primal approach to production analysis and focuses on the nonparametric estimation of the production frontier (see Afriat (1972), Färe, Grosskopf and Lovell (1985)). This approach has also been called Data Envelopment Analysis (DEA) in the management science literature (e.g., Banker et al. (1984)). Note, in contrast, the outer bound  $L$  in (6) requires information on both quantities ( $y$ ,  $-x$ ) and prices ( $p$ ,  $r$ ). Hence, it corresponds to a dual nonparametric approach to production analysis, as developed by Afriat (1972), Hanoeh and Rothschild (1972), Varian (1984), Chavas and Cox (1988), or Cox and Chavas (1990).<sup>6</sup>

Given (5) and (6) as a representation of the bounds on the reference technology  $T$  within the class of admissible sets  $A$ , the evaluations of the distance functions (1) and (2) are straightforward. Obtaining the input distance function  $D_S(y, x)$  in (1) associated with the lower bound  $S$  involves solving the linear programming problem:

$$1/D_S(y_j, x_j) = \min_{\lambda, \delta} [\delta: y_j \leq \sum_{i \in I} \lambda_i y_i, x_j \delta \geq \sum_{i \in I} \lambda_i x_i, \sum_{i \in I} \lambda_i = 1, \lambda_i \geq 0 ], \quad (7a)$$

for all  $j \in I$ . The input distance function  $D_L(y, x)$  in (1) associated with the upper bound  $L$  can be obtained from the solution of the linear programming problem:

$$1/D_L(y_j, x_j) = \min_{\delta} [\delta: p_i' y_j - r_i' x_j \delta \leq p_i' y_i - r_i' x_i, i \in E ], \quad (7b)$$

for all  $j \in I$ .

Similarly, the output distance function  $F_S(y, x)$  in (2) associated with the lower bound  $S$  is given by the solution of the linear programming problem:

$$1/F_S(y_j, x_j) = \max_{\lambda, \delta} [\delta: y_j \delta \leq \sum_{i \in I} \lambda_i y_i, x_j \geq \sum_{i \in I} \lambda_i x_i, \sum_{i \in I} \lambda_i = 1, \lambda_i \geq 0 ], \quad (8a)$$

for all  $j \in I$ . And the output distance function  $F_L(y, x)$  in (2) associated with the upper bound  $L$  is obtained in a similar manner from solving:

$$1/F_L(y_j, x_j) = \max_{\delta} [\delta: p_i' y_j \delta - r_i' x_j \leq p_i' y_i - r_i' x_i, i \in E ], \quad (8b)$$

for all  $j \in I$ .

The results given in (7) and (8) can be used to calculate the Malmquist productivity indexes IM and OM discussed in section II. In the case where  $D_T(y^T, x^T) = F_T(y^T, x^T) = 1$ , these productivity indexes reduce to the ones proposed by Caves et al. (1982b), using  $T$  as the reference technology. They provide nonparametric bounds on the input-based radial productivity measures  $IP = 1/D_T(y, x)$ , as well as the output-based radial productivity measures  $OP = F_T(y, x)$ .<sup>7</sup> This primal-dual nonparametric

approach to productivity measurement is illustrated next with an application to U.S. agriculture.

#### IV) Application to U.S. Agriculture.

Aggregate time series data for the U.S. agricultural sector for the years 1950-1982 are taken from Capalbo and Vo (1987). Each year is treated as an observation on a different representative farm. The data analyzed include quantity indexes (1977 = 1.00) and associated implicit price indexes for 6 agricultural outputs (small grains, coarse grains, field crops, fruits, vegetables, and animal products) and 9 inputs (family labor, hired labor, land, structures, other capital, materials, energy, fertilizers, pesticides, and miscellaneous).<sup>8</sup> Previous work by Chavas and Cox ((1988), p. 307) found evidence that these data are not consistent with the existence of an output aggregator using Varian's nonparametric weak separability test (Varian (1984), p. 588). This suggests the presence of possible aggregation bias in previous productivity analysis based on the use of aggregate output (e.g., such as found in Cox and Chavas (1990)) and motivates a need to generalize previous work to a multi-product context. Similar multi-product arguments with respect to U.S. agriculture (in a parametric context) are found in Antle (1984) and in Shumway (1983).

The analysis of U.S. agriculture productivity presented here is based on estimating the distance functions in (7) and (8). This was done by solving the linear programming problems (7)-(8) using GAMS/MINOS.<sup>9</sup> Our investigation focuses on an intertemporal approach where the reference technology  $T$  involves observations for all time periods, i.e. where  $I$  in (7) and (8) include all years from 1950 to 1982. This implies that the same base technology is used in the estimation of productivity indexes for all years. The resulting indexes can thus be interpreted as cumulative measurements of productivity over time.<sup>10</sup> Also, following Caves et al. (1982b), we will implicitly assume that each observation is technically efficient.<sup>11</sup> As a result, any departure from the frontier reference technology will be interpreted as evidence of technical change.

The input-based radial productivity measures  $IP = 1/D_T(y,x)$  are calculated for both the primal lower bound (S) and the dual upper bound (L). Similarly, the output-based radial productivity measures  $OP = F_T(y,x)$  are computed for each bound. The results give a productivity index consistently equal to 1 for the year 1982:  $IP_{82} = OP_{82} = 1$ . This indicates that the highest productivity in U.S. agriculture was obtained in the last year of the data. Such a finding is consistent with non-regressive technical change (where the production possibility set expands over time). Thus, the reference technology  $T$  is basically the technology available in 1982. It follows that the indexes  $IP_t$  or  $OP_t$  should be interpreted as "true productivity indexes" compared to the 1982 technology. In this context, an increase in  $IP$  or  $OP$  over time means technical progress toward the 1982 technology.<sup>12</sup> Table 1 and Figure 1 present the productivity indexes rescaled to 1 in the year 1950, i.e.  $IP_t/IP_{50}$  and  $OP_t/OP_{50}$ ,  $t = 50, \dots, 82$ . For comparison purposes we also include the total factor productivity index proposed by Christensen and Jorgenson (CJ) (computed using these same data). Recall that this CJ productivity index is appropriate for either: a homogeneous translog transformation function exhibiting Hick's neutral technical change and input/output separability (Diewert (1976)); **or** a homogeneous translog transformation functions where the second order terms for the current period are identical to the base period, reference technology (Caves, et al. (1982a)).

The most striking result of Table 1 and Figure 1 is that all the primal nonparametric productivity indexes are equal to one. That is, the primal measures provide no evidence of technical change in U.S. agriculture over the 1950-82 period analyzed. Quite in contrast, the dual nonparametric productivity measures suggest considerable technical change. They indicate that agricultural productivity has increased by about 70 percent over the period analyzed. They also show a fairly steady increase in productivity from one period to the next, which is consistent with non-regressive technical change. The few years where productivity exhibits a small decline (1957, 1959, 1970, 1978 and 1980) likely reflect weather shocks and other measurement errors in the data. The CJ productivity

index for U.S. agriculture is situated in-between the primal lower-bound index and the dual upper-bound index.<sup>13</sup> However, it is found to be much closer to the dual upper bound and follows a similar pattern.

Note that the dual nonparametric input-based and output-based productivity indexes are not identical: the output-based index tends to be a slightly higher than the input-based index. This indicates that the input and output distance functions are not reciprocal to each other, reflecting a departure from constant return to scale.<sup>14</sup> More specifically, it provides evidence of a technology exhibiting decreasing return to scale.<sup>15</sup>

The difference in the productivity indexes from the primal and dual approaches reflects the existence of a fairly large gap between the primal lower bound and the dual upper bound representation of the underlying technology. In other words, there is a fairly large region of the production set that is consistent with the observations on prices and quantities. This indicates that, while many parametric functional forms (e.g., production, cost and/or profit functions) may provide a good fit to the data, there may be little basis to choose among them.<sup>16</sup>

This gap between the primal and dual bounds reflects the nature of the data as well as the method used for the analysis. As noted above, the primal nonparametric approach generating the lower bound does not require price information. This is advantageous when price information is not available. However, when prices are observed and change over time (as is the case with the time-series data analyzed here), then failure to incorporate this information may imply a significant loss.<sup>17</sup> The dual nonparametric methods directly incorporate this price information while the primal methods do not. For example, while the primal approach only restricts the marginal product of inputs to be non-negative, the dual approach further restricts such marginal products to be consistent with observed relative prices under profit maximization.

In the current context, it appears that the price information can play a crucial role in the

assessment of technology. The dual approach generating the upper bound representation of technology may be more informative than the primal approach, especially in situations where there are significant price variations across observations (as typically found in time series data), . The productivity results presented in Table 1 and Figure 1 illustrate this point well. Alternatively, the primal approach may be viewed as less restrictive than the dual approach in the sense that it does not require imposing any behavioral assumption (e.g. profit maximization<sup>18</sup>) on production data. Thus, our productivity results suggest the existence of important trade-offs between the behavioral assumptions that economists may be willing to make and the informativeness of the associated results.

Our analysis so far has been based on a multiproduct-multifactor technology. Previous dual nonparametric measures of total factor productivity in U.S. agriculture (Cox and Chavas (1990)) have used an aggregate output specification. However, the U.S. data have been shown to be inconsistent with a functional structure exhibiting weak separability in outputs (Chavas and Cox (1988)). To assess the impact of this assumption, we calculated Theil-Törnquist price and quantity indexes for aggregate output and reestimated the productivity indexes based on this aggregate. Table 1 and Figure 2 compare the dual productivity measures for the aggregate output/disaggregate inputs (AODI) against the disaggregate outputs/disaggregate inputs (DODI) results.<sup>19</sup> Comparison of the input (output) based productivity measures for AODI versus DODI indicates that they follow similar patterns and are fairly close in level. The less restrictive DODI specification yields slightly higher levels of productivity for both the input and output radial measures. This suggests that the aggregation bias of wrongly assuming the existence of an aggregate output is not large in this case. Whether this result is generalizable is an open question easily amenable to further testing with nonparametric methodology. Finally, note that both sets of dual productivity indexes in Figure 2 are higher than the CJ productivity index. This may reflect the fact that these productivity measures are based on an upper bound representation of the underlying technology.

## V) Summary and Conclusions.

This paper illustrates and contrasts primal and dual nonparametric methods for computing input-based and output-based distance functions under the assumption of profit maximization. These primal and dual methods generate, respectively, lower and upper bound representations of the technology that is consistent with a finite number of observations on production data (Banker and Maindiratta (1988)). These nonparametric representations, which can fully and flexibly characterize a reference technology, are then used to generate nonparametric bounds on input and output based productivity measures. The dual methods presented generalize the previous nonparametric productivity work (Chavas and Cox (1990); Cox and Chavas (1990)) by allowing for multi-product productivity measurements.

Application of this primal-dual methodology to time series data on U.S. agriculture for 1950-82 indicates the existence of large differences in the nonparametric bounds on the underlying technology. This translates into very different productivity indexes from the primal approach compared to the dual approach. The primal measures (lower bounds) are found to imply no technical change in U.S. agriculture, while the dual measures (upper bounds) indicate important technical progress. This difference is attributed in a large part to the use of price information and the assumption of profit maximization in the dual approach. These results suggest the usefulness of the dual approach to productivity analysis over time, especially when relative price variations are important.

Given the divergence in the output versus input based productivity measures, it appears these data do not support the existence of a constant return to scale production technology. This provides nonparametric evidence that the total factor productivity index proposed by Christensen and Jorgenson (1970) will not be exact for these data. The CJ index is found to lay between the primal lower-bound and the dual upper-bound productivity index for U.S. agriculture, but is found to be closer to the dual index than to the primal nonparametric productivity indexes.

Potential aggregation bias in nonparametric measurement is explored by contrasting dual productivity indexes for an aggregate versus disaggregate outputs specification. In the context of U.S. agriculture, this potential aggregation bias appears to be small. However, caution in generalizing this result should be exercised.

The primal-dual nonparametric methods are easy to implement empirically, requiring the solution of standard linear programming problems. These methods easily handle multiple outputs (6 in the current example) and multiple inputs (9 in the current example). The primal-dual nonparametric representations fully characterize the implicit production technology while imposing a minimum of a priori structure. Hence, these methods are very flexible and convenient tools for applied work on productivity analysis.

Hopefully, this paper will contribute to a better unified understanding of primal and dual nonparametric methodologies. At this point, the relative merits of a primal versus a dual approach appear to depend to a large extent on the nature of the data. Another unresolved problem is how to choose between an input-based and an output-based productivity index when technology departs from constant return to scale. Further research is needed to develop better insights on these issues.

Table 1. Alternative Primal and Dual Nonparametric Total Factor Productivity (TFP) Measures for 1950-82 U.S. Agriculture.

		DODI:				AODI:	
		6 DISAGGREGATE OUTPUTS				AGGREGATE OUTPUT	
		9 DISAGGREGATE INPUTS				DISAGGREGATE INPUTS	
CAPALBO		=====					
& VO		PRIMAL TFP		DUAL TFP		DUAL TFP	
CJ		=====					
TFP		INPUT	OUTPUT	INPUT	OUTPUT	INPUT	OUTPUT
YEAR	INDEX	IP/IP <sub>50</sub>	OP/OP <sub>50</sub>	IP/IP <sub>50</sub>	OP/OP <sub>50</sub>	IP/IP <sub>50</sub>	OP/OP <sub>50</sub>
-----							
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000
51	1.028	1.000	1.000	1.031	1.038	1.029	1.034
52	1.073	1.000	1.000	1.057	1.069	1.059	1.071
53	1.109	1.000	1.000	1.084	1.101	1.085	1.102
54	1.108	1.000	1.000	1.086	1.103	1.085	1.101
55	1.097	1.000	1.000	1.099	1.118	1.090	1.108
56	1.181	1.000	1.000	1.174	1.207	1.164	1.194
57	1.152	1.000	1.000	1.143	1.168	1.141	1.165
58	1.173	1.000	1.000	1.188	1.223	1.180	1.212
59	1.132	1.000	1.000	1.171	1.203	1.157	1.186
60	1.164	1.000	1.000	1.200	1.238	1.188	1.222
61	1.174	1.000	1.000	1.222	1.263	1.202	1.239
62	1.197	1.000	1.000	1.242	1.288	1.221	1.260
63	1.217	1.000	1.000	1.264	1.313	1.240	1.284
64	1.245	1.000	1.000	1.289	1.343	1.264	1.311
65	1.233	1.000	1.000	1.294	1.350	1.269	1.318
66	1.246	1.000	1.000	1.307	1.364	1.284	1.336
67	1.275	1.000	1.000	1.333	1.396	1.310	1.367
68	1.278	1.000	1.000	1.348	1.414	1.324	1.384
69	1.292	1.000	1.000	1.370	1.441	1.344	1.408
70	1.259	1.000	1.000	1.344	1.409	1.319	1.378
71	1.324	1.000	1.000	1.413	1.492	1.383	1.454
72	1.339	1.000	1.000	1.428	1.511	1.398	1.473
73	1.360	1.000	1.000	1.448	1.535	1.419	1.499
74	1.409	1.000	1.000	1.453	1.541	1.416	1.495
75	1.438	1.000	1.000	1.479	1.573	1.455	1.542
76	1.437	1.000	1.000	1.485	1.580	1.457	1.545
77	1.493	1.000	1.000	1.538	1.644	1.512	1.610
78	1.452	1.000	1.000	1.526	1.629	1.497	1.593
79	1.530	1.000	1.000	1.600	1.719	1.573	1.683
80	1.527	1.000	1.000	1.586	1.702	1.561	1.669
81	1.636	1.000	1.000	1.688	1.824	1.662	1.790
82	1.652	1.000	1.000	1.717	1.859	1.691	1.825

SOURCE: Computations by the authors from linear programming solutions to (7a) and (8a) (primal input and output distance functions, respectively) and (7b) and (8b) (dual input and output distance functions, respectively). All productivity indexes are rescaled to 1 in the base year 1950.

Figure 1. Comparison of Primal and Dual Nonparametric Measures of Total Factor Productivity for U.S. Agriculture: 6 Disaggregate Outputs and 9 Disaggregate Inputs (DODI).

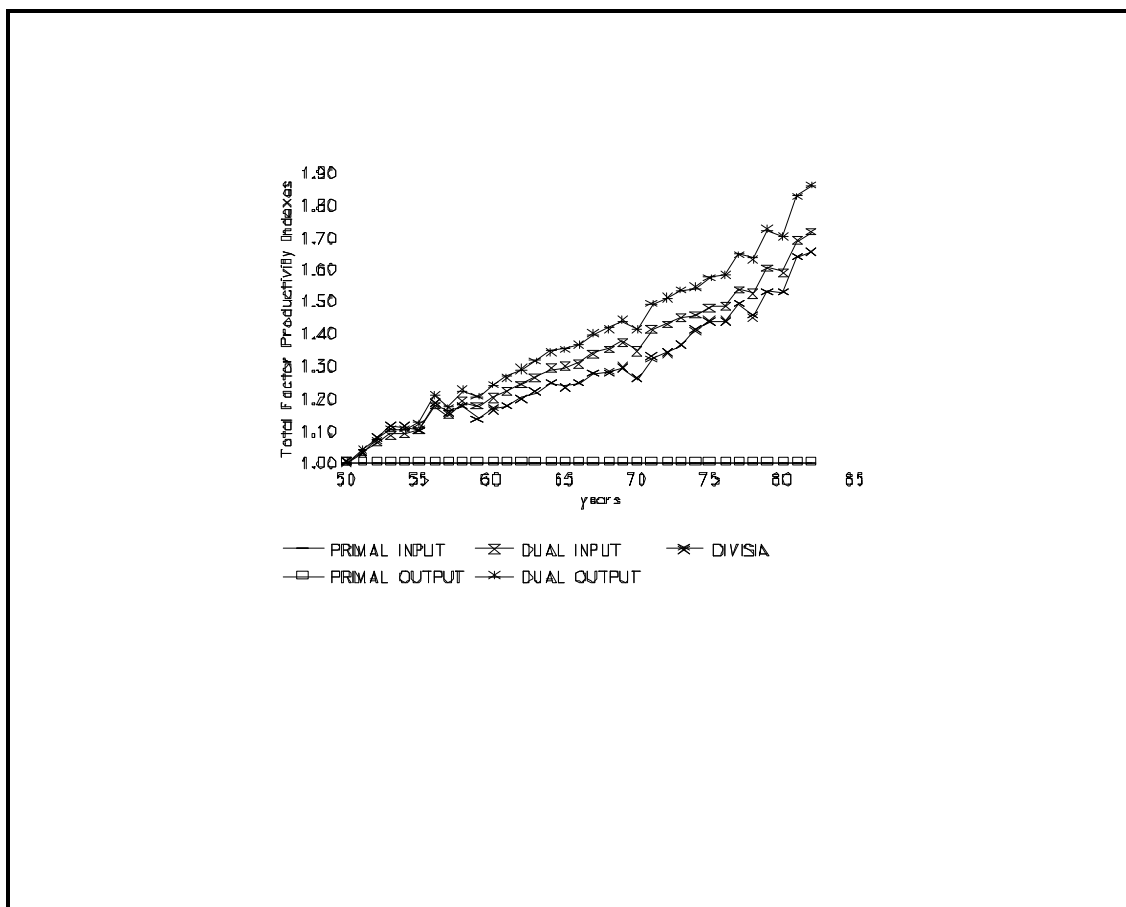
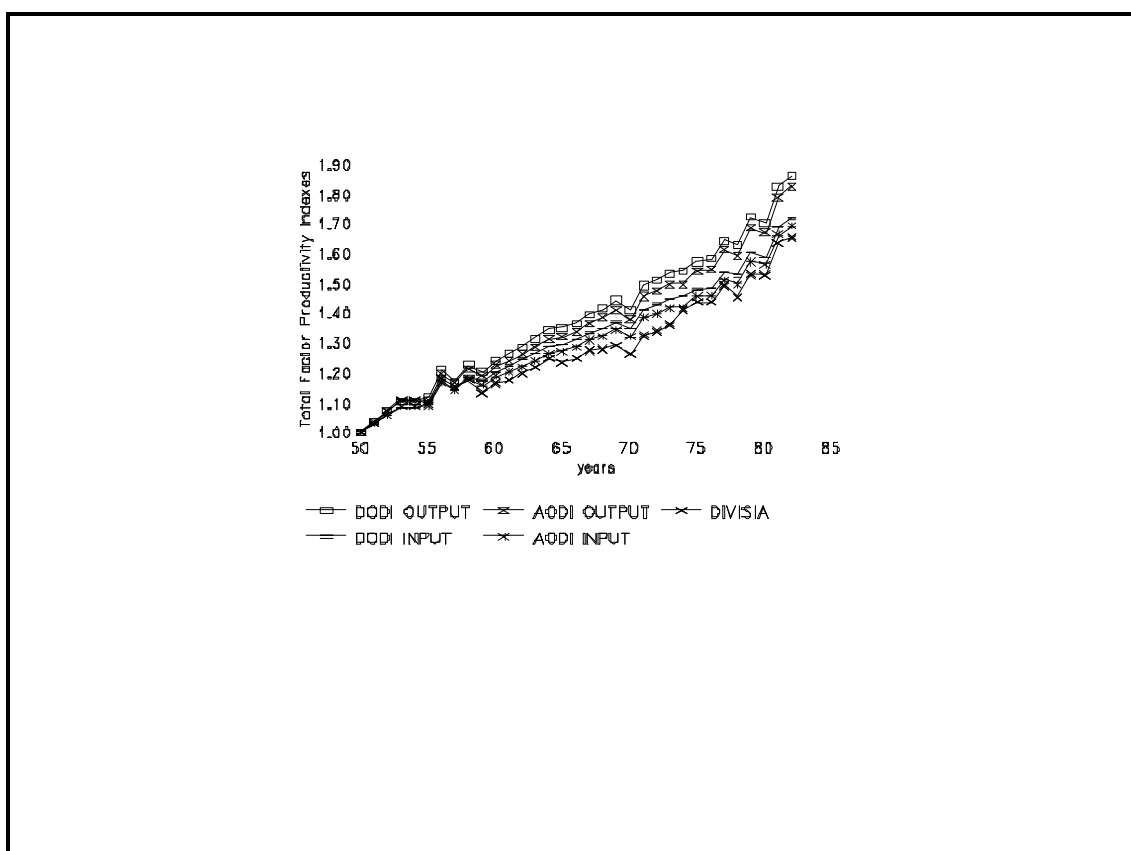


Figure 1

Figure 2. Comparison of Dual Nonparametric Measures of Total Factor Productivity for U.S. Agriculture, 1950-82: Impacts of Aggregate (AODI) versus Disaggregate (DODI) Outputs.



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## Footnotes

1. A set  $T$  is said to be negative monotonic if  $t_1 \in T$  and  $t_2 \leq t_1$  implies that  $t_2 \in T$ . Note that this implies free disposal.
2. The joint analysis of efficiency and productivity based on panel data has been investigated recently by Färe et al. (1992, 1994).
3. Diewert (1976) defined "superlative indexes" as indexes that are exact for a flexible production, cost, or revenue function (i.e. for a functional form that can provide a second order approximation to the true (but unknown) underlying function). He recommended the use of such indexes in economic analysis.
4. An additional condition assumed by Caves et al. (1982b) is that the firm is allocatively efficient, i.e. that it either minimizes cost (for the input-based measure) or maximizes revenue (for the output-based measure). Note that this allocative efficiency assumption is crucial in the derivation of some of Caves et al.'s results.
5. This is motivated in a large part by the nature of the data used in section IV below. For an extension of the analysis based on panel data (allowing a distinction between productivity growth and technical efficiency), see Färe et al. (1992, 1994).
6. Afriat was the first to show that either a primal or a dual nonparametric approach generates a representation of the underlying technology. Specifically, assuming a single product technology, Afriat ((1972), p. 596) derived the primal and dual nonparametric production frontiers (associated with (5) and (6)) to be respectively:

$$\text{PRIMAL:} \quad f_s(x) = \max \{ \sum y_t \lambda_t : \sum x_t \lambda_t \leq x, \sum \lambda_t = 1, \lambda_t \geq 0 \}$$

$$\text{DUAL:} \quad f_L(x) = \min_t \{ y_t + (r_t/p_t)'(x - x_t) \}.$$

7. As noted in section 2,  $D$  and  $F$  can also be interpreted in terms of efficiency indexes. In this context, alternative characterizations of these indexes have been proposed in the literature. This has been done by relaxing the "strong disposability" assumption implicit in (7a) and (8a), and/or by relying on non-radial measurements (see Färe, Grosskopf and Lovell (1985)).
8. For more details concerning these data, please refer to Capalbo and Vo (1987).
9. These solutions were checked for the existence of multiple solutions. As all non-basic variables in the optimal solutions had non-zero dual values, no evidence of multiple solutions was found in the

results presented here.

10. A possible alternative would be using a sequential approach where the reference technology for each year does not involve subsequent years (i.e. where at time  $t$  the set  $I$  in (7) and (8) includes only the years from 1950 to  $t$ ). This would give sequential productivity indexes (e.g. from time  $(t-1)$  to  $t$ ), each using a different reference technology. Although such sequential indexes can be "chained" together to generate cumulative productivity indexes over time, the resulting indexes would in general not be "true" indexes (as defined in section 2) since they do not have a fixed reference technology.

11. Note that this may be a questionable assumption (e.g., see Färe et al. (1992, 1994)). It is motivated in large part by the difficulty of distinguishing between technical efficiency and technical change in the absence of time series - cross section data. Our discussion below should be interpreted in the light of these limitations.

12. However, note that the ratios  $IP_t/IP_{t-1}$  or  $OP_t/OP_{t-1}$  are not to be interpreted as productivity indexes as defined in section II. The reason is that the productivity indexes  $IP$  and  $OP$  use  $T$  as the reference technology, which in general differs from the technology available at time  $t$  (or  $(t-1)$ ).

13. Although finding the CJ productivity index to be between the primal and dual bounds may not be surprising, such comparisons should be made with caution. Strictly speaking, the reference technology implied by the CJ TFP index is not the same as that of the primal and dual nonparametric measures (i.e., the admissible possibility set  $A$  which is consistent with the existence of profit maximization). In addition, constant return to scale and other such a priori functional structure (such as translog) are not required by these nonparametric measures.

14. Note that departure from constant return to scale will in general invalidate the CJ productivity index as an exact index of productivity (Caves et al. (1982b)). In this case, Caves et al. (1982b) have shown how to adjust the CJ productivity index to account for the existence of economies (or diseconomies) of scale.

15. Indeed, profit maximizing behavior leads to unbounded choices under a technology exhibiting global increasing return to scale. This is ruled out by the data.

16. Note that Banker and Maindiratta found tighter bounds than the ones reported here. This indicates that the tightness of these nonparametric bounds clearly depends on the data. Thus, it should be kept in mind that our results may not apply to different industries or different data sets.

17. This is similar to the contrast of estimating a parametric production function directly versus estimating the production function from a dual cost or profit function with explicit incorporation of a behavioral premise (such as cost minimization or profit maximization). In the latter case, behavioral optimization together with price information yield restrictions on the marginal productivity of inputs and outputs.

18. If profit maximization does not hold, then the dual productivity indexes would not be appropriate measures of the rate of technical change. Note that such arguments would apply as well for the Tornquist indexes.

19. The primal AODI results are similar to the DODI results of Table 1 and Figure 1, hence are not presented.