Optimal Management of Drawdown and Saltwater Intrusion in Coastal Aquifers

Jibran Punthakey and Greg Hertzler

Faculty of Agriculture and Environment
University of Sydney

jpun2885@uni.sydney.edu.au greg.hertzler@sydney.edu.au

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Abstract

Coastal aquifers are highly sensitive natural systems that require careful management and adequate planning in order to ensure that they are not over-exploited. The development of effective management strategies is particularly pertinent to coastal communities faced with increasing population pressures. This paper develops a multidisciplinary approach for the optimal extraction of water from a coastal aquifer. The objective is to maximise the net present value of the net economic benefits of pumping, subject to various hydrological constraints. A nonlinear demand function is used to quantify the benefits of water use, and analytical equations of groundwater flow and saltwater intrusion are used to integrate core principles of hydrology within the economic framework. For the optimisation of drawdown levels, a modified version of the Theis (1935) solution is used to calculate drawdown levels. For the prevention of saltwater intrusion, constraints are calculated using the single-potential sharp-interface solution developed by Strack (1976). The model is then applied to a hypothetical coastal community situated on the East coast of Australia. The results confirm the importance of accounting for spatial heterogeneity and temporal effects when modelling extraction from a complex natural system. In addition, a volumetric price of water is recommended as an efficient policy tool for the control and regulation of demand.

Keywords

Groundwater economics; Coastal aquifers; Drawdown; Saltwater intrusion; Optimisation

1. Introduction

Coastal aquifers supply fresh, potable water to coastal communities around the world. These underground water resources are often pristine, uncontaminated and easily accessible. Due to their close proximity to population centres, they are also well positioned and relatively inexpensive to develop. Improper management puts coastal aquifers at risk of over-exploitation due to excessive pumping and contamination by saltwater intrusion (Bear *et al.* 1999). Remediation of contaminated coastal aquifer systems is often both unfeasible and undesirable due to the extensive costs involved. In many cases, the only available alternative is to develop other sources of supply to supplement the reduced availability of fresh groundwater (Post 2005).

Previous studies of groundwater resources stress the need to prescribe sustainable management strategies and solutions. Yet it is not uncommon for economic models to incorporate, whether implicitly or explicitly, various assumptions about the dynamics of aquifer systems. For instance, lumped parameter models, otherwise known as 'single-cell', 'bathtub' or 'milk-carton' models, were often used in lieu of complex and elaborate hydrological flow equations (see Castle and Lindeborg 1960; Kelso 1961; Domenico *et al.* 1968; Brown and Deacon 1972). These studies generally viewed the aquifer as a closed pool, using the total stock of water remaining in the aquifer to represent changes in the state of the resource (Reichard 1987). Lumped parameter models are simplified representations that do not recognise and appreciate the complex nature of the underlying resource. Tacit in the formulation of these models are a number of limiting assumptions: First, the spatial distribution of individual extraction points is considered irrelevant. In other words, at any point in time, the depth to the water table is constant throughout the aquifer. Second, all wells are assumed to withdraw an equal amount of water. In reality, some wells are more productive than others, and are responsible for a higher share of total pumping. Finally, many previous studies also assumed path independence of the resource. This refers to the assumption that the past history of extraction does not undermine one's ability to withdraw water from the aquifer in the present or in the future (Brozovic *et al.* 2002). Relaxing one or more of these assumptions may alter the results and conclusions of previous work.

The use of distributed parameter models in economic studies of groundwater first appeared in a simulation model developed by Bredehoeft and Young (1970). Unfortunately, the increased verisimilitude observed in simulation models came at a cost. Most were only able to compare alternative management scenarios, and could not be used to determine optimal management solutions. Other 'multi-cell' groundwater models tended to be highly inflexible, and were only calibrated to specific groundwater basins (Noel *et al.* 1980; Noel and Howitt 1982; Reichard 1987). At the same time, the hydrological literature was over populated with large-scale numerical models that provided a highly accurate and detailed description of lateral flow within a specific aquifer system. While the majority of numerical groundwater models are spatially, temporally and hydrologically realistic, economists have generally failed to synthesize and combine the central principles of both disciplines. Consequently, most studies are characterised by a disproportionate emphasis on either economics or hydrology.

The optimal control of saltwater intrusion is treated as an independent class of problem in the groundwater literature. Early models were typically site-specific and involved differing management objectives. Curtailment of the saltwater interface was often achieved indirectly, by minimising the volume of intruded saltwater or by setting restrictions on pumping capacity (Benhachmi 2003a). For instance, Shamir *et al.* (1984) embedded a multi-objective linear programming model within a multi-cell model of a coastal aquifer. Owing to limited computational power, a linear function was used to estimate saltwater encroachment. In recent years, equations of hydrodynamic dispersion and molecular diffusion have been used to develop more realistic simulations of solute transport and density-dependent flow. A number of researchers have successfully utilised these equations to develop linked simulation-optimisation models of coastal aquifers (Bhattacharjya and Datta 2005; Arlai *et al.* 2007).

Unfortunately, increased complexity does not go hand in hand with increased flexibility. Models of this nature are expensive and time-consuming to implement, and remain peculiar to a specific district or region.

Curiously, management models of coastal aquifers are largely centred on the control and prevention of saltwater intrusion. In the process, conventional basin-wide management strategies are often neglected. A unique feature of the model presented here is that optimal control of saltwater intrusion takes place in conjunction with the optimisation of drawdown levels. The development of simplified analytical approximations of the saltwater interface has paved the way for optimisation models that can easily be calibrated to different aquifer systems. The sharp-interface solution developed by Strack (1976) is perhaps the most seminal innovation in this field. Following the work of Cheng *et al.* (2000), the formula has been applied to a multitude of economic models and cross-disciplinary investigations (Benhachmi *et al.* 2003a; Benhachmi *et al.* 2003b; Park and Aral 2004; Kastifarakis and Petala 2006). It also forms the basis of the saltwater intrusion modelling constraint presented in this paper.

The purpose of this research is to present a holistic approach to the optimal management and modelling of a coastal aquifer. An effort has been made to integrate the fundamentals of hydrology within an economic framework. Accordingly, the objective is to maximise the net present value of benefits to residential users minus the costs to a water management authority from pumping a coastal aquifer. In addition, many of the limitations of previous studies have been relaxed or eliminated. For instance, the conventional use of a single lumped reservoir to represent the aquifer system is replaced with a set of N spatially distributed wells. Analytical equations, rather than numerical simulations, are used to formulate the model, resulting in increased versatility and ease of computation. Finally, the assumption of path-independence of the resource is eliminated. The model is formulated in discrete time, and optimal pumping rates are derived for each well over a specified time horizon considered relevant by the water management authority.

2. Formulation of the Economic Model

The groundwater management model presented in this paper is designed for communities relying on an underground aquifer system as their sole source of freshwater supply. Given that alternative sources of supply are both costly and limited in availability, the model seeks to prescribe optimal pumping rates for a specified set of N wells operating within the catchment. Although all wells are owned and operated by the local council or water management authority, the model still needs to account for spatial heterogeneity to ensure sustainable management of the natural resource. Furthermore, the model must be both dynamic and path-dependent, to ensure a realistic rendition of the nature of extraction.

The use of optimisation techniques to model groundwater extraction is extensively documented throughout the literature. Typically, models aim to maximise or minimise a particular economic objective, subject to hydrological constraints. However, there is little consensus as to the most suitable formulation of the objective function. A large number of studies favour minimisation of the cost of pumping (Sidiropoulos and Tolikas 2004; Katsifarakis 2008). Others opt for maximisation of the total volume of pumping (Cheng 2000; Katsifarakis and Petala 2006). Several investigations have used the price of water to measure net revenues from the sale of groundwater (Loáiciga *et al.* 2000; Loáiciga 2004). This framework is especially applicable to studies of co-operative exploitation of groundwater resources in irrigation districts. For the most part, however, these alternatives are chosen due to the difficulties associated with quantifying the benefits of groundwater abstraction.

The model presented here aims to maximise the economic value of water use, using various constraints to ensure sustainable exploitation of the coastal aquifer. In each year, the withdrawal of water from the aquifer generates benefits and costs for the local population. It is assumed that daily pumping rates in each well are held constant over a period of 365 days. At the beginning of each year, pumping rates are adjusted to account for increases in population. Thus the model is formulated in discrete time, over a pre-designated time horizon of T years. The objective is to maximise the net present value (NPV) of the benefits (B_t) above costs (C_t) from T years of pumping, at a specified discount rate (r):

$$Max \text{ NPV} = \sum_{t=1}^{T} \frac{B_t - C_t}{(1+r)^t} \tag{1}$$

subject to the constraints:

1. Prevention of over-exploitation at each individual well – in each year (t), the drawdown at well n ($s_t(n)$) must not exceed the maximum allowable limit (s_{max}):

$$s_t(n) \leq s_{\max} \qquad t = 1, \ldots T;\ n = 1, \ldots N \tag{2}$$

2. Prevention of saltwater intrusion at each well – in each year (t), the x-coordinate of the toe of the interface ($x_{toe}[t, y_{stag}(n)]$) must not exceed the x-coordinate of the stagnation point ($x_{stag}(t,n)$):

$$x_{toe}\left[t, y_{stag}(n)\right] \leq x_{stag}(t,n) \qquad t = 1, \ldots T;\ n = 1, \ldots N \tag{3}$$

In each year, the benefits (B_t) arising from groundwater use are a function of the total quantity of water extracted from the aquifer (Q_t). Implicit in this statement is the assumption that marginal benefits are invariant across all wells. Thus, if the quality of freshwater is uniform across all wells, a relationship between B_t and Q_t can be derived.

Although many economic studies of groundwater conceptualise the notion of 'economic benefits', few have attempted to quantify the benefits accruing to water users. Allen and Gisser (1984) estimated both a linear and a nonlinear aggregate demand curve for a continuous time optimisation model of groundwater in the Pecos Basin in New Mexico. Pulido-Velázquez *et al.* (2006) used a similar approach, estimating a nonlinear demand curve for optimisation of conjunctive use of groundwater and surface water in the Adra River Basin in Spain. In this paper, we deviate from previous works by incorporating population growth into an estimated aggregate nonlinear demand curve:

$$Q_t = \alpha(1+g)^{t-1} P_t^{\varepsilon} \tag{4}$$

where:

Q_t is the quantity of water (kL) in demand in time t

α is the scale parameter of the demand function

ε is the price elasticity of demand

g is the annual population growth rate

P_t is the price of water

According to Marshallian surplus theory, the economic value of water can be measured by integrating under the demand curve. However, Figure 1 confirms that the demand for water is highly inelastic. The value of water needed for survival is immeasurable, and consequently the area beneath the demand curve is infinite and cannot be assigned a numerical value.

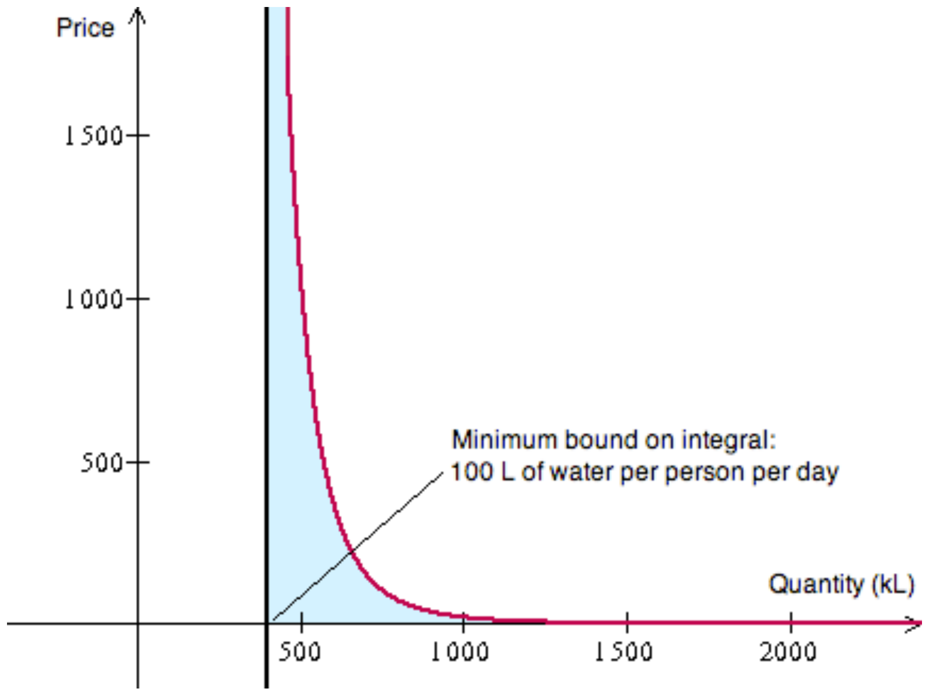

Figure 1: Sample daily water demand curve, ε = -0.173, population 4000.

To solve this problem, the benefit function is redefined in a way that does not alter the model's solution of optimal pumping rates. To be precise, a minimum water use of 100 Litres per person per day is used to form the lower bound on the integral. Thus the shaded area in Figure 1 represents the area used to measure the benefits accruing to water users in a given time period. On the other hand, the area between the y-axis and the vertical 'minimum bound' line represents the economic value of the first 100 litres (or 0.1 kL) of water consumed by each person in the population, and is not included in the calculation of benefits. Hence, the equation for the total benefits derived from water use in year *t* is given by:

$$B_t = \int_{0.1*Pop_t}^{Q_t} P(Q_t)dQ_t = \left[\alpha(1+g)^{t-1}\right]^{\frac{-1}{\varepsilon}} * \left(\frac{\varepsilon}{1+\varepsilon}\right) * \left[Q_t - (0.1 * Pop_t)\right]^{\frac{1+\varepsilon}{\varepsilon}} \tag{5}$$

where Pop_t is the population of the coastal community in time *t*.

Equation (5) demonstrates that the benefits in each period (B_t) can be expressed as a function of the total quantity of pumping in that period (Q_t). However, strictly speaking, the cost function (C_t) cannot be expressed as a function of total pumping (Q_t). This is because the cost of transporting water to the town centre varies across individual wells, and is typically higher for remote and outlying wells. As a result, the per-unit cost of pumping varies for each well operating within the catchment. In addition, marginal pumping costs will also increase as the water level drops in a well, and more energy is required to lift water from deeper in the aquifer. For the purposes of this model, however, these factors are assumed to be inconsequential. The marginal cost of water use (MC) is assumed to be constant. Furthermore, the fixed costs associated with installing wells and pump stations are assumed sunk, are not included in cost considerations. Thus the total cost of pumping in year t is given by:

$$C_t = MC * Q_t \tag{6}$$

Together, Equations (5) and (6) are used to calculate the objective: maximisation of the net present value of the net economic benefit of water.

3. Drawdown Constraint

A confined aquifer is a stratum of saturated porous material bounded from above and below by highly impermeable layers of rock. Due to pressure from the confining layers, the water table (potentiometric surface) will often lie above the upper confining layer of the aquifer (Walton 1970). Continuous pumping or discharge from a well in a confined aquifer creates a localised cone of depression, centred at the well. The dimensions of this cone are dependent on the rate of pumping (Q_t), and the various hydrological parameters that are used to describe the physical properties of the aquifer (Kruseman and de Ridder 2000). Figure 2 illustrates how unrestrained increases in pumping can threaten sustainability of the aquifer system.

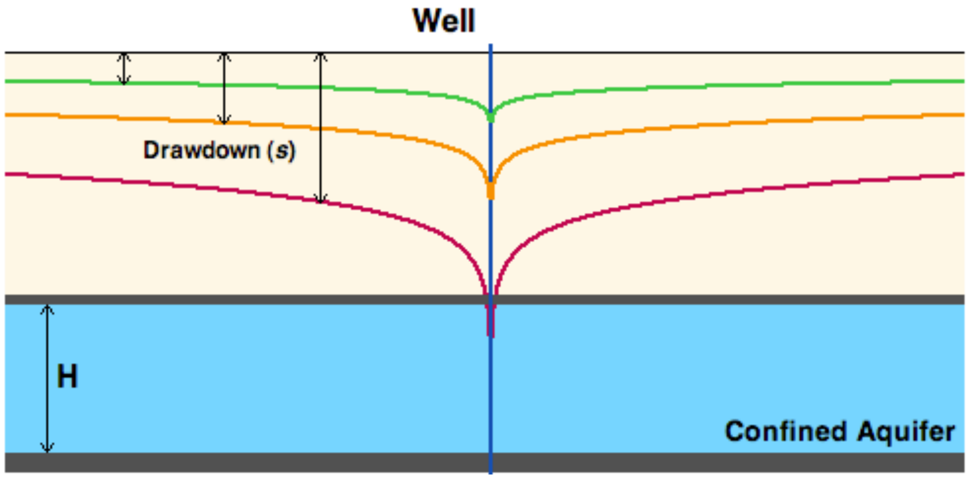

Figure 2: Drawdown cones in a confined aquifer.

In order to prevent exploitation of groundwater resources, optimisation models often impose constraints on the level of drawdown in wells operating throughout the aquifer (Brozovic *et al*. 2002; Loáiciga 2004; Katsifarakis 2008). The two most established and conventional approaches for calculating drawdown levels are Theim's method (1906) and the Theis solution (1935). The model presented in this research paper makes use of an extended form of the Theis solution to compute drawdown levels. The formula incorporates both time and Storativity, and is generally preferred to Theim's steady-state equation (Kruseman and de Ridder 2000). The general form of the unsteady-state Theis solution is written as:

$$u = \frac{r^2 S}{4kHt} \tag{7}$$

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = -\gamma - \ln u + u - \frac{u^2}{2*2!} + \frac{u^3}{3*3!} - \frac{u^4}{4*4!} + \ldots \tag{8}$$

$$s = \frac{Q}{4\pi kH} W(u) \tag{9}$$

where:

r is the horizontal distance from the well to an observation point (m)

S is the aquifer's coefficient of Storativity (dimensionless)

k is the hydraulic conductivity of the porous medium (m/day)

H is the thickness of the confined aquifer (m)

kH is the Transmissivity of the aquifer (m^2/day)

t is the time since pumping started (days)

γ is Euler's constant, approximately 0.577216

Q is the constant rate of pumping or discharge from the well (kL/day)

s is the drawdown measured at a distance r from the well (m)

In theory, equation (9) is only valid if the following conditions are met (Theis 1935):

- The aquifer is infinite in areal extent and of uniform thickness.
- Water is released instantaneously from storage.
- The aquifer is homogeneous, i.e. composed of the same material throughout.
- The aquifer is isotropic, i.e. its properties (hydraulic conductivity, Transmissivity, and Storativity) are equal at all points within the saturated medium.
- Wells penetrate the entire thickness of the aquifer, and are infinitely small in diameter.

Perhaps the most limiting aspect of the Theis solution is the assumption of a constant rate of pumping. In reality, a water management authority may be compelled to increase pumping rates in response to increases in population and increased demand for water. Birsoy and Summers (1980) used the principle of superposition to extend the Theis solution for variable pumping rates. For a series of T pumping rates Q_1, Q_2, ... Q_T effectuated at times τ_1, τ_2, ... τ_T, the drawdown at time $t > \tau_T$ and horizontal distance r from a well is given by the following formulae:

$$u_i = \frac{r^2 S}{4kH(t - \tau_i)} \qquad (10)$$

$$s_t = \frac{1}{4\pi kH}\sum_{i=1}^{T}\left[\left(Q_i - Q_{i-1}\right) * W(u_i)\right] \tag{11}$$

where:

$\tau_1 = 0$ is the point when the well first begins to pump at rate Q_1

$Q_0 = 0$, i.e. we assume that no pumping activity has taken place beforehand.

Another significant shortcoming of the Theis solution is the level of computational power required to calculate the infinite series in the well function *W(u)*. Cooper and Jacob (1946) observed that for small values of *u* (< 0.01), the well function could be approximated by the first two terms in the series:

$$W(u) \approx \ln\left(\frac{0.5615}{u}\right) \tag{12}$$

The Cooper-Jacob approximation reduces the well function to a single monotonically increasing logarithm, lending a remarkable amount of efficiency and convenience to drawdown calculations. From the perspective of a water management authority, it is sufficient to maintain drawdown levels (s_t) within a specified maximum allowable limit (s_{max}) over the entire span of pumping activity. Consequently, by assuming that the rate of pumping in a well will only change once per year ($\tau_t - \tau_{t-1} = 365$), it is sufficient to calculate and control drawdown levels at the end of each year in which pumping activities take place. For a series of *t* pumping rates (Q_1, Q_2, ... Q_t) applied over a period of *t* years, the drawdown at the end of year *t* is determined as follows:

$$u^* = \frac{r^2 S}{4kH * 365} \tag{13}$$

$$s_t = \frac{1}{4\pi kH}\left[Q_1 \ln\frac{t}{t-1} + Q_2 \ln\frac{t-1}{t-2} + \ldots + Q_{t-2}\ln\frac{3}{2} + Q_{t-1}\ln 2 + Q_t \ln\left(\frac{0.5615}{u^*}\right)\right] \tag{14}$$

The development of this formula is noteworthy as it confers a significant amount of tractability to the calculation of drawdown levels. Previously, economic models of groundwater resources have been compelled to impose the restrictive assumption of constant pumping rates (Loáiciga *et al.* 2000; Loáiciga 2004; Sidiropoulos and Tolikas 2004; Katsifarakis 2008). At the very least, equation (14) facilitates the incorporation of hydrological constraints within an economic framework.

When multiple wells are in operation, drawdown cones will overlap. The extent of interference depends on the distance between wells and their respective flow rates. According to the principle of superposition, the composite drawdown at a point in the aquifer is given by the sum of the drawdown from each individual well within the aquifer system (Kruseman and de Ridder 2000):

$$s_t(n) = \sum_{i=1}^{N} s_t(n,i) \tag{15}$$

where:

$s_t(n,i)$ is the drawdown at well n due to pumping activity from well i.

$s_t(n)$ is the composite drawdown at well n due to pumping activity from all N wells operating in the catchment.

The superposition of drawdown cones from two active wells is illustrated in Figure 3:

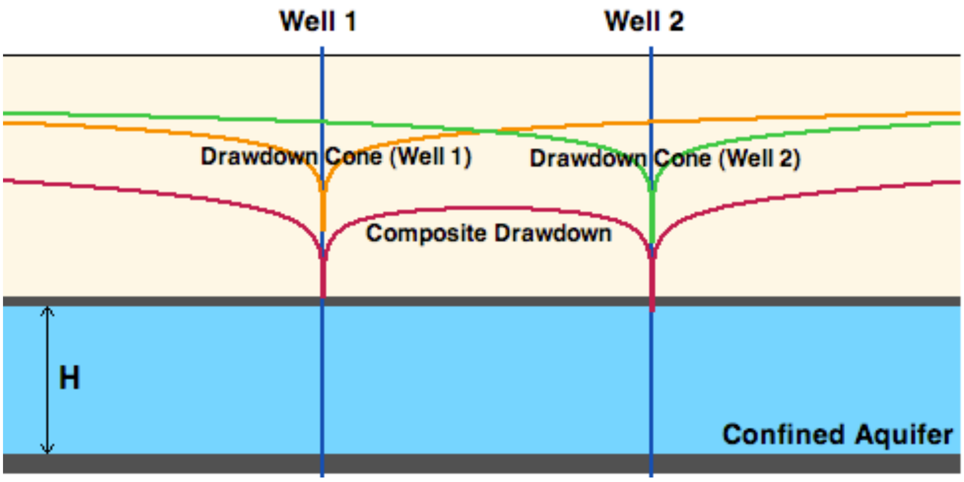

Figure 3: Superposition of drawdown cones in a confined aquifer.

To ensure continued sustainability of the groundwater resource, the drawdown at each well must be maintained below a specified maximum permissible level, determined by hydrological criteria specific to the aquifer (Loáiciga 2004). Pumping beyond this maximum allowable limit (s_{max}) can lead to compression of the confined stratum and dewatering of the aquifer. If no action is taken, the capacity of the aquifer to store and yield water will eventually diminish. Thus at each well, the drawdown constraint is defined as:

$$s_t(n) \leq s_{\max} \qquad t = 1, \ldots T;\ n = 1, \ldots N \qquad (16)$$

4. Saltwater Intrusion Constraint

In coastal aquifers around the world, saltwater intrusion has become a major constraint to the exploitation of underground water resources (Bear *et al.* 1999). The formation of a saltwater interface occurs naturally in coastal groundwater systems that are in hydraulic continuity with the ocean. Due to the differences in density between saltwater and freshwater, any saltwater entering the aquifer is rapidly overlain by freshwater. In its pristine state, a uniform rate of freshwater discharge (Q_h) will counteract the incoming deluge of saltwater, and the position of the interface will be held in equilibrium along the coastline. This acts as a natural mechanism to prevent contamination of freshwater within the aquifer. However, the development and expansion of pumping activities can easily disturb this natural balance, and induce an inward progression of the saltwater interface.

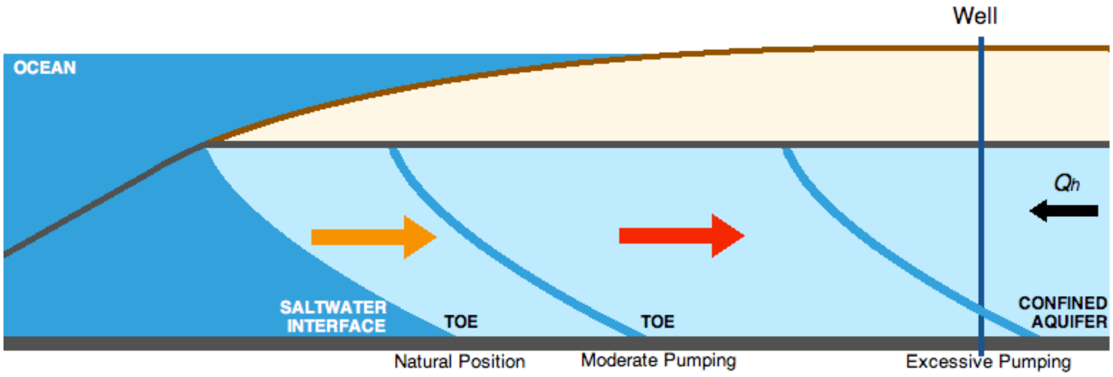

Figure 4: Saltwater intrusion in a coastal confined aquifer.

The remediation of contaminated coastal aquifer systems is extremely difficult and expensive. In the short term, artificial recharge schemes and the installation of injection wells can counteract lateral migration of the saltwater interface. Alternatively, it may be necessary to decommission existing wells, and install new wells in unaffected parts of the aquifer. In the long run, however, population growth will ultimately necessitate the development of alternative, more costly sources of supply (Post 2005). From a management perspective, it is crucial to be able to calculate the position of the saltwater interface, and control pumping rates in order to prevent contamination of wells operating within the aquifer. The use of optimisation is favourable when addressing this category of problem, as it allows for the determination of a socially desirable set of pumping rates from each well within the coastal aquifer system.

Calculation of the position of the toe of the saltwater interface is based on the single potential theory approach. The fundamental assumption underlying this theory is that a single governing equation can be derived to describe lateral flow throughout the coastal aquifer. The equation was originally developed by Strack (1976) to describe the movement of the toe of the saltwater interface in a coastal aquifer with one active pumping well. Using the principle of superposition, Cheng *et al.* (2000) extended the original equation to accommodate multiple (N) pumping wells:

$$\Phi = \frac{1}{2}kH^2\left(\frac{\rho_s - \rho_f}{\rho_f}\right) = -Q_h x + \sum_{i=1}^{N}\frac{Q_i}{4\pi}\ln\left[\frac{(x-x_i)^2 + (y-y_i)^2}{(x+x_i)^2 + (y-y_i)^2}\right] \qquad (17)$$

where:

Φ is the single-value potential, continuous throughout the entire aquifer

k is the hydraulic conductivity of the porous medium (m/day)

H is the thickness of the confined aquifer (m)

ρ_f is the density of freshwater (1.0 g/cm^3)

ρ_s is the density of saltwater (1.025 g/cm^3)

Q_h is the uniform rate of horizontal freshwater discharge (m^2/day)

Q_i is the rate of pumping at well i (m^3/day)

x_i is the x-coordinate of well i (m)

y_i is the y-coordinate of well i (m)

The biggest limitation of equation (17) is that it requires the following assumptions for its solutions to be valid (Strack 1976):

- The confined aquifer is homogeneous, isotropic and of uniform thickness throughout.
- A sharp-interface separates the saltwater and freshwater zones. This approximation is acceptable if the miscible transition zone is sufficiently narrow (Bhattacharjya and Datta 2005).
- Lines of equal potential are approximated by vertical surfaces (Dupuit-Forchheimer assumption). Effectively, vertical flow is neglected and only lateral flow is considered. This reduces the problem to two dimensions.
- The body of saltwater is at rest (Ghyben-Herzberg assumption).
- The Storativity coefficient is excluded, thus the solution is time-independent and steady-state conditions are observed.

Analysis of equation (17) reveals that the movement of the interface toward a pumping well is highly unstable. As the rate of pumping increases, the interface will approach a critical 'stagnation point', after which it experiences a discontinuous jump over and beyond the well. The progression of the interface is illustrated in Figure 5. The image exhibits a top-view sequence of gradually increasing pumping rates. Initially, the well is inactive ($Q = 0\ kL/day$) and the interface lies parallel to the coastline (represented by the thin blue strip along the y-axis). Systematic increases in pumping prompts a lateral migration of the saltwater interface up to the stagnation point, with the final pumping rate ($Q = 800\ kL/day$) representing the case of saltwater intrusion and contamination at the well:

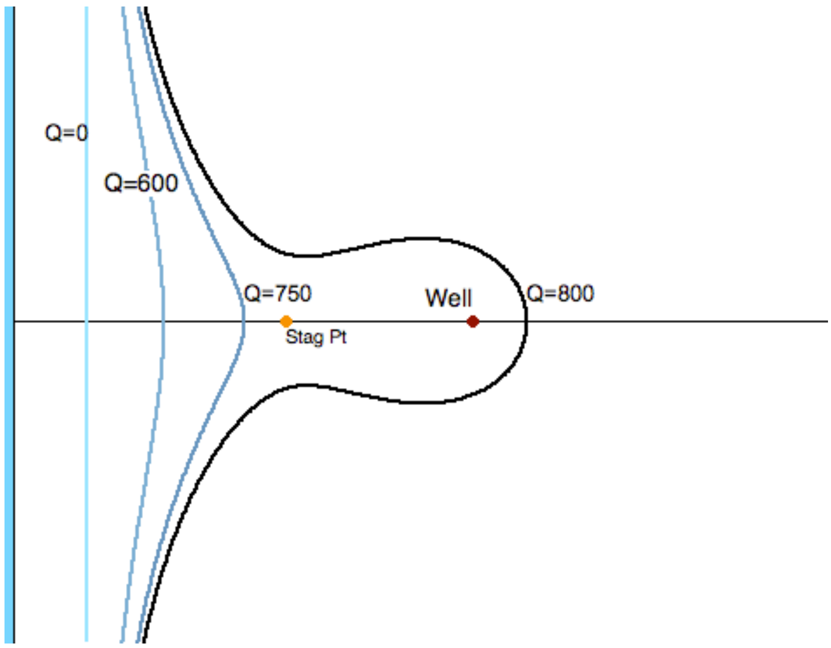

Figure 5: Progression of saltwater interface with various pumping rates.

The design of constraints used to control and prevent saltwater intrusion has a significant impact on efficiency in the optimisation process. Typically, models constrain progression of the saltwater interface by setting the x-coordinate of the toe of the interface to be less than the x-coordinate of the well ($x_{toe} \leq x_n$, $n = 1, \ldots N$). This formulation of the constraint is sufficient to prevent intrusion, and is adopted by numerous optimisation studies (Cheng *et al.* 2000; Benhachmi 2003a; Benhachmi 2003b). However, due to the presence of multiple discontinuities and local maxima, the problem is virtually impossible to solve using conventional gradient-based nonlinear solvers. As a result, saltwater intrusion optimisation models are commonly solved using the Genetic Algorithm (GA) technique, an optimisation tool specifically designed for non-smooth problems (Cheng *et al.* 2000; Benhachmi 2003a; Benhachmi 2003b; Park and Aral 2004; Katsifarakis and Petala 2006).

An alternative approach is to calculate the coordinates of the stagnation points at each well, and set the *x*-coordinate of the toe of the interface to be less than the *x*-coordinate of the stagnation point (Park and Aral 2004):

$$x_{toe}\left[t, y_{stag}(n)\right] \leq x_{stag}(t,n) \qquad t = 1, \ldots T;\ n = 1, \ldots N \tag{18}$$

By reformulating the problem in this manner, optimisation becomes far more efficient as the model is no longer discontinuous. A standard gradient-based nonlinear solver can then be used to solve the model, and optimal pumping rates appear to be considerably better than those found by GA or evolutionary solvers. The downside to this method is that solving for the coordinates of the stagnation points is a lengthy process requiring numerous convoluted calculations. A stagnation point is defined as a point in the aquifer with zero flow in both the *x* and *y* directions. The coordinates of the *N* stagnation points are therefore found by partially differentiating the potential *Φ* with respect to *x* and *y*, and setting the two components of discharge equal to zero (Strack 1976):

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Phi}{\partial y} = 0 \tag{19}$$

Differentiating equation (17) with respect to *x* and *y* yields the following two nonlinear equations:

$$\frac{\partial \Phi}{\partial x} = -Q_h + \sum_{n=1}^{N} \frac{Q_n}{2\pi}\left[\frac{x - x_n}{(x - x_n)^2 + (y - y_n)^2} - \frac{x + x_n}{(x + x_n)^2 + (y - y_n)^2}\right] = 0 \tag{20}$$

$$\frac{\partial \Phi}{\partial y} = \sum_{n=1}^{N} \frac{Q_n}{2\pi}\left[\frac{y - y_n}{(x - x_n)^2 + (y - y_n)^2} - \frac{y - y_n}{(x + x_n)^2 + (y - y_n)^2}\right] = 0 \tag{21}$$

Park and Aral (2004) outlined the procedure for simultaneous solution of equations (20) and (21), using the Newton-Raphson method for two equations and two unknowns. The process requires further differentiation of equations (20) and (21)[1]:

$$\frac{\partial^2 \Phi}{\partial x^2} = \sum_{n=1}^{N} \frac{Q_n}{2\pi} \left[\frac{(x+x_n)^2 - (y-y_n)^2}{\left[(x+x_n)^2 + (y-y_n)^2\right]^2} - \frac{(x-x_n)^2 - (y-y_n)^2}{\left[(x-x_n)^2 + (y-y_n)^2\right]^2} \right] \tag{22}$$

$$\frac{\partial^2 \Phi}{\partial y^2} = -\frac{\partial^2 \Phi}{\partial x^2} \tag{23}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial x}\right) = \sum_{n=1}^{N} \frac{Q_n}{\pi} \left[\frac{(x+x_n)(y-y_n)}{\left[(x+x_n)^2 + (y-y_n)^2\right]^2} - \frac{(x-x_n)(y-y_n)}{\left[(x-x_n)^2 + (y-y_n)^2\right]^2} \right] \tag{24}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial x}\right) \tag{25}$$

The Newton-Raphson method is an iterative method for solving a set of nonlinear equations. The formula is a useful tool because it often converges to a solution very quickly. Beginning with an initial guess of coordinates for the stagnation point (x_k , y_k), a more accurate guess (x_{k+1} , y_{k+1}) may be given by the following equation:

$$\begin{Bmatrix} x^{k+1} \\ y^{k+1} \end{Bmatrix} = \begin{Bmatrix} x^{k} \\ y^{k} \end{Bmatrix} - \frac{1}{D} \begin{bmatrix} \dfrac{\partial^2 \Phi}{\partial y^2} & -\dfrac{\partial}{\partial y}\left(\dfrac{\partial \Phi}{\partial x}\right) \\ -\dfrac{\partial}{\partial x}\left(\dfrac{\partial \Phi}{\partial y}\right) & \dfrac{\partial^2 \Phi}{\partial x^2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial^2 \Phi}{\partial x^2} \\ \dfrac{\partial^2 \Phi}{\partial y^2} \end{bmatrix} \tag{26}$$

---

[1] Equations (20) – (25) are presented here in their most simplified form. They appear different but are mathematically equivalent to the more convoluted expressions derived by Park and Aral (2004).

where:

D is the determinant of the matrix: $\begin{vmatrix} \dfrac{\partial^2 \Phi}{\partial x^2} & \dfrac{\partial}{\partial y}\left(\dfrac{\partial \Phi}{\partial x}\right) \\ \dfrac{\partial}{\partial x}\left(\dfrac{\partial \Phi}{\partial y}\right) & \dfrac{\partial^2 \Phi}{\partial y^2} \end{vmatrix}$

One of the drawbacks of the Newton-Raphson method is that it requires a good initial guess in order to converge to the correct solution. If iterations do not begin from a point in the immediate vicinity of the stagnation point, the method may fail to converge. For the purposes of this saltwater intrusion problem, a point lying adjacent to and on the coastal side of a well is usually sufficiently close to ensure convergence to the desired solution.

Calculation of the toe of the saltwater interface (the left-hand side of the saltwater intrusion constraint) can only take place after the N stagnation points have been located. The y-coordinate of each stagnation point is then used to determine the x-coordinate of the toe in line with the stagnation point. This time, the Newton-Raphson method for one equation and one unknown variable is used to solve for the x-coordinate of the toe:

$$x_{toe}^{k+1} = x_{toe}^{k} - \left[\Phi(x_{toe}^{k}, y_{stag}) \div \frac{\partial \Phi}{\partial x}\right] \tag{27}$$

Once again, a good initial guess is required for the method to converge to the correct solution. The x-coordinates of the interface are then compared to the respective x-coordinates of the stagnation points. If the interface lies between a stagnation point and the coastline, the saltwater intrusion constraint is not binding at that well and there may be potential to increase pumping rates without risking contamination.

5. Results

In this section, the optimisation model is applied to an aquifer supplying water to a hypothetical coastal community located along the East coast of Australia. Benefits are measured using an aggregate water demand function for residential users in Sydney, Australia (Grafton and Ward 2008). Due to cultural and geographical similarities, it is assumed that the preferences of water users in the coastal community are identical to preferences of Sydney residents.

Solutions to the model are obtained using the Risk Solver Platform (Version 10.0) developed by Frontline Systems, Inc. The software is embedded into Microsoft Excel and is a robust and versatile tool for the solution of conventional optimisation problems. The standard GRG nonlinear engine is most adept at solving this category of problem, however the evolutionary solver can also be used with varying results. Due to the large number of variables, functions, and dependencies, the model must be coaxed to the optimal solution. This is achieved by solving for the optimal pumping rates in each individual year (*1,…T*), and using this solution as a starting point for the search for the global optimum. As such, it is not entirely clear that the solutions found are globally optimal. However, due to binding constraints, there is very little room for improvement in the given solutions, and if they are not globally optimal, they are certainly very close.

5.1 Baseline Scenario

The optimisation model has been tested under a number of different scenarios and objectives. For all of the experiments performed in this section, management occurs over a pre-designated time horizon of 20 years. It is assumed that due to the cumulative effect of discounting, extending the time horizon beyond this point will not significantly alter the results of the model. On the other hand, the location of wells relative to the coastline and relative to one another has a marked effect on the optimal pumping rates simulated by the model. However, owing to the potentially infinite variety of different well arrangements, the number of wells in operation and their locations (*x* and *y* coordinates) are also held constant across all simulations. Parameters used in the baseline scenario are given in Table 1:

General Model Parameters		
Time horizon (*T*)	20 years	
Number of wells (*N*)	5	
Objective Function Parameters		
Scale parameter (α)	1676.71	
Price elasticity of demand (ε)	-0.173	
Initial population (Pop_1)	4000	
Population growth rate (*g*)	4 % p.a.	
Marginal cost of pumping (*MC*)	$2.35/kL	
Discount rate (*r*)	5 % p.a.	
Aquifer Parameters		
Storativity (*S*)	0.0001	
Hydraulic conductivity (*k*)	12 m/day	
Thickness of confined aquifer (*H*)	20 m	
Transmissivity (*kH*)	240 m^2/day	
Maximum drawdown (s_{max})	10 m	
Density of saltwater (ρ_s)	1.025 g/cm^3	
Density of freshwater (ρ_f)	1.0 g/cm^3	
Horizontal freshwater discharge	-1.2 m^2/day	
Well Coordinates		
Well I.D. (*n*)	***x***	***y***
1	600	0
2	360	60
3	1350	300
4	900	-150
5	315	-300

Table 1: Model parameters (Baseline Scenario).

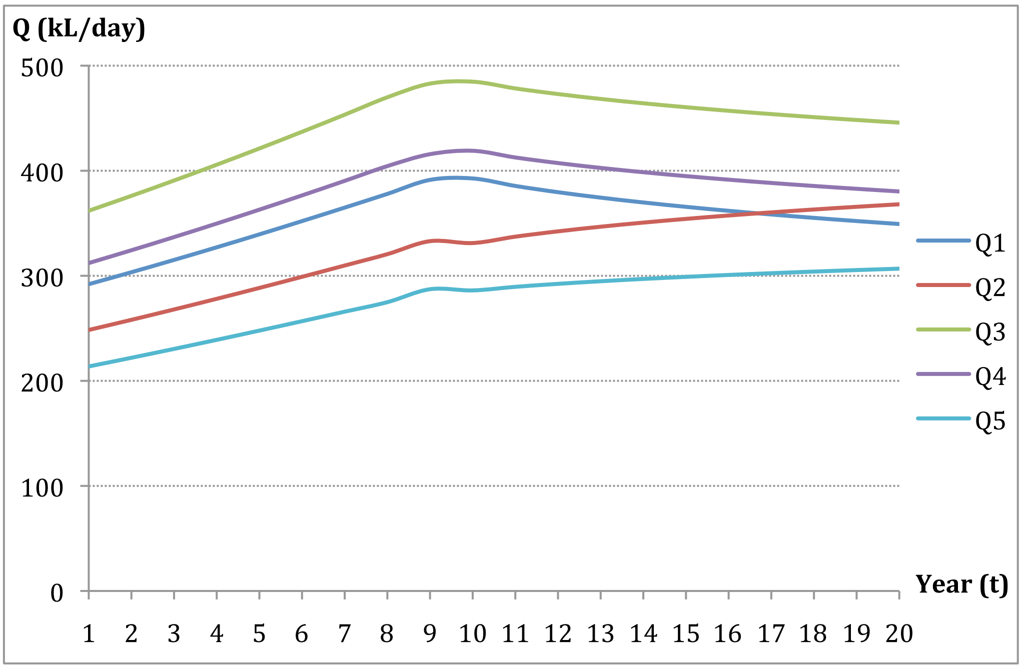

Figure 6: Optimal pumping rates (Baseline Scenario).

The solution of optimal pumping rates for wells 1-5 (Figure 6) reveals a number of interesting details. First, the potential for future exploitation of the aquifer is limited. For the first 10 years, population growth spurs rising pumping rates at all 5 wells. In year 9, however, wells 2 and 5 are constrained by saltwater intrusion. These wells lie closest to the coast and are the first to be threatened by the incoming saltwater wedge. Figure 8 illustrates the evolution of the saltwater interface over the 20-year period. Effectively, wells 1, 3 and 4 are shielded by wells 2 and 5, and are never at risk of contamination by saltwater intrusion.

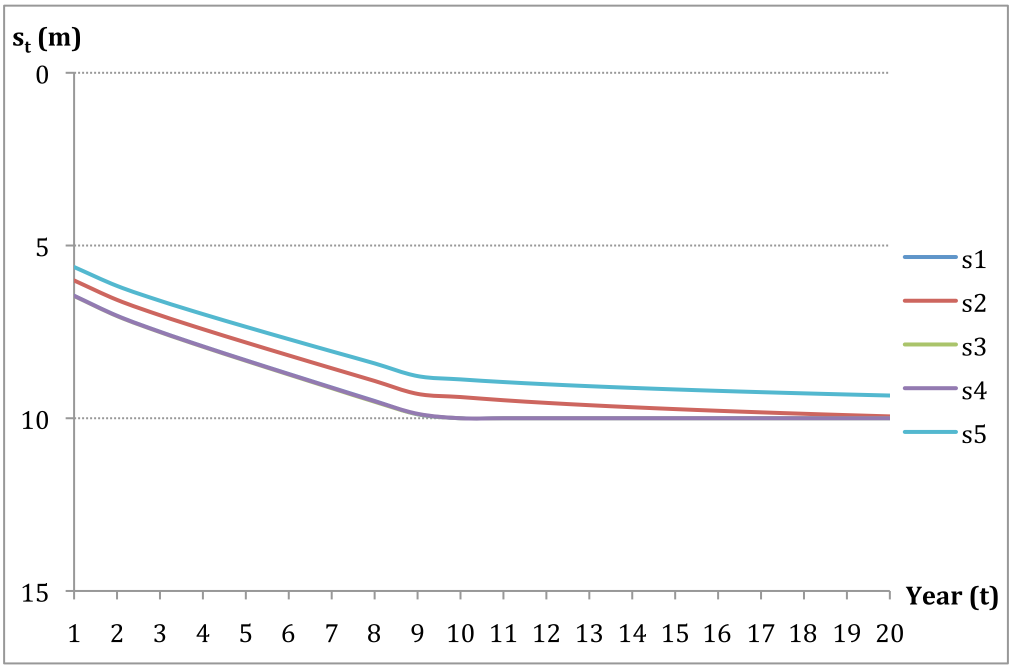

Figure 7: Drawdown levels (Baseline Scenario).

Figure 7 shows that the drawdown constraint is binding at wells 1, 3 and 4 from year 10 onwards (the axis is displayed in reverse to give the appearance of declining water levels). Beyond this point, the evolution of optimal pumping rates is largely dictated by the interaction of the drawdown and saltwater intrusion constraints. The drawdown constraint is a path-dependent solution. For this reason, pumping rates at wells 1, 3 and 4 gradually decrease over time, approaching their individual steady-state levels. It is interesting to note that although the saltwater intrusion constraint is a steady-state solution, pumping at wells 2 and 5 can still evolve over time in response to the transient nature of the drawdown constraint. Consequently, wells 2 and 5 experience steady growth in pumping rates from year 10 onwards. It is expected that drawdown levels at wells 2 and 5 will eventually reach the maximum allowable limit of 10 metres, after which pumping rates will begin to decrease. This does not take place during the 20-year time horizon; rather, it is predicted from the trends observed in the current set of results.

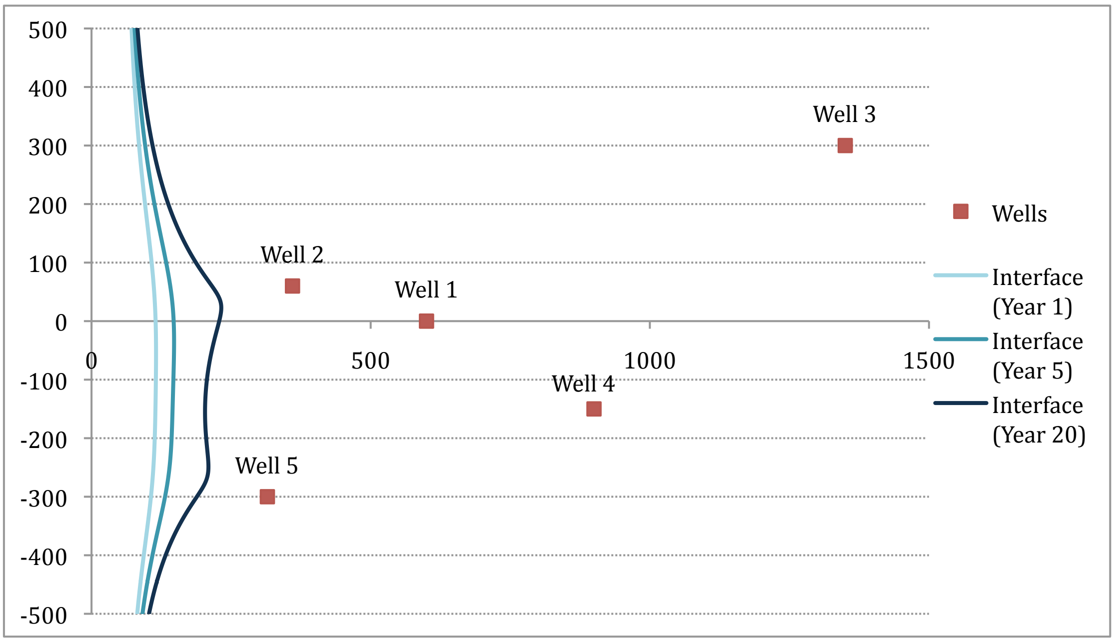

Figure 8: Saltwater interface progression (Baseline Scenario).

A number of interesting additional observations can be made from the relative distribution of pumping. For example, well 5 is somewhat isolated from the other wells, yet it is the lowest yielding well in the catchment. The most likely explanation for this is that it is situated too close to the coastline. Well 2 is only 45 metres further inland, and it yields the second-lowest volume of water for 16 out of the 20 years. On the other hand, well 1 is the most centrally located well, and is the most affected by well interference due to overlapping drawdown cones. In spite of this, it only supplies 26.5 kL/day less than well 4 (on average). This suggests that when choosing the location of a new well, proximity to the coastline will have a far greater influence on pumping capacity than positioning relative to other wells. This is exemplified by the rate of withdrawal observed at well 3. Well 3 is the most remotely located well, and is by far the most productive in the catchment.

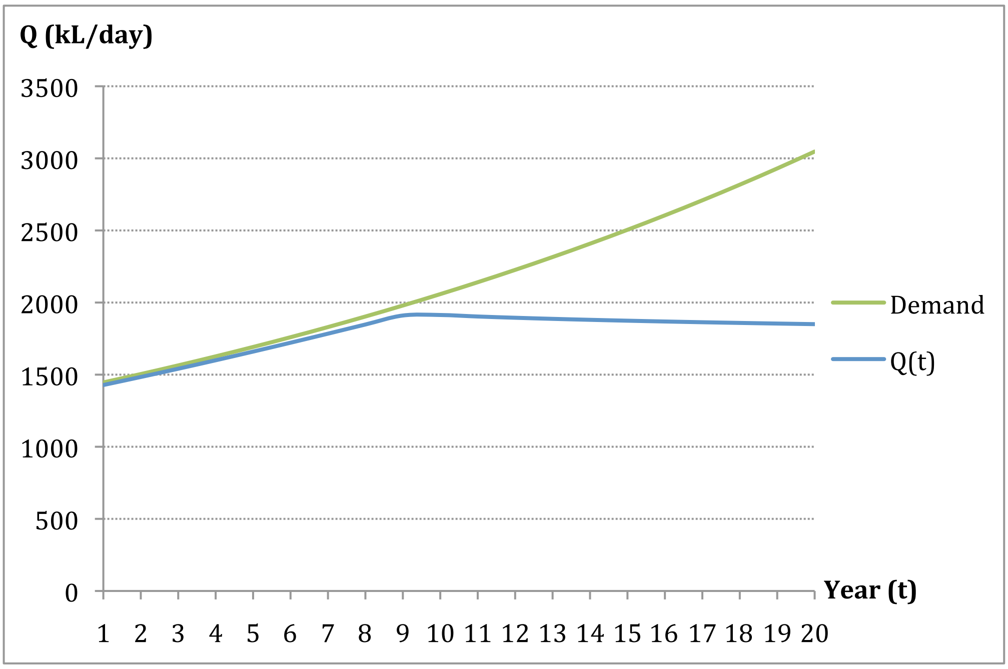

Figure 9: Daily water demand vs. actual consumption (Baseline Scenario).

Figure 9 illustrates the quandary faced by the water management authority in year 10: in spite of the efforts taken to sustainably manage the coastal aquifer, population growth is unremitting and the aquifer only has limited storage potential. The increasing gap between total daily water demand (green) and total daily water consumption (blue) reflects the rising scarcity cost of water due to increasing supply shortages. There is also a small, barely observable gap between the two curves during the first 10 years. This gap arises as a result of the inter-temporal nature of the optimal solution: a portion of water consumption must be sacrificed in early years to allow for greater consumption in the future. The vertical distance between the two curves represents the volume of water foregone by the coastal community. Thus, over the entire 20-year time span, per-capita water demand is fixed at 362 litres per day. In year 1, each person consumes an average of 357 litres of water per day, sustaining a mere 1.25 % reduction in demand. By year 20, however, per-capita water consumption is projected to fall to 220 Litres per day. In other words, the aquifer will only be able to supply 61 % of demand, and the water management authority will have to pursue the development of other supply alternatives. In the intermediary stage, the administration may find it useful to charge a volumetric price of water in order to control demand. If we interpret the water demand curve as a marginal valuation curve, the total volume of water used in a particular time period can be used to determine an optimal pricing schedule to restrict demand (Grafton and Ward 2008). The recommended volumetric price scheme generated by the baseline results scenario is illustrated in Figure 10:

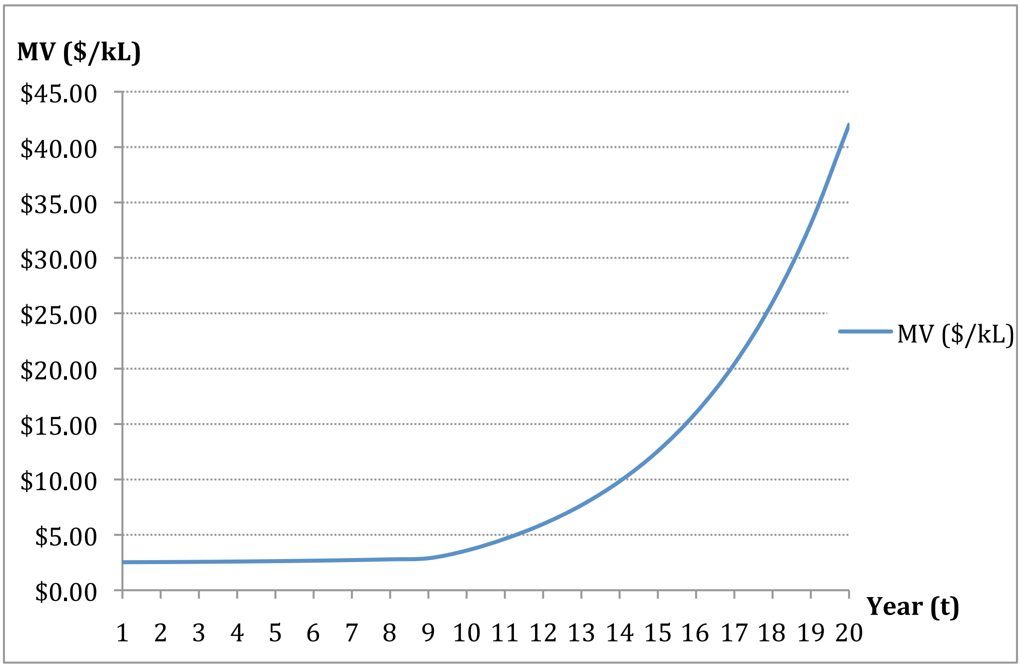

Figure 10: Marginal value of water per kL (Baseline Scenario)

Curiously, water utilities often favour the use of demand rationing, marketing campaigns, mandatory water restrictions, and other economically inefficient policies for controlling demand (Hughes *et al.* 2008). The use of water restrictions in particular can entail significant welfare losses to society, due to the heterogeneity observed across marginal valuations of water users. In many cases, volumetric pricing may be a far more appropriate and effective policy tool. For example, Grafton *et al.* (2009) found that water users faced with a volumetric price will consume approximately 25% less water than unmetered users. In addition, a volumetric price is easy to incorporate into optimisation and management models.

5.2 Scenario 2

In this experiment, the parameters used in the Baseline Scenario are held constant, and the effects of a myopic management strategy are investigated. Thus, instead of maximising the net present value of net economic benefits over a period of 20 years, the water utility chooses to maximise the benefits above costs in each individual time period. The new objective function is formulated below:

$$Max\,Z_t = B_t - C_t \qquad t = 1, \ldots T \tag{28}$$

The objective function presented here aims to model a truly shortsighted management strategy, where the potential benefits of abstraction in future years are completely neglected. It is important to note that due to the drawdown constraint, the problem is still path-dependent, and present-day extraction decisions continue to impact on the availability of water in the future.

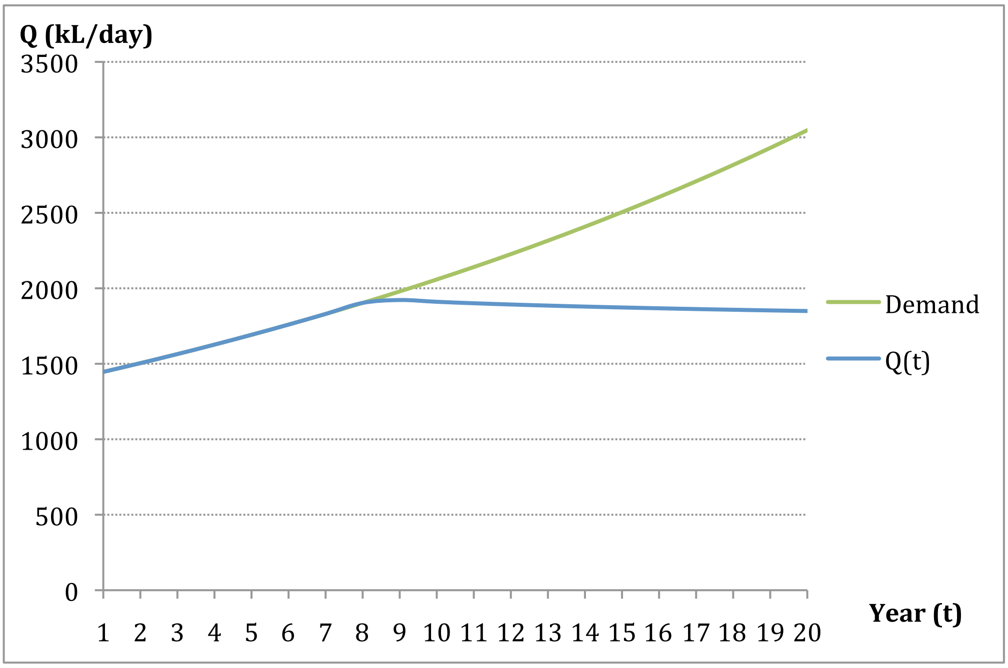

Figure 11: Daily water demand vs. actual consumption (Scenario 2).

The graphs of optimal pumping rates, drawdown levels, and the saltwater intrusion barrier are not presented here because they are virtually identical to the results of the baseline scenario. The difference between the two scenarios can be observed by comparing Figures (9) and (11). In Scenario 2, there is no observable gap between demand and consumption during years 1 to 8. By disregarding the future, the water utility is able to exactly meet demand until year 9, when the saltwater intrusion constraint becomes binding at wells 2 and 5. The NPV calculated from the objective function in Scenario 2 is \$5,683,050.00, and the NPV from the Baseline Scenario is \$5,683,081.66. The use of a lower bound on the integral in the benefit function (B_t) means that these NPVs do not include total benefits. However, the difference between the two NPVs can serve to quantify the change in welfare of the new objective. The results indicate that pursuing the objective of 'maximum social benefit' will only generate an additional benefit of \$31.66 in present-value terms. Due to the considerable difficulty and uncertainty associated with estimating future demand for water, it may be more appropriate for the water management authority to ignore the future and pursue the myopic management objective.

5.3 Scenario 3

Population growth is the driving force behind water demand in the Baseline Scenario. In this trial, the population growth rate (*g*) is reduced to 2 %. All other model parameters are maintained at the same levels observed in Table 1. The results of the experiment are given below:

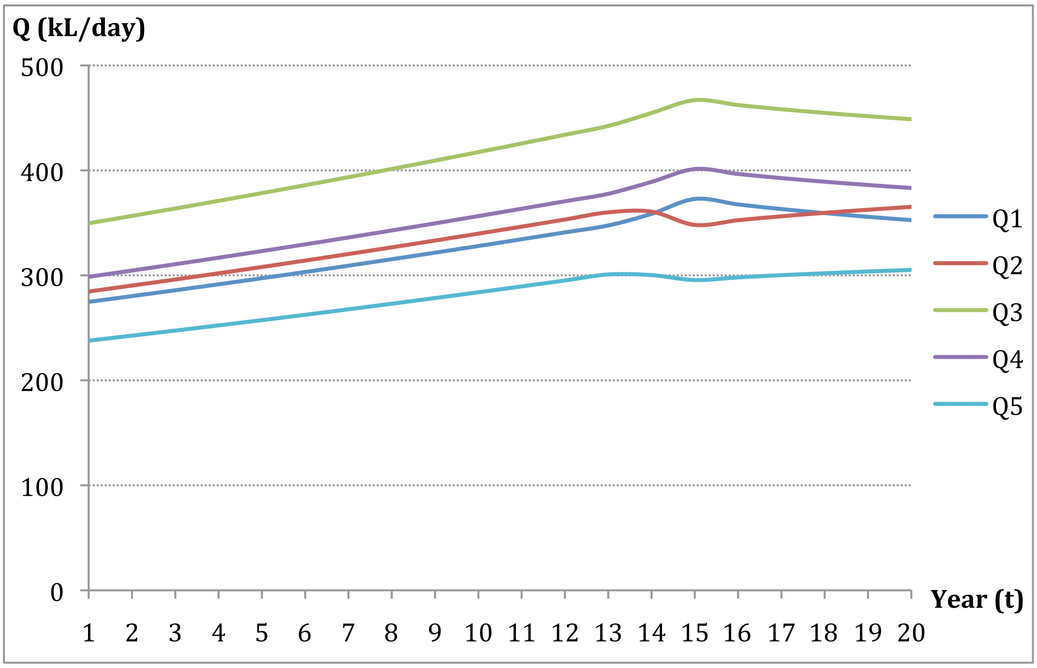

Figure 12: Optimal pumping rates (Scenario 3).

The optimal pumping rates displayed in Figure 12 demonstrate that the change in the population growth parameter (*g*) has a marked effect on the results of the model. With 2 % population growth, the aquifer is able to sustain increases in demand until year 14, when the saltwater intrusion constraint is activated at well 2. By year 15, hydrological constraints are binding at all 5 wells in the catchment. Wells 2 and 5 are constrained by saltwater intrusion, and wells 1, 3 and 4 are constrained by the maximum allowable drawdown of 10 metres.

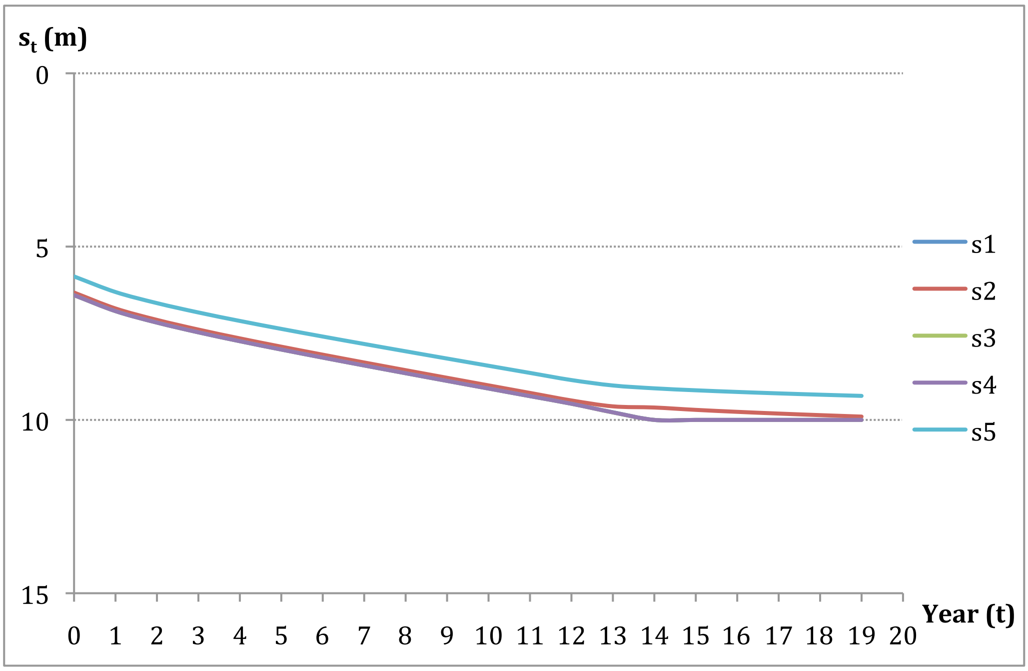

Figure 13: Drawdown levels (Scenario 3).

The temporal evolution of drawdown levels appears in Figure 13. The well coordinates have not changed, so the relative distribution of pumping across wells is similar for both sets of results. Once again, the lowest pumping rate is observed in well 5 due to its close proximity to the coastline, and well 3 is the highest yielding well due to is remote location. One peculiar aspect of the results is the crisscrossing of pumping rates between wells 1 and 2. Initially, the pumping rates observed at well 2 are higher than the pumping rates at well 1. This is because no hydrological constraints are binding over the first 13 years. As a result, the distribution of pumping among wells is somewhat inconsequential, and has almost no impact on the final objective function value. The first crossover in the pumping rates occurs due to a drop in pumping at well 2, caused by the activation of the saltwater intrusion constraint. After year 15, drawdown levels at wells 1, 3 and 4 reach the maximum limit of 10 metres, and pumping rates begin to fall in unison. This decline reduces the threat of saltwater intrusion and allows pumping rates at wells 2 and 5 to increase in response. The second crossover can be explained by the interaction between the drawdown and saltwater intrusion constraints.

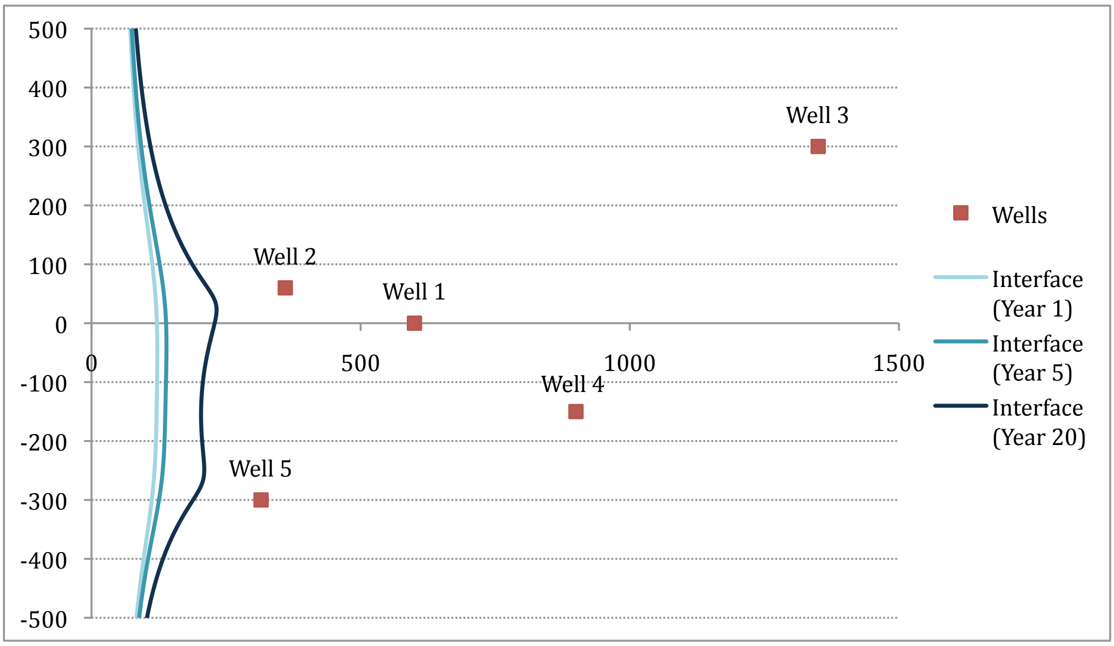

Figure 14: Saltwater interface progression (Scenario 3).

The lateral progression of the saltwater wedge is illustrated in Figure 14. The initial (year 1) and final (year 20) positions of the interface are virtually identical when compared with the results of the Baseline Scenario (Figure 8). However, the position in year 5 is visibly different. Due the lower rate of population growth, pumping in year 5 is only 1564 kL/day, compared to 1660 kL/day in the Baseline Scenario. Consequently the interface in Figure 14 has only experienced a minor inward shift.

The increased longevity of the aquifer as a sole source of supply to the coastal community is exemplified in Figure 15. The volumetric price required to regulate demand for water remains only slightly higher than the marginal cost of supply for the first 14 years. After year 14, increasing scarcity triggers an exponential rise in the marginal value of water. However, the population growth rate is only 2 %, and the rate of increase is far less than that observed in the Baseline Scenario (Figure 10).

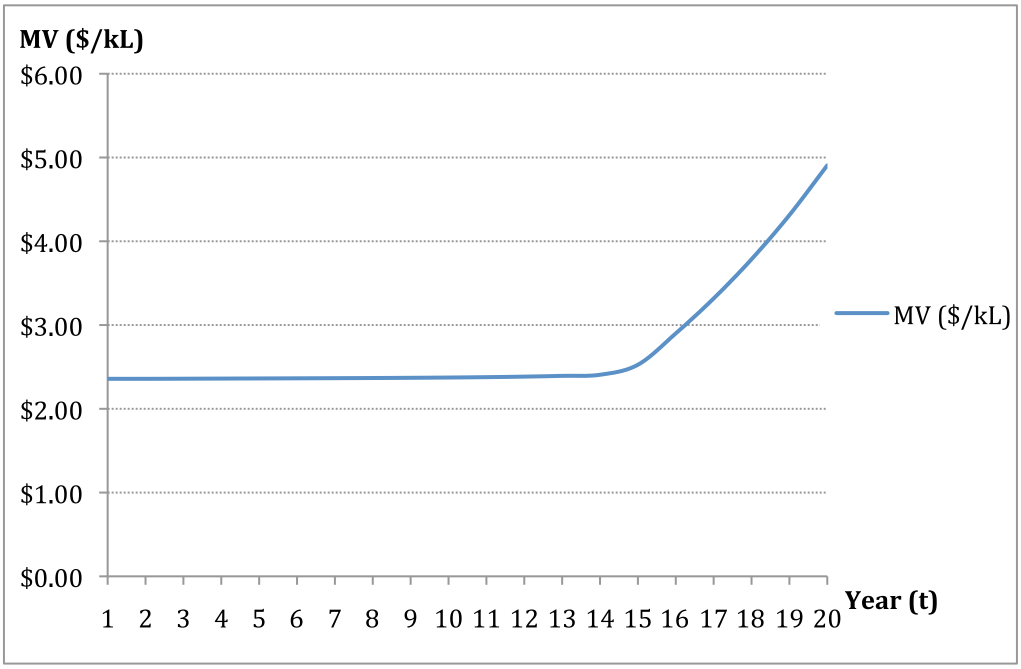

Figure 15: Marginal value of water per kL (Scenario 3).

6. Conclusions and Further Work

This study has developed an analytical framework for the management of a coastal aquifer. The three scenarios outlined in the previous section serve to demonstrate the robust and versatile nature of the framework. Furthermore, they also confirm the importance of adopting a multidisciplinary approach in the modelling of groundwater resources.

Numerous limitations found in the literature were relaxed or eliminated in the formulation of this model. The optimal pumping rates derived in the results of the baseline scenario demonstrate significant variations across wells and over time. This confirms the need to account for spatial heterogeneity, temporal effects, and varying rates of abstraction when developing a basin-wide groundwater management model. Furthermore, significant interaction was observed between the drawdown and saltwater intrusion constraints. This attests to the importance of controlling for both factors in an optimisation model.

Future development of this model should address four further limitations. Allowing the marginal cost of pumping to vary across wells and to increase in line with drawdown is more realistic than the assumption of a constant marginal cost of extraction. Second, many coastal towns are subject to seasonal fluctuations in demand for water, due to the influx of tourists during the peak summer season. Incorporating these factors into the model is likely to alter the results presented here. Third, the true demand curve for a groundwater resource is usually unknown, and quite difficult to estimate. Predicting future growth in demand is often very difficult and highly uncertain. Therefore optimal management of groundwater resources may not be possible in the real world. It is possible that a myopic strategy (not controlling for future demand for water) will be preferred to an optimal control strategy based on an estimated demand curve (Allen and Gisser 1984). Finally, the model presented here is deterministic, yet in real life aquifer parameters are not known with certainty. Benhachmi (2003b) provides a framework for a stochastic optimisation model controlling for saltwater intrusion.

7. Acknowledgements

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