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Optimality, Hartwick's Rule, and Instruments of Sustainability Policy and Environmental Policy

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Abstract. We consider a closed, constant-technology, capital-resource economy with resource stock amenity value, which would otherwise aim for conventionally, PV-optimal development that maximises the present value of utility using a constant discount rate. In this economy, we calculate the decentralised policy instruments needed to achieve continuously zero net investment, and hence (by Hartwick's rule) sustainability in the form of constant utility. We also calculate the environmental policy needed to internalise the resource's amenity value: its natural form is a subsidy on holding the stock. The sustainability policy comprises this stock subsidy (needed if sustainability is to be maximal, though the subsidy will be at a different level from that for environmental policy alone); and a consumption tax which ultimately falls towards a 100% subsidy. The latter gives agents the incentive needed to invest when the return on capital is less than the utility discount rate. Neither a resource flow tax nor the resource stock subsidy on its own has any power to achieve sustainability. As a preliminary, we clarify some confusion in the literature about the relationship between PV-optimality and Hartwick's Rule, using an exact solution for illustration.

[1] I thank three anonymous referees for helpful comments.

1. Introduction

Hartwick's rule - that continuously zero net investment in human-made capital and natural resources in a dynamically efficient economy results in continuously constant utility - has been extensively developed since its first appearance in Hartwick (1977). Variants have included allowing for renewable resources, multiple non-renewable resources, international trade, and the disutility from cumulative resource degradation (Hartwick 1978a, 1978b, 1995 and 1997, to cite just a small selection). (Without disutility from degradation, the rule results in constant consumption as well as constant utility.) A general form of the rule was stated by Dixit, Hammond and Hoel (1980), and of the converse (that constant utility implies zero net investment) by Withagen and Asheim (1999). In the 1990s, much interest was shown in zero net investment (and hence constant utility) as a well-defined if restrictive prescription for the otherwise ill-defined concept of sustainable development. For example:

> "What should each generation give back in exchange for depleted resources if it wishes to abide by the ethic of sustainability? ... we owe to the future a volume of investment that will compensate for this year's withdrawal from the inherited stock." (Solow 1993, p170, p171)

But only a little attention has been given to what Solow called the "concrete translation of sustainability into policy". By this is meant here the laws and incentives that governments can use as decentralised policy *instruments* to intervene in markets, and induce us as individuals to achieve the ethical goals that we collectively desire as a society. Becker (1982, pp179-180) suggested that Hartwick's rule could be enforced by governments permitting borrowing and lending of consumption at an interest rate equal to the (varying) utility discount rate that supports constant utility, but he did not investigate the detailed mechanisms of such a policy.

Howarth and Norgaard (1992) considered the interaction of intergenerational equity and a dynamic environmental externality in an overlapping generations model, but without a specific goal of constant utility. So the question of what intervention will achieve zero net investment, in a representative-agent economy with capital, resources and environmental externalities, remains relatively unexplored. This is despite the considerable amount of empirical work done on measuring net investment since pioneering work such as by Pearce and Atkinson (1993).

The question has no precise answer until one clarifies the dynamic objective that individuals in a market economy would pursue without any imposed policy goal. Here we assume that the objective would be conventional "PV"-optimality: maximising the present value of utility using a constant discount rate (PV). We also assume that government chooses a policy which results in constant rather than PV-optimal utility. Why the present-value-maximising agents of the economy would elect such a government is a central though under-researched question in the economics of sustainability, but not one pursued further here.[2]

Before giving an explicit model of policy to achieve zero net investment, we first clarify in Section 2 some occasional technical confusions in the literature which run counter to our results. They come in two main forms. First, it is sometimes claimed that momentarily zero net investment gives

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2. One could simply appeal to the argument in Marglin (1963, p98) that "... the Economic Man and the Citizen are for all intents and purposes two different individuals". More satisfying would be to develop the informal idea in Daly and Cobb (1989, p39) and Howarth and Norgaard (1993, p351) that sustainability may be seen as a partly-public good because of the sexual intermixing of bequests across successive generations.

constant utility at the same moment. Second, it is sometimes suggested that a zero net investment rule can be added to a PV-optimal economy, without requiring any policy intervention, or causing any change in the economy’s path. Neither holds except in the special case where the PV-optimal path has constant utility anyway. It is important to counter the second claim because it suggests that intergenerational equity can somehow be a free lunch, requiring correct accounting procedures using the "right prices", but no real policy choices which change quantities (of consumption or resource use) as well as prices in real markets. A numerically explicit counterexample of the conflict between constant consumption and PV-optimality is given in Section 3.

Section 4 sets out a formal model of policy intervention, using a closed, constant-technology, capital-resource, representative-agent economy, with production and utility functions fully differentiable, and utility dependent on the resource stock as well as consumption. The last point allows the model also to consider the difference between individually and socially PV-optimal development, with environmental policy seen as moving from the former to the latter. The model’s simplifying assumptions undoubtedly limit the policy relevance of its results: the representative agent framework obscures the interaction of separate generations; the "weak" sustainability assumption of full differentiability avoids the key issue of the limits to substitutability; and in the twenty-first century, the only closed economy of interest is the global one. Nevertheless, the results shed some light on the kind of "Rawlsian conviction" which might be needed to achieve constant utility.[3] The

3. "...I can see no force - other than ‘Rawlsian conviction’ - that will steer a market economy to constant consumption and the right [initial rate of resource depletion]. ‘Rawlsian conviction’ here means something more than current legislation; there needs to be a kind of social contract to bing the next Congress, and the next." (Solow 1974)

required policy is a consumption tax or subsidy, while social efficiency requires a subsidy on the resource stock to internalise its amenity value, whether or not constant utility is a goal. Section 5 concludes, and stresses the need for further work to include two key features of modern economies not included in the analysis here: technical progress and international trade.

2. Clarifying technical confusions about Hartwick's rule

We note below a number of side remarks made in the literature that suggest that either:

(A) Momentarily zero net investment implies momentarily constant utility; or

(B) It is generally possible for net investment to be continuously zero (and then utility is constant) on a PV-optimal path, with no policy intervention or departure from the PV-optimal path.

It is not difficult to disprove these suggestions. Asheim (1994, pp262-5) comprehensively explained the inaccuracy in (A). A heuristic disproof of (B) is that a PV-optimal path is already uniquely determined without any regard for the value of net investment, so it is impossible to add an extra, zero-net-investment condition while staying on the PV-optimal path. Nevertheless, we will develop a formal framework of analysis. It inevitably differs in detail from the frameworks used by the authors we cite, but it will serve both to make our points here, and as a basis for the rest of the paper. The suggestions (A) or (B) are not central points in the works cited here, being more like side remarks. Clarification nevertheless seems useful, since the works are either a highly-cited classic (Solow 1986); or ones appearing since Asheim (1994). The latter show both the persistence of technical misunderstandings of the zero-net-investment rule, and an associated, more general belief that sustainability need not represent a radical departure from conventional, constant-discount-rate optimality.

We consider a constant population, closed economy like that in Krautkraemer (1985, p159) where the inputs to a twice continuously differentiable, constant technology, production function $F(K,R)$ are a non-depreciating, human-made capital stock $K(t)$, and the depletion rate $R(t)$ of a non-renewable resource stock $S(t)$ with zero extraction costs. Production is divided linearly between consumption $C(t)$ and capital investment $\dot{K}(t)$. The infinitely lived representative agent has a twice continuously differentiable, instantaneous utility function $U(C,S)$, and a general utility discount factor $\phi(t)$.[4] Adding in typical conditions on the partial derivatives (denoted by subscripts) of $F(.)$ and $U(.)$, the optimal development path (not yet "PV"-optimal because we have not restricted the discount rate $-\dot{\phi}/\phi$ to be constant) then satisfies:

$$\left.\begin{array}{l}
\max\limits_{C,\,R} \int_0^\infty U[C(t),S(t)]\phi(t)dt \\
\text{s.t. } \dot{K} = F(K,R) - C \quad \text{and} \quad \dot{S} = -R \\
K(t),\ C(t),\ S(t),\ R(t) \geq 0, \text{ all } t \\
K(0) = K_0 > 0;\ S(0) = S_0 > 0 \\
F_K,\ F_R > 0,\ F_{KK},\ F_{RR} < 0 \\
U_C,\ U_S > 0,\ U_{CC},\ U_{SS} < 0,\ U_{CS} > 0;\quad U_C \to \infty \text{ as } C \to 0 \\
\eta(C) := -U_{CC}C/U_C > 0
\end{array}\right) \qquad [1]$$

Assuming that an internal solution path exists for this problem, Appendix 1 shows that the socially optimal path (that accounts for all effects

4. The points made here could be made more generally *[and probably will be, in a revised version of this paper]* using the result $\dot{H} = (-\dot{\phi}/\phi)(H-U)$ of Aronsson, Johansson and Lofgren (1997, p31), where H is the current value Hamiltonian of the optimisation problem.

on utility) always obeys these Ramsey and Hotelling rules:

$$\dot{U} = [\{F_K-(-\dot{\phi}/\phi)\}U_C - R(U_{CS}+\eta U_S/C)]\ C\ /\ \eta; \qquad [2]$$

or if $\phi(t) = e^{-\rho t}$, $\rho>0$ constant, a case denoted by *, [2] becomes

$$\dot{U}^* = [(F^*_K-\rho)U^*_C - R^*(U^*_{CS}+\eta^* U^*_S/C^*)]\ C^*\ /\ \eta^*; \text{ and} \qquad [3]$$

$$\dot{F}_R/F_R = F_K - U_S/(U_C F_R) \text{ for any path of } \phi(t). \qquad [4]$$

The Hotelling rule [4] is obeyed by any solution to [1], including one that maximises constant utility. It shows how the resource's amenity value $U_S/(U_C F_R)$ lowers the effective rate of return that has to be earnt from the growth rate $\dot{F}_R/F_R$ of the resource price. Since $F_{RR}<0$, this also lowers the resource depletion rate.

Since the discount factor $\phi(t)$ is unspecified in the general Ramsey rule [2], the latter can be compatible with the zero net investment rule

$$\dot{K}(t) = F_R(t)R(t) \quad \text{for all } t, \qquad [5]$$

which gives constant utility, $\dot{U} = 0$ for all t, confirming Hartwick's rule (see Appendix 1 again).

Suggestion (A) follows if one mistakenly assumes that zero net investment at one moment, which may indeed happen somewhere on a PV-optimal path, allows one to take the time derivative of [5] ($\ddot{K} = \dot{F}_R R + F_R\dot{R}$) at the same moment. Suggestion (B) is false because the conventional, PV-optimal Ramsey rule [3] for the utility path is generally incompatible with the continuous zero net investment rule [5]. That is, if $\phi = e^{-\rho t}$, it is generally impossible for the utility change $\dot{U}$ to be both the required PV-optimal form [3], and at the same time zero as a result of zero net investment. If it is unclear whether an author is claiming that continuously, or momentarily, zero net investment gives constant utility on a PV-optimal path, then it is correspondingly unclear whether suggestion (A) or (B) is being made, but it must be one of them.

What, then, of recent literature? Lozada (1995, p142) clearly made suggestion (A) when he talks of "...the well-known 'Hartwick rule': zero value of investment implies constant consumption..." when referring to an instant rather than to all time. Aronsson, Johansson and Lofgren (1997, p101, in our notation) clearly made suggestion (B): "...introducing Hartwick's rule, which tells us that [net investment is zero] for all *t* along an optimal path implies that utility (consumption) is constant for all *t*."[5] Aronsson and Lofgren (1998, p213) contains a similar statement. Both fail to say what an extraordinary coincidence it would be if PV-optimal net investment actually is zero. Neumayer (1999, p151,224) is ambiguous - his text mentions the "for ever" requirement, but his algebra does not - but he is clearly making one suggestion or the other.

Suggestion (B) was also made by the originators of Hartwick's rule, in Solow (1986) and Hartwick (1997). Solow's paper is a little informal, but it is a citation classic and should be discussed as it reads, not as it may have been intended to read. On p146 he started with PV-optimality, "an economy that acts so as to maximize the present value of consumption..." with a constant interest rate r (i.e. maximising $\int_0^\infty C(t)e^{-rt}dt$ rather than $\int_0^\infty U(t)e^{-\rho t}dt$). But on p147 he wrote: "Now suppose that [net investment] = 0 from some date on. This is Hartwick's rule." No explanation was given for this. For net investment to suddenly become zero requires the introduction at "some

5. A similar statement on their p35 mentions "the policy of investing resource rents" (i.e. zero net investment). Their meaning of "policy", as a rule chosen by the whole of society, differs from our meaning of policy as decentralised incentives created by government to persuade the rest of society to move to zero net investment.

date" (say T) of an unexpected policy intervention (as analysed in our Section 3) to induce all resource rents to be invested in reproducible capital after T. Since the economy is closed, this policy will cause a step change at T in all quantities and prices − consumption, investment, the resource rents to be invested − and the interest rate after T may be varying (as it does in Solow' 1974 original constant consumption path). This undermines Solow's interpretation of net national product on a constant utility path as being a constant interest rate times a constant ("maintained intact") stock.

Hartwick (1997) suggested that a (continuously) zero net investment rule leads to constant utility in the context of PV-optimal development, but did not note that such development produces a utility path which is generally not constant. He used a PV-optimal control problem to derive the Hotelling-like asset price rules used in the proof, but the rules also come from a maximisation exercise like [1] above with general, unspecified discounting.[6] Hartwick partly recognised a problem by stating (p514) that "It is only when current asset prices are 'taken from' a constant consumption program that they will work in the zero net investment criterion", and a similar idea is found in Solow (1993, p168). However, this generally will not work, as will be shown at the end of Section 3.

3. An exact example

Two key points above were that generally, an economy's PV-optimal and zero-net-investment paths are distinct, and zero net investment and constant utility occur at different times on the PV-optimal path. Here we

6. Such an explicit exercise would also show that if there is capital depreciation at rate δ, then F_K in the Hotelling rule needs to change to $F_K - \delta$. Zero net investment then still results in $\dot{U} = 0$, not $\dot{U} < 0$ as stated in Hartwick (1977, p974).

describe here a simplified version of the economy in [1] with specific functional forms, for which exact analytic solutions are available to illustrate these points. It has

$$F(.) = K^{\alpha}R^{1-\alpha},\quad U(.) = C^{1-\alpha}/(1-\alpha),\ (0.5<\alpha<1), \text{ and } \phi(t) = e^{-\rho t}.$$

The Ramsey rule [3] is then

$$\dot{C}^*/C^* = (F^*_K - \rho)/\alpha, \qquad [6]$$

and Appendix 2 shows that the PV-optimal paths of consumption and net investment are

$$C^*(t) = (\rho^2 S_0/\alpha^2)\ [J(t)]^{1/(1-\alpha)}\ e^{(-\rho/\alpha)t} \quad \text{and} \qquad [7]$$

$$\dot{K}^*(t) - F^*_R(t)R^*(t) = (\rho S_0/\alpha)\ \{[\alpha^2 - \rho J(0,t)]/\alpha\}\ [J(0,t)]^{\alpha/(1-\alpha)}\ e^{-(\rho/\alpha)t}$$

$$\text{where } J(x,y) := [\alpha K^*(x)/\rho S^*(x)]^{1-\alpha} + (1-\alpha)y \qquad [8]$$

Figure 1. PV-optimal paths of economy with $F=K^{0.7}R^{0.3}$, $U=C^{0.3}/0.3$, $K_0=S_0=1$, $\rho=0.1$.

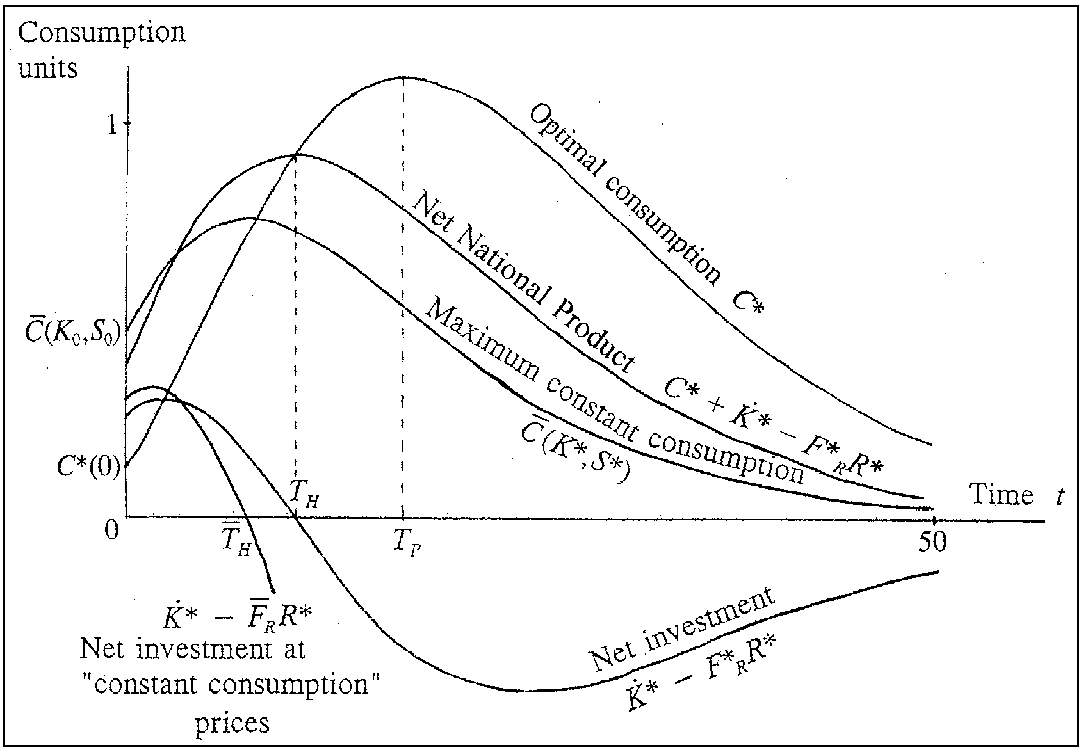

Figure 1 shows these numerical levels of consumption C^*, net investment $\dot{K}^* - F^*_R R^*$, consumption plus net investment $C^* + \dot{K}^* - F^*_R R^*$ (= net national product NNP), "net investment at constant consumption prices" (explained below) and "maximum constant consumption" (ditto), for the parameter values $\alpha = 0.7$, $K_0 = S_0 = 1$ and $\rho = 0.1$.

If the initial capital/resource stock ratio (K_0/S_0) is small enough, then from the time derivative of [7], consumption (and hence utility) is momentarily constant when $t = T_P := [\alpha/\rho - (\alpha K_0/\rho S_0)^{1-\alpha}]/(1-\alpha) > 0$; and from [8], net investment is zero when $t = T_H := [\alpha^2/\rho - (\alpha K_0/\rho S_0)^{1-\alpha}] / (1-\alpha) > 0$. And $\alpha<1 \Rightarrow T_H < T_P$ (as in Figure 1), so that constant consumption and zero net investment do not occur at the same time.

Moreover, using "asset prices corresponding to a future constant consumption path", as suggested by Hartwick (1997, p514), will generally not work if these prices are combined with PV-optimal quantities. An obvious price to use is the resource price that would hold at the start of the Solow (1974, p39) *maximum constant consumption* path, which itself starts from the capital and resource stocks $K^*(t)$ and $S^*(t)$ reached on the PV-optimal path at any time t. This is $\bar{C}(t,t') = \bar{C}[K^*(t),S^*(t)] = \alpha\{[K^*(t)]^{2\alpha-1}[(2\alpha-1)S^*(t)]^{1-\alpha}\}^{1/\alpha}$ for all $t' \geq t$,[7] as shown on Figure 1 for all $t \geq 0$.[8] Denoting this resource price by $\bar{F}_R(t,t)$, we can combine it with

7. The utility discount factor which makes this constant consumption path maximize the generalized present value in [1] is $\phi(t) = \{1+[(1-\alpha)/\alpha]\bar{C}t\}^{-\alpha/(1-\alpha)}$.

8. Note that in general $\bar{C} \neq$ NNP. This illustrates the error of general comments such as "under idealised conditions Hicksian income [consumption plus capital accumulation] and Fisherian income [maximum sustainable level of consumption] are identical" (Nordhaus 2000, p259), which persist despite Asheim's (1994) correction. It would take special conditions, such as a small open economy facing a constant,

investment and depletion rates on the PV-optimal path to give net investment at "constant consumption" prices, $\dot{K}^*(t)-\bar{F}_R(t,t)R^*(t)$ instead of $\dot{K}^*(t)-F^*_R(t)R^*(t)$. This is also shown in Figure 1, using the formula derived in Appendix 3:

$$\dot{K}^*(t) - \bar{F}_R(t,t)R^*(t)$$
$$= (\rho S_0/\alpha)\ \{[2\alpha-1-\rho\, J(t,t)]/(2\alpha-1)\}\ [J(t,t)]^{\alpha/(1-\alpha)}\ e^{-(\rho/\alpha)t}. \qquad [9]$$

[9] becomes zero at $t = \bar{T}_H := [(2\alpha-1)/\rho-(\alpha K_0/\rho S_0)^{1-\alpha}]/(1-\alpha)$, a time earlier than T_H, since $(1-\alpha)^2 > 0$ means $2\alpha-1 < \alpha^2$. The intuition is that because resource depletion is eventually, for large enough t, slower at the start t of a possible maximum consumption path than on the PV-optimal growth path, the resource price is eventually greater ($\bar{F}_R(t) > F^*_R(t)$), so that $\dot{K}^*-\bar{F}_R R^* < \dot{K}^*-F^*_R R^*$ at such a t.

4. Sustainability policy and environmental policy

We consider here the nature of sustainability policy intervention and environmental policy intervention in the capital-resource market economy described in Section 2. These policies are here given precise definitions, which are much more limited than the wide range of meanings used in public debates. *Sustainability policy* means intervention that causes zero net investment for all time and hence sustainability in the form of constant utility, $\dot{U} = 0$. As stated earlier, exactly why the representative individual would support such intervention, given that she maximises PV in her private choices, is something we do not explore here. *Environmental policy* means bringing about fully PV-optimal development, by internalising the amenity value ($U_S/U_C > 0$, when expressed in consumption units) of the resource

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exogenous interest rate, for NNP and maximum sustainable consumption to be the same. How big the difference between the two measures is empirically, deserves further research.

stock S. These two definitions maintain a distinction which is often lost now that "sustainable" has become a loosely-defined buzzword almost synonymous with "environmentally desirable". Both policies start from a market economy where the representative agent pursues individually PV-optimal behaviour, by ignoring the effect of his own resource depletion on the amenity value of the total stock S. The U_S term will thus be absent from the relevant first order condition involving $\partial H/\partial S$ in the solution of the optimal control problem.

The instruments available to the government are chosen here to allow pursuit of either policy goal, and to be moderately realistic. So we move beyond the approach noted in Section 2 where society is assumed to be able to decree a direct "policy" of zero net investment. Also, we do not directly investigate Becker’s (1982) idea, that the government can "enforce" the utility discount factor $\phi(t)$ underlying zero net investment by allowing borrowing and lending at the interest rate $-\dot{\phi}(t)/\phi(t)$, though our result will suggest a way of achieving an equivalent outcome. Our policy instruments are specific tax rates of $\tau_C(t)$ on consumption, $\tau_K(t)$ on capital, $\tau_R(t)$ on resource depletion, and $\tau_S(t)$ on the resource stock. The "tax" rate τ_S may well be negative, i.e. a subsidy to encourage holding a larger resource stock. Net revenues from the taxes are returned to agents as a lump sum Ω, which is negative (i.e. a lump sum tax) if net revenues are negative. Some "Rawlsian conviction" would indeed be needed to set the path of these tax rates credibly for the rest of time, but in common with most optimal control modelling, we do not explore what constitutional innovations this might require in a democratic society.

The representative agent's maximisation problem [1] then becomes

$$\underset{C,\,R}{\text{MAX}} \int_0^\infty U[C(t),S(t)]\phi(t)dt$$

s.t. $\dot{K} = F(K,R) - C - \tau_C C - \tau_K K - \tau_R R - \tau_S S + \Omega$

and $\qquad \dot{S} = -R \qquad\qquad [10]$

with other conditions on $U(.)$, $F(.)$, etc being as before in [1].

The necessary first order conditions for an interior solution to [10] then give (see Appendix 4) these Ramsey and Hotelling rules for the solution (denoted $\hat{}$) of this economy:

Policy-induced, individually PV-optimal path

$$\dot{\hat{U}} = [\{\hat{F}_K - \rho - \tau_K - \dot{\tau}_C/(1+\tau_C)\}\hat{U}_C - \hat{R}(\hat{U}_{CS} + \hat{\eta}\hat{U}_S/\hat{C})]\hat{C} \;/\; \hat{\eta} \qquad [11]$$

$$(\dot{\hat{F}}_R - \dot{\tau}_R - \tau_S)/(\hat{F}_R - \tau_R) = \hat{F}_K - \tau_K \qquad [12]$$

For convenience we copy here the corresponding rules from Section 2 for the socially PV-optimal and net-investment-zero paths:

Ramsey rule [3] for socially PV-optimal path

$$\dot{U}^* = [(F^*_K - \rho)U^*_C - R^*(U^*_{CS} + \eta^* U^*_S/C^*)]\; C^* \;/\; \eta^* \qquad [13]$$

Ramsey rule for zero-net-investment (Hartwick rule) path

$$\dot{\bar{U}} = 0 \qquad [14]$$

Hotelling rule [4] for both socially PV-optimal and zero-net-investment paths

$$\dot{F}_R/F_R = F_K - U_S/(U_C F_R) \qquad [15]$$

It turns out to be simpler to consider environmental policy first, and then sustainability policy. Before doing so, note that all quantities used to define the policies are as measurable as any "green accounting" quantities defined solely in consumption-denominated units. For example, U_S/U_C would typically be measured in dollars per ton. It could be computed from the same kinds of (practically very difficult!) non-market valuation exercises used for conventional green accounting. So in this particular case, and we suspect more widely, one can question the view expressed by Pearce, Markandya and Barbier (1989, p49) that utility-based definitions of sustainability are impractical because involve unmeasurable quantities.

4.1 Environmental policy

Comparing [11] and [12] with [13] and [15] shows that there is no unique set of tax paths $\tau_C(t)$, $\tau_K(t)$, $\tau_R(t)$ and $\tau_S(t)$ that make the individually PV-optimal path coincide with the socially PV-optimal path. However, the net effect $-\tau_K - \dot{\tau}_C/(1+\tau_C)$ of capital and consumption taxes in [11] needs to be zero, so both tax rates may as well be zero, rather than non-zero and cancelling each other out. This then leaves $(\dot{F}_R - \dot{\tau}_R - \tau_S)/(F_R - \tau_R) = \dot{F}_R/F_R + U_S/(U_C F_R)$ as the problem to be solved by resource taxes τ_R and τ_S. The simplest solution is $\tau_R = 0$ and $\tau_S = -U_S/U_C$. The natural environmental policy is therefore just *a resource stock subsidy* τ_S *equal to the resource stock's amenity value:*

$$\tau_S = -U_S/U_C, \quad \tau_R = 0; \quad \tau_C = \tau_K = 0 \qquad [16]$$

4.2 Sustainability policy

Comparing [11] and [12] with [14] and [15] shows that there is also no unique combination of tax paths, which the government could use to induce the representative PV-maximizing individual to make utility constant forever with the maximum possible value of U. But there is a natural combination. Let us first make an additional assumption that in the absence of policy, the individually PV-optimal return on capital F_K (the competitive interest rate) eventually falls below the utility discount rate ρ:

On [11] with $\tau_C=\tau_K=\tau_R=\tau_S=0$, $\lim_{t\to\infty}F_K =: \mu$, $0 \le \mu < \rho$. [17]

This is plausible, given the assumptions in [1] of no technical progress, a non-renewable resource (hence $\lim_{t\to\infty}R = 0$), and $F_{KR} > 0$. It means that falling utility will eventually occur on path [11] in the absence of policy, since the resource amenity term $-R(U_{CS}+\eta U_S/C) < 0$ from [1].

Note that *with no capital or consumption incentives, the resource depletion and resource stock incentives* $\tau_R(t)$ *and* $\tau_S(t)$ *are powerless to achieve constant utility*. This is because they do not appear directly in [11]. The only way they can make $\dot{U}=0$ is by raising the resource flow R, and thus raise the return on capital $F_K(K,R)$. Even if this works − and it may not, because it also makes the amenity term $-R(U_{CS}+\eta U_S/C)$ more negative − such a non-vanishing R can be sustained for only a finite time by a finite stock S_0.

As for a capital tax, $\tau_K = U_S/U_CF_R > 0$ could equate the Hotelling rules [12] and [15], but it would worsen the problem of declining utility in [11]. So to make $\dot{U} = 0$, the natural sustainability policy is to leave τ_K zero, and simply add a consumption tax schedule to the resource stock subsidy identified in [16]:

$$\left.\begin{aligned} &\tau_S = -U_S/U_C, \quad \tau_R = 0 \\ &\dot{\tau}_C/(1+\tau_C) = -\,[(\rho-F_K) + R(U_{CS}+\eta U_S/C)/U_C], \quad \tau_K = 0 \end{aligned}\right) \qquad [18]$$

Though it is not immediately obvious from the above, the resource subsidy τ_S, along with all other variables, will generally differ in size between the environmentally optimal and sustainable paths achieved by policies [16] and [18]. This echoes the central observation in Howarth and

Norgaard (1992) that environmental valuation procedures cannot exist in a vacuum, independently of policies for sustainability. The changing τ_C in [18] changes the effective utility discount rate that an individual uses, so this is a way of achieving Becker's idea of manipulating this rate. From [1], $-R(U_{CS}+\eta U_S/C)/U_C \leq 0$ always, and from [17], $-(\rho-F_K) < 0$ eventually, so $\dot{\tau}_C/(1+\tau_C) < 0$ eventually. And as shown in Appendix 5, the eventually falling consumption tax τ_C must ultimately be a *100% subsidy:*

$$\lim\big|_{t\to\infty}\dot{\tau}_C/(1+\tau_C) < -(\rho-\mu) < 0 \quad \Rightarrow \quad \lim\big|_{t\to\infty}\tau_C = -1 \qquad [19]$$

The ultimate extremity of this policy instrument reflects the difficulty of persuading people to make capital investment equal to resource rents, when they would normally look for a return of at least ρ on their investment, but the marginal return on capital investment is dwindling towards zero.

Environmental and sustainability policy instruments [16] and [18] are thus distinct.[9] Any socially efficient solution, whether PV-optimal or sustainable, needs a subsidy to internalise the social amenity value of the resource stock. This done, sustainability policy itself entails only a time-varying consumption tax. However, both environmental and sustainability policies eventually require subsidies here, which would be difficult politically because of the lump sum taxes needed for financing subsidies.

5. Conclusions

We have analysed here a certain type of sustainability policy in a closed, representative agent, market, capital-resource economy, where the resource

9. Contrast this with the steady-state models of Pezzey (1992, Sections 7 and 8), where sustainability and environmental policies were just different, and possibly overlapping, strengths of the same incentive to conserve the resource.

stock has amenity value. By this we meant the time paths of policy instruments needed to bring about maximum constant utility, and hence zero net investment by Hartwick's rule, in an economy where individuals would otherwise aim for conventionally (PV)-optimal development. At the same we analysed the environmental policy needed to cause individuals to follow a socially PV-optimal path. As a preliminary we clarified that, in general, Hartwick's rule (and hence constant utility) cannot simply be added to PV-optimal development paths as a "policy", despite some literature suggesting this. In general, Hartwick's rule means departing from PV-optimality, and the choice between PV-optimality, or sustainability in the form of constant utility, is unavoidable.

Environmental policy in our economy was found to be a subsidy on the resource stock equal to its current amenity value. Sustainability policy consists of both this subsidy (otherwise sustainability will not be maximal), and a consumption tax profile which effectively converts the representative agent's utility discount factor into whatever yields constant utility as an individually optimal result. On their own, resource incentives are powerless to achieve constant utility. Assuming that the return to capital is ultimately less than the original discount rate, the consumption tax eventually falls to become a 100% subsidy, reflecting the difficulty of persuading people to invest when PV-optimally they would be disinvesting. The problems of measuring the amenity values needed to make all these policies operational are no less, but no more, daunting than for any non-market valuation problem such as "green accounting".

Many theoretical extensions to the simple model used here could and should be tried, so as to understand sustainability policies better. Obvious features to include would be resource extraction costs; environmental effects on production; trade; and technical progress. The last two are very important, since trade could simplify the sustainability problem if the economy is small enough for exogenous world prices to apply throughout it; and technical progress could well remove the sustainability problem altogether. Where sustainability policies do still exist, this paper's results suggest that they will always be quite distinct from purely environmental policies. More research is also needed on why present-value-maximising individuals would vote for a government that enacts sustainability policies. Finally, it is also necessary, though obviously difficult, to quantify this theoretical work and apply it somehow to empirical policy situations.

APPENDICES

Appendix 1. The socially PV-optimal path of the general economy

The current value Hamiltonian for an interior solution of the maximisation problem [1] in Section 2 is

$$H = U + \omega\dot{K} + \mu\dot{S} = U(C,S) + \omega[F(K,R)-C] - \mu R \qquad \text{[A1.1]}$$

where $\omega(t)$ and $\mu(t)$ are the respective co-state variables. Assuming that the solution exists, it will satisfy the first order conditions

$$\partial H/\partial C = 0 = U_C - \omega \quad \Rightarrow \quad \omega = U_C \qquad \text{[A1.2]}$$

$$\partial H/\partial R = 0 = \omega F_R - \mu \quad \Rightarrow \quad \mu = U_C F_R \qquad \text{[A1.3]}$$

$$\partial H/\partial K = -\dot{\omega} - (\dot{\phi}/\phi)\omega = \omega F_K \;\Rightarrow\; \dot{U}_C = [(-\dot{\phi}/\phi) - F_K]U_C \qquad \text{[A1.4]}$$

$$\partial H/\partial S = -\dot{\mu} - (\dot{\phi}/\phi)\mu = U_S$$

$$\Rightarrow\; d(U_C F_R)/dt = (-\dot{\phi}/\phi)U_C F_R - U_S \qquad \text{[A1.5]}$$

Then [A1.4] $\Rightarrow$ $[(-\dot{\phi}/\phi)-F_K]U_C = U_{CC}\dot{C} - U_{CS}R$

$$\Rightarrow \; \dot{C} = [(-\dot{\phi}/\phi)-F_K]U_C/U_{CC} + RU_{CS}/U_{CC}$$

$$= [F_K-(-\dot{\phi}/\phi)-RU_{CS}/U_C]C/\eta \qquad \text{[A1.6]}$$

$$\Rightarrow\ \dot{U} = U_C\dot{C} - U_S R$$
$$= [\{F_K - (-\dot{\phi}/\phi)\}U_C - R(U_{CS} + \eta U_S/C)]\ C\ /\ \eta, \qquad \text{which is [2];}$$

and [A1.5] $\Rightarrow\ \dot{U}_C F_R + U_C\dot{F}_R = (-\dot{\phi}/\phi)U_C F_R - U_S$

$$\Rightarrow\ \dot{F}_R/F_R = (-\dot{\phi}/\phi) - U_S/(U_C F_R) - \dot{U}_C/U_C, \quad \text{which using [A1.4]}$$
$$= F_K - U_S/(U_C F_R) \qquad \text{which is [4].}$$

Hartwick's rule

Zero net investment forever, $\dot{K} = RF_R \ \Rightarrow\ \ddot{K} = \dot{R}F_R + R\dot{F}_R$ [A1.7]

and [4] $\Rightarrow R\dot{F}_R = RF_R F_K - RU_S/U_C = \dot{K}F_K - RU_S/U_C$ [A1.8]

so $\dot{U} = U_C\dot{C} - U_S R$

$= U_C(\dot{F} - \ddot{K}) - RU_S$, which using [A1.7]

$= U_C(\dot{F} - \dot{R}F_R - R\dot{F}_R) - RU_S$, which using [A1.8]

$= U_C(\dot{F} - \dot{R}F_R - \dot{K}F_K)$

$= 0$ as required.

Hartwick's rule is compatible with optimal growth only if the discount factor $\phi(t)$ is chosen such that [2] also reduces to $\dot{U} = 0$.

Appendix 2. The PV-optimal path with specific functional forms

If $F(K,R)$ has constant returns to scale, it can be written in the intensive form $f(x) = [F(K,R)]/R$, where $x := K/R$. Hence (Dasgupta and Heal 1974, p11):

$$\dot{x} = \sigma(x)f(x) \quad \text{for all } t \text{ on any efficient path} \qquad \text{[A2.1]}$$

where $\sigma(x)$ is the elasticity of substitution. Here we have $F/R = K^{\alpha}R^{-\alpha} \Rightarrow f(x) = x^{\alpha}$, $\sigma = 1$, and [A2.1] is $\dot{x} = x^{\alpha}$. Hence the Ramsey rule [6] is (dropping the *'s for neatness) $\dot{C}/C = (\alpha x^{\alpha-1} - \rho)/\alpha = \dot{x}/x - \rho/\alpha$. Following Pezzey and Withagen (1998, p524), we integrate this to

$$C(t)/C(0) = [x(t)/x(0)]e^{-(\rho/\alpha)t}. \quad \text{[A2.2]}$$

which with $\dot{R} = R(\dot{K}/K - \dot{x}/x) = (\dot{K} - Rf)/x = -C/x$ gives

$$\dot{R} = [C(0)/x(0)]e^{-(\rho/\alpha)t}. \quad \text{[A2.3]}$$ [10]

Integrating [A2.3] and using the obvious transversality condition that $S(t) \to 0$ as $t \to \infty$, we find

$$S(t) = S_0 e^{(-\rho/\alpha)t} \text{ and } R(t) = (\rho/\alpha)S_0 e^{(-\rho/\alpha)t}. \quad \text{[A2.4]}$$

Hence $x_0 := x(0) = K(0)/R(0) = \alpha K_0/\rho S_0$. Inserting this, $x(t) = [x_0^{(1-\alpha)} + (1-\alpha)t]^{1/(1-\alpha)}$ (equation (1.36) from Dasgupta and Heal (1974), from integrating $\dot{x} = x^\alpha$), and $J(t,t)$ from [8] into $C = -x\dot{R}$, gives:

$$C(t) = (\rho^2 S_0/\alpha^2)\ [J(t,t)]^{1/(1-\alpha)}\ e^{(-\rho/\alpha)t} \quad \text{which is [7].}$$

Lastly, $K = Rx$ and $F_R R = (1-\alpha)F$ give

$$K(t) = (\alpha/\rho)C(t) \Rightarrow \dot{K}(t) = (\rho S_0/\alpha)\ (1 - \rho J/\alpha)\ J^{\alpha/(1-\alpha)}\ e^{-(\rho/\alpha)t} \quad) \quad \text{[A2.6]}$$
$$F_R R = (1-\alpha)K^\alpha R^{1-\alpha} = (\rho S_0/\alpha)\ (1-\alpha)\ J^{\alpha/(1-\alpha)}\ e^{-(\rho/\alpha)t}. \quad)$$

Hence net investment

$$\dot{K} - F_R\dot{R} = (\rho S_0/\alpha)\ \{[\alpha^2 - \rho J(t)]/\alpha\}\ [J(t)]^{\alpha/(1-\alpha)}\ e^{-(\rho/\alpha)t} \quad \text{which is [8].}$$

Appendix 3. "Constant consumption prices" for the Appendix 2 economy

We denote by $\bar{X}(t,t')$ the value of variable X at time t' ($\geq t$) on the constant consumption path which departs from the PV-optimal path *at time* t (≥ 0) with stocks $K^*(t)$ and $S^*(t)$. $\bar{X}(t,t')$ may change as t' does; and by

continuity, $\bar{K}(t,t) = K^*(t)$ and $\bar{S}(t,t) = S^*(t)$. From Solow (1974, p39), the maximum constant consumption level itself is

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10. The equation given in Dasgupta and Heal (p17) for $\dot{R}$ when $\eta \neq \alpha$ is incorrect.

$$\bar{C}(t,t') = \alpha\{[K^*(t)]^{2\alpha-1}[(2\alpha-1)S^*(t)]^{1-\alpha}\}^{1/\alpha}. \qquad [A3.1]$$

and other variables are as follows.

Output $\bar{F} := F(\bar{K},\bar{R}) = \bar{K}^{\alpha}\bar{R}^{1-\alpha}$, and

$\bar{C} = \bar{F} - \dot{\bar{K}} = \bar{F} - \bar{F}_R\bar{R} = \alpha\bar{F} \;\Rightarrow\; \bar{F} = \bar{C}/\alpha$, also constant

$\dot{\bar{K}} = (1/\alpha-1)\bar{C} \;\Rightarrow\; \bar{K}(t,t') = K^*(t)+(1/\alpha-1)\bar{C}(t'-t),\;\; t' \ge t.$

Together with $1/\bar{R} = \bar{K}^{\alpha/(1-\alpha)}/\bar{F}^{1/(1-\alpha)}$ and then using [A3.1], this gives

$\bar{F}_R \;= (1-\alpha)(\bar{K}/\bar{R})^{\alpha} = (1-\alpha)(\bar{K}/\bar{F})^{\alpha/(1-a)} = (1-\alpha)(\alpha\bar{K}/\bar{C})^{\alpha/(1-a)}.$

which at the starting time t is

$= (1-\alpha)\mathbf{[}\alpha K^*(t)/\alpha\{K^*(t)^{2\alpha-1}[(2\alpha-1)S^*(t)]^{1-\alpha}\}^{1/\alpha}\mathbf{]}^{\alpha/(1-\alpha)}$

$= (1-\alpha)[K^*(t)^{(1-2+1/\alpha)\alpha/(1-\alpha)}/(2\alpha-1)S^*(t)$

$= (1-\alpha)K^*/(2\alpha-1)S^*.$

Inserting $K^*(t)$ from [A2.6] and $S^*(t)$ from [A2.4] then gives

$\bar{F}_R(t,t) \;= (1-\alpha)(\rho S_0/\alpha)[J(t,t)]^{1/(1-\alpha)}e^{(-\rho/\alpha)t'} \,/\, (2\alpha-1)S_0e^{-(\rho/\alpha)t'}$

$= (1-\alpha)(\rho/\alpha)[J(t,t)]^{1/(1-\alpha)} \,/\, (2\alpha-1).$

Combining this with $\dot{K}^*$ from [A2.6] and R^* from [A2.4] finally gives

$\dot{K}^*(t) - \bar{F}_R(t,t)R^*(t)$

$= (\rho S_0/\alpha)(1-\rho\, J/\alpha)\, J^{\alpha/(1-\alpha)}e^{-(\rho/\alpha)t} - (1-\alpha)(\rho/\alpha)J^{1/(1-\alpha)}(\rho/\alpha)S_0e^{(-\rho/\alpha)t}/(2\alpha-1)$

$= (\rho S_0/\alpha)\, J^{\alpha/(1-\alpha)}e^{-(\rho/\alpha)t}\;\; [1 - \rho\, J/\alpha - (1-\alpha)(\rho/\alpha)J/(2\alpha-1)]$

$= (\rho S_0/\alpha)\; \{[2\alpha-1-\rho\, J(t,t)]/(2\alpha-1)\}\; [J(t,t)]^{\alpha/(1-\alpha)}\; e^{-(\rho/\alpha)t}.$ which is [9].

Appendix 4. Tax policy intervention in the individually PV-optimal, general economy

To save repetition, we omit here the hat (^) overscripts denoting the policy-induced solution path. Specific taxes are paid (in units of the consumption good) at a rate $\tau_C(t)$ on consumption, $\tau_K(t)$ on the capital stock, $\tau_C(t)$ on resource depletion and $\tau_S(t)$ on the resource stock, and revenueneutrality is achieved by lump sum refunds Ω. The representative individual then sees the output budget constraint as

$$F = C + \dot{K} + \tau_C C + \tau_K K + \tau_R R + \tau_S S - \Omega.$$

The undiscounted Hamiltonian for the PV-maximisation problem is then

$$H = U(C,S) + \omega[F(K,R) - (1+\tau_C)C - \tau_K K - \tau_R R - \tau_S S + \Omega] - \mu R$$

with first order conditions

$$\partial H/\partial C = 0 \;\Rightarrow\; \omega(1+\tau_C) = U_C \qquad [A4.1]$$

$$\partial H/\partial R = 0 \;\Rightarrow\; \mu = \omega(F_R - \tau_R) = U_C(F_R - \tau_R)/(1+\tau_C) \qquad [A4.2]$$

$$\partial H/\partial K = -\dot{\omega} + \rho\omega = \omega(F_K - \tau_K) \qquad [A4.3]$$

$$\Rightarrow \; \dot{\omega}/\omega = \rho + \tau_K - F_K = \dot{U}_C/U_C - \dot{\tau}_C/(1+\tau_C) \qquad [A4.4]$$

$$\partial H/\partial S = -\dot{\mu} + \rho\mu = -\omega\tau_S = -U_C\tau_S/(1+\tau_C) \qquad [A4.5]$$

Because we are calculating the individually PV-optimal path, in [A4.5] U_S is ignored. It is the amenity cost of total resource depletion, and so not perceived by an individual agent in choosing their own resource level.

$$[A4.4] \Rightarrow \quad \dot{U}_C/U_C = -\eta\dot{C}/C - RU_{CS}/U_C = \rho + \tau_K - F_K + \dot{\tau}_C/(1+\tau_C)$$

$$\Rightarrow \quad \dot{C} = [F_K - \rho - \tau_K - \dot{\tau}_C/(1+\tau_C) - RU_{CS}/U_C]C/\eta \qquad [A4.6]$$

$$\Rightarrow \; \dot{U} = U_C\dot{C} - U_S R = [\{F_K - \rho - \tau_K - \dot{\tau}_C/(1+\tau_C)\}U_C - R(U_{CS} + \eta U_S/C)]C/\eta$$

which is [11].

U_S is not ignored here, because each individual's utility is affected by the amenity value of the total resource stock S.

$$[A4.2] \Rightarrow \quad \dot{\mu}/\mu = \dot{U}_C/U_C + (\dot{F}_R - \dot{\tau}_R)/(F_R - \tau_R) - \dot{\tau}_C/(1+\tau_C)$$

$$[A4.4] \Rightarrow \quad \dot{\mu}/\mu = \rho + \tau_K - F_K + (\dot{F}_R - \dot{\tau}_R)/(F_R - \tau_R) \qquad [A4.7]$$

$$[A4.5], [A4.2] \Rightarrow \quad \dot{\mu}/\mu = \rho + \tau_S\omega/\mu = \rho + \tau_S/(F_R - \tau_R) \qquad [A4.8]$$

$$[A4.7], [A4.8] \Rightarrow \quad (\dot{F}_R - \dot{\tau}_R - \tau_S)/(F_R - \tau_R) = F_K - \tau_K$$

which is [12].

Appendix 5. The need for an asymptotically 100% subsidy on consumption

To prove: If $-\dot{\tau}_C/(1+\tau_C) > 0$ and bounded away from zero after some time, then $\lim_{t\to\infty}\tau_C = -1$. *Proof:* the subsidy rate $\tau_C > -1$, or else an individual's desired consumption would be unbounded. Hence $\dot{\tau}_C < 0$, to make $-\dot{\tau}_C/(1+\tau_C) > 0$. So $\lim_{t\to\infty}\tau_C = -1+z$ for some finite $z \geq 0$, and $\lim_{t\to\infty}\dot{\tau}_C = 0$. But then $\lim_{t\to\infty}[-\dot{\tau}_C/(1+\tau_C)] = 0/z$, and $\lim_{t\to\infty}[-\dot{\tau}_C/(1+\tau_C)] > 0$ by assumption. Hence z=0.

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