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A Survey of Methods for Determining A Planning Horizon

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In multi-period farm planning problems involving uncertainty, the usual case in the real world, planning must involve a continuous process of re-evaluation. In this dynamic situation it is necessary to know the number of periods to include in the decision model to ensure optimality. This is referred to as the planning horizon. In practice, planning models used are frequently simplifications of reality so the ability to determine the planning horizon under a range of situations is an important facet of planning. This paper contains a survey of methods of determining the horizon and offers comments on the approach necessary to resolve the cases for which methods have not been developed.

1 Introduction

Farm planning in the real world should usually account for the non-certain[1] and dynamic environment in which farms exist. Consequently decision models must, in theory anyway, be stochastic and multi-period in nature. Decisions made for other than the first period may never be implemented due to changing conditions. Effectively, planning should be regarded as a continuous process so it is pointless to construct decision models containing more periods than are necessary to ensure first period decisions are optimal. The number of periods that needs to be included in the model to ensure this is generally called "the planning horizon" in normative planning theory. This discussion contains a review of the problem of determining what is an appropriate planning horizon and of methods of calculating this horizon. Use of the correct horizon is a pre-requisite to obtaining optimal decisions as if too few periods are considered, sub-optimal first period actions will occur, whereas, on the other hand, if a greater number of periods than are necessary are used planning costs will not be minimized. Hence an optimal planning horizon exists. This horizon can constantly change in length as conditions change and should be constantly updated as re-planning continues.

In reality formal continuous planning probably never occurs. Most re-planning involves intuitive adjustments to formally or informally made long term plans. The reasons for this include the lack of readily available formal data, the extreme complexity of multi-period stochastic models and the lack of proof indicating that formal planning has a pecuniary advantage over intuitive reasoning. This means that most formal planning utilizes simplified models so it is important to consider methods of determining the planning horizon for a range of situations including cases involving the assumption of certainty in the variables.

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[1] A term used to encompass both risk and uncertainty.

Unfortunately, for some cases it is not possible, at least under current knowledge, to determine "the" planning horizon. Nor is it possible to find what can be called "a" planning horizon where this is regarded as a total number of periods which is at least as great as "the" planning horizon.

It is only in recent years that any real recognition has been given to the question of a planning horizon in theories of a firm. This has stemmed from the non-acceptance in the formal theories of the dynamic (both in a non-certain and time sense) nature of the planning and implementation situation. Even currently it is only in specialized cases (*e.g.*, Hillier & Lieberman 1967) that the question of a planning horizon is discussed in detail. Usually modern texts (*e.g.*, Naylor and Vernon 1969; Baumol 1972) make only passing reference to the problem indicating the need for a general theory in this area. Similarly, agricultural research workers have tended to ignore the planning horizon problem. For example Renborg (1970), in reviewing concepts and models of firm growth, does not refer to the question at all, and Day (1977), in discussing the literature on economic optimization, also fails to mention the problem. Frequently, where a decision on the number of periods to include in a multi-period model is necessary an arbitrary decision is made (*e.g.*, White 1959; Throsby 1962; Cocks 1965; Scobie 1967; Clark and Kumar 1978).

The review contains, firstly, a more detailed discussion on what is meant by "the" planning horizon. Secondly, methods of determining a planning horizon for a range of planning situations are reviewed and, finally, some comments are offered on where a general theory might lie. Reference is made to work outside the agricultural economics literature as there has tended to be a greater emphasis on the planning horizon problem in other fields. Likely reasons for this are that many general business problems are less complex and the larger size of firms in the secondary and tertiary sectors means a very real reward to the successful development of operational models is possible.

2 Planning Horizon Concepts

While the main discussion revolves round a normative horizon, it should be noted that reasons why farmers do in fact use a particular planning horizon have been proposed. The horizon used can be regarded as the positive horizon and it is probably determined in an intuitive way without a knowledge of dynamic decision theory.

Shackle (1961) suggests entrepreneurs, when considering investment decisions, estimate the maximum loss possible from a system for different periods. As future periods get "less certain", this will increase. Shackle maintains the maximum loss is based on what the decision maker would be "very surprised" could occur in any eventuality (called the "focus of loss"). Once this maximum loss, which increases with time, reaches a level equal to the maximum loss the decision maker is prepared to accept, the horizon is cut off for planning purposes. Similar concepts are defined for minimum gain situations. While the ideas were developed specifically for capital investment situations, they clearly have potential implications to all forms of forward planning (Haring 1961). Subjectively estimated distributions may be given a greater range with time so that maximum losses are exceeded and minimum gain requirements not met[2].

[2] The "focus-loss" concept has also been applied to static planning. An example is given by Boussard and Petit (1967).

Shackle considers the existence of a positive horizon is a reaction to non-certainty. This is also stressed by Svennilson (1938) and Naylor and Vernon (1969). Svennilson uses a slightly different approach to Shackle by suggesting that at some future time the decision maker loses confidence in his own ability to anticipate outcomes and so cuts off the horizon. Naylor and Vernon make the point that alternative investments may have different "degrees of non-certainty" and so in estimating their worth different horizons are used for each. Similarly, Brownlee and Gainer (1949) noted that farmers had greater confidence in their ability to predict technical outcomes rather than price outcomes so that plans were based on technological considerations. In these cases the period over which plans are made could well be that necessary, in the farmer's opinion, to ensure the technical success of the operation.

Returning to the question of a normative horizon, Modigliani and Cohen (1961) discuss the idea in some detail. They note that planning must consider future periods as first period decisions may affect the opportunities open to a firm in later periods. Accordingly definitions of relevant and irrelevant parameters are made which then lead to defining a planning horizon. Their sufficient conditions for a parameter to be irrelevant are—

> "A parameter p of a given future constraint is conditionally irrelevant within some stated range if and only if the optimal value of every component of the first move (first period decisions) is unchanged, no matter what value p might take in the stated range; it is unconditionally irrelevant if the stated range includes all *a priori* admissable values of p."

Conversely, a relevant parameter is one which does affect the first period decisions. While the definition is in a constraint context, Modigliani and Cohen also discuss relevance with respect to the objective function. Essentially their second theorem states that where the total payoff can be expressed in terms of two sub-outcomes and the first largely depends on first period decisions and the second only on later decisions, then decisions relating to the second pay-off component can be ignored provided decisions relating to the first component do not affect opportunities of obtaining the second component. Thus, a separable objective function is a pre-requisite.

Besides defining conceptual relevance they also introduce practical irrelevance. A parameter is said to be practically irrelevant where either implementation inaccuracies mean the parameter is not significant, or where the additional gains occurring from allowing for the parameter are small, or where the solving costs associated with its inclusion do not warrant its inclusion.

Noting that "plans are not decisions about future courses of action", the theorem on irrelevance lead Modigliani and Cohen to state that "the latest time period for which plans are made can be called the relevant planning horizon". Effectively they are saying that decisions must be made regarding which parameters are relevant so that the nearest future period in which all parameters are conditionally irrelevant can be isolated. The time up to and including the previous time period then becomes the relevant planning horizon.

In terms of an operational system for determining the planning horizon, the ideas of relevancy provide only broad principles and are only capable of direct use in a limited number of particular cases. Such cases occur where resources available for productive uses at some future period are in no way affected by any possible actions during the preceding periods or, through initial simple exploration, it is clear that the possible optimal decision set will not be influenced by previous actions. These cases seldom occur. Cases where the objective function is separable are common but the problem of later period physical actions being affected by first period actions means this separability cannot be exploited. Thus, while Modigliani and Cohen's principles provide a general conceptualization of the problem, particularly the idea of conditional irrelevancy, other methods must be developed for solving the planning horizon problem.

3 Methods Used in Determining a Planning Horizon

3.1 A Simple Theory

Planning in a dynamic and non-certain situation is conceptually simple where the marginal value products (MVPs) of all resources that might be held at the end of the first period are known. It is only necessary to determine the decisions which maximize the sum of the return from the first period plus the total value of resources held at the end of the period given the initial state or bundle of resources. The problem, of course, is that seldom are the MVPs known. Furthermore, doubt surrounds resource valuation theories, particularly in non-certain and intertemporal situations (see, for example, Upton 1976; Layard and Walters 1978, chp. 12). However, investors do in fact make some kind of assessment and the availability of, at least, expected present worth figures is likely to be useful in this context.

To overcome the lack of knowledge about the MVPs at the end of the first period a number of workers have developed specialized myopic search techniques for a variety of cases. Essentially the requirement is for techniques giving the MVPs in some period, or at least for putting narrow bounds on the MVPs, without the need to consider the total time period the business might potentially operate. In some cases the total value of resources of some state may dominate all others and this may be discernable without actually calculating the MVPs.

The purpose of this section is to consider the various cases for which solutions are obtainable. In reviewing the cases it is useful to categorize the types of planning problems and to consider each in turn. In all cases it is assumed the total horizon or time period over which a firm is prepared to stay in business is greater than a planning horizon. If this is not the case the planning horizon equals the total horizon.

Cases are classified according to two factors, the first being whether certainty exists, or is assumed to exist, with respect to all future prices, costs and technology. Any case in which at least one variable is non-certain is classified as a non-certainty case provided use of expected values does not effectively reduce it to a certainty case. The second factor relates to whether conditions in each future period are changing relative to previous periods. If prices, costs and technology, whether the certain values or the density functions, change from period to period, the case is called the non-stationary case and vice-versa. It is also assumed the objective function does not change for the stationary case to exist.

Within some combinations of these two factors it is also necessary to introduce two sub-classifications. The first relates to whether the maximum time span of any investments possible enables particular investments to be repeated within the effective total horizon. The possibility of repeatability depends on the type of investments possible where an investment in this context refers to the purchase of any resource no matter what its expected maximum life is. Repeatability is also related to the concept of a stable policy. A stable policy is defined as a set of decisions which are sequentially implemented in identical form. The length of period to which the set of decisions applies can vary and effectively can involve any number of sub-periods. Thus, for repeatability the time span of any investments must be short enough to enable the stable policy to be applied several times within the total horizon (the exact number will depend on the case). If repeatability is not possible a stable policy cannot exist. Further, as one year is frequently used as the accounting period, in assessing investments with a time span greater than a year the form of the consumption function is important, particularly with respect to time preference (the particular objective function used in any case implies a specific consumption function).

The second factor relates to whether it is physically possible to instantaneously adjust the system to any other state within the total set of possible states. Whether a non-constant cost or return is associated with the change is also important.

3.2 The Case of Certainty and Stationarity

If repeatability exists and a large number of repetitions are possible it is not necessary to consider the total horizon since a simple myopic search system is possible (*e.g.*, at the start of the growing season on a cropping farm where previous crops do not influence future responses). The problem is to find the optimal stable policy and then to implement it immediately. Determining the optimal policy involves the use, for example, of a static linear programming model where the length of the production systems are fixed. Where the duration of the production period is variable the optimal policy consists of repetitions at the time maximising net present worth (where marginal present worth equates with average net worth per period).

If time and cost are associated with changing from the current state to the starting state of the optimal stable policy then initial period decisions may not be the stable policy decisions. Thus, it is necessary to consider a number of periods. The problem consists of determining the optimal stable policy and then, given the current state, setting up a model with a sufficient number of periods such that the decision set of the last period is the same as the stable policy. To test for this condition it is clearly necessary to predetermine the stable policy. Effectively, the stable policy provides MVPs. Once the resource valuations obtained for the last period of the planning model are the same as those in the optimal policy sufficient periods have been considered. Grinold (1971) rigorously develops a proof of this system based largely on a linear model. This case assumes, of course, that the objective function does not change and that there are a sufficient number of periods for the MVPs to converge.

Where repeatability is not possible, the concept of an optimal stable policy cannot be used. Included in this case are problems where repeatability can occur but the number of repetitions are insufficient for the apparently stable policy to be, in fact, optimal even towards the end of the total horizon. This will occur where the starting state is sufficiently different from the starting state of the stable policy to make it sub-optimal to use the policy. This will occur where continual additional investment or growth is possible and desired.

Attempts to find a myopic search system in this case have rested on the turnpike idea. Tsukui (1966, p. 396) explains the turnpike concept in the following statement:

> ". . . suppose a balanced growth path (the turnpike) of the stock of goods is uniquely determined in a closed reproduction system. Then the efficient time-path of stocks, starting from any given common initial stocks and attaining in the terminal period (N) a Pareto-optimum in the possible set of stocks, will have the following properties:
>
> (a) if N is sufficiently large, all efficient paths of stocks may stay outside of a properly selected neighbourhood of the turnpike *for at most* a certain period N_0 determined independently of N.
>
> (b) all efficient paths of stocks remain consecutively in the neighbourhood of the turnpike except for certain period at the start and the termination."

Radner (1961) was one of the first workers to prove the existence of a turnpike. With respect to the planning horizon problem, the turnpike concept says that, in a growth situation, no matter what the starting state and the desired ending state, the optimal policy consists of making for the turnpike and staying on this until somewhere near the total horizon. Thus, provided a sufficient number of periods are considered to ensure that the system has reached the turnpike, the optimal first period decision will have been determined.

Boussard (1971) has applied the concept to farm planning. The assumptions necessary, however, are very limiting. Boussard assumes there are no constraints other than the initial cash supply and that a linear consumption function exists. This means a given proportion of each period's income is invested in capital stocks giving a growth situation. Using this simple function the objective is to maximize the total value of assets held at the end of the total horizon since if terminal assets are maximized so will consumption in each period. (In that the only capital good used was land, problems of depreciation are not considered). To overcome the problem of determining the correct valuations, Boussard turned to the turnpike concept and attempted to rely on the fact that if the total horizon is long enough it is not necessary to actually know the optimal final state. The problem is to ensure that a sufficient number of periods is included in the model (a linear programming formulation was used) so that it can be guaranteed that the turnpike has been reached. However, the number of periods necessary may give matrix size problems so Boussard notes that provided the correct values are placed on the possible ending states then the model can be truncated. His procedure was, therefore, to set up the model for a limited number of periods and to solve for the optimal solution with the land held at the end of the last period valued at subjectively estimated maximum and minimum values. If the first period solution was the same for both solutions it was assumed the first period action must be the optimal first move in approaching the turnpike. In agricultural investment it is doubtful whether farmers exhibiting a linear consumption function and, more importantly, facing investment situations that are not constrained, do in fact exist. This means the approach has little practical use. Further, it relies on subjective valuations so it cannot be proved that the first period decision is the optimal first period move to attain the turnpike.

Boussard attempts to introduce non-certainty through using Shackle's (1961) "focus of loss" concept. In that some of the alternative within-period actions may not satisfy the minimum loss requirement, he notes that the action set is reduced and therefore it may take longer to reach the turnpike. The conclusion is that the planning horizon will be shorter as with fewer alternative actions less periods will be needed in the model to give the same first period decision for the maximum and minimum valuations.

3.3 The Case of Certainty and Non-Stationarity

Given the expectation that conditions will continually change, the concept of a stable policy cannot be used in a myopic search technique. In this case a common approach is to solve the problem with a range of planning horizons and, if the first period solution is constant, to accept this as being optimal. The work of Rae (1970) and Byrne and Healy (1969) are two examples of this approach. (This approach can be used in any of the cases). Rae, in using a model of an horticultural property, found that the first period decision set did not change when the planning horizon was altered from four to five years and so accepted the first period solution as being optimal. Byrne and Healy came to the same conclusion after comparing the first period solution to a sheep replacement problem from eight and sixteen year period models. Use of this approach cannot, however, guarantee an optimal first period decision as there is no proof that adding additional periods will not, at some stage, lead to a change in the first period decision.

The only objective myopic search techniques developed involve the general inventory planning problem. While this only involves one product it will be discussed as it is an example of how specific conditions can lead to an easily identified planning horizon using a limited search. It is also the problem for which the first objective planning horizon rules were developed (Modigliani and Hohn 1955). Of the work in this area, that of Charnes, Dreze and Miller (1966) is used as it also introduced a non-certainty case and is therefore referred to later.

For the deterministic case Charnes *et al.* consider the problem of determining the optimal quantity (X_j) of a good to purchase and sell (Y_j) in time period j given a warehouse of known capacity (B) and known future prices and costs (P_j and C_j independent of quantity) over a total horizon of N periods. The initial inventory is designated h_o. Where β_N is the per unit value of any inventory on hand at the total horizon, the problem is to find X_j and Y_j values which

maximize $f_N + \beta_N h_N$

subject to

$$\left.\begin{array}{ll}\text{(i)} & Y_j \leqslant h_{j-1} \\ \text{(ii)} & h_j \leqslant B \\ \text{(iii)} & X_j,\ Y_j \geqslant 0\end{array}\right\} \quad j = 1,2, \ldots N$$

where:

$$f_j = \sum_{i=1}^{j} (P_i Y_i - C_i X_i) = f_{j-1} + P_j Y_j - C_j X_j$$

$$h_j = h_o + \sum_{i=1}^{j} (X_i - Y_i) = h_{j-1} + X_j - Y_j$$

Thus, sales occur from inventory and must not exceed inventory at the end of the previous period and inventory at the end of a period must not exceed warehouse capacity. Considering the last period, due to the linear price, cost and terminal value functions, it is profitable to go to the extremes, depending on the conditions of buying and selling that the inventory level and the warehouse capacity allow. For example, if $P_N \leqslant C_N \leqslant \beta_N$ it is profitable to keep any inventory on hand and to purchase to build up the terminal inventory. Thus, given these conditions, the optimal decision is to set $Y_N = 0$ and to purchase sufficient to make up the inventory to B. Similarly, the possible relationships between P_N, C_N and β_N lead to distinct decision rules. Charnes *et al.* summarize these in a table which is given below:

Event	Optimal Decision		Terminal Assets		Value of Terminal Assets ($f_N + \beta_N h_N$)
	X_N	Y_N	h_N	f_N	
1. $\beta_N \leqslant P_N, C_N$	O	h_{N-1}	O	$f_{N-1} + P_N h_{N-1}$	$f_{N-1} + P_N h_{N-1}$
2. $C_N < \beta_N, P_N$	B	h_{N-1}	B	$f_{N-1} + P_N h_{N-1} - C_N B$	$f_{N-1} + P_N h_{N-1} + B(\beta_N - C_N)$
3. $P_N \leqslant C_N < \beta_N$	$B - h_{N-1}$	O	B	$f_{N-1} - C_N(B - h_{N-1})$	$f_{N-1} + C_N h_{N-1} + B(\beta_N - C_N)$
4. $P_N < \beta_N \leqslant C_N$	O	O	h_{N-1}	f_{N-1}	$f_{N-1} + \beta_N h_{N-1}$

It will be noted that in all four cases the value $f_N + \beta_N h_N$ is a linear function of initial last period assets (f_{N-1}, h_{N-1}) and of B. Thus, using β_{N-1} and q_{N-1} as the coefficients of h_{N-1} and B respectively, the following equality holds:

$$f_N + \beta_N h_N = f_{N-1} + \beta_{N-1} h_{N-1} + q_{N-1} B$$

The table indicates that β_{N-1} and q_{N-1} will have specific values depending on the price, cost and value relationships. Effectively, β_{N-1} is the implicit value of a unit of inventory carried into period N and q_{N-1} is the 'evaluator' of a unit of warehouse capacity. Examination of the table indicates:

$$\beta_{N-1} = \max\,[P_N, \min\,(C_N, \beta_N)]$$

Furthermore, it can be shown that this relationship holds recursively for N-2, N-3 ..., j, ... 2. Thus, the table can be used to give the optimal decision provided N is replaced by N-1, and so on. Further:

$$f_{N-1} + \beta_{N-1} h_{N-1} = f_{N-2} + \beta_{N-2} h_{N-2} + q_{N-2} B$$

so that for some period j:

$$f_N + \beta_N h_N = f_j + \beta_j h_j + B \sum_{i=j}^{N-1} q_i$$

where

$$\beta_j = \max\,[P_{j+1}, \min\,(C_{j+1}, \beta_{j+1})]$$

and

$$q_j = \max\,[0, (\beta_{j+1} - C_{j+1})]$$

Given the β_j, the table indicates the optimal values of X_j and Y_j. However, β_j depends on future periods so to develop a myopic search technique it is necessary to see whether β_j can be determined from a limited search. From

(a) $\beta_j = \max P_{j+1}, \min (C_{j+1}, \beta_{j+1})$

and

(b) $\max (C_{j+1}, P_{j+1}) \geqslant \beta_j \geqslant P_{j+1}$

(that is unit value of inventory lies within the bounds of the purchase and sale prices in the following period) if:

(i) $P_{j+1} > C_{j+1}$, then $\beta_j = P_{j+1}$

or

(ii) $P_{j+1} \geqslant \max (C_{j+2}, P_{j+2})$, then $P_{j+1} \geqslant \beta_{j+1}$ and $\beta_j = P_{j+1}$

or

(iii) $P_{j+1} < C_{j+1} < P_{j+2}$, then $P_{j+1} < C_{j+1}\ \beta_{j+1}$ and $\beta_j = C_{j+1}$

In all other cases β_j cannot be uniquely determined without reference to more periods. These results occur as, given specific conditions, β_j depends only on the next period's conditions in the case of (i), and on the next two periods in the case of (ii) and (iii).

The planning horizon significance of the β_j determining rules is that if some period j exists such that β_j is uniquely determined by examining the next two periods, then an optimal first period decision can be made without recourse to periods beyond $j + 2$. If the above conditions do not occur further periods must be considered.

This inventory model has been considered in detail as it clearly demonstrates the principle that in some cases a limited search may uniquely determine the valuations and so lead to a planning horizon. Whether this will occur depends on the nature of the problem. Necessary and sufficient planning horizon conditions have been developed for other types of inventory problems within the certainty-non-stationarity case. They all depend on the various price relationships, which are relatively simple to explore for the single product situation. Examples are (a) Eppen, Gould, Pashigian (1969) who consider the problem of deciding on period production levels of a single product to meet known demands where set-up costs, holding costs and production costs vary with time, (b) Kunreuther and Morton (1973) who consider a similar problem but with an additional cost being related to the differences in production levels in each period (production smoothing), and (c) Lieber (1973) who considers the problem where backlogging can occur at a known cost in each period.

3.4 The Case of Non-Certainty and Stationarity

If the total horizon is long enough for repeatability the concept of an optimal stable policy can be used. With non-certainty that cannot be reduced to a certainty equivalent form, an optimal stable policy consists of a set of decision rules rather than a single policy. For each of the condition sets that can occur, one of the decision rules applies. Given instantaneous adjustment is possible, the myopic procedure is to determine the optimal stable policy and to immediately implement this in the first period. Shapiro (1968) considers such a problem for cases where an "unique optimal stationary strategy" exists. He uses the turnpike concept by noting that, provided the total horizon is long enough, the infinite horizon stable policy can be used in the initial periods.

Where instantaneous adjustment is not possible, the more likely case, the transition problem of reaching the optimal stable policy must be considered. Essentially, the myopic system of determining the stable policy and then setting up a model with a sufficient number of periods to give a final period solution the same as the stable policy can be used. Thus, *a priori* knowledge of the stable policy indicates when a planning horizon has been reached. Due to the non-certainty, the actual solution to the stable policy problem is not simple in most cases. Burt and Allison (1963) provide a typical example of the methods used.

Burt and Allison consider the Markovian decision problem of whether to plant a crop of wheat or to leave the field fallow in a semi-arid region. Actions and events in the immediately preceding year (wheat or fallow and rainfall) determine the soil moisture level at planting. Response in the current period depends on this moisture level and the rainfall. The decision on whether to plant or fallow depends on soil moisture and the rainfall probabilities as well as the prices and costs. For a given starting soil moisture level and decision there are a set of probabilities for the possible end of season soil moisture levels. Therefore, for a particular soil moisture level decision rule, a matrix of transition probabilities is defined, each row of which holds for one of the starting soil moisture levels. If R is defined as a column vector with components being the one period return for each starting state, β the discount factor, P the matrix of transition probabilities and $f(n)$ a column vector with components being the present worth of following the given policy for each starting state to infinity, then:

$$\text{(i) } f(n) = R + \beta P f(n-1) = R + \beta PR + \beta P f(n-2)$$
$$= R + \beta PR + \beta^2 p^2 R + \beta P f(n-3)$$
$$= (I + \beta P + \beta^2 p^2 + \ldots + \beta^{n-1} p^{n-1}) R$$

and

$$\text{(ii) for } n \longrightarrow \infty$$
$$f(n) = (I - \beta P)^{-1} R$$

Thus, the present worth vector of a constant policy over infinity can be determined by solving this single matrix equation.[3] Each component of the vector is the present worth of following a particular policy for one of the soil moisture states. In order to solve the transition problem, and so obtain the first period decision, Burt and Allison set up a dynamic programming formulation for the actual starting state for a limited number of total periods. This was solved and re-solved with additional periods until a constant policy was implemented over the last few periods. Then using the suggested constant policy to obtain $f(n)$ above, and therefore the state values, the original problem was effectively resolved using the derived values as the ending state valuations. If the same constant policy occurs, the first period decision is optimal no matter how many additional periods might be used and thus a planning horizon determined.

Where the problem features do not permit repeatability, the optimal stable policy cannot be used. The only methods used in this case have been to either use the total horizon or to use a subjectively determined system. One method is to add periods to the model until it appears the first period solution is stable. There can be no guarantee, however, that the addition of further periods will not alter the solution. Another method is to argue that the market

[3] Provided the Markov chain has 'ergodic states' (see Hillier and Lieberman 1967, p. 413).

determines correct marginal value products of resources making up the ending state and, therefore, to use a limited number of periods with the ending values being set at the market values, assuming they can be predicted in some way. Trebeck and Hardaker (1972), in considering the problem of spatial diversification of beef production, use this latter approach in a model involving several periods totalling one year. However, in most cases the market valuations do not reflect the true marginal value products to an individual farmer. There can be many reasons for the discrepancy including the fact that farmers' objectives, fixed factors, total horizons, and managerial ability vary. Market factors such as "cost plus" pricing and rigidities can also lead to discrepancies between the MVPs and actual market prices. Accordingly the use of market prices in valuing resources held at the end of a period is unlikely to reflect the true productive value and so may lead to a sub-optimal first period decision set.

3.5 The Case of Non-Certainty and Non-Stationarity

This is likely to be the most common case assuming the model used represents reality. The non-stationary characteristic means a optimal stable policy will not exist. In some cases, however, the extent of change between periods and over the total horizon will be sufficiently minor to put them into the non-certainty/stationary case.

Solving approaches must either use the total horizon or use planning horizon rules which guarantee a planning horizon can be isolated. Due to the non-stationarity, workers involved in such problems have had to search for special conditions giving a planning horizon. The only cases where necessary and sufficient conditions have been isolated are the single product inventory problems. In other cases the subjectively assessed approximate methods of Trebeck and Hardaker (1972) and Rae (1970) have been used.

The inventory problem discussed by Charnes *et al.* (1966) is a case where limited planning horizon rules can be proved given specific forms of non-certainty are introduced into the problem. The method used in devising the rules is similar to that used in the certainty case. It is assumed that the product purchase and sale prices in any period follow a joint density function but that once a particular period is reached the prices are known with certainty. They also consider the case where future price distributions depend on current prices (serial dependence). Effectively, the system involves taking a limited number of periods with the terminal β_j set at either plus or minus infinity. If the two $E(\beta_1)$ derived in each case give values which satisfy conditions similar to those in their certainty decision rule table (section 3.3), then the optimal first period purchase and sale decisions are uniquely determined. If not, the number of periods is increased. With price non-certainty the simple certainty rules cannot be used as they rely on known prices. Other examples of inventory type problems are given by Veinott (1968), who considers a production scheduling problem where demands are stochastic and in which more explicit horizon rules can be determined, and Symonds (1962) who considers a similar problem in which backlogging can occur (but at a cost).

3.6 Combinations of the Cases

Cases may exist, at least approximately, in which there are groups in which any one of the cases defined may occur. It is possible, for example, for a decision maker to make estimates of future conditions which reflect non-certainty and non-stationarity for the initial periods and non-certainty and stationarity for the remaining periods. Depending on the case it may be possible to use the stable policy concept. The work of Hopkins (1971), in solving the equipment replacement and capacity expansion problem of a single-product firm, provides an example. He considered the certainty case in which non-stationarity eventually gave way to stationary conditions. This meant that valuation could be obtained using the stable policy approach for the periods beyond the non-stationary section of the total horizon. This valuation was then used to value terminal stocks at the end of the initial non-stationary section of the horizon.

4 Conclusion

The survey of solving methods indicates that under stationarity it is possible to determine the optimal first period decision without explicitly determining "the" planning horizon. For the more generally realistic non-stationary case planning horizon rules have been developed for a limited number of relatively simple problems. As Charnes *et al.* (1966) note (p. 308), "no generally applicable methodology has yet been devised . . . for locating horizons". As many problems fall within the non-certainty/non-stationary case there is a need for exploring objective methods of determining the planning horizon. Furthermore, even cases where stationarity enables an optimal first period decision to be uniquely determined, a method of determining the planning horizon may remove the need for estimating the stable policy where this is a complex problem. Faced with any particular problem, however, a decision is required on whether simplifications are justifiable which will then enable, for example, the stable policy concept to be used where its determination is relatively simple.

As the problem of incorporating a planning horizon into continuous planning appears to revolve around the valuation of resources, (the dual problem) future work aimed at developing methods for the unresolved cases should consider valuation methods. Emphasis on the associated primal approach would appear to be less rewarding as valuation information provides a convenient and simple link between periods in a multi-period planning framework.

Systems that give MVPs, or perhaps put narrow bounds on them, without having to take into account the total horizon are required. The inventory problem quoted (Charnes *et al.* 1966) is a perfect example. Of course, whether formal continuous planning incorporating any methods that might be developed will give superior payoffs compared with subjective and simplified methods is another question. This assessment must await new developments.

Systems to estimate MVPs must rely on the relationships between MVPs and the parameters of the problem as well, of course, on a clear understanding of the relevant objective function and therefore the basis for valuations.[4] The MVP of inventory in the simple certainty case depended on specified price relationships. Similarly the MVPs in realistic problem situations will depend on defined relationships between prices, resource availabilities, technological considerations and so on. It is these relationships which must be researched in future so that MVPs can be predicted.

[4] As pointed out earlier, there are many unresolved questions in capital theory.

Research into the development of theories and methods for solving the planning horizon and dynamic planning problems will raise many challenges. As the real world is dynamic and non-certain the problems of quantifying objectives and formulating forecasts under these conditions must be overcome if directly useful theories and models are to be developed. The alternative is to continue using subjective approaches. In fact some workers maintain that there will never be an alternative to subjective estimation and planning. At the individual farm level, information and planning costs will be important factors in resolving these issues as, also, will be the major interests of farmers. Harle (1974) maintains that farmers are more concerned with technical achievements than questions of profit because of, at least in part, the difficulties of profit planning in a non-certain environment. It seems clear, however, that until more research is carried out many of these issues cannot be settled. While research in these areas will not be easy it is useful to remember that many problems to which reasonable answers are currently available were either not thought of, or were regarded as major stumbling blocks, by the early farm management and agricultural economics workers.

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