Risk, Utility and the Value of Information in Farmer Decision Making

Derek Byerlee and Jock R. Anderson*

A model is developed from decision theory for evaluating probabilistic information, especially for decision makers who are risk averse. The value of information to such a decision maker is disaggregated into mean and variance effects. It is shown that the degree of risk aversion of the decision maker may have important effects on the value attached to the information; however, there is not necessarily a positive correlation between risk aversion and the value of information, since the decision to acquire new information is itself often a risky decision. The concepts and procedures are illustrated by application to a fodder conservation decision with rainfall forecast information.

Decision making in agriculture, whether by farmers or agricultural policy makers, is inevitably done in an environment of uncertainty. To reduce this uncertainty, decision makers expend resources to obtain better information about the future. Farmers expend time and resources to attend extension meetings, to monitor the media, etc., partly for the purpose of obtaining information about weather, prices and new technologies. Governments spend large sums of money on such services as extension, meteorology and price forecasting to provide farmers with better information. Although, in each case, decisions must be made on how much and what type of information should be obtained or provided, most analyses of agricultural decision making have focused on deriving optimal rules for making decisions under existing uncertainty, rather than on rules for making decisions to obtain new information to reduce this uncertainty.

Microeconomic analysis and evaluation of additional information in farmer decision making under risk are addressed herein. The decision theoretic approach (Anderson, Dillon and Hardaker (ADH) 1977) is extended to provide a model for evaluating information in the important case where the decision maker's preferences are not neutral with respect to risk. Both theoretical models of information acquisition (e.g. Cato and Gibbs 1973; Schlaifer 1969; Szidarovskzy and Bogardi 1976; Marks 1980) and applications in agriculture (e.g. Love 1963; Byerlee and Anderson 1969; Doll 1971; Halter and Dean 1971;

*University of New England.

Baquet, Halter and Conklin 1976) have generally been limited (a major and comprehensive exception is Drynan (1977)) to the assumption that decision makers maximise expected returns regardless of risk—an assumption contrary to emerging evidence on farm management decision making (e.g. Officer and Halter 1968; and other references of ADH 1977, ch. 4). The approach developed here is illustrated through application to the evaluation of long-range rainfall forecasts in a decision to hold drought fodder reserves in livestock production. Particular attention is focused on the effect of the decision maker's attitude to risk on the value imputed to this rainfall information.

The theoretical model for analysing and evaluating information is developed in three parts. In the first stage, the decision maker is assumed to be indifferent to risk and hence a profit maximiser. The incorporation and evaluation of information under this restricted assumption is treated extensively in the literature and is only briefly reviewed here. Second, a nonlinear preference (utility) function is introduced and a method developed for evaluating information in this more general case. Finally, this model is reformulated in terms of an expected value and variance (E-V) framework. This allows some insights into the effects of information on both expected profits, the only concern of a profit maximiser indifferent to risk, and variance of profits, which has some relevance to decision makers with risk preferences.

Value of Information with Profit Maximisation

Consider a simple decision problem in which profits, π are given by the function

(1) $$\pi = \pi(x, \theta),$$

where x is an input into the production process, θ is a random event with probability distribution $h_o(\theta)$ and $\partial^2\pi/\partial x\partial\theta \neq 0$.[1] The decision maker chooses x^*_o which maximises expected profit, i.e., $\partial E_o[\pi]/\partial x = 0$.[2] Let $\mu^*_o = E_o[\pi(x^*_o, \theta)]$, be this maximum expected profit.

Assume now that the decision maker is provided with a predictor defined here as a process of generating new information about the event θ. Suppose this predictor yields a prediction P_k providing new information in the form of a posterior distribution $h_k(\theta)$ such that $\sigma^2_k(\theta) < \sigma^2_o(\theta)$, i.e., the variance of the uncertain event, θ, is reduced by the new information. The decision maker then chooses x^*_k such that $\partial E_k[\pi]/\partial x = 0$. Expected maximum profit at this point is $\mu^*_k = E_k[\pi(x^*_k, \theta)]$. However, the expected profit of using the prior optimal action x^*_o, given the new information in the posterior distribution $h_k(\theta)$, is given by μ'_k where[3]

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[1] This type of profit function could result from either price uncertainty or technological uncertainty. In the case of price uncertainty, the functional form is $\pi = p_y(\theta)f(x) - p_x x$ where p_y is the uncertain output price and p_x the price of inputs. Alternatively, for technological uncertainty, the profit function is $\pi = p_y f(x, \theta) - p_x x$ where θ is an uncertain and uncontrolled input such as rainfall (Byerlee and Anderson 1969).

[2] Throughout, expectations are denoted by E preceding the variable or function with the subscript on E denoting the probability distribution used in computing the expectation. In this case, the expectation is computed with distribution, $h_o(\theta)$. The random argument of expectation and variance operators is enclosed in square brackets.

[3] Note particularly that expected profits of using the prior optimal action are evaluated not with respect to the prior distribution but with respect to the posterior distribution which includes all the information available on the outcome of θ.

(2) $$\mu'_k = E_k[\pi\ (x^*_o, \theta)].$$

The value of the prediction, V_k, is then the increase in expected profits from using the posterior optimal action, x^*_k. That is,

(3) $$V_k = \mu^*_k - \mu'_k.$$

The set of posterior optimal actions, x^*_k, for each prediction, P_k, is a Bayes' strategy. If the predictor generates predictions, P_k, with probability distribution, $z(k)$, the value of the *predictor*, V_z, is given by

(4) $$V_z = \int \mu^*_k z\ (k)\ dk - \mu^*_o,$$

which is the difference between expected profits using the predictor and expected profits with prior information, and can also be shown to be the expected value of the prediction, $E_z[V_k]$.

Value of Information with Utility Maximisation

The analysis of the previous section can be generalised by introducing the risk preferences of the decision maker through a von Neumann Morgenstern utility function, $U = U(\pi)$. The decision maker then maximises expected utility under prior information with $\partial E_o\ [U\ (\pi)\ ]/\partial x = 0$ and under posterior information with $\partial E_k\ [U\ (\pi)\ ]/\partial x = 0$. Let this maximum expected utility for the posterior optimal action, x_k^* be $U^*_k = E_k\ [U\ (\pi(x^*_k, \theta))]$ and expected utility of the prior optimal action given posterior information be $U'_k = E_k\ [U(\pi(x^*_o, \theta))\ ]$. The value of information analogous to the profit maximising case will be represented by the increase in expected utility, $U^*_k - U'_k$. Clearly, this measure of information value is of limited use since utility is unique only up to a linear transformation and comparison of the relative value of information from different sources and to different decision makers cannot be made.

To overcome these problems, the value of information is converted into a monetary measure by defining it as the maximum price the decision maker could pay for that information and remain as well off (in utility terms) as he would be if he did not have this information. In the case in which the information is produced and consumed privately, this definition amounts to a demand price for the information and, summed over all producers, provides a demand schedule for the information. Alternatively, if the public sector provides the information, as is often the case in agriculture, then the definition provides a key component of measures of economic surplus that may be used in the evaluation of public goods of this type (Lerner 1963).

Using this definition, V_k, the value of the prediction, P_k, is given by the solution to the equation

(5) $$E_k\ [U(\pi\ (x, \theta) - V_k)] - E_k[U\ (\pi\ (x^*_o, \theta)\ )] = 0,$$

subject to the constraint that expected utility is maximised, i.e.

(6) $$\partial E_k\ [U\ (\pi\ (x, \theta) - V_k)]/\partial x = 0.$$

That is, the profit function is reduced by V_k until maximum expected utility is equal to the expected utility of using the prior optimal action. In a similar manner, the value of the predictor, V_z, is given by the solution to the equations

(7) $$\int E_k \left[U \left(\pi \left(x, \theta \right) - V_z \right) \right] z \left(k \right) dk - E_o \left[U \left(\pi \left(x_o^*, \theta \right) \right) \right] = 0,$$

subject to the set of constraints $\partial E_k \left[U \left(\pi \left(x, \theta \right) - V_z \right) \right] / \partial x = 0$ for all k in the range $z \left(k \right) > 0$.

Finally, the value of perfect information is important in calibrating the value of information against its potential value in a decision problem. The value of a prediction, V_θ, that perfectly predicts θ is given by

(8) $$U \left(\pi \left(x, \theta \right) - V_\theta \right) - U \left(x^*_o, \theta \right) = 0,$$

subject to $\partial \pi \left(x, \theta \right) / \partial x = 0$, since, without uncertainty, profits are maximised. If x^*_θ is the optimal action for this perfect prediction of θ, the value of the perfect predictor, $V^\theta z$, is given by

(9) $$E_o \left[U \left(\pi \left(x^*_\theta, \theta \right) - V^\theta z \right) \right] - E_o \left[U \left(x^*_o, \theta \right) \right] = 0.$$

Value of Information in E-V Space

The value of information for a decision maker with a nonlinear utility function is more clearly portrayed in terms of the effect of the information on the expected value and variance of profits. In the restricted (theoretically unsatisfactory but analytically convenient) case that the decision maker has a quadratic utility function, $U = \pi + b\pi^2$, expected utility may be expressed as

(10) $$E \left[U(\pi) \right] = \mu + b\mu^2 + b\sigma^2,$$

where μ and σ^2 are the expected value and varience of profits, respectively. This form of expected utility provides the familiar E-V indifference curves shown in Figure 1 for the typical case of risk aversion.

The E-V frontier of the expected value, $E_k \left[\pi \left(x, \theta \right) \right]$ and variance, $E_k \left[\pi \left(x, \theta \right) - E_k \left[\pi (x, \theta) \right] \right]^2$ of profits for different levels of x and with posterior information provided by prediction, P_k, is represented by OP in Figure 1. Point A is then the point of maximum expected utility with expected profits $\mu^*_k = E_k \left[\pi(x^*_k, \theta) \right]$ and variance, $\sigma^*_k{}^2 = E_k \left[\pi \left(x^*_k, \theta \right) - \mu^*_k \right]^2$. Let expected value, μ'_k and variance $\sigma'_k{}^2$ of using the prior optimal action given the posterior information be represented by point C with co-ordinates, $\mu'_k = E_k \left[\pi \left(x^*_o, \theta \right) \right]$ and $\sigma'_k{}^2 = E_k \left[\pi \left(x^*_o, \theta \right) - \mu'_k \right]^2$.

If the decision maker pays V_k for the information, the E-V frontier is moved vertically to $O'P'$ where the distance AB equals V_k in Figure 1.[4] The new optimal action is at point D. At point D, however, the decision maker is indifferent (a) to using the prior optimal action (point C) and (b) to using the posterior information with price V_k (point D). Hence, by definition, V_k is the value of the prediction.

The value of information, V_k can now be divided into two parts by triangulation, such that

(11) $$V_k = \left(\mu^*_k - \mu'_k \right) - a \left(\sigma^*_k{}^2 - \sigma'_k{}^2 \right),$$

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[4] If the decision maker pays V_k, expected profits are $E_k[\pi - V_k] = E[\pi] - V_k$ and variance is unchanged since variance is given by $E_k[\pi - V_k - E[\pi - V_k]]^2 = E_k[\pi - E[\pi]]^2$. Hence the E-V frontier is moved vertically.
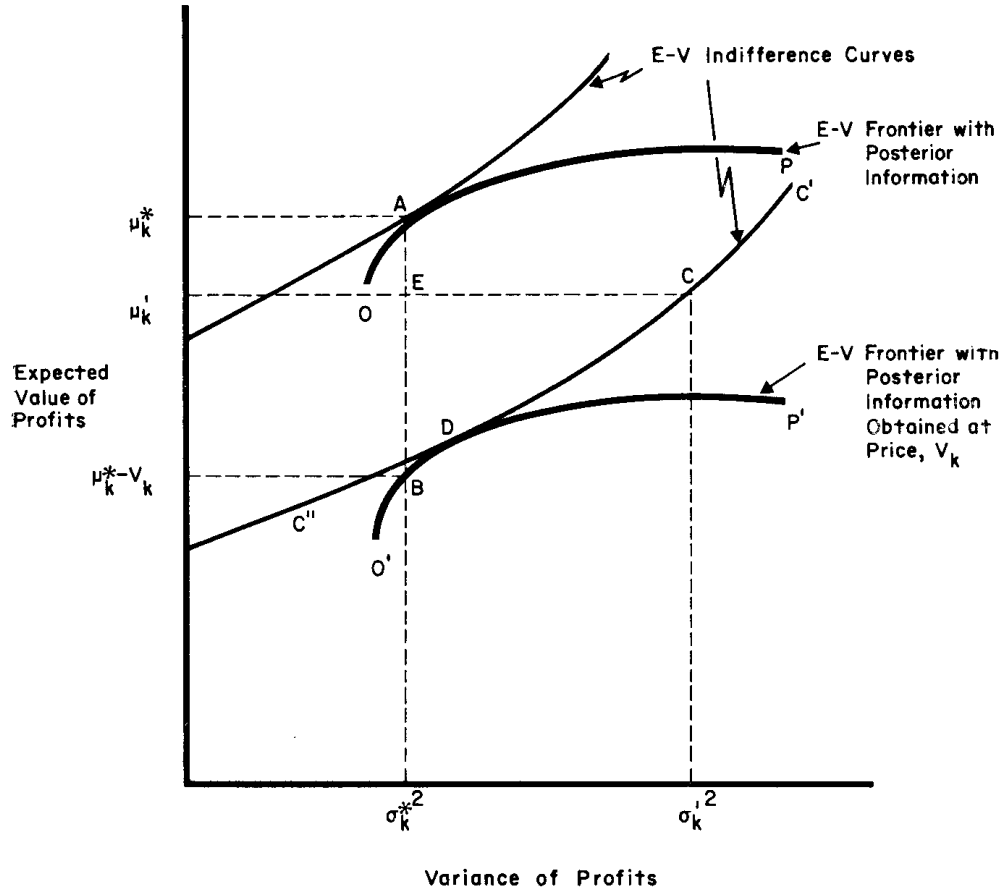
Figure 1: Value of a Prediction in *E-V* Space

where a is the slope of the line BC, and is positive for a risk averse decision maker. If we can approximate a by the slope of the indifference curve, $-b/U'$, we arrive at equation (12)[5]

(12) $$V_k = (\mu^*{}_k - \mu'{}_k) + b(\sigma^*{}_k{}^2 - \sigma'{}_k{}^2)/U',$$

where U', is the marginal utility along the indifference curve and, by definition, is positive. Thus the value of information consists of two components: (a) the change in expected profits, $\mu^*{}_k - \mu'{}_k$ and (b) a function of the change in variance, approximated by $b(\sigma^*{}_k{}^2 - \sigma'{}_k{}^2)/U'$. If posterior information increases expected profits and reduces variance, the variance component is positive if the decision maker is risk averse, ($b < 0$), negative if he is risk prone ($b > 0$) and zero if he is risk neutral ($b = 0$). Other combinations of mean and variance effects are possible in Figure 1 which depicts the situation faced by a risk averse

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[5] The slope of the indifference curve is $-b/(1 + 2b\mu)$ and, in the case of a quadratic utility function, $dU(\mu) = 1 + 2b\mu$. The closeness of the approximation depends on the slope of the indifference curve, the shape of the E–V frontier, and the difference between the prior and posterior optimal action. In the empirical example reported later in this paper, this approximation procedure evaluated at $\mu'{}_k$, resulted in errors of less than one per cent in estimating the value of the information, within the extremes of risk preferences represented in the farmer population (see Byerlee 1968, pp. 105–9).

decision maker. If the prior optimal action is represented by C', the mean effect is negative and the variance effect is positive and larger than the mean effect. With the prior action at point C'', the variance effect is negative and the mean effect is positive and larger than the variance effect.

The value of a predictor can be computed in E-V space in a similar manner. However, in computing variance for a predictor, the notions of preposterior mean and variance must be introduced to allow for two types of uncertainty faced by a decision maker in acquiring information. First, given new information provided by a particular prediction, the decision maker still faces uncertainty except in the special case of a perfect prediction. Second, the decision maker faces uncertainty because he does not know which prediction will eventuate before he purchases the predictor. It can be shown (Byerlee 1968, pp. 99–100) that the preposterior mean, $\bar{\mu}$, and the preposterior variance, $\bar{\sigma}^2$, can be expressed as,

$$(13) \qquad \bar{\mu} = E_z\ [\mu^*_k],$$

$$(14) \qquad \bar{\sigma}^2 = V_z\ [\mu^*_k] + E_z\ [\sigma^*_k{}^2],$$

where $E_z\ [\mu^*_k]$ and $V_z\ [\mu^*_k]$ are the expected value and variance of the expected profit, μ^*_k, of the posterior optimal action over the distribution $z(k)$ and $E_z\ [\sigma^*_k{}^2]$ is the expected value of the posterior variance. The components of preposterior variance are then clearly identified as uncertainty remaining after the information is obtained, $E_z\ [\sigma^*_k{}^2]$ and uncertainty about which prediction will occur, $V_z\ [\mu^*_k]$. The value of a predictor is then a function of the extent to which it affects (a) the preposterior mean and (b) each component of preposterior variance.

In the case of a perfect predictor, the value of information can be represented diagramatically in Figure 2. If OP is the E-V frontier with prior information and U_0 is the decision maker's indifference curve, the prior optimal action is at point A with utility U^*_0. Let the preposterior mean and variance of the Bayes' strategy resulting from the use of the perfect predictor be at point B. Then the value of the predictor V^p_z is given by the distance BC in Figure 2, since at point C the decision maker is indifferent between using prior information and paying amount V^p_z for the use of a perfect predictor. Figure 2 also shows that the value of this perfect predictor is only BD for a risk-prone decision maker with indifference curve U_1. It is also clear that there is a decision maker with indifference curve U_m where the value of the predictor is a maximum.[6]

Again, it can be shown that the value of the predictor can be disaggregated as in equation (15) into a component resulting from an increase in profit and a component resulting from a change in variance, and, approximating the slope of AC by the slope of the indifference curve, we get

$$(15) \qquad V^p_z = (\bar{\mu} - \mu^*_o) + b\ (\bar{\sigma}_o{}^2 - \sigma^*_o{}^2)/U'.$$

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[6] Note that, in the case of perfect information, all decision makers choose the same Bayes' strategy regardless of risk preferences, with preposterior mean and variance of profits at point B.
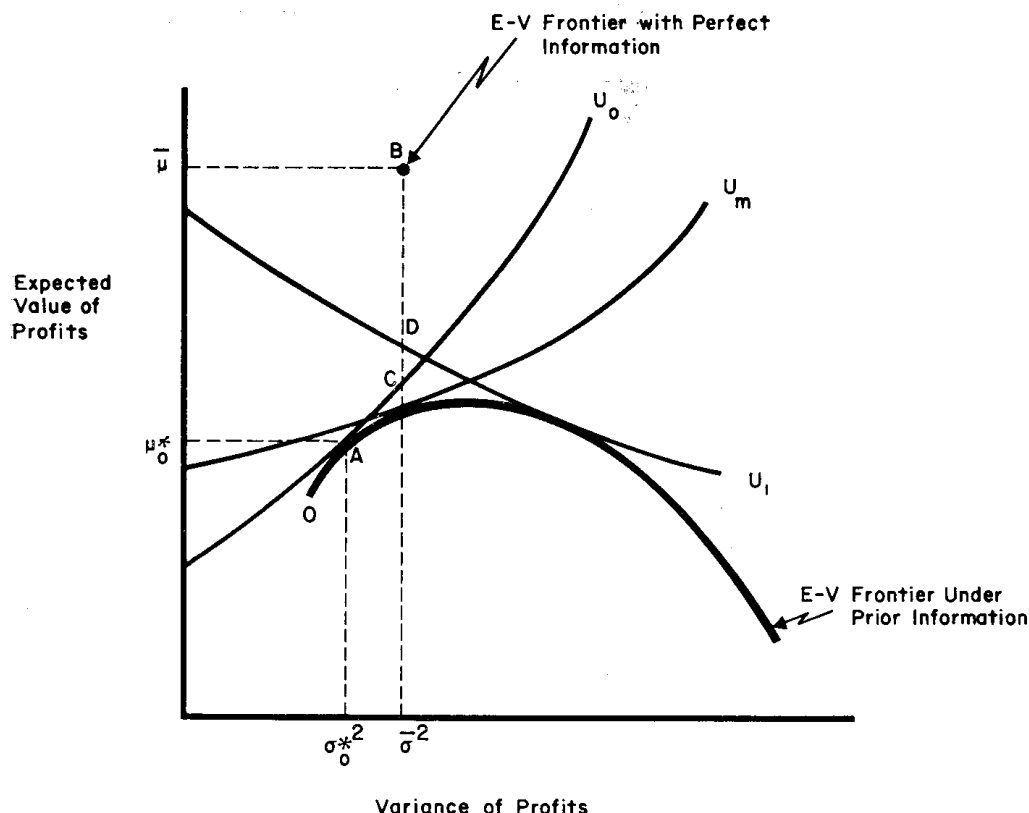
Figure 2: Value of a Perfect Predictor in E-V Space.

Application of the Model to a Fodder Conservation Decision Problem

The model developed above is illustrated by evaluating the usefulness of a long-range rainfall forecaster at one time initiated in Australia.[7] Such a predictor clearly has potential value to a range of both agricultural and non-agricultural decisions sensitive to rainfall. In this example, the evaluation is restricted to the farm management decision on how much fodder to conserve for livestock feeding in drought years. This problem is an important test of the value of the rainfall information, since livestock production is a major industry characterised by high variability in production due to periodic droughts.

A simple inventory model for fodder demands was chosen following Officer and Dillon (1965). The variables of the model are:

C_1 = nondrought acquisition cost of fodder;

C_2 = penalty cost of fodder if stocks are consumed and fodder is bought during the drought at higher prices (i.e., $C_2 > C_1$);

C_3 = salvage value placed on excess reserves at the end of the planning period and which is less than C_1 because of quality deterioration;

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[7] An earlier evaluation of this forecaster in nitrogen fertilizer use in wheat assuming profit maximisation is provided by Byerlee and Anderson (1969).

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h_o (θ) = prior probability distribution of fodder demands in time of drought;

h_k (θ) = posterior distribution of fodder demands given rainfall prediction, P_k;

U (C) = quadratic (dis)utility function of the form $C + bC^2$ as a function of costs, C, where for risk averters, $b > 0$, because the function is expressed in costs;

L = length of the planning horizon; and

x = the level of fodder conserved, the decision variable of the model.

The settings of the variables used in the model are representative of wool production in the New England region of New South Wales. The relationships among the cost variables C_1, C_2 and C_3 are known from past drought experience. The utility function used in the model was varied since a major objective was to explore the effect of decision makers' preferences on the value attached to rainfall information.[8] Prior and posterior probability distributions on rainfall and fodder demands were represented through piecewise polynomial approximation of cumulative frequency distribution functions following procedures developed by Byerlee and Anderson (1969). The posterior distributions $\{h_k$ (θ), $k = 1, 2, \ldots 6\}$ of fodder demands were derived from posterior distribution of rainfall, m_k (r), through the equation

(16)
$$h_k (\theta) = \int g (\theta|r) m_k (r) \, dr,$$

where g ($\theta|r$) is the probability of fodder demands given annual rainfall. r.[9]

Costs of fodder conservation, C are given by

(17)
$$C = C (x, \theta) = \begin{cases} C_1 x + C_2 (\theta - x) \text{ for } x < \theta < L \\ C_1 x - C_3 (x - \theta) \text{ for } 0 < \theta < x, \end{cases}$$

with expected value, μ, and variance, σ^2, given, respectively, by

(18)
$$\mu = C_1 x + C_2 \int_x^L (\theta - x) \, h (\theta) \, d\theta - C_3 \int_0^x (x - \theta) \, h (\theta) \, d\theta,$$

(19)
$$\sigma^2 = \int_x^L [C_1 x + C_2 (\theta - x) - \mu]^2 \, h (\theta) \, d\theta$$
$$+ \int_0^x [C_1 x - C_3 (x - \theta) - \mu]^2 \, h (\theta) \, d\theta.$$

The model is then solved as a cost minimisation or, in this case, a disutility minimisation model, by setting [10]

(20)
$$\partial E [U (C (x, \theta))] / \partial x = 0,$$

where, because a quadratic utility function is assumed,

(21)
$$E [U (C (x, \theta))] = U (\mu) + b\sigma^2.$$

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[8] The range of variation was based on elicitation of utility functions of farmers in the area by Robert R. Officer (Officer and Halter 1968) and subsequently by Derek Byerlee.

[9] $g(\theta|r)$ was determined subjectively through reviewing with an experienced farmer one-hundred years of daily rainfall records to determine the number of months of each year where drought fodder reserves would be needed.

[10] The cost minimisation problem is analogous to the profit maximisation case discussed earlier and becomes identical if $(-U)$ is maximized where U in this case is disutility.

The prior optimal action, x_o^*, was computed using the prior distribution, h_o (θ). The calculations were repeated with the set of posterior distributions $\{h_k$ (θ), $k = 1, 2, \ldots 6\}$ provided by the predictor to give a Bayes' strategy, $\{x_k^*, k = 1, 2, \ldots 6\}$.

Using the model developed earlier, the value of information was computed for the following categories of information:

(a) the value V_k of each rainfall prediction $\{P_k, k = 1, 2, \ldots 6\}$;

(b) the value V_z of the rainfall predictor;

(c) the value $V_{z_R}^P$ of a perfect predictor of annual rainfall; and

(d) the value $V_{z_\theta}^P$ of a perfect predictor of fodder demands.

Value of predictions

The prior optimal action and the posterior optimal actions given each rainfall prediction are reported in Table 1 for decision makers with varying attitudes to risk. As expected, more risk-averse decision makers conserve more fodder, and all decision makers reduce fodder holdings where high rainfall is predicted and increase holdings when low rainfall is predicted. Table 2 shows the value of each rainfall prediction. Extreme rainfall predictions produce greatest value for all decision makers. However, for a given prediction, the value of information varies substantially with the risk attitudes of the decision maker. Comparison of Tables 1 and 2 reveals that, when the difference between the prior optimal action and the posterior optimal action (i.e., $x_o^* - x_k^*$) is large, the value of information is highest. This explains the fact that the most risk averse[11] decision maker (i.e. $b = 0.00667$) attaches greatest value to the prediction of a wet year since, under prior information, he would hold large reserves and foreknowledge of a wet year enables him greatly to reduce reserves. In fact, given that most farmers are risk averse, the most valuable prediction will be one that predicts a wet year rather than a drought.

Table 1: *Prior and Posterior Optimal Levels of Fodder Reserves for Farmers with Varying Risk Preferences*

Utility function[a] $b \times 10^2$		Prior optimal reserves x^*_o	Posterior optimal reserves, x^* with prediction, P_k					
			P_1	P_2	P_3	P_4	P_5	P_6
			Very dry	Dry	Slightly dry	Slightly wet	Wet	Very wet
			Months of reserves					
Risk averse	0.667	9.05	11.21	10.66	9.69	6.27	4.45	2.79
	0.333	8.76	11.19	10.60	9.53	6.04	4.23	2.61
	0.167	8.35	11.16	10.51	9.28	5.75	3.99	2.45
Risk neutral	0.000	6.81	11.04	10.06	8.20	4.92	3.38	2.12
Risk prone	−0.033	5.92	10.89	9.61	7.36	4.54	3.14	2.00
	−0.067	4.51	8.95	7.52	5.58	4.00	2.82	1.87

[a] The utility function is of the form, $U = C + bC^2$.

[11] Note that since disutility is being minimised, risk averse decision makers have a positive b coefficient in contrast to the profit and utility maximisation model discussed earlier.

Table 2: *The Value of the Rainfall Predictions*

Utility function $b \times 10^2$		Value V_k of prediction, P_k					
		P_1	P_2	P_3	P_4	P_5	P_6
		Very dry	Dry	Slightly dry	Slightly wet	Wet	Very wet
		$ per farm*a*					
Risk averse	0.667	319	105	8	133	350	510
	0.333	387	130	12	127	337	509
	0.167	488	167	17	118	315	478
Risk neutral	0.000	870	295	37	70	212	349
Risk prone	−0.033	1022	324	41	41	152	278
	−0.067	1201	189	39	7	66	142

a Here and elsewhere in the results the currency values are 1968 A$.

In interpreting these results, it is important to understand that the most valuable predictions occur relatively infrequently. For example, the very dry and very wet predictions occur only with 0.02 probability. On the other hand, the less valuable predictions P_3 and P_4 together occur with a probability of about 0.5. There was also about a 0.4 chance that no useful information is provided by the predictor, in which case the decision maker uses prior information.[12]

Value of predictors

The values of various predictors are shown in Table 3. In general, the rainfall predictor only realises about 25 per cent of the value of a perfect predictor of annual rainfall. In turn, a perfect predictor of annual rainfall is only about half of the value of a perfect predictor of fodder demands, reflecting the fact that the seasonal distributions of rainfall are more important than annual rainfall in determining drought conditions.

The value of all predictors varies substantially with risk attitude. Three observations can be made with respect to the relationship between risk attitudes and the value of the predictors in this example. First, risk averse decision makers here attach more value to the information than decision makers who are risk prone, although it is not true, as might be expected intuitively, that the more risk averse a decision maker is, the more value he attaches to the information. This is in accord with Hilton's (1981) Theorem 2 finding that there is no general monotonic relationship between the degree of risk aversion and the value of information. Second, over the range of risk attitudes seemingly most relevant to the local farmers, $0 < b < 0.00667$, the value of the predictor is

[12] The posterior distribution of rainfall generated in these cases was judged not significantly different from the prior distributions to warrant further evaluation.

quite insensitive to the extent of risk aversion. Third, in each case, there is one decision maker who attaches greatest value to the predictor. This is consistent with our earlier observation in Figure 2 that there is a particular decision maker with indifference curve, U_m where the value of the predictor is a maximum.

Table 3: The Value of Predictors

Utility function	$b \times 10^2$	Value of rainfall predictor (V_z)	Value of perfect rainfall predictor (V^P_{zR})	Value of perfect predictor of fodder demands ($V^P_{z\theta}$)
			$ per farm	
Risk averse	0.667	55	208	542
	0.333	58	254	550
	0.167	60	286	555
Risk neutral	0.000	59	312	520
Risk prone	−0.033	52	304	480
	−0.067	37	210	358

The risk associated with using additional information is demonstrated by these results. The value of the predictor to the most risk averse farmer is $55 but, if he pays this amount, and prediction P_3 eventuates with a value of $8 (Table 1), he makes a loss. Alternatively, if prediction P_6 eventuates, he gains substantially. In the case in which the information is provided free to the farmer by a public agency, this risk is born by the public agency, and a more useful measure of the value of the predictor to the farmer is the average value of the predictions, $E_z[V_k]$, which in this case was computed as $61, somewhat higher than the value of the predictor.

Expected values and variances

The value of the rainfall information is portrayed in Table 4 by means of the effect that each prediction has on the expected value and variance of costs. The expected cost is generally reduced by using the rainfall information. However, for half of the predictions the decision maker chooses a posterior optimal action with higher variance than the prior optimal action. Following equation (11), the value of each prediction is decomposed into the effects due to (a) change in costs and (b) changes in variance. In most cases, the effect on expected costs is absolutely a larger component of the information value than the effect on variance.

Table 5 shows the preposterior mean and variances of the rainfall predictor and of a perfect predictor of fodder demands. Preposterior variance is disaggregated according to equation (15) into two components. First, there is the expected posterior variance which is less than the prior variance and, in the case of the perfect predictor, is zero. This variance is a measure of the risk remaining after the farmer has received the information. Second, the risk associated with the fact that the decision maker does not know which prediction will eventuate before he purchases the information is given by the variance of expected costs of the posterior optimal actions. This variance is relatively high for the perfect

predictor. The combined effect of these two variances is the preposterior variance which is higher than the prior variance even for the perfect predictor. When the values of the predictors are decomposed into mean and variance effects, the variance effect is negative because of the higher preposterior variances. Again, however, the variance effect is absolutely less important than the mean effect.

Implications for policy

The value of the rainfall information in a particular decision to individual farmers in a given region has been sketched. Such values require aggregation over a number of decision problems and regions, and the costs of supplying the information must also be accounted for in a detailed policy evaluation.[13] Nonetheless, this type of microeconomic analysis does have some implications for policy for producing and promoting such predictions.

Although the apparent overall (expected) value of the rainfall information is not high (about $50 a year for a "representative" farmer), some of the predictions produced by the rainfall predictor have potentially high payoff, particularly where extremes of rainfall are predicted. This analysis suggests that foreknowledge of a wet year would be particularly valuable to a risk averse farmer in his decisions on the level of drought reserves. Since the major cost of the information appears to be in its dissemination to farmers, policy makers could vary the amount spent on dissemination according to the particular prediction to hand. Finally, the evaluation of two hypothetical perfect predictors indicates that the potential value to development of more accurate long-range rainfall information may be high, suggesting there may be rewards to further research on improving the accuracy of the forecasts, and on providing more detail on a seasonal basis.

Finally, for policy analysis, aggregate market effects must be considered. Assuming no impact on factor markets, the supply curve will shift downward by up to V_z, the value of the rainfall predictor (Hayami and Peterson 1972). The extent of the aggregate shift would naturally depend on the proportion of producers who both believe and respond to the forecast (Smyth 1973). Total producer and consumer surplus generated is then of the order of up to (V_z/q) $(Q_1 + Q_0)/2$, where Q_1 and Q_o represent the equilibrium levels of output with and without the new information, respectively, and q is the output level of the representative producer.[14] The division of this surplus between producers and consumers will depend on the price elasticies of demand and supply, as in the case of research benefits (Lindner and Jarrett 1978), and the trading situation for the commodities concerned (Edwards and Freebairn 1981). In the empirical example considered here, an additional complication arises from probable effects on input prices. With all producers having access to the same rainfall information and with an inelastic supply of fodder over farmers' planning horizons, the difference between C_2, the penalty cost of fodder in a drought and C_1, the nondrought acquisition cost of fodder, would be reduced. This would correspondingly reduce the value of the information. However, the conservative aspect of the illustrative analysis is that benefits have been attributed to the impact of the forecast on only one farm management decision whereas, in practice, several decisions may be analogously advantaged.

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[13] In addition, the approach here measures the potential value of information since, in the absence of rainfall information, farmers would acquire information from monitoring actual weather conditions and change their actions accordingly (Eisgruber 1973).

[14] Indicative values for the case study are $q = 10t$, $Q_o = Q_1 = 700\,000\ t$ of wool, implying a total gain in surplus of up to $3.5 m.

Table 4: *Posterior Expected Value and Variance of Costs and the Components of the Value of a Prediction for a Risk Averse Farmer*[a]

Variable	Unit	Symbol	Posterior Information					
			P_1	P_2	P_3	P_4	P_5	P_6
			Very dry	Dry	Slightly dry	Slightly wet	Wet	Very wet
Expected Costs of Fodder Demand								
Expected cost of posterior optimal action	$	μ^*_k	4,596	3,767	3,097	1,847	1,189	639
Expected cost of prior optimal action	$	μ'_k	4,801	3,768	3,066	2,077	1,610	1,194
Variance of Costs								
Variance with posterior optimal action (x10⁻³)	^{2}	σ^{*2}_k	1,016	1,581	1,702	1,222	681	302
Variance of prior optimal action (x10⁻³)	^{2}	σ'^2_k	2,145	2,377	1,950	813	447	194
Components of Value of Prediction								
Value of prediction, P_k	$	V_k	319	105	8	133	350	510
Value due to decrease in expected cost	$	$\mu'_k - \mu^*_k$	205	1	−31	230	421	555
Value due to change in variance	$	$-a(\sigma^{*2}_k - \sigma'^2_k)$	114	104	39	−97	−71	−45

[a] Utility function assumed to be $U = C + 0.00667C^2$.

Table 5: Preposterior Expected Value and Variance of Costs and Components of the Value of a Predictor for a Risk Averse Farmer[a]

	Unit	Symbol	Prior information	Rainfall predictor	Perfect predictor of fodder demands
Expected Cost of Fodder Demands					
Expected cost of prior optimal action	$	μ^*_o	2,443	....	....
Preposterior expected costs of using information	$	μ	....	2,366	1,767
Variance of Costs					
Variance using prior optimal action	$^2	σ^{*2}_o	1,890	....	....
Expected posterior variance	$^2	$E_z[\sigma^{*2}_k]$	....	1,595	0
Variance of expected costs of posterior optimal actions	$^2	$V_z[\mu^*_k]$	....	406	2,175
Total preposterior variance of using new information	$^2	$\bar{\sigma}^2 = E_z[\sigma^{*2}_k] + V_z[\mu^*_k]$	....	2,001	2,175
Components of Value of a Predictor					
Value of prediction	$	V_z	....	55	542
Value due to reduction in costs	$	$\mu^*_o - \bar{\mu}$	....	77	599
Value due to change in variances	$	$a(\sigma^{*2}_o - \bar{\sigma}^2)$	....	-22	-57

[a] Utility function assumed to be $U = C + 0.00667C^2$.

Conclusions

A theoretical model was developed that enables monetary evaluation of information for the general case in which the decision maker has non-neutral risk preferences. That is, the value of additional information was defined as the amount of money that a decision maker could pay for the new information and remain as well off as he would be by using the prior information. In a case in which the risk preferences of a decision maker are a function only of the expected value and variance of profits, it was shown that the value of information has two components due to (a) a change in expected profits and (b) a change in variance of profits. The importance of the latter term is related to the degree of risk aversion of the decision maker.

Implicitly, it is often assumed that additional information reduces variance in a decision problem, and therefore, has more value to a risk averse decision maker. An important finding of the theoretical and empirical analysis of this paper is that variance may not be reduced by new information since there are two types of risks associated with a decision problem. First, when the decision maker has received a particular piece of information he is still faced with some risk, measured by the posterior variance. However, by definition, this risk will be less than using prior information and, in the case of perfect information, is zero. Second, the decision maker, in making the decision to purchase a particular information generating process, does not know *a priori* what information will be forthcoming, and the decision to purchase information is therefore a risky decision. For example, if the decision maker decides to purchase a prediction and the particular information forthcoming results in the same action he would have used with prior information, he will have lost from the decision to purchase this information. This additional source of risk which occurs for both perfect and imperfect information may help explain the failure of decision makers to obtain information, even when the monetary returns appear to exceed the costs.

The empirical application of the model to the evaluation of long-range rainfall information revealed that risk attitudes may play an important role in the value that a decision maker places on information. The value of individual rainfall predictions varied substantially with the risk attitude of the decision maker. However, the value of the rainfall predictor as a whole was fairly insensitive to risk attitudes. In this example, the risk averse decision makers attached more value to the information than the risk prone but, among risk averse decision makers, the value of information declined slightly with the degree of risk aversion, as a result of the higher preposterior variance associated with the use of information.

Risk aversion may have an important bearing on both private and public decision making on production and acquisition of information. The model presented here provides a framework for analysing the role of risk aversion in the use of information.

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