Estimating Research Benefits when there is Input and Output Substitution: An Applied Analysis

Loris Strappazzon, Mark Eigenraam, Rukman Wimalasuriya and Gary Stoneham.

Economics Branch, Victorian Department of Natural Resources and Environment

Level 13, 8 Nicholson St, East Melbourne, 3002, Victoria, Australia

For Correspondence: Loris.Strappazzon@nre.vic.gov.au, (Ph) 61 3 9637 8353

(Fax) 61 3 9637 8115

Abstract

In this paper, we develop a new method for estimating the producer benefits of technical change, taking account of input or output substitution. Our approach is consistent with a profit function approach to benefit estimation, and can be used to measure the benefits of either a 'cost-reducing', or 'output-expanding', technical change. Our approach combines farm-level linear programming models with index numbers. We use three case studies to illustrate our approach under varying circumstances: (1) when there is no input or output substitution; (2) when there is predominantly output substitution and (3) when there is predominantly input substitution.

Keywords: Economic Surplus; Profit Function; Economies of Scope; Input/Output Substitution; Benefit Cost Analysis; Linear Programming

1 Introduction

In this paper we present a method for estimating the *ex ante* benefits of on-farm technologies. We then compare our method to others presented in the economic literature, and show that it has some desirable properties.

Most of the authors on benefit-cost analysis have focused on economic surplus as the appropriate measure of net-benefits (see, for example, Alston, Norton and Pardey 1998; and Mishan 1988). The evaluation of agricultural research has received considerable attention in this literature (see, for example, Mullen *et al*. 1988; Lemieux and Wohlgenant 1989; and Voon 1992).

A common approach by many of these authors has been to model technical change—from (say) research—as a downward shift of a single commodity supply function. The aggregate supply function shifts downward because *each firm* has either (i) increased output, holding inputs constant; or (ii) reduced inputs holding output constant. In other words, the aggregate supply function shifts down because firms have improved their productivity.

Alston *et al.* (1998) differentiate between several stages of a firm's adjustment to a technical change. For our purposes there are two important stages: (i) when a firm has implemented a new technology, but its processes, input and/or output mix, remain constant—Alston *et al.* call this k_1; and (ii) when a firm has adjusted its processes, input and/or output mix to better accommodate the new technology—which Alston *et al.* call k_2.

Alston *et al.* argue that in order to measure the benefit of a technical change, then the focus should be on k_2; only after firms have adjusted to a new technology can its full worth be known. For example, if a farmer introduces a higher yielding grain-legume variety, he may alter his crop rotation (and hence output mix) to incorporate more of the higher yielding grain-legume. The measure of such a change would be called k_2. In contrast, k_1 would be the measure of change when a farmer uses the higher yielding grain legume without adjusting the output mix.

There has been little discussion in the literature about methods for estimating k_2. Pannell (1999) has made the general argument that farm-level models are valuable because they allow an analyst to take account of interactions that occur between different outputs. These interactions may occur because of substitution and competition for resources, and possibly through *biological* complementarity or competition. In an earlier study, Griffith *et al.* (1995) estimated a quasi-k_2 for a lamb technology using a combination of gross margin (GM) analysis, and linear programming. Griffith *et al.* make the same points as Pannell: that linear programming allows the analyst to model on-farm interactions using a coherent framework.

That these interactions are important in agriculture is obvious from the fact that many farm systems are multi output; for example, grains, mixed grains-livestock or mixed livestock farms, and all have the ability to accommodate technical change with output substitution.

From an economic point of view, there is a large incentive for a farmer to produce multiple outputs when production is characterised by economies of scope (e.o.s). Where there are e.o.s, it is cheaper for a firm to produce multiple outputs (in some combination) rather than specialising in these outputs (Baumol, Panzar and Willig, 1988). Chavas and Aliber (1993) provide some empirical support for the existence of economies of scope in agriculture.

The implication of e.o.s is that a farmer will concentrate on the least cost way of producing a *group* of products; he will concentrate on the *farming system* rather than a specific crop within the system[1]. Farm-level linear programming (LP) models can readily model this multi-output system. Farm-level LPs can also take account of input substitution in a single-output dominated firm, such as dairying (where e.o.s are less important, but substitution is still very important).

In this paper, we propose a new method for (mainly) the *ex ante* estimation of the benefits from technical change: we use LP models in conjunction with index numbers to measure farm-level productivity improvements, k_2. The method can, to some extent, take account of on-farm adjustments that accommodate a technical change. Therefore, we can get closer to estimating k_2. We then use the calculated *farm-level* productivity improvement to value the benefits of a technical change. Our method can be used to estimate the benefits of both a 'cost-reducing' and 'output-expanding' technical change.

In the next section we explain how LP models can be used to take account of farm responses to technical change, and we explain how our measure relates to others proposed in the literature. We present three empirical applications in Section 3. In the final section, we provide a summary and some future research needs.

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[1] There could also be advantages to multi product firms in terms of risk reduction, see Hanly and Cheung (1998).

LP models and farmers' adjustment to new technology

In this section, we will describe how LPs can be used to measure a farm-level productivity change that takes account of farmers' adjustment—in terms of output and input mix—to a new technology.

The system we describe below was developed within the Victorian Department of Natural Resources and Environment (NRE); and has, more recently, been used by the Department of Primary Industries and Resources South Australia. Mostly, the system has been used for *ex ante* evaluation, but in some cases it has been applied to ex-post evaluation as well.

A problem for any economist evaluating research is to translate the outcomes of a proposed scientific project into an economic change. In terms of LPs, the economist needs to alter one or more of the technical coefficients in the model to estimate k_2. Often, using an LP is an easy way to model a technical change because the economist can alter the LP parameters *directly*; LPs imbue many of the biological processes and technical constraints that exist in a farming system.

After altering one or more LP parameters we calculate the effect that this has on farm-level productivity using a Fisher index (Fisher, 1923) in a simple four-part procedure:

1) solve the LP for its optimal solution (the 'without' scenario)—report the inputs and outputs in an 'input-output table';

2) alter one or more parameters in the LP according to the technical change being evaluated;

3) re-solve the LP for its new optimal solution (the 'with' scenario)—report new input-output table; and

4) calculate the productivity change with input-output information, holding either the input level, or output level, constant.

This last step requires some explanation. Often when re-solving the LP, it will change the level of inputs *and* outputs. For example, if we increase the yield of pasture in a dairy model, the LP's 'with' solution will contain an increased level (index value) of outputs *and* inputs. However, to get an accurate measure of the shift in the production function, we need to hold either outputs or inputs constant.

We can do this simply enough by understanding the production function of the LP in the 'without' scenario. This production function is constructed by re-solving the LP at different output levels, and mapping input usage. We then use regression techniques to fit this function. In essence, this is the 'without' production function. Any of the 'with' scenarios are then compared to the 'without' production function.

Once we have calculated the farm-level productivity change, it is a simple matter to calculate the aggregate benefit of technical change. If the farm level productivity change is calculated as an increase in outputs holding inputs constant, this can be multiplied by the *relevant* value of production. If the farm-level productivity change is calculated as a decrease in inputs holding output constant, this can be multiplied by the *relevant* total variable costs (see the Appendix for a simplified mathematical explanation).

The above procedure provides a partial equilibrium estimate of the change in short-run gross margin or GM (total receipts minus variable costs), that Just, Hueth and Schmitz (1982) argue is an estimate of producer welfare. Our procedure is similar to that used by Cooke and Sundquist (1993), who measured the benefits from productivity improvement using a Fisher cost-efficiency index. However, Cooke and Sundquist applied their method *ex post*, whereas our application (below) is *ex ante*. In an *ex ante* application, an LP is useful because its objective function (risk neutral gross margin maximisation) allows us to predict the reaction of a farm to some technical change. Further, since we use partial equilibrium analysis, we ignore problems associated with changes in factor prices; this was a key concern for Cooke and Sundquist who focused on estimating an industry's average cost curve excluding rent.

Griffith *et al.* (1995) undertook an *ex ante* estimation of the reduction in farm average cost from a 'large lean lamb' technology. However, the authors measured the average cost in the with and without scenarios at different output levels. Theoretically a cost reduction should be measured holding the level of output constant, which—as explained above—is our approach.

We can further explain our procedure using Figure 1 [2]. Initially, we have a straight line supply function, S_0. The price of output, which we assume is determined in the world market, is P. The quantity produced is Q_0. Some quantity of variable inputs, v, are used to produce output; and each unit of input costs p_v. In Figure 1, v_0 units of variable inputs are used to produce Q_0[3].

[2] Note that the case we have drawn here is what Martin and Alston (1997) call the 'elastic' situation.

[3] Note that Figure 1 is drawn for a single commodity. We are often dealing with many outputs in our approach. We consider the implications for our technique in terms of estimating conventional producer surplus in the next Section.

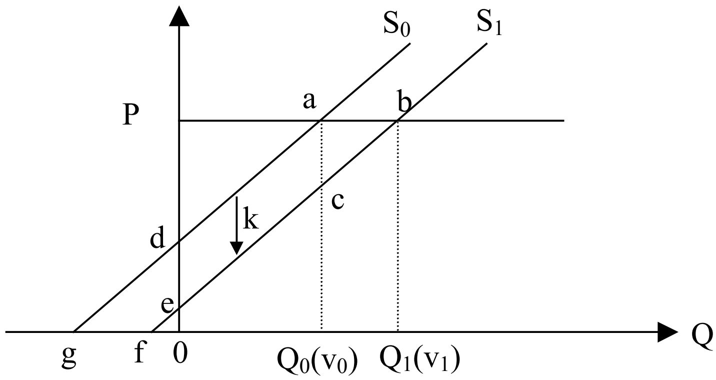


Figure 1: The downward shift of the supply function due to technical change

A technical change shifts S_0 downwards, by *k*, to S_1. We assume that this is a parallel shift. The standard measure of producer surplus from this change would be *abed*. However, Martin and Alston (1997) argue in favour of a profit function approach that would, for an output-expanding technical change, measure the area, *abfg*—which is equal to area abQ_1Q_0.

If a technical change increases output, but inputs remain constant, then we can calculate the value of this benefit, B_{opi}, as:

$$B_{opi} = E_0 P Q_0,$$

where E_0 is the productivity change in this scenario, and PQ_0 is the total revenue from producing Q_0. E_0 is estimated taking into account the output-expanding technology and output substitution. B_{opi}, as calculated here, is equal to what Martin and Alston call the output-expansion benefit, area *abfg* (see the Appendix Part (i)).

Conversely, if the productivity change is calculated by holding outputs constant, and reducing inputs, then we could calculate the value of such a change, B_{ipi}, as:

$$B_{ipi} = E_i p_v v_0,$$

where E_i is the productivity change resulting from an input-reducing technical change, and $p_v v_0$ measures the total variable cost of producing Q_0.

This is equal to area *aced* in Figure 1 (see the Appendix Part (ii)). In other words, this is the standard measure of producer surplus truncated at the initial level of output; it does not take account of the increase in output that may be induced by a productivity improvement. Again, this corresponds with the profit function measure which Martin and Alston (1997, p. 151) call a short-run 'cost-reducing technical change'.

In the case where technical change affects outputs only, or inputs only, then our measures of the gross annual research benefit—B_{opi} and B_{ipi} respectively—will be exactly the same as a change in gross margin using the standard approach. However, in many circumstances, it would be difficult to calculate a standard gross margin change. For example, if a project aimed to improve cow feed conversion efficiency (considered below), then an economist may find it hard to determine exactly how a farm would respond. To do so, he may need to take several steps: to understand biological factors (such as the amount of energy used by cows, and the amount of energy contained in each food source); to decide how these relationships would affect the on-farm decision making, etc. However, the further we take this logic, the closer we get to mimicking the solution of a liner programming approach.

Our approach has some advantages—relative to a gross margin or single commodity supply curve approach— however there is a problem: we now have two estimates of the benefit, B_{opi} and B_{ipi}. Martin and Alston (1997) argue that a profit function approach (which we are consistent with) could be rationalised in several ways (Martin and Alston, pg 151). However, we are hesitant about using their explanations. For example, using B_{opi} implies that we are interpreting the area *gdef* in Figure 1 as economic benefit. It is unclear to us what this area represents. Further, it seems to us that the situations where a problem requires an input-reduction or output-expansion approach is not clear-cut. Our method allows the estimation of both, and even if the initial change on a farm were (say) an input reduction (for example, a reduction in the number of crop sprays) then this could still be translated into an output expansion (an increase in the output quantity index).

If our method were only being used to rank a list of projects, then it could be argued that, as long as B_{opi} and B_{ipi} gave consistent results, the 'absolute' value of the benefits doesn't matter. However, on many occasions, decision-makers will want to know the absolute value of the benefit of a program or project. For example, where decision-makers are performing cross-program comparisons, and one of these programs doesn't involve input or output substitution. It seems to us this is a good topic for further research.

2.1 Estimating Producer Surplus When there is Economies of Scope

Figure 1 is drawn for a single output, yet we have argued our method is applicable in multi-output situations. Figure 1 is still useful for expository purposes: the quantity-price axes could be thought of as measuring index values. However, if the axes represent indices, we do not have a direct measure of area the abQ_1Q_0 from Figure 1. Rather, we have a measure of its value relative to the initial position—in terms of total revenue area $0PaQ_0$, or total variable cost area $0daQ_0$.

To measure conventional surplus, area *abed* in Figure 1, we would require commodity-specific supply curves. However, E_i is based on multi-output situations and cannot be used with commodity-specific supply curves. This proposition can be extended to the general case where e.o.s exist: when there are e.o.s, we cannot assign total (or hence marginal) costs over all ranges of output, hence it is not possible to identify a commodity-specific supply curve (Carlson 1974).

Other analyses have tackled multi-product firms by modelling each product separately. These include an empirical application by Mullen *et al.* (1988); and a theoretical framework by Just, Hueth and Schmitz (1982, pp. 189-191). However, neither of these tackles the allocation of costs to single-commodity supply curves in the face of economies of scope.

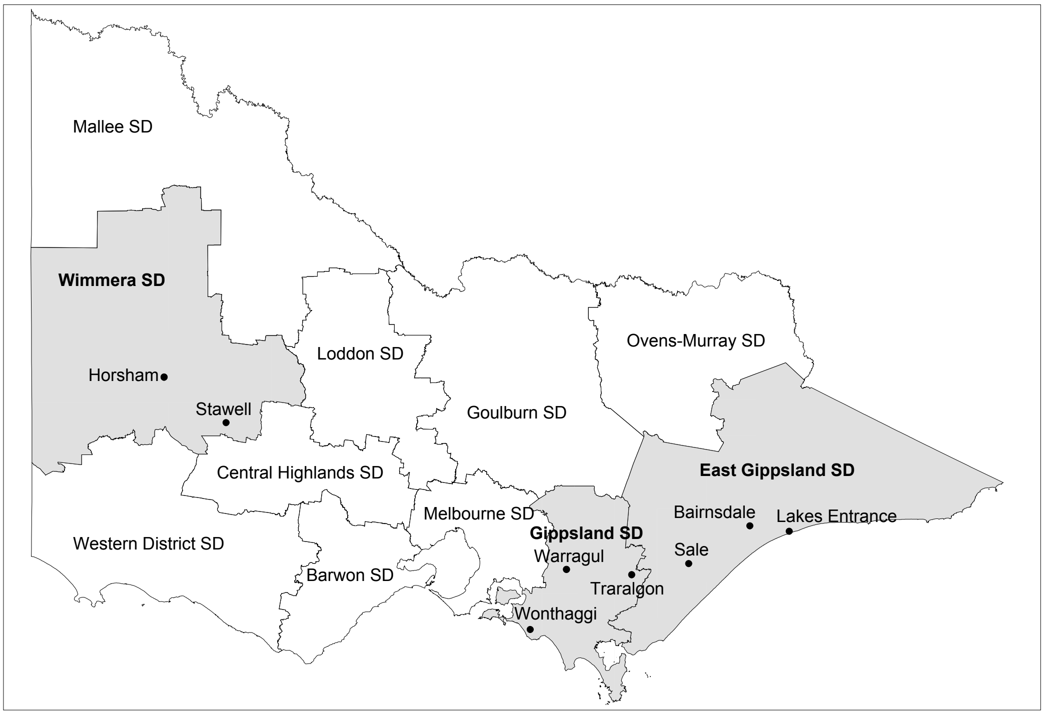
3 Applying the Method

In this section, we present three case studies that apply the above method: two of a crop farming system; and one of a dairy farming system. We calibrated each model using five-year average yield (estimated from Australian Bureau of Statistics, 2001) and price data (Australian Bureau of Agricultural and Resource Economics, various years) from 1994-5 through 1998-9.

We focus on these three case studies to illustrate different aspects of our methodology: the first example corresponds to a simple input-only change (with no accommodating input or output substitution); the second example involves substitution amongst outputs; and the third example—which has a dominant single output (milk)—involves input substitution. First, the models are validated in the base case, in other words, we derive the 'without' scenario for each case study, then we examine the case studies.

3.1 Model Validation

We consider the benefits of the projects in case studies 1 and 2 (grains) for the Wimmera region of Victoria; and the benefits of the project in case study 3 (dairy) from the Gipplsand region (see Figure 2).



Note: SD stands for Statistical Division.

Figure 2: Location of the Wimmera and Gippsland regions in Victoria

3.1.1 Validating the Grains Model

We establish a 'theoretical Wimmera' based on two farm types, one where farmers use lentils in their five-part rotation, and one where farmers use canola—some farmers do not grow lentils due to constraints such as soil type.

Table 1 gives a comparison of the theoretical and actual.

Table 1: The theoretical Wimmera region with two main groups of farmers

	Farmer type 1: Canola growers	Farmer type 2: Lentil growers	Actual Wimmera
Cropping rotation	Oil-Cer-Leg-Cer-Cer	Leg-Cer-Leg-Cer-Cer	n.a
Area under cereals (ha)	246,438	273,386	638,515 (81%)
Area under grain legumes (ha)	82,146	182,258	264,403 (100%)
Area under oilseeds (ha)	82,146	-	82,146 (100%)
Total GVP ($ million)	146	153	344

Note: Oil=oilseed crop; Cer=cereal crop; Leg=grain legume crop, n.a not applicable, figures in brackets indicate the modelled proportion of the actual.

3.1.3 Validating the Dairy Model

Table 2 shows some basic statistics regarding the Gipplsand region. We derive an estimate of the quantity of milk using the total revenue, implied Gipplsand dairy area, and yield of milk per hectare. Our implied Gippsland milk production is 1,589 million litres, which is close to the corresponding estimate from the Australian Dairy Corporation: 1,695 million litres.

Table 2: GVP and TVC data used in Case Study 3

GVP from milk and meat from dairying in Gippsland ($ million)	468.22
Revenue from milk in the model ($/ha)	4,585
Revenue from meat in the model ($/ha)	287
TVC in the model ($/ha)	1,911
Implied area estimated (ha)	96,112
Implied quantity Gippsland milk (million litres)	1,589
Australian Dairy Corporation Estimate Gippsland milk (million litres)	1,695

Case Study Results

Briefly our three case studies are:

Case Study 1, a new wheat variety that requires less herbicide

Case Study 2, a solution to a chickpea disease that would require reduced spraying

Case Study 3, an increase in feed conversion efficiency of dairy cows.

The results are given Table 3.

Table 3: Results from the Three Case Studies

	CASE STUDY 1		CASE STUDY 2		CASE STUDY 3
	Canola growers	Lentil Growers	Canola growers	Lentil Growers	Dairy Farmers
Input Reduction, E_i, (%)	2.04	2.00	2.26	2.05	4.53
TVC ($ million)	60.52	68.40	60.52	68.40	183.65
B_{ipi} ($ million)	**1.23**	**1.37**	**1.37**	**1.40**	**8.32**
Output Expansion, E_o, (%)	1.38	1.48	2.02	1.32	2.35
GVP ($ million)	146.05	153.04	146.05	153.04	468.22
B_{opi} ($ million)	**2.02**	**2.27**	**2.95**	**2.02**	**11.00**
GM change ($/ha of wheat)	5.00	5.00			
Implied area of cereals (ha)	246,438	273,386			
GM benefit ($ million)	**1.23**	**1.37**			

3.2.1 Discussion

Table 3 demonstrates that the gross margin calculation for Case Study 1 is the same as our input-reduction method (as stated in Section 3, and shown in the Appendix). This is due to the fact that the only change on-farm is the input reduction; this is a k_1-type supply shift.

Case Study 2 is an application of our technique where there is output substitution: the new technology allows farmers to reduce the cost of spraying for a chickpea disease, hence some farms substitute towards chickpeas in their crop choice. This Case Study would be particularly difficult to analyse using either a standard gross margin approach, or a single-commodity supply curve analysis: it would be difficult to predict how a reduction in crop sprays would alter a crop rotation, taking account of cross-crop interactions.

Case Study 3 is an application of our technique to the case where there is input-substitution: after an increase in cows' feed conversion efficiency, the dairy model substitutes out of turnips, and towards pasture. An analysis of Case Study 3 would be very difficult using a standard gross margin approach. The main reason is that the analyst would have to predict the response of farmers to the change in feed conversion efficiency. Using our model, we can do this by letting the altered biological relationships, in conjunction with a profit maximisation assumption, drive on-farm substitution. It is unclear how this could be modelled without a framework to coherently integrate such information.

Alternatively, our dairy example could be analysed using a single-commodity supply curve approach, which has been common in the economics literature. The economist would probably assume that all dairy-farm costs were allocated to a joint product—milk-meat—produced in fixed proportions. This circumvents the problem apparent in grains, where output substitution was important to the analysis. The economist could then proceed by estimating producer surplus. However, he or she would still not be able to take account of on-farm input substitution. He would have to ignore this altogether, or estimate it using a simplified version of our approach.

4 Summary and Further Research

4.1 Summary

Alston *et al.* recommended measuring the benefits of research—derived from the use of a new technology—by taking account of the adjustments that each firm makes to a new technology: changes in firm processes, output mix, and input mix.

In this paper, we have taken up that challenge for the case of broadacre agriculture, using an approach that is consistent with that proposed by Pannell (1999), and that is similar to that used previously by Griffith *et al.* (1995). We have demonstrated how farm-level models can be used to measure the benefits of a new technology. Farm-level models have an advantage over single-commodity supply analysis in that they take account of on-farm changes that accommodate a new technology. We showed that, theoretically, this approach fits comfortably into the previous literature on research evaluation.

We have couched the benefits of using farm-level models—for research evaluation—in terms of economies of scope. When there are economies of scope, a farmer will be concerned with his *multi-product* profit function, rather than a single-product profit function. The latter has been the focus of evaluation in much of the economic literature to date.

In the first case study we represented a technical change by a simple input reduction, with no accommodating change in other inputs, or in the farm's output mix. We showed that in this case, our approach is equivalent to a gross margin analysis. However, gross margin analysis is more difficult to apply when farm adjustments accommodate a technical change. We examined two such situations: one where a new crop variety is accommodated by a change in crop rotations; and one where an increase in cow energy conversion efficiency is accommodated by input substitution.

4.2 Further research

Our farm-level models could be altered to better account for on-farm changes that accommodate a new technology. There are several ways to do this, which we consider below.

First, our method enables the user to calculate the benefits of—what Martin and Alston (1997) have termed—an 'output-expanding' or 'cost reducing' technical change: B_{opi}, and B_{ipi}. However, these two benefit measures can be calculated for any given technical change. We have not examined the situations under which one is preferred to the other.

Second, our farm-level models often do not relate inputs to outputs in a functional way. Instead, many of the input levels are fixed for a given output. For example, in our grains model the optimisation procedure (maximising receipts less variable costs) chooses the best mix of outputs, but does not substitute amongst inputs. Clearly we would expect grain farmers to substitute inputs, as well as outputs in reaction to new research methods. If our models related inputs in a functional manner then they would (ostensibly) give us a more accurate depiction of farmers' reactions to new research, and hence better estimates of research benefits.

Third, and related to our first point, we could improve the ability of our models to take account of quality (particularly of the outputs). This relates to both types of our farm-level models: (i) those with multiple outputs such as grains; and (ii) those dominated by a single output such as milk. The key to estimating the benefits of quality change using a productivity index—as in our system—is to disaggregate outputs and inputs as much as possible (Star 1974; Craig and Pardey 1996). In the output case, for example, our grains model does not have a relationship between farming practices and wheat quality. To do this, our model would need several different wheat types and, given a technical change, the model would need to choose a wheat type (or several types) using assigned probabilities. One of the key restrictions to doing this at the moment is data availability.

Better data would also enable us to improve the accuracy of our initial rotations. Currently, we use yields based on ABS data, and allow our model to choose an 'optimal' base rotation. We then check whether the model's initial rotation makes sense when compared to data of the aggregate production quantity, and area, for the region. Currently, we do not have data on farmers' rotation choices, or their reasons for these choices. Such data would allow us to more accurately disaggregate farms into groups that use different rotations. Farmers might choose different rotations because of, for example, intra-regional variations in soil quality.

Finally, our approach *implies an* elasticity of supply for some group of farm outputs; when a farm-level model is affected by a technical change, it will choose a new output mix, and hence change the farm's output-quantity index. Our approach takes account of the fact that many farms produce multiple outputs, and/or make input substitutions. However, the implied quantity-index expansion is based on our farm-level models' biological parameters, and their objective function (risk neutral gross-margin maximisation). The accuracy of our estimated quantity expansion is only as good as the quality of these assumptions.

5 Acknowledgements

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6 Appendix

In this Appendix we give explanations of why we think our method is consistent with a profit function approach to benefit estimation. We use the simplest index possible; one that measures changes in a single output or input. In reality, indexing procedures are useful for multi-output or input situations. However, our approach makes the explanation relatively easy to understand, so it serves expository purposes, rather than providing formal mathematical proofs.

Part (i): Benefit Estimation for an Output-Expansion Productivity Improvement, and its Connection to the Profit Function Approach

We need to show that the value of a productivity change multiplied by the value of output is equal to area abQ_1Q_0 in Figure 1. It is obvious that:

$$abQ_1Q_0 = P(Q_1 - Q_0) = B \qquad (1)$$

Our task is to show that B_{opi}, as given in the text, is equal to B.

If outputs change and inputs do not, then an index of productivity change (for the simple single-output case) is given by E_0, which can be written as:

$$E_0 = \frac{Q_1 - Q_0}{Q_0}.$$

Therefore, our measure is:

$$B_{opi} = \left(\frac{Q_1 - Q_0}{Q_0}\right).P.Q_0$$

$$B_{opi} = (Q_1 - Q_0)P \qquad (2)$$

which is the same as the value of B given in (1).

Part (ii) Benefit Estimation for an Input-Reducing Productivity Improvement, and its Connection to the Profit Function Approach

We need to show that area *aced* in Figure 1 is equal to the value of productivity change (E_1), multiplied by total variable costs. For the simple case where there is only one variable input, then we can write the total variable cost function as:

$$TVC_j = p_v v_j(Q)$$

where $j = a \text{ or } b$, and $v_j(Q)$ represents an inverse production function.

Writing marginal cost as *MC*, then we have $MC = p_v v'(Q)$, where $v'(Q)$ is the derivative of the inverse production function with respect to output.

At Q_0, we have two supply cures S0 and S1, which we can interpret as marginal cost curves, MC_a and MC_b respectively ($MC_b < MC_a$ because MC_b has been shifted down due to a technical change). Therefore, the vertical difference between the marginal cost curves is:

$$k = MC_a - MC_b > 0$$

$$k = p_v v_a^{'}(Q) - p_v v_b^{'}(Q)$$

$$k = p_v [v_a^{'}(Q) - v_b^{'}(Q)]$$

In other words, since the price of the variable input is assumed constant, the absolute value of the k shift—at a given level of output Q_0—represents the difference in the marginal input required to produce the Q_0 th unit. Since k is constant from 0 through Q_0 (we assumed a parallel shift) then:

$$aced = kQ_0$$

$$aced = p_v [v_a^{'}(Q) - v_b^{'}(Q)] Q_0 .$$

But $[v_a^{'}(Q) - v_b^{'}(Q)] Q_0$ is equal to the total reduction in variable input units from the technical change. Therefore:

$$aced = p_v (v_0 - v_1) . \qquad (3)$$

Our measure of benefits is $E_1 . v_0 . p_v$, which can be written:

$$B_{ipi} = -\left(\frac{v_1 - v_0}{v_0}\right).v_0 . p_v$$

$$B_{ipi} = -(v_1 - v_0) p_v = (v_0 - v_1) p_v \qquad (4)$$

which is the same as the area given by (3).

Note that if we multiply E_1 by the value of production, we have:

$$B_z = \left(\frac{v_1 - v_0}{v_0}\right) PQ_0 \qquad (5)$$

We can compare B_z to the output-expansion approach by rewriting (5) as:

$$B_z = \left(\frac{v_1 Q_0}{v_0} - Q_0\right) P$$

which we can compare to (2) by asking if $\frac{v_1 Q_0}{v_0}$ is equal to Q_1. If we rearrange, we can (more intuitively) ask, is $\frac{v_1}{v_0}$ equal to $\frac{Q_1}{Q_0}$? This will not be the case if the production function exhibits diminishing marginal returns to the variable factor. Specifically, if there are diminishing returns, then the input ratio will be greater than the output ratio; the estimate using the input reduction (5) will be greater than using the output expansion in (2). Therefore we cannot multiply the input-reduction productivity change by the value of production, and expect to get the same answer as when we multiply the output-expansion productivity change by the value of production. More succinctly:

$$E_1 PQ_0 \neq E_0 PQ_0 .$$

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