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A Review of the Arts of Supply Response Analysis

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This paper classifies methods for estimating agricultural output supply responses to price into two broad categories (1) programming and (2) econometric. The latter category is further subdivided into two-stage procedures, directly estimated supply systems and directly estimated single commodity models. In Part 1 the theoretical basis of these methods is considered, while Part 2 explores their strengths, weaknesses and potentials from an empirical standpoint.

Introduction

The object of this paper is to review the state of the arts of estimating the quantitative supply response of agricultural commodities to changes in product prices, input prices, and other relevant measurable aspects of the changing environment for agricultural production. There are precedents (for example, Nerlove and Bachman 1960; Heady *et al* 1961; Cowling and Gardner 1963; F.A.O. 1971; and Colman 1978). The conclusions of these studies have stood the test of time comparatively well, but changes in the general economic climate (notably the much higher rates of inflation in the 1970's and early 1980's), further increases in computing power and methods, and developments in the theory of production and supply suggest the need for some reappraisal of the merits of different approaches to supply response analysis.

In a strict theoretic sense it is inappropriate to employ the term supply response to characterize the studies and methods under review. It would be more appropriate to use the terms output or production response. For, in elementary economic theory, supply is defined as the amount of a commodity offered for sale in a particular market during a specific time interval at the prevailing values of prices and any other relevant conditioning variables; the supply response function or surface indicates supply levels for all alternative combinations of values of the explanatory variables. Supply in this sense for unprocessed agricultural commodities could be taken to mean the amount of produce offered into the domestic wholesale market by producers, stockholders and importers. This is markedly different from the concept of supply employed in the studies reviewed here. They typically focus upon the level of actual or potential (where studying livestock inventories, or cumulated plantings of trees in the case of perennial crops) *output* on farms. Since producers themselves may be major holders of stocks, particularly in the case of cereals, it is not possible in general to equate production with actual supply onto the wholesale market. While these observations may, at first glance, appear to be purely

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pedantic or semantic, the fact is that the capacity for analysts to employ the neoclassical theory of the firm to assist in quantitative estimation of output responses rests entirely upon the fact that it is agricultural production which is made the focus of study and not supply in the broader sense. Models which explain quantities released into the market include, in addition to output response, functions to explain net changes in stocks and net trade flows.

For the purposes of the paper methods of analysing supply response are classified into four categories. A broad distinction is drawn between programming and econometric approaches with the latter subdivided into three. These three subclasses are (1) what are here called two-stage procedures, (2) directly estimated systems, and (3) directly estimated single commodity models. This classification, while having inevitable similarities to those adopted by others (*e.g.*, Schmidt 1982) is specifically chosen to facilitate examination of the role of economic theory in the different approaches, and to help assess their empirical problems and properties. Part 1 of the paper is devoted to an examination of the theoretical underpinning of the different categories of models, and Part 2 to an assessment and critique of their virtues and limitations.

1. Theory of the Firm as the Basis for Output Response Analysis

In the context of empirical estimation, theory about the behaviour of economic decision makers is used, via restrictions imposed upon (1) the form of systems of equations, (2) the form of equations themselves, and (3) upon parameter values, to force the results to conform to the theory and thus to "squeeze" appropriate results from the data. Application of theory in this way has had its most conspicuous success in the complete systems approach to estimating final consumer demand functions. This is due to the fact that, despite the problems associated with demand for consumer durables and habit formation, it has proved generally acceptable to treat aggregate level demand for different categories of commodity as a problem of achieving a short-run equilibrium allocation of a fixed level of disposable income. Despite the continued efforts of analysts to develop comparable complete systems approaches for supply response a lesser degree of success has been achieved in supply analysis. This largely results from the much more complex dynamics of supply, which necessitates treating the short-run allocation of resources and output. within the context of a long-run framework, and from the absence of any single binding constraint upon production which is analogous to disposable income. Consequently approaches to estimating output responses vary considerably in the extent to which they depend upon the theory of the firm to produce results. There are also divergences of opinion among analysts as to the benefits of relying upon the theory to overcome estimational difficulties.

1.1 Programming Models

Programming models, typically of the linear variety, have exercised an appreciable intellectual sway over agricultural supply analysts. In part this is attributable to the fidelity with which the empirical process follows the steps prescribed by theory for deriving the output and input levels which maximize the profit of a firm with a given production technology.[1] The basic procedure adopted

[1] The other merits will be discussed in section 2.1 below.

is equivalent to pursuing Route 1 in Figure 1 for the case where the exact form of the production technology is assumed. It involves constructing a complete linear model to describe the production system of each of a number of reference (or typical) farm types. This amounts to specifying a set of linear, additive production functions for each possible output the farms might produce, and also specifying bounds on resource availabilities. An objective function is specified which is usually a straightforward profit function, but which may also make allowances for other objectives such as aversion to risk (as for example in Pomareda and Samayo 1979). Well-tried linear programming procedures are then applied to solve for the profit maximizing output and input levels for each reference farm which are consistent with exogenously specified production technology, and product and input prices.

By solving the problem repeatedly for different sets of prices (*i.e.*, by variable-price programming) supply-price relationships can be established for each commodity and reference farm. If it is assumed that farmers operate to maximize their profits, and if sufficient information is available about the number of farms in the population corresponding to each reference group, then it is possible to scale up and aggregate the supply-price functions for the individual farms to obtain what in terms of microeconomic theory can be defined as market level supply response relationships. Because this market level relationship is built up in this way from farm level relationships obtained by iterative solution, this approach to supply response is sometimes described as a synthetic approach.

It should however be noted that these supply response relationships are typically of a partial character and trace out the schedule of output values for particular products associated with a schedule of output or input price values. No attempt is made to summarise in a formal functional statement the multidimensional response surface between outputs and prices which is implied by the simulated numerical results. In fact, because of the stepped and irregular nature of response schedules derived from linear programming models, such a characterization is not practicable. What is occasionally performed, (*e.g.*, Shumway and Chang 1977), is to econometrically fit functions to the data scheduling the partial derived relationships between, say, an output and its own price, which provides a limited cross-sectional view of the output to price response surface.

This approach to supply response estimation, the advantages and disadvantages of which are discussed in Section 2.1, has been pursued in many studies including Cowling and Baker (1963), Sheehy and McAlexander (1965), Frick and Andrews (1965), Tweeten *et al* (1968), Eyrendson (1972), Miller and Heady (1973), Sahi and Craddock (1975), Wicks *et al* (1978) and Thompson and Buckwell (1979).

1.2 Econometric

1.2.1 Two-Stage Procedures

In this class of procedures output response relationships are not obtained by direct econometric estimation. They are derived (in a second-stage) by algebraic manipulation, imposing profit maximizing marginal conditions on results obtained by econometric estimation in the first-stage. Because, according to the principles of duality (see Fuss and McFadden 1978; or Blackorby *et al* 1978), there is a direct equivalence between the production and cost, and production and profit functions, any one of these three functions could be econometrically estimated in the first-stage and used to derive supply response parameters.[2]

An outline of the duality relationships within the neoclassical theory of the firm is shown in Figure 1. The duality relationship between the production and cost functions arises because it has been established, that, for every regular production function, profit maximization dictates that the minimum cost function has corresponding regular properties which reflect the same underlying technical conditions of production; and vice-versa. Thus for any regular production function $q = f(\underline{x}, \underline{z})$, there exists, given output (q), input prices $(\underline{r})$ and the level of fixed inputs $(\underline{z})$, a dual minimum cost function $c = c(q, \underline{r}, \underline{z})$. This duality also applies in the multiple-output case where the technological relationship between outputs and inputs is represented by a transformation function.

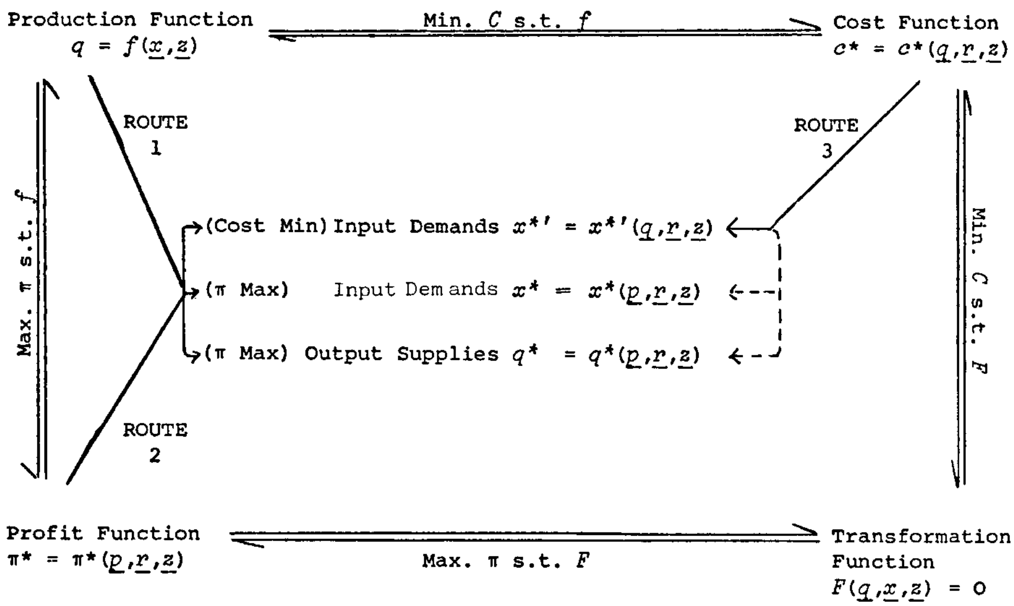
A similar reciprocal correspondence exists between the production (and transformation) function and profit function, such that with profit maximization there exists a dual relationship between a regular production function $q = f(\underline{x},\underline{z})$ and a profit function $\pi = \pi(p,\underline{r},\underline{z})$, where p is the price of the product. The significance of this duality is as Lau and Yotopoulos have stated (1972, p. 11), "McFadden has shown there exists a one-to-one correspondence between the set of concave production functions and the set of convex profit functions. Every concave production function has a dual which is a convex profit function, and vice-versa. Hence, without loss of generality one can consider only profit functions in the empirical analysis of profit maximizing, price-taking firms". It is this relationship, that dual functions all contain the same basic information, which creates the possibility that the output supply and input demand functions can be derived from any of the dual functions.

A number of studies of agriculture have employed what in Figure 1 is labelled Route 1, in which single-commodity or aggregate production functions are estimated from cross-section farm data or (less commonly) from time series, and used to generate output supply response functions. These include the milk supply response studies of Schuh (1957), Kadlec (1960) and Gossling (1964), and the aggregate supply response studies of Griliches (1963) and Rosine and Helmberger (1974).

In these studies the production function was of the Cobb-Douglas form, and the procedure for obtaining the supply function followed those outlined by Heady and Tweeten (1963), and Dayal (1969). These involved imposing upon the production function the first-order profit maximizing conditions that the marginal rates of substitution between inputs were equated to their inverse price ratios, and that marginal production cost was equated to the given

[2] While there is an exact dual relationship for the Cobb-Douglas and Constant Elasticity of Substitution (C.E.S.) forms of these functions, Burgess (1975) has shown that this is not true for the more general and flexible transcendental logarithmic form. This is discussed below in section 2.2.1.

Figure 1: *Relationships between Supply Functions and Other Functions in the Theory of the Competitive Firm*



Key:

⇆ = a dual relationship.

→ = denotes an algebraic procedure described in the text.

$\underline{q}$ = vector of outputs.

$\underline{p}$ = vector of output prices.†

$\underline{x}$ = vector of inputs.

$\underline{r}$ = vector of input prices.

$\underline{z}$ = vector of fixed input quantities.

π = profit.

C = cost.

s.t. = subject to

* = profit maximising or cost minimising level.

† In the case of a production function, and of its associated dual and derived functions, the vectors $\underline{q}$ and $\underline{p}$ contain only one element.

† In the case of a production function, and of its associated dual and derived functions, the vectors q and p contain only one element.

marginal revenue (p). The resulting equations are solved for the profit maximizing input levels, given values of $\underline{p}$, $\underline{r}$ and $\underline{z}$. These input demand functions can then be substituted back into the production function to obtain an estimate of the supply function expressed in terms of the same exogenous variables.

The development of more flexible functional forms, such as the constant-elasticity-of-substitution (C.E.S.) and transcendental logarithmic, which are progressively more general forms subsuming the Cobb-Douglas function, has broadened the scope for pursuing Route 1. However, recent use of this approach to obtain agricultural supply response estimates has been minimal, although there have been studies of aggregate national supply based on flexible production functions such as those by Berndt and Wood (1975) and Burgess (1975). Instead, agricultural economists have recently exhibited a preference for pursuing the dual Routes 2 and 3, because it has proved more convenient and useful to estimate the duals of the production function.

Derivation of agricultural supply response estimates via Route 2, from a profit function, has been extensively performed by Lau and Yotopoulos and their collaborators (1972, 1976[3], 1979[3]). In this series of studies the procedure is based upon the estimation of a Cobb-Douglas, normalized profit function for a single output production process from cross-section farm survey data. The normalization procedure involves deflating all price variables in the profit function by the output price, which saves one degree of freedom as the product price term on the right hand side collapses into the intercept term. According to the Hotelling-Shepherd Lemma, the negative of the derivative of the profit function with respect to the normalized price of an input, r_j is the (profit-maximizing) demand for that input, *i.e.*, $\frac{\delta \pi'}{\delta r_j} = D(\underline{r},\underline{z})$. These equations, in conjunction with the profit function itself, enable expressions to be obtained for the share of input j in normalized profit—their general form is $R_j = r_j'.D(\underline{r}', \underline{z})/\pi(\underline{r}',\underline{z})$. These input share equations, expressed in terms of the parameters of the assumed profit function, represent an alternative statement of the dual profit maximization problem and provide an additional source of information about the parameters to be estimated. As a consequence the estimation procedures generally adopted involve joint estimation of the profit function and the input share equations with cross-equation restrictions on the parameters. This, plus additional parameter restrictions to reflect linear homogeneity of the profit function in prices and fixed inputs, provides a powerful role for neoclassical firm theory in determining the estimated parameters of the system.

Once the system of equations has been estimated in this way, the input demand functions can be obtained using the Hotelling-Shephard Lemma. These can then be substituted into the normalized profit function which can then be solved for a supply function expressed in terms of product price (p), input

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[3] Referenced under Yotopoulos, Lau and Lin 1976, and Yotopoulos and Lau 1979.

prices ($\underline{r}$) and quantities of fixed inputs.[4] In addition to the single-commodity applications by Lau and Yotopoulos and their colleagues (1972, 1976, 1976), the basic procedure has been applied by Sidhu and Baanante (1981) who estimated the system of translog profit function plus input share of profit equations to examine wheat supply response in the Punjab. In a further extension to a multiple output case for sheep and wool, crops and beef cattle in Australia, McKay *et al* (1982) have also estimated a system of share equations plus translog profit function. In this case the implicit underlying technical relationship between inputs and outputs is a transformation function, rather than a production function, and the estimated profit function is its dual.

Arising from the duality principle, there also exists the possibility of pursuing Route 3 in Figure 1 to obtain an estimate of supply response behaviour. However, as Figure 1 indicates, the dual cost function includes the quantity produced (q) as an argument rather than the output price ($\underline{p}$). Consequently the derivatives of the estimated cost function with respect to input prices ($\underline{r}$) directly yield the so-called compensated demand functions $\underline{x} = x'(\underline{q}, \underline{r}, \underline{z})$. These do not contain output price on the right-hand side, and the term compensated refers to the fact that they are cost-minimizing demands for an exogenously given supply. These functions provide a good deal of information about the technical relationships of substitution between inputs, and about the input-price responsiveness of demand for them. Indeed, those studies which have adopted this approach such as Binswanger (1974), Lopez (1980) and Ray (1982) have been primarily concerned with estimating the Allen elasticities of substitution between factors. They have not attempted to derive estimates of supply response to output price; although this is theoretically possible through the expedient of assuming profit maximizing behaviour, which will be characterized by setting marginal cost of production equal to produce price. This then gives the desired relationship between output supply and product price *i.e.*, the supply function $q = q(\underline{p},\underline{r},\underline{z})$. Similarly we can obtain profit maximizing input demands if we wish to do so.

In the studies by Binswanger (1974) and Lopez (1980) CES cost functions are estimated for an implicit aggregate single-commodity supply function. Ray (1982) in his study estimated the more flexible translog form of cost function for a case in which two outputs, crops and livestock, were separately identified. He found that in order to achieve satisfactory results from his aggregate time-series data it was necessary to jointly estimate the full dual system including cost, input share of cost, and output share of revenue equations. This is exactly analogous to the use of share equations in the full profit function system of dual equations.

Those extensions, whereby mutliple output supply problems can be analysed employing dual profit or cost function systems, represent a significant increase in flexibility over the use of the production function with its restriction to one product. It is true that, in the context of the general symmetry displayed

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[4] Where a non-normalized variable profit function is estimated, the product price enters as an explicit argument, and the supply function can be obtained directly by differentiating and setting $\frac{\delta\pi}{\delta\rho} = \underline{q}$.

by the theoretical relationships in Figure 1, the possibility exists for deriving supply functions from estimates of transformation functions which are multi-product, multi-input expressions. Studies have been undertaken to estimate agricultural transformation functions, notably those by Mundlak (1963, 1964) and El Issawy (1970) for two output, two input cases using cross-section data. However the estimational problems with even such limited dimensions are such that development of this approach has been inhibited and preference given to estimating the duals of the transformation function.

The theoretical relationship between the optimal output and input levels also permits a further two-stage variant in which a supply function is algebraically derived from a directly estimated set of input demand functions. Rayner (1970) has shown that the own-price supply response elasticity of output equals the weighted sum of the input demand elasticities with respect to product prices, where the weights are the shares of estimated equilibrium total cost for each input. Empirical studies which have adopted this procedure have, in the first stage, all econometrically estimated the necessary input demand elasticities from time-series data. They have included Tweeten and Quance (1969), Rayner (1970) and Colman and Rayner (1971).

The class of two-stage procedures relies upon a maintained hypothesis of profit maximization to derive market level supply response and input demand functions from the production, cost, profit or input demand functions estimated in the first-stage. Either time-series or cross-sectional data can be employed, and the duality property enables the choice of which system of dual equations to estimate to be adjusted to the data available. Also, as the recent studies with multiple input and outputs in particular indicate (McKay *et al* 1982; Ray 1982), the dual systems of equations are highly flexible, and can assist the data to yield a good deal of useful information.

1.2.2 Directly Estimated Supply Response Systems

At the same time that progress has been made in developing the dual profit and cost system approaches to indirectly estimating supply response functions, work has also been conducted using the neoclassical theory of the firm to generate restricted systems of directly estimable supply functions. A seminal work in this development was that of Powell and Gruen (1968). The basis of their approach is acceptance of an agricultural production possibility frontier, or isofactor curve, which is determined by the assumed fixity of inputs within the annual time periods of the time-series analysis. And as they note (p. 186), "The emergence of the symmetry postulate in demand analysis was inevitable once indifference curves had been discovered. Equally, and for mathematically the same reasons, symmetry of supply responses follows inexorably once the production frontier has been accepted as a legitimate tool of analysis." This parallel between demand theory and Powell and Gruen's supply theory is a clear one. In demand theory a fixed amount of consumer's income is allocated to the purchase of "goods" in such a way as to maximize utility. In the supply theory a fixed bundle of factors are allocated to the production of products in such a way as to maximize profits.

In this approach the production possibility curve for the N products produced in a region was assumed to display a constant elasticity of transformation (CET) into products, where the pairwise elasticity of transformation is defined (analogously to the elasticity of substitution of factors) as the ratio of the proportional change in the product ratio to the proportional change in the marginal rate of product transformation. It was further assumed that there existed a system of equations in which the supply of each product is expressed as a linear function of the expected prices of all products.[5] It is clear however that since the linear supply system "is not globally compatible with the CET transformation schedule, the pragmatic virtues of both systems might be captured by establishing a local correspondence between them. This is tantamount to choosing the most convenient linearization of the CET model, and should not be construed as anything more" (Powell and Gruen 1968, p. 319). This fusion of the two systems is achieved by imposing the CET conditions upon the cross-price parameters (for the effects of changes in expected prices upon supplies) at arbitrary (mean) values of expected prices and of outputs. It also involves imposing the profit maximizing condition of equating the marginal value product of the fixed (and therefore conglomerate) factors in alternative uses. Manipulation of the resulting equations produces directly estimable restricted forms in which supply of each product is a function of expected prices and mean levels of both outputs and expected prices. These CET transforms of the linear supply equations enable the complete set of N^2 supply-price response coefficients to be obtained from only $\frac{1}{2}N(N-1)$ estimated parameters.

The CET assumption is more restrictive than at first it appears to be, in the sense that it actually involves constraining all pairwise partial transformation elasticities to be equal. To circumvent this crude symmetry, Dixon *et al* (1976), and Vincent *et al* (1977, 1980) have devised a somewhat more general estimable supply system of a related type. This involved employing a suggestion put forward by Hannoch (1971) for a more general form of the CES production function, namely the "constant ratio of elasticities of substitution homothetic" (CRESH) production function. Extending the principle of this generalization to the CET possibility frontier Vincent *et al* (1980) have employed the "constant ratio of elasticities of transformation homothetic" (CRETH) production frontier.

Another limitation of the original Powell and Gruen CET system is that all inputs were taken to be fixed (Gardner 1979) and also not product-specific, whereas in reality input and output levels are typically jointly determined and at least some factors are product specific. To allow for this, Vincent, Dixon and Powell have suggested a larger CRESH/CRETH system in which the production system is described in terms of two joint processes. In the first, the CRESH production function determines the aggregate production frontier which can be achieved employing a set of non-product-specific variable inputs (selected at the beginning of the production period) and the fixed inputs; in the second, the CRETH transformation function describes the allocation of the aggregate output between the multiple products of the agricultural sector,

[5] In this, and the CRESH/CRETH model referred to below, expectations are assumed to be determined adaptively, and the adaptive expectation coefficients are decided by trial and error rather than direct estimation.

where products are valued net of specific inputs. Again, imposing profit maximizing assumptions on the joint CRESH/CRETH system and solving the appropriate Lagrangean profit function (Dixon, Vincent and Powell 1976, pp. 13-9); deriving the first-order maximum conditions; imposing linearity[6] in the endogenous variables; and algebraically manipulating the system to eliminate the unobservable Lagrangean multipliers; produces an estimable set of supply and input demand functions. Because of the restrictions imposed the large number of coefficients in the final reduced-form supply and input demand functions are complex non-linear functions of only a small number, $N + M$, of parameters, where N is the number of products and M the number of variable inputs. In the theoretical case cited by Dixon *et al* (1976, p. 38) for a problem where $N = 7$, $M = 5$ and there are two fixed inputs, 99 reduced-form coefficients are the product of only 11 parameters. This is a clear reflection of the use of theory to squeeze from the data a large amount of information which would otherwise almost certainly be unobtainable. In this case it enables the generation by one-stage estimation of full matrices of own—and cross-price responses to expected product and input prices for both supply and input demand. Admittedly it requires a special non-linear, iterative full-information maximum likelihood estimation package, and the imposition of the theory necessitates several compromizes and simplifications, but it is nevertheless a challenging and stimulating development in the arts of supply response analysis.

1.2.3 Directly Estimated; Partial, Commodity Supply Models

The majority of agricultural supply response studies fall into this class. They involve direct estimation of supply functions from time-series (or possibly pooled cross-section and time-series) data. Most of them are of a single commodity type, but the class also includes models in which several supply functions are independently estimated, and those in which a system of supply equations for some subset of all commodities is simultaneously estimated. One of the essential characteristics of this class of models, in the circumstance where agriculture is typified by multiple outputs is that they are of a partial nature. This severely limits the role which the theory of the firm can play in the specification and estimation of the models, and there is no scope for employing the profit maximizing conditions as restrictions upon the supply equations.

That is not to say that there is no theoretical underpinning for such studies. There is, but it is of an *ad hoc* nature and derives largely from the fact that these studies are based on time-series data in which supply response is measured at an aggregate, (*i.e.*, market), level. To cope with the fact that agricultural supply responses to any particular stimulus may be spread over a number of successive data periods various theories have been developed to explain the dynamics of supply.

Since production in agriculture is not instantaneous, and is in any case dependent upon past investment decisions, the production observed in any period tends to be affected greatly by decisions taken in the past. These may

[6] This again has to be done in an approximate, local way by fixing the product and factor share expressions in the right hand side of the estimated equations at their mean values.

be a function both of economic condition prevailing at the time key decisions were taken (*e.g.*, to sow a crop, or to buy in more cattle) and of expectations about future conditions. It is therefore hardly surprising that one of the largest bodies of theory relating to supply response is that relating to the formation of expectations and to the derivation of appropriate functional forms, variables, and estimating procedures to incorporate the various postulated expectations generators. Developments in this area have proceeded from the theories of Koyck (1954) and Nerlove (1956) through flexible distributed lags of the Almon (1965) type to rational expectations (Muth 1961).

Incorporation of expectational variables for prices, revenue or profits into supply functions represents a short-cut (or *ad hoc*, or reduced-form) method of allowing for the role of investment in supply response. This is also the case in those studies which invoke either (1) the partial adjustment "theory" of Nerlove, (2) some variable to allow for the influence of changes in economic uncertainty, such as in Just (1974) and Traill (1978), or (3) asymmetric supply responses. In other studies, however, the role of investment is brought to the centre of the stage and plays a direct role in explaining production. This is particularly true in models of livestock product and perennial crop output response in which the principal response functions explain the stocks of (investment in) various categories of livestock or of trees and bushes. Supply is explained as a function of these inventories through what are essentially yield functions.[7]

2. Critique of the Alternative Supply-Price Response Estimation Procedures

Self-evidently the intended use of empirically estimated/generated supply functions is to produce conditional projections or forecasts, where a forecast is defined as the most probable conditional projection, and where expected prices are among the most important conditioning arguments. However this is not to say that the required informational output of such functions is always of the same specific type; indeed there may be a number of different emphases.

If the condition that a forecast should be verifiable is to be met it follows that where forecasts are required the output statements must be dated. This imposes the need for a specific dynamic structure in the supply response model such that values of the output variables can be projected for nominated future periods. This requirement also applies in all projection exercises in which the time-path of the system's development is of interest.

It is conceivable that in other circumstances the objective might be to assess the consequences of a hypothetical policy change upon a number of

[7] The same model structure is found in annual crop output response models in which there are separate functions for the harvested area and for the yield.

different variables and/or groups of economic factors. In such cases time-dating of the projections may be of secondary importance, and the total policy impacts as revealed through undated long-run total effects may be considered quite suitable. It is also likely to be especially important in these circumstances that the different variables projected should be consistent with one another, and this may necessitate the use of a jointly determined consistent system of equations.

While it may be obvious that the objective of empirical supply response models is to assist in making projections and forecasts, there are variations in the way they are employed. In some cases the estimated form of the model is transformed directly into a projection tool, possibly through the addition of some identity and definitional equations. In other cases however summary measures of the estimated response parameters are extracted from the model, *e.g.*, in the form of elasticities, and these may be used in some other *ad hoc* structure or manner to produce projections. In this case a good deal of the information contained in the estimated model about the dynamics of supply response may be discarded.

2.1 Programming—Representative Farm Approach

The term representative farm approach (RFA) has been coined from a paper by Sharples (1969). In this he simplifies the sketch of the procedure presented in section 1.1 above into five elements: (1) Stratify all farms within a region into homogeneous groups, (2) define a representative farm for each stratum, (3) derive supply functions for each farm, (4) aggregate the supply functions, and (5) remove the model's simplifying assumptions and adjust the results accordingly in order to make predictions or prescriptions. The major problems of adopting the RFA approach coincide fairly well with these five separate stages, but before spelling these out it is worth indicating why this technique is so attractive to agricultural economists.

One attraction is the close correspondence between the steps of the procedure and the theoretical steps taken in the neoclassical theory of supply; both entail starting with a known technology at the firm level and aggregating up to the market level. A more fundamental attraction stems from the capacity to handle the complex of inter-relationships arising from the multi-product nature of the farm-firm. The programming procedure solves for optimum level of outputs and inputs in a way which takes full account of the competition between products for limited resources. Because it is also possible to impose institutional, and in principle farmer's preference, constraints upon the model, it is argued that there is no theoretical reason why highly realistic farm models cannot be constructed. Within the constraint set it is possible to include constraints for available area and to allow for the need for crop rotations. This permits the RFA approach to take account of important technical factors affecting supply response to a degree not possible in alternative supply response analyses.

Thus the procedure is capable, in principle, of taking account at the farm level of the effects upon supply of all product prices, all input prices, and all relevant institutional, technological and physical restrictions. This is something not adequately accomplished by any alternative procedure. For example, one of the disadvantages of the typical single commodity time-series approach to supply is that in practice input prices often have to be omitted from the analysis, as also do the prices of many competing products.

The developments of recursive (Heady 1961, Ch. 5) and dynamic linear programming (Johnston 1965; Throsby 1964), have made it possible for the full dynamic aspects of supply to be allowed for in the RFA approach (although the majority of applications continue to be of the static linear variety). Psychological and real constraints upon the ability and willingness of farmers to adjust rapidly to their perceived supply optimum can thus be reflected in recursive programming by the use of flexibility constraints. These constraints operate by restricting the level of given activities in any year (or analysis period) to a function of their level in the previous period. In this way a specific procedure for dating the solution variables can be introduced, and can provide estimates of the specific (dated) path of adjustment to the long-run equilibrium position. This opens up the possibility of using the programming approach for forecasting since it permits verifications. Further strengthening of this link arises from the multi-period optimization solutions permitted by recursive and dynamic linear programming. These make it possible for investment activities to be incorporated into the activity set. Consequently it is possible for short-run supply decisions to be related to a long-run strategy of investment decisions. In this setting it is logical to argue for setting the flexibility constraints so that once investment has been undertaken there are different supply responses to upward and downward price movements. In this way it will be possible to generate a supply function for upward price movements and a separate one for downward price changes. However it should be said that there are appreciable difficulties in estimating the appropriate flexibility constraints to use (see for example, Miller 1972), and that a strong element of personal judgement is required in obtaining suitable values.

Perhaps the most valuable strengths of the RFA approach arise from its properties (a) that optimal output is related to all product and input prices, and (b) that the set of possible production alternatives allowed by the model may be larger than that chosen at any particular constellation of prices. Given N products and M inputs, property (a) entails that the method makes allowance for $N \times N$ effects of product prices on outputs, $M \times M$ effects of input prices on input use, and $N \times M$ effects of product prices on input use, all in a fully consistent manner. The word "effects" has been chosen deliberately here, rather than say coefficients or elasticities, since the individual quantity-price relationships are not likely to be smooth or continuous and hence cannot be summarized by any single parameter. At the very least the partial (equilibrium long-run) supply responses will exhibit abrupt changes of slope and a stepped characteristic, as portrayed, for example, in the study by Zepp and McAlexander (1969). More fundamental irregularities in the multi-dimensional supply response surface may result from property (b) as a reflection of threshold price combinations at which new products enter the optimal set and others drop out.

Not only are programming models comprehensive in the above sense, but they have also been extended in a number of other directions which have been fully reviewed in the recent paper by Norton and Schiefer (1980). Among the classes of extension they list is that of the multi-regional cost-minimizing production model. Of more importance are the derivatives of the spatial equilibrium models developed by Takayama and Judge (1964, 1971), including the large-scale price-endogenous models which determine the levels of supply, demand and trade which maximize the combined economic "surplus" of producers and consumers. One of the largest and best-known of these programming models is the CHAC model for Mexico (Duloy and Norton 1973). This and other programming models of national agricultural and food sectors emphasize the considerable (economic) descriptive detail which this class of model can handle.

The comprehensiveness of the method is a considerable potential virtue in policy impact analysis in which quite radical changes in conditions are postulated, since any results will remain fully consistent with the resource and technical constraints imposed.[8] Short-cut models, of the single-commodity one-stage econometric variety, are often severely handicapped by their partial nature (*i.e.*, this failure to allow for all input and output price effects) for projection exercises which involve postulating values of explanatory variables well outside the variable range in the estimation period. Similarly, the econometric basis of the two-stage procedures limits their validity for extreme conditions, and they as yet have not been developed to deal with the degree of product and input disaggregation manageable by programming models. While, as comments below signify, programming models also have some difficulties in coping with macro-economic and other feedback, their consistency properties reduce some of the dangers of applying them to rather extreme values of the explanatory variables.

Despite all the positive features listed, the RFA approach does encounter a number of problems. One major area of difficulty revolves around obtaining a suitable classification of farms to permit the reference strata to be defined in such a way that aggregation bias is minimized in step 4 of the RFA procedure. As Buckwell and Hazell (1972, p. 133) note in conclusion to their paper on the aggregation problem "it has been shown that unbiased aggregate supply estimates are only attainable if very stringent homogeneity criteria are applied in the classification of farms. In addition to the well-known results for static models, it is also necessary that farms within a group grow in proportional ways and have identical rates of technological innovation", They are forced to conclude that in practice it is not possible to devise stratification procedures which totally eliminate aggregation bias, but they are able to offer constructive suggestions for reducing it which do not simply require increasing the number of reference farms.

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[8] It should of course be noted that the price response relationships derived from both the RFA and two-stage approaches are complex functions of the underlying technical parameters of the physical production system.

There are of course difficulties in defining the activities and constraints to be specified for each representative farm once the stratification procedure has been decided upon; these are problems in addition to the inevitably costly one of collecting basic data at the farm level. For example, while it may be possible to take account of personal preferences, *e.g.*, refusal to milk cows, when programming an individual farm, it is difficult to decide how to reflect these in programming a farm to represent a whole group of individual farms. Further, there are difficulties in defining the activity set; what alternative activities not employed by the representative farm in recent economic circumstances should be included as possibilities to be considered under future conditions?

There are quite a lot of significant problems to be dealt with under Sharples' (1969) fifth stage "to remove the model's simplifying assumptions and adjust the results accordingly." One simplifying assumption which cannot be wholly adjusted for is that farmers maximize profits; this is despite the possibilities for adjusting the objective function to allow for risk aversion. The criticisms noted earlier by Wipf and Bawden (1969), that this assumption leads to overestimation of supply responsiveness, are certainly of relevance to the RFA, and were supported by the results of an interesting study by Zepp and McAlexander (1969). In this study the authors compared the supply prediction performance of alternative models and found that both recursive and linear programming models generated larger prediction errors than a time-series regression model. However, a later study by Shumway and Chang (1977) challenges this conclusion. They devised an indirect way for generating crop supply forecasts from a standard linear-programming (LP) model, to compare with forecasts from single commodity econometrically estimated functions. Their results, assessed for the supply of some fifteen crops in California, were that the prediction errors for the LP based forecasts were higher in 1974 but considerably lower in 1975, from which they conclude (p. 335) "little evidence is found to support the Wipf and Bawden, and Tweeten and Quance observations that conditionally predictive estimates tend to be less reliable predictors than the positive estimates of supply". However there are questions about the Shumway and Chang methodology; the "positive" supply functions are of a rather naive variety, and the procedure for generating forecasts from the LP model involves applying econometric estimation to data generated by parametric price programming. This latter procedure was necessary to convert undated LP output statements into a dateable form, but the theoretical justification for the procedure adopted is by no means self-evident and it serves to emphasize the problem of associating a specific time period with any response coefficient derived from programming models.

Problems relating to the simplifying assumptions of the RFA method which are amenable to modification, at some cost, are those which arise because it is not possible to assume either that certain things which are fixed at the farm level are fixed at the aggregate level, or conversely that things variable at the farm level are variable at the aggregate level. For example it is perfectly reasonable to assume prices of inputs and outputs to be unaffected by the decisions of each individual reference farm, but at the market level increased supply will tend to decrease product price and force up the price of inputs. Similarly while it is reasonable to allow land purchase and sale activities at the firm level it is not reasonable to allow aggregate solutions which reflect land buying by all classes of farms. It is therefore necessary to devise an interactive system in which the optimal aggregate output and input demand solutions for an initial set of prices are fed into a system of price-quantity equations to determine new factor and product prices. These are then fed back into the representative farm models and the process is repeated until convergence is achieved between the initial and terminal vectors of prices.

In addition to the problems already mentioned with the RFA approach there is the need to allow for changing farm size and structure, to allow for technical change, and to accommodate non-linear functions. The demands on data and research manpower required to solve all the problems attendant in developing such a complete and complex supply model as the RFA model lead this writer to the conclusion that a short-cut solution is desirable for most problems.

2.2 Econometrics

2.2.1 Two-Stage Procedures

As a method for estimating agricultural supply responses Route 1, starting with the production function, has considerable limitations. In the first place, as Kehrberg has pointed out (in Heady *et al.* 1961, Ch. 7) only in the trivial cases in which products are either completely independent in terms of factor competition or are joint products produced in fixed proportions by the multiproduct firm, might single-commodity production functions be accepted as a sound theoretical basis for estimating supply functions. This criticism does not of course apply to the decidedly non-trivial case of aggregate supply, but for individual products it is generally valid since in most agricultural systems all products compete for capital and labour and all crops compete for the available land.

Another difficulty specific to the production function is that of simultaneous bias. It is accepted that in reality the levels of inputs and outputs are jointly and simultaneously determined in the light of exogenous economic circumstances. Hence to treat the level of inputs as exogenous determinants of output is not wholly appropriate. Estimating profit or cost functions overcomes this problem and is one reason explicitly given by Lau and Yotopoulos (1972) for preferring to estimate profit functions. While overcoming the particular problem arising from simultaneous determination of inputs and outputs, the problem of simultaneity between prices and profit remains in those cases where the object of study is the whole agricultural sector (as opposed to the farm, as in cross-sectional studies). Among the additional factors cited by Lau and Yotopoulos in support of their preference is that it permits allowance to be made, when cross-sectional data is used, for the fact that different farms receive and pay different prices.

In many of the recent studies estimating profit or cost functions, *e.g.*, McKay *et al* (1982), Binswanger (1974), Lopez (1980) and Ray (1982) a major interest has been in input demand and substitution. The tractability of these approaches in providing this information, plus their additional merits indicated above, doubtless helps explain why studies employing these approaches to agricultural input demand and supply response now outnumber those based on production functions. An additional factor in their favour, following the work of McKay *et al* (1982) and Ray (1982) is the ability of the full dual profit and cost function system to handle multiple-output, multiple-input systems. This capacity, to consistently estimate output supply and input demands for all variable quantities in some agricultural system, is an appreciable advantage.

That said, there appear to be limitations on the degree of product and input disaggregation which the dual profit and cost-function systems can conveniently handle. McKay *et al* (1982) employed three output and three variable input categories, while Ray (1982) employed only two outputs and five variable inputs. For the latter case Ray indicates that to obtain acceptable results the whole set of restrictions from the theory of the firm needed to be imposed. That is to say the full system, comprising the cost equation, input shares of cost equations, and output shares of revenue equations, with its cross-equation restrictions on parameters had to be estimated. This emphasises how important the maintained hypotheses of profit maximization and cost minimization are in these two-stage approaches. In most cases the data require a strong squeeze from the theoretical restrictions to generate results which are deemed acceptable. In these circumstances it is clearly important to test the acceptability of the maintained hypotheses. Care is taken to do this in the series of studies based on the profit function presented by Yotopoulos and Lau (1979). In each case the null hypothesis, that the unrestricted estimates of the parameters of the profit function are consistent with profit maximization, is tested, and in almost every case the hypothesis is not rejected using an F-test. Unfortunately failure to reject a hypothesis represents a weak test, and the probability of accepting a false hypothesis is high with conventional confidence intervals. Thus no clear impression emerges as to quite how powerfully the theory is squeezing the data.

An important issue in this class of supply response studies revolves around the choice of functional form to estimate in the first-stage of the procedure. Wipf and Bawden (1969) working with production functions have demonstrated the great sensitivity of the supply response elasticities computed in the second stage of the procedure to the form of function assumed in the first. For example, they obtained own-price elasticities of supply for turkeys in the U.S.A. of 3.21 from a logarithmic production function, −1.905 from a quadratic form, and 27.003 from a square-root function. Similar extreme variations were obtained for broilers and hogs. Their conclusions are firm indicators as to why this particular indirect approach to supply response has not found much acceptance. They state (p. 177):

> "A. Output predictions obtained from the derived supply approach do not exhibit a consistent magnitude or direction of bias. They range from slight under-estimates to extreme over-estimates of actual output, with the latter being most prevalent.
>
> B. Both casual observation and directly estimated supply equations show firms to be less responsive to price changes than indicated by supply elasticities derived from production functions.
>
> C. Elasticities and output predictions based on derived supply functions appear over-sensitive to changes in the length of run."

Wipf and Bawden's conclusions should be no surprise, especially as regards the tendency for over-estimation of supply responses. This would seem to be inevitable in models which assume that the agricultural sector adjusts timelessly to an economic optimum in an environment unconstrained by uncertainty, preferences of farmers, and so on. This capacity for over-estimation of behavioural responses is also shared by the linear programming approach.

With the introduction of more flexible functional forms the force of Wipf and Bawden's argument is undoubtedly diminished. The Cobb-Douglas function is a special case of the CES function which in turn is a special case of the translog function. Thus there is no necessity to maintain restrictive hypotheses about the degrees of homogeneity and homotheticity; by estimating a translog function the data can be allowed to determine those, subject to the effects of any other restrictions imposed on the estimation process. Nevertheless this increase in flexibility does not wholly banish problems regarding choice of function. As Burgess (1975) indicates, unlike the Cobb-Douglas and CES functions, the translog form is not self-dual. He compares time-series estimates from a translog cost function for U.S. aggregate output with those of a translog production function, where these are estimated both independently and jointly with the appropriate input share-of-cost equations. The input demand and factor substitution elasticities obtained differ substantially between the two sets of estimates, and Burgess concludes (p. 119) "It is rather disconcerting, both from the standpoint of applied econometrics and from the standpoint of making predictions for public policy, to find substantial differences in parameter estimates arising from what would appear to be rather minor differences in specification."

An important fact to note in connection with the functions associated with the two-stage procedures is that they are all specified in a static form, and the algebraic procedures for deriving the supply or input demand functions assume that producers act as profit maximizers, as if in conditions of price certainty. Even where the prices are labelled as expected prices, it is the profit maximizing level of supply with the respect to these expectations which is derived. It is, however, obvious and generally accepted that the nature of the agricultural production process, with its long lags between initiating production and selling the product, creates a dynamic environment of decision making under price uncertainty. In addition farmers' knowledge of relevant technology and prices is likely to be imperfect. In these circumstances actual output at any point in time is virtually certain to deviate from the economic optimum and it is therefore unlikely that the true supply curve can be adequately represented by the profit maximizing output solution of a static production, cost or profit function. In principle it may be conceded that there exist implicit dynamic production, cost and profit functions which are consistent with dynamic supply functions. However, these have not been formally elaborated and the fact remains that empirical exercises in this two-stage category are static ones and that their results can only be applied in a comparative static way. Hence they have limitations for generating dated forecast statements; it is true that the choice about which "variables" to treat as variable and which as fixed does implicitly determine the time scale of any computed supply responses, but this cannot readily be identified in order to permit verification of the projections.

2.2.2 Supply Systems Models

In view of the comments already set out in section 1.2.2. above and of a recent comprehensive critique of the CRESH/CRETH system by Vincent (1982), discussion here will be brief. The outstanding merits are that in a one-stage econometric procedure using time-series data the CRESH/CRETH approach generates estimates for the influence of all input and output prices on supply and input demand which are consistent with underlying resource constraints. It is true that the programming approach also achieves this, but whereas the programming approach (a) involves assuming technical parameters of the production functions, (b) has to find a method of deriving market level responses from underlying firm responses, and (c) does not permit any specific summary algebraic description of the multi-dimensional response surface, the CRESH/CRETH approach overcomes all of these difficulties. Estimation occurs at the market level, the implicit technical coefficients are directly estimated from actual market level data, and total (reduced-form) supply response coefficients are directly obtainable.

Inevitably there are some problems still outstanding with the procedure and these are well set out by Vincent (1982). In particular, in common with all the methods, other than the single-commodity, single-stage one, the dynamics of response are not elaborated. The model is in fact a static one and no specific time lags are associated with what are identified as short and long-run responses. Moreover the strong suspicion remains that, if one thinks in terms of the "short-run" being the same calender length for each output and input, there must be some bias and distortion of the relative response elasticities as a result of the imposition of timeless substitution between the primary factors of production and between the outputs. Other questions also arise such as (a) what is the extent of the bias which arises from the method of linearization when the model is used for projection outside the range of the data used in estimation?, and (b) despite the device of treating most product specific inputs as excise taxes, are there not product-specific elements left in fixed capital, land and even labour (the primary inputs) which limit the acceptability of the assumption that they are non-product-specific and fully substitutable with one another? Despite these questions the method's potential advantages make it a valuable addition to the armoury of techniques. More widespread application and validation will reveal in time its true merits.

2.2.3 Directly Estimated Single Commodity Supply Models

In this approach to supply response analysis no attempt is made to build up the various parameters of supply functions from the technical parameters underlying them. Instead the behavioural parameters are obtained directly from statistical analysis of historical time-series data. The data which are the object of this analysis are the directly observed series of supplies in the past, and the statistical analysis tries to find an explanation of these in terms of a set of explanatory variables chosen on the basis of economic theory and knowledge of the technical conditions of production.

The theory of production and the firm makes a negligible contribution to this form of analysis. It is usually limited to taking a simple, general (and, in effect, reduced) form of the derived output-supply function, in which output is assumed to be a function of product and input prices. However, whereas the theory underlying the exposition in Figure 1 usually assumes prices to be given and derives undated equilibrium relationships between output and the known prices, the one-stage approach has to confront immediately the requirements of explaining observed, dated supplies (which are unlikely to be at profit maximizing levels) as functions of the variables chosen as exogenous. Because some agricultural products, mainly crops, are produced discontinuously only at certain periods of the year some output variables are recorded in the published statistics with an annual periodicity (or six-monthly if there are two harvests a year); but other products, including all livestock products other than wool, are produced continuously and are aften studied on a monthly basis, although the possibility exists for weekly or even daily observations.

As already noted, because transformation of inputs into output takes time, output at a point in time is the outcome of prior investment decisions of such forms as allocating resources to preparing and planting a field to a particular crop, or deciding to retain a heifer for milk production. In fact these past decisions are likely to be the dominant determinants of current output and there may be (in the case of crops in particular) only limited scope for very short-run output adjustment to prices prevailing at the time output is realized.[9] Consequently in one-stage procedures what is done is to attempt to find a way of explaining output in a particular period as a function of the factors determining key investment decisions. This may be done explicitly in cases where crop output is seen as depending upon the number of hectares (trees) planted and upon yield, or where livestock output is explained in terms of the numbers of particular classes of livestock held on farms and of the yield or offtake of these animals. Alternatively a reduced-form approach may be employed in which output is related directly to the factors assumed to influence investment.

Nearly all the partial theoretical contributions to the specification of econometric supply response models can be seen to represent alternative ways of trying to explain the underlying investment decisions. This is made absolutely explicit in those studies in which supply of a single commodity is determined via a specific investment model. It is an approach which has been found to be essential in studying the supply of perennial crops and is well exemplified by the coffee supply studies of Ady (1968), French and Matthews (1971) and Wickens and Greenfield (1973). While there are basic similarities between these three studies it is the latter which adopts the most comprehensive investment approach, in which the number of bushes of each vintage is explained. Actual output is a function of potential output and current economic factors; potential output is a function of the stock of bushes of all dated vintages; and the stock of bushes of each vintage is a function of those expected prices which determined the expected discounted return from investing in planting bushes. Hence actual output in year t is a dependent, in a specific (rationally) functional way, upon coffee prices in each of n past years, where n exceeds 30. Thus their investment based model establishes a link with rational expectation models.

[9] The capacity to decrease supply through non-harvesting of a crop always exists, and it is scope for upward adjustment which is obviously tightly constrained.

Another good example of adopting an explicit investment approach is Rayner's (1975) model of U.K. milk supply in which milk supply is explained entirely as the product of the factors determining past investment decisions relating to (a) retention of heifer calves as potential replacements, (b) recruitment of heifers to the milking herd, and (c) culling cows. The model might be criticized for not allotting some role to current economic values, and for the retention of some *ad hoc* elements especially in the handling of price expectations. Nevertheless it is an instructive attempt to supplant *ad hoc* reduced-form approaches to supply by something theoretically sounder.

There is no doubt that dynamic analysis of livestock product supply responses is exceptionally complex. The reasons for this are that for cattle, sheep and pigs "a given animal at a given time may be viewed as (a) a finished good, (b) a good in process, or (c) a piece of fixed capital" (Hildreth and Jarrett 1955, p. 21). These three characteristics apply more forcefully to female animals, and taken together they imply the need for simultaneous equation systems to explain output and inventories. For it is evident that current prices will affect the number of animals supplied for slaughter and hence all other aspects of livestock supply, while in turn, in the absence of policy measures to rigidly fix prices, prices themselves will be affected by current supplies, and hence there will be interdependency between supplies and current prices. On occasions time-series analysts have been tempted to sweep this problem aside by arguing that, within (say) a quarter, producers have only limited economic freedom to accelerate or retard slaughterings and that supply may be 90 to 95 per cent predetermined. This argument, however plausible superficially, does not seem wholly satisfactory since 5 to 10 per cent adjustments in supply are not of negligible significance, and examples of simultaneous models which have rejected it are too numerous to quote.

Watson (1970) has gone so far as to argue that, in livestock models, the complexities arising from the underlying investment decisions are such that in time-series regression analysis no satisfactory explanation of supply in terms of prices is likely to be possible, and consequently that life-cycle models are to be preferred in which supply is explained by a simulated model of the dynamic biological interractions within the relevant livestock population. In his view this arises because the relationship between any exogenous price and the desired and actual levels of the capital stock are not likely to be constant; the effect of a given price change on livestock numbers may differ between one period and another. While this may well be true, the fact is that the underlying problem is one of correctly identifying the way in which producers form expectations about the relevant explanatory variables and in which they respond to maximize their welfare over time; clearly if farmers do respond differently at different times to the same price change it is because this change is not the sole influence on their expectations. This serves to emphasize the fundamental connection between the incorporation of price expectations into supply models, the lagged role of investment decisions upon supply, and the consequent dynamic nature of supply responses to price. Models which simply adopt an *ad hoc* specification in which price expectations are assumed to directly influence supply are short-cutting the need to specify the investment model underlying supply response behaviour. There are in fact many studies which have adopted this general approach. In some cases the coefficients determining price expectation may be directly estimated along with other parameters from functions which assume Nerlovian adaptive expectation or some Almon lag scheme. Alternatively, to reduce estimational difficulties, the price expectation generating equation may be exogenously assumed to reflect a specific numerical adaptive expectation or Almon scheme, or, as in a number of recent studies (*e.g.*, Gardner 1976; Peck 1976), it may be assumed that prices on futures markets are a direct reflection of expectations.

There are also a number of other interesting partial theories of how supply response functions should allow for the role of investment. One of the crudest of these is the well-tried Nerlovian (1958) partial adjustment mechanism[10] which reflects the hypothesis that there is inertia, arising from investment adjustment costs and technical constraints, which results in the response to any change in economic stimuli being spread over a number of time periods in a geometrically declining way. A more sophisticated adjustment is where some allowance is made for the influence of price risk upon investment and supply; in these cases it is not just changes in the mean expected values of prices which are assumed to be important, but also changes in the variance of these expectations which is usually captured by incorporating terms for the sum of squared deviations of price variables from their moving averages (Behrman 1968; Just 1974; Traill 1978). Insofar as these terms often only sum deviations over the previous three or five production periods, their use as measures of changed risk seems rather questionable and it is quite likely that in some studies they are capturing the effects of general price inflation. Nevertheless the concept is an interesting one, as also is the notion that supply response coefficients will not be the same for upward price movements as downward ones. This asymmetric response concept is again rooted in a partial theory of investment in which the low salvage value of assets already invested is assumed to restrict the influence of downward price movements, and where price increases only begin to increase production once any spare capacity has been eliminated (see Traill, Colman and Young 1978). It is instructive that among the first applications of asymmetric supply response were those by Arak (1969) and Saylor (1974) for coffee since there is no flexibility to switch coffee trees to other uses and their salvage value is very low relative to their acquisition cost. Other applications include that by Gemmill (1978) to sugar supply, by Traill *et al* (1978) to onion supply, and by Hartman (1974) to egg supply.

So much attention has been devoted to different approaches for imparting dynamic behaviour into the one-stage class of supply response models, because this is their key characteristic. It renders them far more suitable to the task of generating conditional dated projections of the impact of price changes than any of the alternative methods. Their partial nature may represent something of a limitation, but there are many multi-commodity, multi-level models in which the supply sector is built up of what are in effect separately estimated single commodity models. Large scale models of this type include Agriculture Canada's Food and Agriculture Regional Model (FARM), the U.S.D.A.'s Grain and Oilseed Model (GOL), and the Wharton Agricultural Model of the

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[10] The number of studies of this type reviewed by Askari and Cummings (1977) runs into the hundreds, and there are many studies they omit.

U.S.A. The degree of stability which can exist in the supply sectors of such models is not imposed by restriction but derives from the consistency of the underlying data; although it may not prove robust when projections well outside the estimation range are required.

Furthermore the partial approach allows a good deal of flexibility to develop appropriate dynamic structures and it obviates the need to impose similar structures on widely dissimilar products as seems to occur in the systems approach. Indeed for perennial crops, some horticultural crops, and for supplementary livestock enterprises such as pigs and poultry there may be very little competitive interaction with other products. For these a single commodity approach may be entirely suitable, and the versatile nature of the approach can be exploited.

Given that it is the identification of the impacts of prices (which are often manipulated by policy) upon supply which is the focus of attention in this paper, it is disappointing in the one-stage models that insufficient attention is often given to the form in which price variables should enter supply equations. All too often product and input prices enter as completely independent variables. However, the implication of the theory of the firm is that supply depends upon the profitability of production, and in the multiple product case that it depends upon relative profitability. While it is quite difficult to achieve suitable definitions and measures of enterprise profitability, it is quite clear (Colman 1972, pp. 49–50) that a supply model for wheat which includes as its only price variables the price of wheat and the price of barley entering independently, cannot in general be said to have taken account accurately of changes in the relative profitability of wheat and barley. Only if costs of production of the two crops can be assumed to move together, and if yields remain constant, can the ratio of the price of wheat to the barley price be assumed to provide a suitable measure of profitability change. In general yields and cost vary independently, and simple price ratio variables or independently entering price variables to not fulfill their intended function. Unfortunately very few attempts are made to build the complex relative profit variables needed for this type of analysis, and input costs are often ignored completely.

A critique of the econometric approach to supply response analysis would be incomplete without the briefest of acknowledgements of some inherent limitations of multiple regression analysis. Only a few general points will be made, and some of these of course apply also to all the procedures reviewed with the exception of the programming approach. Firstly the estimation procedure requires that the number of times-series observations exceeds the number of explanatory variables—and the larger the difference the better. However it cannot be readily assumed that behavioural parameters remain unchanged for long periods of time. For prediction purposes it is necessary to have estimates of current behavioural responses, not those from the distant past, and this creates pressure to cut down the length of time series, which in turn creates pressure to reduce the number of explanatory variables. In fact most estimated supply equations include only a small subset of the variables which could theoretically be assumed to affect supply; this is most notably, and unfortunately revealed in the highly restricted number of cross-price effects typically estimated with this class of model. That this is so reflects not only pressure on degrees of freedom, but also the inability of regression analysis to clearly distinguish the partial influence of separate variables when these variables change through time in a correlated way, which time-series variables tend to do.[11] In the presence of such multicollinearity standard errors of estimates tend to be high. The parameter estimates also tend to be unstable and very sensitive to possible model misspecification so that the addition or elimination of one variable in the explanatory set may have a marked effect on the estimated parameters of the other variables. There do exist ways of reducing these effects of multicollinearity, such as by taking first differences of all the variables, expressing some of the explanatory variables in ratio form, or by employing ridge-regression techniques. Nevertheless, the fact remains that multiple regression analysis cannot measure the true underlying relationship (based on technical production relationships) in supply functions but merely the degree of association revealed by the movements in the time-series. In these circumstances supply-price responses estimated by multiple regression techniques cannot automatically be accepted as useful for policy impact analysis, and in general a great deal of judgment based on outside (*a priori*) knowledge is used to establish the acceptability of estimates obtained by these means. If such *a priori* information does exist there is a clear case for imposing it as restrictions upon the supply function and employing some form of restricted or Bayesian estimation technique.

It is clear from the foregoing incomplete critique that there exist major problems with the time-series regression approach to single-commodity supply response analysis. However, this is also true of its competitors, and it remains the case that aggregate time-series analysis is the most used and preferred of the methods. The most significant factors in its favour are that it operates directly upon the aggregate supply data which are the object of interest for projection purposes, and that it handles dynamic adjustments to supply in ways in which the other procedures do not. It is the simplest of the procedures in terms of estimational methods and data requirements. That it entails a smaller number of steps to generate supply response coefficients of the appropriate type for projection purposes also confers on it an advantage over the two-stage and programming procedures, and minimizes the capacity for specification errors to accumulate through successive stages. Finally, and perhaps most tellingly, it is a technique which has shown itself capable of generating acceptable and useful results.

3. Final Observations

It is apparent that each alternative method for empirically estimating agricultural supply response has its own particular merits, and that the choice of method in every instance should depend largely upon the decision-making process which the estimates are intended to aid. The choice may be influenced by pragmatic considerations such as the data, personnel and time available for the study, and upon computing facilities.

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[11] In the two-stage and supply systems approaches the imposition of restrictions implied by profit maximizing, cost minimizing theory helps overcome these problems, and aids the statistical process to extract theoretically acceptable parameter estimates.

Where the intended use of the results is for comparatively short-run forecasting of the supply of some subset of products, directly estimated functions using one-stage econometric procedures on market level time-series may well be preferred. This is also true for longer term forecasting of the output of supplementary enterprises (such as poultry) or of perennial crops where, typically, there is limited substitutability for other outputs. However, where forecasting supplies (at specific future dates) is less important, but where the objective is sector-wide agricultural policy impact analysis the need for consistency and comprehensiveness of the projections may dictate the use of one of the other three approaches, namely (1) a programming model based on reference farms, (2) a directly estimated supply system or (3) a two-stage profit or cost function system.

In the context of the highly complex and variable nature of agricultural supply, aggregate market data tend to have a rather limited information content. For forecasting this may not prove a serious drawback; provided that well-defined, time-related, regular behaviour can be shown to exist, empirically useful supply functions may be estimable. For policy analysis purposes, however, it may be necessary to impose rigorous theoretical restrictions upon the data to extract the required information (say, about the complete set of own and cross-price effects of input and output prices upon supply response and input demand). In programming models these restrictions take the form of imposing Leontief-type, input-output production technology plus constant returns to scale, and assuming profit maximization. In the directly estimated supply-system and two-stage approaches continuous substitutability of inputs and outputs and restrictions are imposed through the choice of functional relationships combined with the assumption of profit maximization or (where a cost function system is estimated) of cost minimization.

A cost of imposing these restrictions in both the programming and econometric procedures is that the dynamics of supply response are suppressed, and the estimated responses indicate changes from one equilibrium position to another. For forecasting purposes this is obviously a limitation. The gains, however, are that these methods make allowance in a theoretically consistent way for the technical/economic relationship between all inputs and outputs specified in the models. In the case of the programming models the extent of product and input disaggregation can be high, and, as already noted, the production options can include some which have not been employed previously but which have a potential for inclusion. These are useful properties for policy impact analysis, but they have to be balanced against the cumbrousness of the procedure and its strongly normative character. Because of the assumption employed the econometric systems and two-stage approaches also have strong normative elements, but these are tempered by the positive elements imparted by the data from which they are estimated. At present a key limitation of these types of models, apart from their static nature, is the rather limited degree of product and input disaggregation they can handle.

A factor limiting product and input disaggregation in the econometric approaches is the reduction in the number of statistical degrees of freedom as the number of parameters increase for any given length of time-series. The recent studies by Ray (1982) and McKay *et al* (1982), however, have demonstrated the feasibility of greater disaggregation, and the possibility exists that this might be extended further by adopting systems involving a set of profit or cost functions related in some hierarchial way. It has also been mentioned that implicitly there exist dynamic production, profit, cost and transformation functions. Conceivably progress might be made towards finding approximations to these which would enable more realistic dynamic response relationships to be estimated using variants of the two-stage procedures.[12]

Many analysts who continue to favour the "single-commodity" econometric approach to supply response do so because of reservations about the descriptive validity of attributing profit maximizing or cost minimizing behaviour to agricultural producers. They prefer not to impose this assumed behaviour on the data in the manner employed in the other methods. Nevertheless, restricted estimation techniques make it possible to incorporate other maintained hypotheses or prior information into commodity supply models. One particular type of information which deserves more attention within *ad hoc* supply systems relates to aggregate agricultural supply response to prices. Comparatively little attention has been paid to this, even within those programming and supply systems models from which such estimates could be derived. From a policy point-of-view an understanding of aggregate supply response is of profound importance. If direct estimates of this were available with respect to price variables it would be possible to impose restrictions when estimating systems composed of single-commodity supply equations. Extending this principle to other types of *a priori* information would permit the properties of single-commodity models to be improved without sacrificing their ability to capture the underlying dynamics of response.

[12] One study which has pointed the way along this path is that by Denny, Fuss and Waverman (1979).

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