# POLICIES FOR THE MANAGEMENT OF WEEDS IN NATURAL ECOSYSTEMS: A DYNAMIC PROGRAMMING APPROACH

**Doreen Odom*, Oscar Cacho*, J.A. Sinden* and G.R. Griffith*[+]**

***Graduate School of Agricultural and Resource Economics,**
**University of New England, Armidale,**
**And [+] NSW Agriculture, Armidale**

## Abstract

Environmental weeds are plants that invade natural ecosystems and are considered to be a serious threat to nature conservation. Environmental weeds have been implicated in the extinction of several indigenous plant species, and they also threaten ecosystem stability and functional complexity. Historically, emphasis has been placed on chemical control, manual pulling of small plants, excluding tourists and feral pig control measures. Recently, biological control has been introduced to control weed infestations. These methods of control have been applied alternatively, with little consideration of the long- term effectiveness. As the threat from environmental weeds is becoming more fully recognised, an integrated, strategic, ecological and economical approach to weed management is needed.

A deterministic dynamic programming model is developed for this purpose in this paper. A case study for the environmental weed *scotch broom* is presented, to assess the ways in which this approach can address the policy issues that face the community in the management of an environmental weed. The model takes account of the weed population dynamics and thirty-two combinations of control developed from the five basic control measures. The dynamic programming model is developed for three different cases, first with weed density as the state variable, second, with weed density and seed bank as state variables and third, with weed density and seed bank as state variables and with a budget constraint for the control variables. Results are presented and policies for managing weeds in natural ecosystems are recommended.

*Key words: environmental weeds, scotch broom, policies, weed management, dynamic programming.*

**Paper presented to the 46th Annual Conference of the Australian Agricultural and Resource Economics Society, Canberra, February 2002**

**Project Funded by the CRC for Weed Management Systems**

# 1. Introduction

Weed invasions have long been recognised as posing major problems in agricultural and pastoral systems, but the study of exotic plants within natural ecosystems is still in its infancy (Humphries *et al*. 1991).

The economics of management strategies for Scotch Broom (*Cytisus scoparius* (L.) Link, hereafter referred to as broom) is the focus of this paper. The ecology of broom is well understood in both its native (Memmott *et al*. 1993; Paynter *et al*. 1998, 2000) and exotic range (Williams 1981; Smith & Harlen 1991; Smith 1994; Downey & Smith 2000; Parker 2000; Sheppard, Hodge & Paynter 2000).

Broom is an exotic leguminous shrub, native to Europe, which invades pastoral and woodland ecosystems and adjoining river systems (Hosking, Smith & Sheppard 1998; Sheppard & Hosking 2000) in cool, high rainfall regions of southeastern Australia. Broom plants can survive for over 20 years and form dense closed thickets. Flowering is in spring, seed set (the only method of reproduction) is in summer, and buds remain dormant over winter.

Broom has invaded 10,000 hectares (Waterhouse 1988) of eucalypt woodland at Barrington Tops National Parks in New South Wales, where it forms dense stands that have significant impacts on vegetation structure, flora and fauna. In addition, within the park broom blocks the tracks and prevents access to watercourses. Broom now poses a serious threat in many regions including the Australian Alps National Parks in New South Wales and Victoria, and in western Tasmania. It has also been recorded around Perth, Western Australia. The total area infested in Australia is estimated currently to be over 200,000 hectares and is still spreading (Hosking *et al.* 1998). Broom is considered to be one of the major environmental weeds of temperate Australia. Broom is also an invader in other regions such as New Zealand and western North America, where it is considered to be serious pest plant (Parsons & Cuthbertson 1992; Hosking *et al.* 1998).

Physical control of broom is complicated by its large, long-lived soil seed bank, and biological control agents have yet to have any significant impact in Australia. At present the control measures undertaken by the Barrington Tops National Park management includes; excluding tourists, pulling out small plants manually, applying herbicide, feral pigs control and biological control. These methods of control are applied alternatively, with little consideration of the long- term effectiveness.

The aim of this paper is to answer some of the policy questions of relevance to the management of the broom problem in Barrington Tops National Park. In addressing this problem, the paper has been organised as follows: we first review policy issues surrounding the management of broom in natural ecosystems. Then we develop a dynamic optimisation model for broom management. We present results for the unconstrained model and the constrained model. Finally we discuss the implications of the results and suggest some conclusions.

## 2. Policy issues surrounding the management of broom in natural ecosystems

National Park Services manage most of the nations natural ecosystems. In the Australian states, national parks and wildlife services are funded from general government revenue and any income, which they receive, is paid into that revenue. They are therefore, dependent entirely on the political process for funds.

Given the non-commercial nature of public conservation and recreational land, political factors play a much greater role in determining effort and methods used to control weeds on public land than on private land (Hartley and Tisdell 1981).

Due to deficiencies in political mechanisms and the presence of market failure weed control and management on public conservation and recreational land is unlikely to be optimal from an economic viewpoint. On conservation lands in particular, there may be insufficient control of weeds because of lack of funding.

Conservation pressure groups in Australia in recent years seem to have concentrated political action on increasing the area set aside for national parks but have not exerted as much pressure for increased funding of park management.

Within this broad context, policy issues surrounding the management of broom can be summarised as follows,

- the budget available for park management is limited,
- there is uncertainty of future funding, and
- methods of controlling broom are limited due to government rules/regulations.

These issues pose the following kinds of policy questions.

- What areas should the National Park and Wildlife Service concentrate on, in controlling broom? Should areas with endangered species be controlled first?
- What are the benefits of a continuous budget for the coming periods/years?
- What combination of controls will best meet agency objectives in controlling broom?
- Is biological control worth doing?
- Is eradication strategy worth pursuing?

In approaching these policy questions, a deterministic dynamic programming model for broom management has been developed and is described in the following section. The model takes account of broom population dynamics, combinations of control measures, two state variables (weed density and the seed bank) and a budget constraint.

## 3. A dynamic optimisation model for broom management

Following land use on Barrington Tops, it is assumed that, a tract of land of 80,000 hectares is presently used for biodiversity protection, recreation and livestock production (Odom *et al.* 2001). We have omitted watershed protection as one of the uses for the land. From the aspect of broom management the land can be defined in terms of four variables; the fraction of sites occupied by broom, the fraction of sites that are unsuitable for broom establishment, the fraction of open sites (areas suitable for broom but not yet colonised), and the average number of viable seeds per site. These variables describe the initial state of the land. It also must be assumed that the same inputs could be applied to the whole area (Odom *et al.* 2001).

The three outputs of the Park are measured and valued as follows. Biodiversity is measured in terms of number of species preserved, recreation quantity in terms of number of group visits, and agricultural output is measured as percentage of potential yield (Odom *et al.* 2001). The net annual benefit obtained from the area in time $t$ ($B_t$) is defined as:

$$B_t = B_{bio}(w_t) + B_{rec}(w_t) + B_{agr}(w_t) - u_t \cdot c_u \qquad (1)$$

where, $B_{bio}$ $B_{rec}$ and $B_{agr}$ are the benefits (as prices x quantity) provided by each of the three outputs, namely biodiversity, recreation and agriculture. The values of the outputs are functions of weed density ($w_t$), with $dB_j/dw_t < 0$ for all $j = bio, rec, agr$. The last term in the equation represents the costs of broom control, where *(u)* is a vector of control measures and *(c)* is a vector of per unit costs of control.

Net benefit is maximised by choosing control measures each period that maximises the present value of a stream of annual net benefits, given the initial state.

## 3.1 Control measures

For simplicity, the only costs considered are those of weed control and these depend on the control method used. Six control options are possible, and a number identifies them:

0. no control
1. exclude tourists
2. pull out manually
3. apply herbicides
4. control pigs
5. biological control

In the model, the particular control applied is represented by a 1x5 vector of zeros and ones, a zero in a given position indicates no control, while a one indicates that the corresponding control is being applied. For example, $u$ = [1 0 0 1 0] indicates that both tourist exclusion (1) and pig control (4) are being undertaken. There are 32 control strategies as shown in Table 1. The first row represents no control and the remaining rows are the control methods (1…5) as described above, followed by 26 combination of controls each represented by a row of the matrix. The cost of control is calculated by multiplying the control vector $u_t$ by the (5x1) cost vector $c_{\boldsymbol{u}}$.

## 3.2 Population dynamics

The dynamics of broom population growth are introduced through the difference equation (state transition equation):

$$w_{t+1} = w_t + f\left(w_t, u_t\right) \tag{2}$$

where, $w_t$ is the weed density in the present period and $w_{t+1}$ is the weed density in the next period. The function $f(\bullet)$ represents the biological model to simulate the spread of broom from Rees and Paynter (1997). In this model there are four state variables: weed density, sites unsuitable for colonisation, sites open for colonisation, and the size of the seed bank. The parameter values for this simulation model are presented in Table 2, and the initial conditions of the area are presented in Table 3.

The transition of a given tract of land from an unsuitable to a suitable site for broom depends on the probability of disturbance ($p_{dist}$), which is affected by factors such as presence of tourists and wild pigs. The simulation model operates with four state variables and hence contains four differential equations. At this stage, only one of those state variables, weed density ($w_t$), is relevant in the economic model, because this is the factor that directly affects biodiversity, recreation value and agricultural output.

The control methods directly affect four biological parameters the probability that a site is disturbed ($p_{dist}$), the probability that a seedling survives the first year ($p_s$), the probability that a seed is retained in the parental site ($f_h$), and weed density ($w_t$). The effects of the control methods on these parameters are shown in Table 4, and their composition is now explained.

Values of the parameters are presented as proportion from the base values shown in Table 2. The estimation of these parameters was based on the logical relationship between the control method and the parameter, ie. where the parameter is expected to increase or decrease with a particular control. Integrated weed management options and the effects of treatment on the biology of broom were constructed from a basic lifecycle diagram of scotch broom and associating treatments to the various stages.

When control methods were combined, the effects on the parameters were estimated from two assumptions. If the controls affect different stages of the weed life cycle, then the parameter values were added. If the controls affect the same stage of the weed life cycle, then the parameter values were added in a partially additive manner.

Parameter values of 1.0 indicate no effect, these values appear in the first row for the no control option, and also control options 1 (excluding tourist) and 4 (pig control) which are directly related in reducing the spread but no effect on the weed density. For example, the second row (excluding tourist) reduces ($p_{dist}$) by 0.2 and ($p_s$) by 0.33, but increases ($f_h$) by 1.23 per cent.

## 3.3 Maximisation of net benefits

The objective of the analysis is to choose a sequence of decision variables or management inputs $(u_t)$ that maximises the present value of a stream of annual net benefits, given the initial state.

The problem of maximising the net present value of the stream of benefits obtained from the area over a planning horizon of $T$ years is solved through dynamic programming. The recursive equation with one state variable is:

$$V_t(w_t) = \max\left[B_t(w_t, u_t) + \delta V_{t+1}(w_{t+1})\right] \tag{3}$$

where $B_t$ is the one-period return (as in equation 1) and $\delta$ is the discount factor $(1+r)^{-1}$ for the given discount rate $r$ (Taylor & Duffy,1994). The first term in this equation represents benefits in the present year and the second term represents benefits from the future. The recursive solution of (3) is executed from $t = T$ to $t=1$, subject to the state transition equation (2).

The values of biodiversity and recreation output are described by the function:

$$v_j = P_j \frac{V_{\max j}(x_{\min j} - w_t)}{k_{mj} + (x_{\min j} - w_t)} \text{ ; for } j = bio, rec \tag{4}$$

where, $(v)$ is the production rate, $(P)$ is the price, $(V_{\max})$ is the maximum number of species preserved or maximum number of recreational visits, $(x_{\min})$ is the weed concentration, and $(k_m)$ is the half-saturation constant (Cacho 2000).

The value of agricultural output is described by the function:

$$v_{agr} = P_{agr} V_{\max agr} \left(1 - \exp\left(k_{magr}(x_{\min j} - w_t)\right)\right) \tag{5}$$

where, $(v)$ is the production rate, $(P)$ is the price of agricultural output, $(V_{\max})$ is the maximum potential yield, $(x_{\min})$ is the weed concentration, and $(k_m)$ is the half-saturation constant. The parameter $x_{\min}$ determines the intercept on the horizontal axis, $k_m$ determines the slope of the curve and $V_{\max}$ determines the intercept on the vertical axis.

Agricultural output is measured as percentage of potential yield, the price of this output is estimated by multiplying the gross margin per hectare times the number of hectares in pasture. The output parameters are presented in Table 5 and the control costs are presented in Table 6.

Values of the parameters $V_{\max}$, $x_{\min}$ and $k_m$ were estimated in consultation with National Parks and Wildlife Service staff and were also based on research by Panetta and James (1999). The prices of outputs ($P_j$) were obtained from three different sources. The authors set the benefits of biodiversity protection, at a basic value of

$100,000 to represent one species' worth. They then tested the solutions for the effect of changes in this value. Benefits for recreation in terms of number of visits were obtained from Sawtell (1999) and confirmed by Tier (2001) research on Barrington Tops. Prices for agricultural output, in terms of gross margins, were obtained from NSW Agriculture, and were prepared by Davies (2000).

## 3.4 The state transition equation

Assumptions regarding broom population growth, and the effect of control methods on biological parameters and state variables, affect the state transition equation, which is represented by a simulation model. The state transition equation for each of the first six control options (including no control) is presented in Figure 1.

The 45$^{o}$ dotted line represents the steady state for any given population density ($w_t$) at a given time $t$. Points below this reference line represent strategies that will cause broom density to decrease, whereas points above the line represent strategies that will cause density to increase. The only line falling above the line is no control. All control methods cause $w_t$ to decrease over time.

## 3.5 The inclusion of the seed bank

The dynamic optimisation model is further extended to include the seed bank as the second state variable in the analysis. Broom population dynamics and the seed bank population dynamics are now introduced through difference equations:

$$w_{t+1} = w_t + f(w_t, s_t, u_t) \quad (6)$$

$$s_{t+1} = s_t + g(w_t, s_t, u_t) \quad (7)$$

The functions $f(.)$ and $g(.)$ represent the biological model to simulate the spread of broom (Rees and Paynter 1997), where $s_t$ is the seed density (per square metre) in time $(t)$.

The recursive equation is now expressed as

$$V_t(w_t, s_t) = \max\left[B_t(w_t, u_t) + \delta V_{t+1}(w_{t+1}, s_{t+1})\right] \tag{8}$$

The recursive solution of (8) is executed from $t = T$ to $t = 1$, subject to the state transition equations (6) and (7).

The model was solved for a planning horizon (T) of 45 years. The numerical deterministic Dynamic Programming technique was implemented in the Matlab (Mathworks 1999) program with the discount rate of 6 per cent. The choice of the discount rate was based on the recommended rates by the Australian government taking into consideration the emphasis on time-preference principle with the suggested rates ranging from 4 to 7 per cent (Sinden & Thampapillai 1995).

The model was solved for the base case parameters of Tables 1 to 6, with no constraint on the budget available to control weeds. An extended version of the model was also solved by incorporating the constraint

$$u_t.c_u \leq K \qquad \text{for all } t = 0,...T \tag{9}$$

where the term on the left is the annual cost of control and $K$ is the budget available. The model was solved for a budget constraint of $K = 50{,}000$.

# 4. Results

## 4.1 Unconstrained by the budget

In the process of obtaining the optimal solution, the model was allowed to run for a maximum time of 45 years and the results provide the optimal state paths for both the weed density (Figure 2) and the seed bank (Figure 3).

As Figure 2 illustrates, the initial weed density was specified at 0.1. It also shows the optimal weed density path from the present year to year 45 with integrated weed management. With the application of control measures, the weed density is expected to drop sharply from the present year to year 11. Density then stabilises and oscillates in a narrow range to year 45, at a weed density level of 0.0201- 0.0207. The results imply that it is not possible to reduce the weed density to zero. Practically this is true, due to the large, long-lived soil seed bank any movement on top of the soil can activate germination, natural disasters like floods and fire can also activate germination.

Starting with an initial seed density of 50 seeds per square metre, as indicated by Figure 3, the seed density is expected to increase sharply to year 2, to a seed density level of 332 seeds per square metre. Between year 2 and year 3 the change is very minimal but after year 3 a sudden drop occurs to year 12 where the seed density is about 114 seeds per square metre. Thereafter, it stabilises at a range of 121 to 132 seeds per square metre up to year 45.

It is very interesting to compare Figures 2 and 3. Before the steady state has been reached, the seed density first increases then decreases as the weed density decreases. After attaining the steady state, as the weed density decreases the seed density increases and as the seed density decreases the weed density increases. By year 45 the weed density is left on a decreasing state while the seed density is on the increasing state. Hence, reducing the weed density does not mean the seed bank reduces as well.

### 4.1.1 The optimal state transition

As mentioned in section 3.4, the assumptions regarding broom population growth, and the effect of control methods on biological parameters and state variables affects the state transition equation.

The role of the optimal state transition in this case is to provide a “package” that could be used to tackle the problem each year depending on the levels of the weed density and the seed bank. The optimal state transition results are presented in two stages. First the optimal state transition for the weed density is presented when the seed density is low and when the seed density is high (Figure 4). Second, the optimal state transition for the seed density is presented with low levels of weed density, and then with high levels of the weed density.

The optimal state transition for the weed density shows a decrease in the weed density in both cases, as shown by Figure 4. The two curves illustrating the weed density in the next state for both higher and lower levels of the seed density are below the $45^0$ dotted line. The control measures have been more effective on the weed density with low levels of the seed density than the weed density with high levels of the seed density. This is shown by the distance of the curves from the dotted line.

The optimal state transition for the seed density as shown by Figure 5, indicates a decrease in the seed density when the weed density is low. This is shown by the curve below the $45^0$ dotted line. When the weed density is high, the seed density shows an increase in the seed density level up to 2,317 seeds per square metre, then the seed density level starts to decrease. This implies that control measures have been effective from this point.

In this case, the only control measure which targets the seed bank is biological control. Bio-control measures takes time to work, and the optimal state transition pattern shows that the seed bank did not respond fully to the control measure until at a density level of 2,317 seeds per square metre.

Control measures seem to be more effective with low levels of weed density than high levels as shown by the curves in Figure 5. This is because the low weed density curve is below the $45^0$ dotted line while the high weed density curve is mostly above the dotted line with a small portion appearing just under the $45^0$ dotted line.

### 4.1.2 The decision rule

The optimal state transition equation generates the decision rule for broom control. Table 7 presents the decision rule for different levels of the weed density. As the results show, in areas where the seed density is low and the weed density is also low, the optimal strategy is 1, which means no control is the optimal strategy. For areas where the seed density is low but the weed density is medium density (0.287) to high-density (0.501), the optimal strategy is number 24. In fact, a combination of manual pull, herbicide application and biological control is the optimal strategy in reducing the weed density level to 0.2054 and 0.3579 respectively.

For areas with high seed density levels and low weed density levels (0 to 0.141), the optimal strategy is number 16, which means a combination of pig control and biological control is the optimal strategy. Areas where the seed density is high but the weed density is medium dense (0.287) to high (0.501), the optimal strategy is number 28. This means a combination of manual pull, herbicide application, pig control and biological control is the optimal strategy, which reduces the weed density to 0.2344 and 0.3831 respectively.

The decision rule derived from the seed bank optimal state transition is presented in Table 8. The results shows that for areas with zero weed density and zero seed density, the optimal strategy is number 1, which means no control is required. In areas where the weed density is low and the seed density is also low, the optimal strategy is number 5, for a seed density level of 4 seeds per square metre. This means that pig control is the optimal measure to reduce the level to 2 seeds per square metre. With low seed density levels of 250 and 594 seeds per square metre, and low weed density levels the optimal strategy is 6, this means biological control is the optimal measure to be used to reduce the seed density levels to 125 and 297 seeds per square metre respectively.

For areas with high weed density levels, and low seed density levels the optimal strategy is 24, which means a combination of manual pull, herbicide application and biological control is the optimal control strategy to be used in order to keep the seed density at lower levels. Areas with high weed density but seed density is medium dense of 2,317 seeds per square metre to high density of 3,350 seeds per square metre, the optimal strategy is 28, which is a combination of manual pull, herbicide application, pig control and biological control.

The optimal control options for different levels of the weed density and the seed bank are summarised in Table 9. The weed density levels are presented vertically in the first column, while the seed density levels are presented horizontally. For each of the weed density level there is a corresponding seed density level and the Dynamic

Programming technique computes the optimal control strategy for each level. Strategies are defined in Table 1.

The net present value (NPV) of the optimal solution obtained from the unconstrained model is $ 210,180,000 for the planning horizon, at a total cost of $1,027,611 which has been discounted annually to the present value.

## 4.2 Constrained by the budget

The second run of the Dynamic Programming model was then carried out with a budget constraint of $50,000. This amount is used for illustration purposes only as the model has been designed to accommodate any size for the budget constraint. Results are presented in the same way as the unconstrained model so that the two sets of results can be compared.

The results of the optimal state paths for both the weed density and the seed bank are both opposite to the optimal state paths from the unconstrained model. With the inclusion of the budget constraint, the optimal weed density path (Figure 2) indicates a decrease in the weed density for a very small proportion of about 0.02 between years 0 and 3. There after the weed density increases to a level of 0.16 by year 45. This occurs because control measures are limited by the constraint to what can be afforded and as a result the weed density increases.

The optimal seed density path under budget constraint (Figure 3) also indicates an increase in the seed density unsteadily, with a drop in year 6 and 10 and increased steadily thereafter to a seed density level of 878 seeds per square metre.

The density paths of the constrained model results in an increase in both the weed density as well as the seed density. The effectiveness of the control measures shows very little on the weed density path in the constrained model unlike the unconstrained model. On the other hand, the effectiveness of the control measures on the seed density does not show up at all in the constrained model like how the unconstrained model shows a reduction in the seed density. The constrained model proves that the biological control aimed at reducing the seed bank could not be afforded by the available budget.

### 4.2.1 The optimal state transition under a budget constraint

In this case the role of the optimal state transition is to provide a "package" that could be used to tackle the problem each year depending on the budget available. The optimal state transition for the weed density is presented in Figure 6. The first curve under the $45^0$ line represents the weed density under the budget constraint and the second curve represents the unconstrained weed density.

The optimal state transition for the weed density shows a decrease in the weed density in both cases (Figure 6). Although the unconstrained weed density curve shows more reduction of the weed density than the constrained curve. Therefore the control measures are more effective in reducing the weed density in the unconstrained model, because the budget constraint limits the control measures to be used.

The optimal state transition for the seed density as shown by Figure 7, indicates an increase in the level of seed density when a budget constraint is imposed. This is shown by the constrained curve being above the unconstrained curve and the $45^0$ line.

### 4.2.2 The decision rule under a budget constraint

The generated decision rule for different levels of weed density with low levels of the seed bank for both the constrained model and the unconstrained model is presented in Table 10. For areas with low seed density and low levels of weed density the optimal strategy is number 1 with a budget constraint. Areas with a low (0.060) to medium (0.287) weed density at the same low levels of the seed density the optimal strategy to be used is 11 under the budget constraint. In this case, only 83% of manual pull and herbicide application can be used to reduce the weed density to 0.0479 and 0.2290 respectively. For areas with low seed density levels and high weed density levels (0.501) the optimal strategy is 4, which means only herbicide application is the optimal method to reduce the weed density to 0.3919.

The decision rule derived from the seed bank optimal state transition is presented in Table 11. The presented results are for areas with high levels of weed density.

For areas with high levels of weed density and low levels of seed density the optimal control strategy is 4, this means the optimal method which can be afforded by the budget is herbicide application.

In areas with high weed density and a medium dense (0.250) to high seed density (3.350), the optimal control strategy, which can be afforded is strategy 11. This means 83% of manual pull and herbicide application is the optimal measure. The response of the seed density is the same as above except for the last three levels of the seed density, which shows a slight decrease in the seed density.

The optimal control options for different levels weed density and the seed bank under a budget constraint are summarised in Table 12. The optimal control options under a budget constraint are different from the optimal control options of the unconstrained model. Only a few of the control measures can be applied at a full proportion. Most of the control measures are limited by the budget constrained in such a way that only a fraction of the combinations in the strategies can be afforded. The description of the control strategies, which can not be applied fully, is presented in Table 13.

The net present value (NPV) of the optimal solution obtained from the constrained model is \$ 203,170,000 for the planning horizon, at a total cost of \$ 695,371, which has also been discounted annually to the present value.

## 5. Discussion and Conclusion

This paper has presented an application of a deterministic Dynamic Programming model for broom management. The results of the unconstrained version of the model indicates that, both the weed density and the seed bank can only be reduced up to an optimal level, where a steady state is achieved. It can also be noted that reducing the weed density alone does not necessarily mean the seed bank reduces as well. Hence, combination of controls which target the weed density and the seed bank are important, and as we have seen, the control methods are more effective on areas with low levels of both the weed density and the seed bank. In addition, the decision rule derived from the optimal state transition for both the weed density and the seed bank indicates that biological control appears as the optimal control measure in 77 optimal control options out of 81 (Table 9).

In the optimal solution, the annual costs range from a minimum value of $15,000 to a maximum value of $136,848, with most costs being around $76,848, which exceeds the budget constraint value by about 50%. Thus, the budget constraint saves about $330,000 but results in a reduction in net benefits of about $7,000,000. It was also noted that, biological control was hardly ever affordable under the budget constraint.

In this paper, we have assumed that we are certain of all the parameters (Tables 2, 3, 5, and 6) used in the model. However, risks are considered to be an important aspect in our research problem, although this was not taken into account due to the complications of the model. Another limitation is that sensitivity analysis of the effectiveness of biological control has not been undertaken at this stage of the model although it is considered important as an extension to the model. Odom et al. (2001) conducted a sensitivity analysis of the initial parameters in a model with one state variable (weed density) with control measures applied alternatively, and the results showed that herbicide prices and herbicide effectiveness were most sensitive to changes in parameter values. In addition the value of biodiversity was tested and the results showed that, the predicted biological systems stayed stable after $100,000.

The model has been able to answer some of the policy questions outlined in Section 2. The benefits of having a continuous budget for the coming years can be accommodated by switching the budget value in the model to obtain optimal solutions. The model indicates that the best combinations of controls, which will meet the objectives of the agency in controlling broom, are strategy 16, for areas with low weed density and high levels of seed density; strategy 24, for areas with low levels of seed density and high levels of weed density; and strategy 28, in areas where both the weed density and the seed density are at high levels. The model also indicates that biological control is worth undertaking, as it appears in all the strategies, which meets the agency objectives, and in almost all-optimal control options of the unconstrained results. As the results showed, an eradication strategy is not worth pursuing, since it is not possible to reduce both the weed density and the seed density to zero.

# 6. References

Cacho, O. J. (2000). *Introduction to Bioeconomics: Study Guide*. Unpublished Lecture Notes, University of New England, Armidale.

Davies, L. (2000). Farm Enterprise Budgets. NSW Agriculture, Orange.

Downey, P. O. & Smith, J. M. B. (2000). Demography of the invasive shrub Scotch broom (*Cytisus scoparius* (L.) Link) at Barrington Tops, NSW: insights for management. *Austral Ecology*, 25, 477-485.

Hartley, K. and Tisdell, C. (1981). *Micro-Economic Policy*, Wiley, Chichester

Hosking, J.R., Smith, J.M.B. & Sheppard, A.W. (1998). *Cytisus scoparius* (L). In F.D. Panetta, R.H. Groves & R.C.H. Sheppard (eds), *The Biology of Australian Weeds*, Vol.2, R.G. & F.J. Richardson, Melbourne, 77-88.

Humphries S. E., Groves R. H. & Mitchell D. S. (1991). Plant invasion of Australian ecosystems: a status review and management directions. *Kowari* 2, 1-134.

Memmott, J., Fowler, S. V., Syrett, P. & Hosking, J. R. (1993) What makes broom a major problem? *Proceedings of the Brighton Crop Protection Conference* pp753-758. British Crop Protection Council, Farnham, UK.

Odom, D.I., Cacho, O., Sinden, J.A. & Griffith, G.R. (2001). Strategies for controlling weeds in natural ecosystems: a dynamic optimisation approach, paper presented to the 45th Australian Agricultural and Resource Economics Society Conference, Adelaide, January.

Panetta, F.D. & James, R.F. (1999). Weed control thresholds: a useful concept in natural ecosystems. *Plant Protection Quarterly 14 (2), 68-76.*

Parker, I. M. (2000). Invasion dynamics of *Cytisus scoparius*: a matrix model approach. *Ecological Applications*, 10, 726-743.

Parson W. T. & Cuthbertson E. G. (1992). *Noxious Weeds of Australia*. Inkata Press, Melbourne.

Paynter, Q., Fowler, S.V., Memmott, J. & Sheppard, A.W. (1998). Factors affecting the establishment of *Cytisus scoparius* in southern France: implications for managing both native and exotic populations. *Journal of Applied Ecology*, 35, 582-595.

Paynter, Q., Fowler, S.V., Memmott, J. Shaw, R.H. & Sheppard, A.W. (2000). Determinants of broom (*Cytisus scoparius* (L.) Link) abudance in Europe. Plant *Protection Quarterly*, 15, 149-155.

Rees, M. & Paynter, Q. (1997). Biological control of Scotch broom: modelling the determinants of abundance and the potential impact of introduced insect herbivores. *Journal of Applied Ecology*, 34, 1203-1221.

Sawtell, A.T. (1999). Estimating the recreation use value of Barrington Tops National Park, BAg Econ. Thesis, University of New England, Armidale.

Sheppard, A.W. & Hosking, J.R. (eds) (2000). Broom management. *Plant Protection Quarterly*, 15, 134-186.

Sheppard, A.W., Hodge, P. & Paynter, Q. (2000). Factors affecting broom regeneration in Australia and their management implications. *Plant Protection Quarterly*, 15, 156-161.

Sheppard, A.W., Hodge, P., Paynter, Q. & Rees, M. (2001). Factors affecting expansion and regeneration of exotic broom stands (*Cytisus scoparius)*, in Australia. Unpublished manuscript, CSIRO, Canberra.

Paynter, Q., Downey, P.O. & Sheppard, A.W. (2001). Age structure and growth of *Cytisus scoparius* in native and exotic habitats and control of exotic weedy populations. Unpublished manuscript, CSIRO, Canberra.

Sinden, J. A. & Thampapillai, D. J. (1995). *Introduction to Benefit-Cost Analysis*. Longman, Melbourne.

Smith, J.M.B. & Harlen, R.L. (1991). Preliminary observation on the seed dynamics of broom (*Cytisus scoparius*) at Barrington Tops, New South Wales. *Plant Protection Quarterly* 6, 73-78.

Smith, J.M.B. (1994). The changing ecological impact of broom (*Cytisus scoparius*) at Barrington Tops, New South Wales. *Plant Protection Quarterly* 9 (1), 6 -11.

Taylor, R.C. & Duffy, P.A. (1994). *Learning Dynamic Programming*. Unpublished Lecture Notes, Auburn University, Alabama.

Tier, F. (2001). Estimation of the relative values of biodiversity protection and recreation in natural ecosystems on Barrington Tops, NSW, BAg Econ. Dissertation, University of New England, Armidale.

Waterhouse, B.M. (1988). Broom (*Cytisus scoparius*) at Barrington Tops, New South Wales. *Australian Geographical Studies* 26, 239-248.

Williams, P.A. (1981). Aspects of the ecology of broom (*Cytisus scoparius)* in Canterbury, New Zealand. *New Zealand Journal of Botany*, 19,31-43.

**Figure 1. The state transition equation**

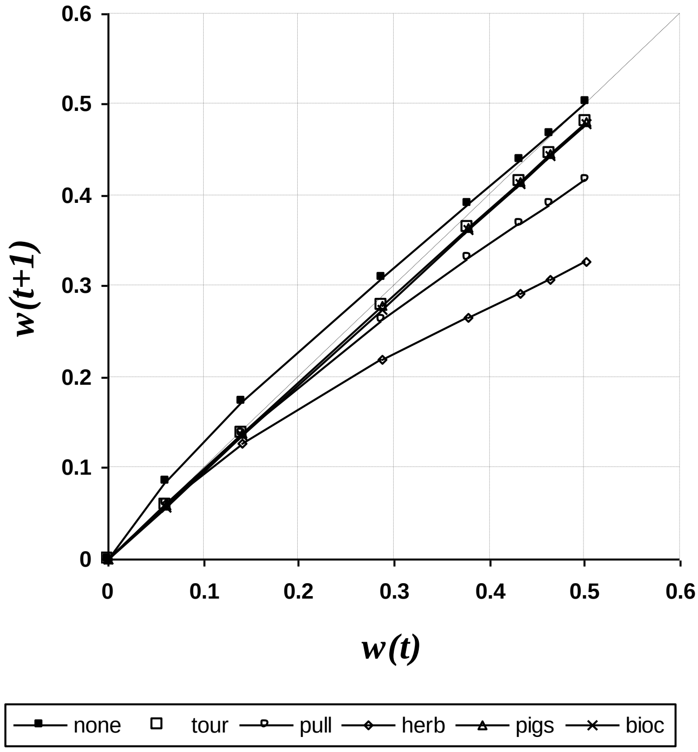


**Figure 2. The optimal weed density path**

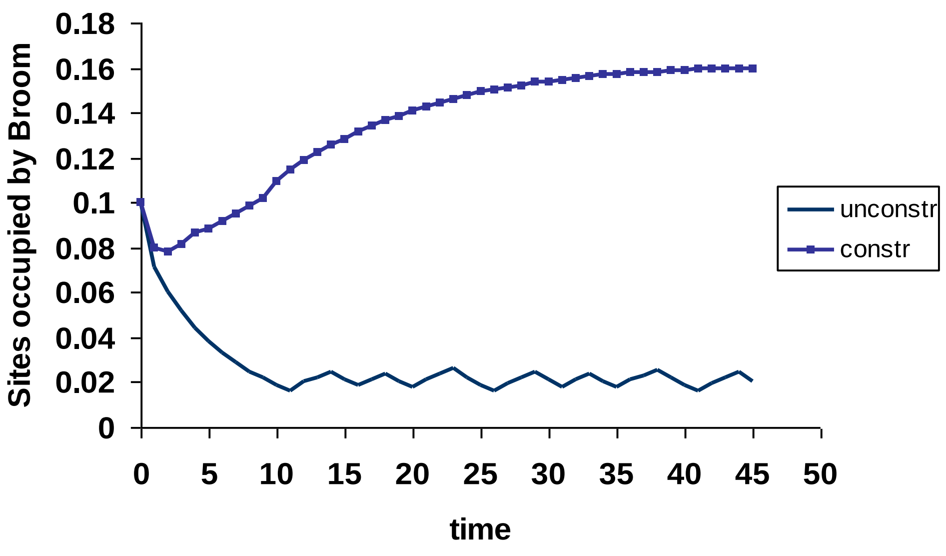

**Figure 3. The optimal seed density path**

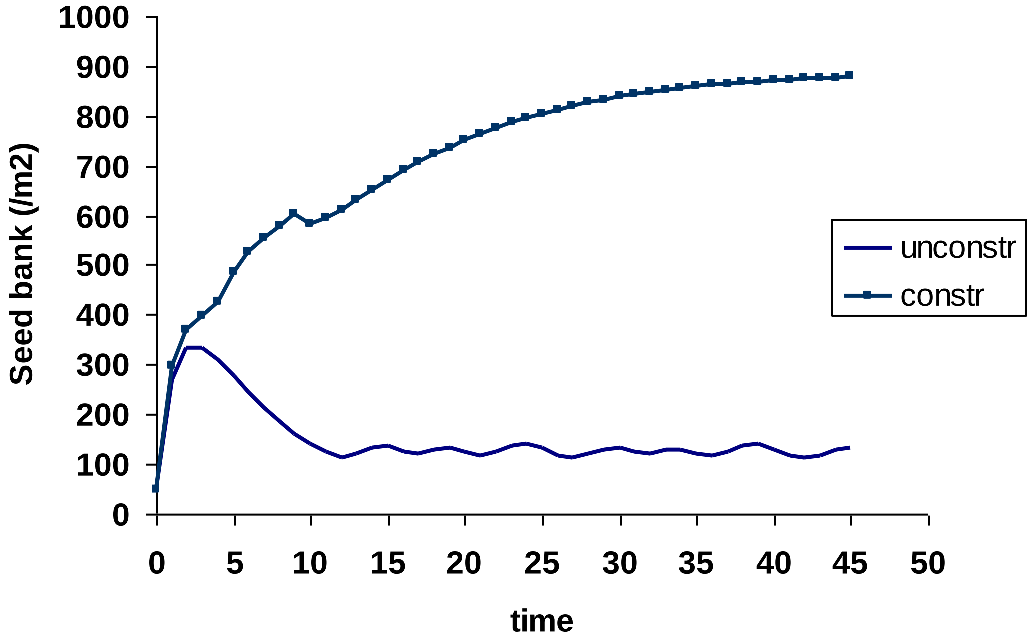


**Figure 4. Optimal state transition for the weed density**

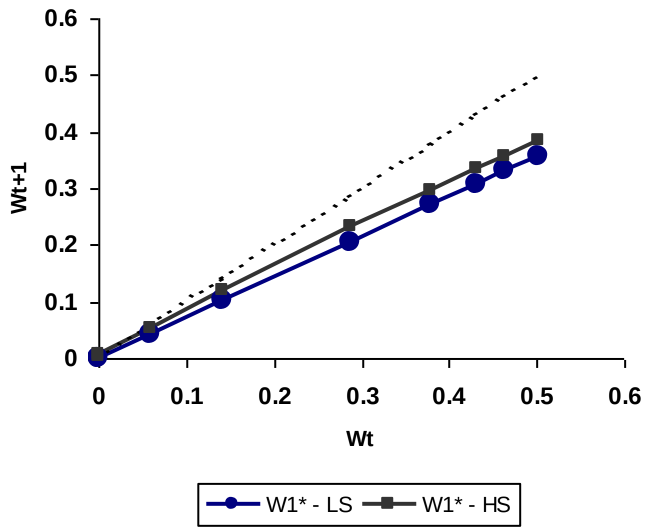

**Figure 5. Optimal state transition for the seed bank**

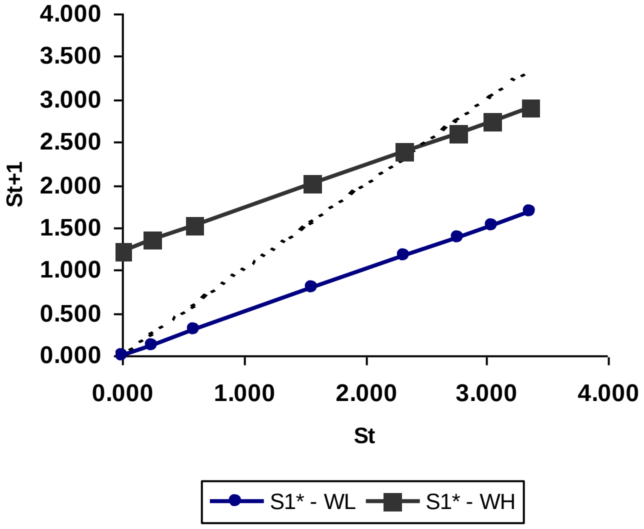


**Figure 6. Optimal state transition for the weed density under a budget constraint**

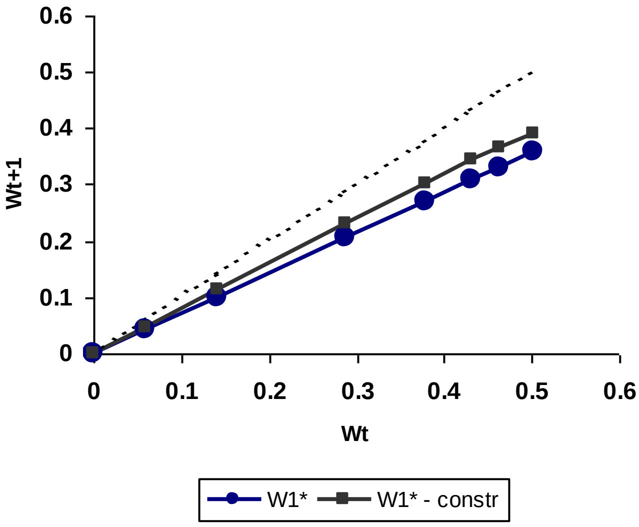

**Figure 7. Optimal state transition for the seed bank under a budget constraint**

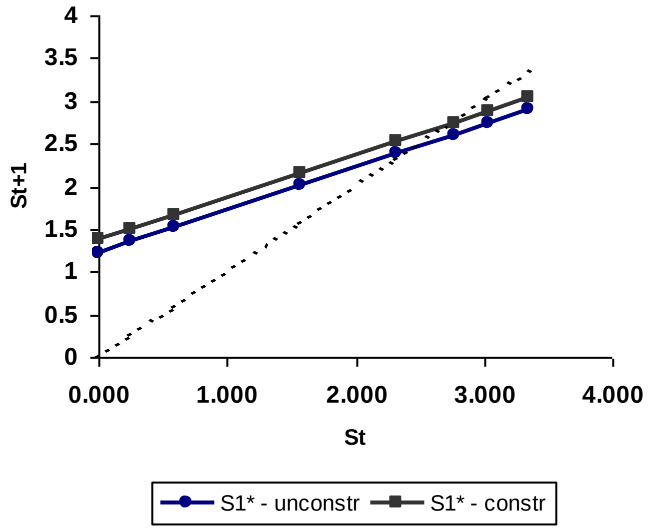


**Table 1: The matrix of control measures, by management strategy**

| Strategy | exclude tourist | manual pull | herbicide application | pig control | biological control |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 |
| 6 | 0 | 0 | 0 | 0 | 1 |
| 7 | 1 | 1 | 0 | 0 | 0 |
| 8 | 1 | 0 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 0 | 0 | 1 |
| 11 | 0 | 1 | 1 | 0 | 0 |
| 12 | 0 | 1 | 0 | 1 | 0 |
| 13 | 0 | 1 | 0 | 0 | 1 |
| 14 | 0 | 0 | 1 | 1 | 0 |
| 15 | 0 | 0 | 1 | 0 | 1 |
| 16 | 0 | 0 | 0 | 1 | 1 |
| 17 | 1 | 1 | 1 | 0 | 0 |
| 18 | 1 | 1 | 0 | 1 | 0 |
| 19 | 1 | 1 | 0 | 0 | 1 |
| 20 | 1 | 0 | 1 | 1 | 0 |
| 21 | 1 | 0 | 1 | 0 | 1 |
| 22 | 1 | 0 | 0 | 1 | 1 |
| 23 | 0 | 1 | 1 | 1 | 0 |
| 24 | 0 | 1 | 1 | 0 | 1 |
| 25 | 0 | 0 | 1 | 1 | 1 |
| 26 | 0 | 1 | 0 | 1 | 1 |
| 27 | 1 | 1 | 1 | 1 | 0 |
| 28 | 0 | 1 | 1 | 1 | 1 |
| 29 | 1 | 0 | 1 | 1 | 1 |
| 30 | 1 | 1 | 0 | 1 | 1 |
| 31 | 1 | 1 | 1 | 0 | 1 |
| 32 | 1 | 1 | 1 | 1 | 1 |

**Table 2: Biological parameters**

| Parameter | Value | Description |
|---|---|---|
| $p_{dist}$ | 0.05 | probability that a site is disturbed |
| $p_g$ | 0.04 | probability that a seed becomes a seedling |
| $p_s$ | 0.3 | probability that a seedling survives the first year |
| $P_d$ | 0.5 | probability that a seed is lost from the seedbank (decay) |
| $A_{min}$ | 3 | minimum age for reproduction of broom |
| $A_{max}$ | 20 | maximum plant age |
| $F$ | 5300 | seed production per site (numbers per square metres) |
| $f_h$ | 0.73 | probability that seed is retained in the parental site |
| $p_{so}$ | 1.0 | probability that site becomes suitable for colonisation after senescence |
| $f_r$ | 0.6 | fraction of broom plants that are reproductive |

Source: Rees & Paynter (1997)
Downey & Smith (2000)
Sheppard, Hodge, Paynter & Rees (2001)
Paynter, Downey & Sheppard (2001)

**Table 3: Initial conditions of the area**

| Variable | Fraction |
|---|---|
| Area occupied by broom | 0.125 |
| Sites that are unsuitable for broom | 0.4 |
| Sites that are suitable for broom | 0.6 |
| Areas open for colonisation | 0.475 [a] |

[a] Areas suitable for broom but not yet colonised

**Table 4: Matrix of control effects**

| Strategy | Control | $P_{dist}$ | $P_s$ | $f_h$ | $w_t$ |
|---|---|---|---|---|---|
| 1 | 0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 2 | 1 | 0.2 | 0.33 | 1.23 | 1.0 |
| 3 | 2 | 1.4 | 0.33 | 0.55 | 0.8 |
| 4 | 3 | 1.6 | 0.3 | 0.27 | 0.6 |
| 5 | 4 | 0.2 | 0.67 | 1.23 | 1.0 |
| 6 | 5 | 0.2 | 0.07 | 0.04 | 0.6 |
| 7 | 1,2 | 1.3 | 0.17 | 0.96 | 0.8 |
| 8 | 1,3 | 1.5 | 0.14 | 1.1 | 0.6 |
| 9 | 1,4 | 0.1 | 0.51 | 0.62 | 1.0 |
| 10 | 1,5 | 0.1 | 0.3 | 1.21 | 0.6 |
| 11 | 2,3 | 0.9 | 0.18 | 0.42 | 0.5 |
| 12 | 2,4 | 1.3 | 0.51 | 0.96 | 0.8 |
| 13 | 2,5 | 1.3 | 0.3 | 0.53 | 0.5 |
| 14 | 3,4 | 1.5 | 0.52 | 1.1 | 0.6 |
| 15 | 3,5 | 1.5 | 0.27 | 0.25 | 0.4 |
| 16 | 4,5 | 0.1 | 0.64 | 1.21 | 0.6 |
| 17 | 1,2,3 | 0.8 | 0.02 | 0.8 | 0.5 |
| 18 | 1,2,4 | 1.2 | 0.34 | 0.34 | 0.8 |
| 19 | 1,2,5 | 1.2 | 0.13 | 0.94 | 0.5 |
| 20 | 1,3,4 | 1.4 | 0.36 | 0.5 | 0.6 |
| 21 | 1,3,5 | 1.4 | 0.15 | 1.08 | 0.4 |
| 22 | 1,4,5 | 0.0 | 0.47 | 0.6 | 0.6 |
| 23 | 2,3,4 | 0.8 | 0.36 | 0.82 | 0.5 |
| 24 | 2,3,5 | 0.8 | 0.15 | 0.4 | 0.3 |
| 25 | 3,4,5 | 1.4 | 0.49 | 1.08 | 0.4 |
| 26 | 4,5,2 | 1.2 | 0.47 | 0.94 | 0.5 |
| 27 | 1,2,3,4 | 0.7 | 0.19 | 0.21 | 0.5 |
| 28 | 2,3,4,5 | 0.7 | 0.32 | 0.8 | 0.3 |
| 29 | 3,4,5,1 | 1.3 | 0.32 | 0.46 | 0.4 |
| 30 | 4,5,1,2 | 1.1 | 0.31 | 0.32 | 0.5 |
| 31 | 5,1,2,3 | 0.7 | 0.02 | 0.8 | 0.3 |
| 32 | 1,2,3,4,5 | 0.6 | 0.16 | 0.19 | 0.3 |

**Table 5: Output parameters**

| Parameter | Biodiversity | Recreation | Agriculture |
|---|---|---|---|
| $V_{max}$ | 130 | 15000 | 1.2 |
| $k_m$ | 0.18 | 0.3 | -2.0 |
| $x_{min}$ | 0.6 | 0.6 | 0.9 |
| $p$ | 100,000 | 138 | 1,680,000 |

**Table 6: Control costs**

| Method | Cost ($/year) |
|---|---|
| 1. exclude tourists | 5,000 |
| 2. manual pull | 15,000 |
| 3. apply herbicide | 45,000 |
| 4. control pigs | 15,000 |
| 5. biological control | 76,848 |

**Table 7: The decision rule for different levels of weed density**

| Initial weed density ($w_{t}$) | Optimal control strategy | Final weed density with low seed density levels | Optimal control strategy | Final weed density with high seed density levels |
|---|---|---|---|---|
| 0 | 1 | 0 | 16 | 0.0049 |
| 0.001 | 1 | 0.0009 | 16 | 0.0058 |
| 0.060 | 24 | 0.0430 | 16 | 0.0542 |
| 0.141 | 24 | 0.1010 | 16 | 0.1207 |
| 0.287 | 24 | 0.2054 | 28 | 0.2344 |
| 0.378 | 24 | 0.2703 | 28 | 0.2977 |
| 0.432 | 24 | 0.3088 | 28 | 0.3352 |
| 0.464 | 24 | 0.3316 | 28 | 0.3574 |
| 0.501 | 24 | 0.3579 | 28 | 0.3831 |

**Table 8: The decision rule for different levels of seed density**

| Initial seed density ($S_t$) ($x10^3$ /$m^2$) | Optimal control strategy | Final seed density with low weed density levels | Optimal control strategy | Final seed density with high weed density levels |
|---|---|---|---|---|
| 0.000 | 1 | 0.000 | 24 | 1.2217 |
| 0.004 | 5 | 0.002 | 24 | 1.2238 |
| 0.250 | 6 | 0.125 | 24 | 1.3467 |
| 0.594 | 6 | 0.297 | 24 | 1.5187 |
| 1.571 | 16 | 0.786 | 24 | 2.0072 |
| 2.317 | 16 | 1.159 | 28 | 2.3802 |
| 2.770 | 16 | 1.385 | 28 | 2.6067 |
| 3.041 | 16 | 1.521 | 28 | 2.7422 |
| 3.350 | 16 | 1.675 | 28 | 2.8967 |

**Table 9: Optimal control options**

| Weed | Seed bank | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| density | 0 | 4.2 | 250 | 594 | 1571 | 2317 | 2770 | 3041 | 3350 |
| 0.000 | 1 | 5 | 6 | 6 | 16 | 16 | 16 | 16 | 16 |
| 0.001 | 1 | 5 | 6 | 6 | 16 | 16 | 16 | 16 | 16 |
| 0.060 | 24 | 24 | 6 | 6 | 16 | 16 | 16 | 16 | 16 |
| 0.141 | 24 | 24 | 28 | 24 | 16 | 16 | 16 | 16 | 16 |
| 0.287 | 24 | 24 | 24 | 25 | 24 | 28 | 28 | 28 | 28 |
| 0.378 | 24 | 24 | 24 | 28 | 24 | 28 | 28 | 28 | 28 |
| 0.432 | 24 | 24 | 24 | 28 | 24 | 28 | 28 | 28 | 28 |
| 0.464 | 24 | 24 | 24 | 28 | 24 | 28 | 28 | 28 | 28 |
| 0.501 | 24 | 24 | 24 | 24 | 24 | 28 | 28 | 28 | 28 |

**Table 10: Decision rule for the weed density under a budget constraint**

| Initial weed density ($w_t$) | Optimal control strategy | Final weed density unconstrained | Optimal control strategy | Final weed density with $50,000 constraint |
|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 |
| 0.001 | 1 | 0.0009 | 1 | 0.0009 |
| 0.060 | 24 | 0.0430 | 11 | 0.0479 |
| 0.141 | 24 | 0.1010 | 11 | 0.1125 |
| 0.287 | 24 | 0.2054 | 11 | 0.2290 |
| 0.378 | 24 | 0.2703 | 11 | 0.3015 |
| 0.432 | 24 | 0.3088 | 11 | 0.3446 |
| 0.464 | 24 | 0.3316 | 4 | 0.3645 |
| 0.501 | 24 | 0.3579 | 4 | 0.3919 |

**Table 11: Decision rule for the seed density under a budget constraint**

| Initial seed density ($S_t$) ($x10^3$ /$m^2$) | Optimal control strategy | Final seed density unconstrained | Optimal control strategy | Final seed density with $50,000 constraint |
|---|---|---|---|---|
| 0.000 | 24 | 1.2217 | 4 | 1.3842 |
| 0.004 | 24 | 1.2238 | 4 | 1.3863 |
| 0.250 | 24 | 1.3467 | 11 | 1.4860 |
| 0.594 | 24 | 1.5187 | 11 | 1.6580 |
| 1.571 | 24 | 2.0072 | 11 | 2.1465 |
| 2.317 | 28 | 2.3802 | 11 | 2.5195 |
| 2.770 | 28 | 2.6067 | 11 | 2.7460 |
| 3.041 | 28 | 2.7422 | 11 | 2.8815 |
| 3.350 | 28 | 2.8967 | 11 | 3.0360 |

**Table 12: Optimal control options under a budget constraint**

| Weed density | Seed bank 0 | 4.2 | 250 | 594 | 1571 | 2317 | 2770 | 3041 | 3350 |
|---|---|---|---|---|---|---|---|---|---|
| 0.000 | 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 0.001 | 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 0.060 | 11 | 11 | 17 | 5 | 5 | 5 | 5 | 5 | 5 |
| 0.141 | 11 | 11 | 11 | 17 | 10 | 10 | 10 | 10 | 10 |
| 0.287 | 11 | 11 | 11 | 17 | 11 | 11 | 11 | 11 | 11 |
| 0.378 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 0.432 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 0.464 | 4 | 4 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 0.501 | 4 | 4 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |

**Table 13: Control strategy description under a budget constraint**

| Strategy | exclude tourist | manual pull | herbicide application | pig control | biological control |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 |
| 10 | 0.6109 | 0 | 0 | 0 | 0.6109 |
| 11 | 0 | 0.8333 | 0.8333 | 0 | 0 |
| 17 | 0.7692 | 0.7692 | 0.7692 | 0 | 0 |